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ICMI STUDY 25

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PREFACE

We both are deeply committed to working collaboratively with teachers and were therefore delighted when ICMI President, Jill Adler and Secretary General, Abraham Arcavi invited us to serve as co-chairs for ICMI Study 25: Teachers of Mathematics Working and Learning in Collaborative Groups. The primary aims of Study 25 are to report the state of the art in the area of mathematics teacher collaboration with respect to theory, research, practice, and policy; and to suggest new directions of research that take into account contextual, cultural, national and political dimensions. Soon after we accepted the invitation, we collaborated with Professors Adler and Arcavi to constitute an International Program Committee (IPC) with expertise in the diverse forms that teacher collaboration can take and the breadth of issues associated with understanding and supporting teacher collaboration. The IPC met in Berlin in December 2018 to develop the ICMI Study 25 Discussion Document, which was finalized and distributed internationally in February 2019. The Study Document served as a call for papers for the ICMI Study 25 Conference, which was hosted by the Instituto de Educação, Universidade de Lisboa in Lisbon in February 2020.

Teachers work and learn together in a diverse set of formal and informal groupings, including teams, communities, schools, teacher education programs, professional development courses, and local and national networks. Because there are different ways of understanding the nature of teacher collaboration and its consequences, the call for proposals encouraged contributions representing multiple theoretical perspectives and a variety of methodological approaches. We solicited contributions from teachers as well as researchers, to ensure that teachers' voices are given prominence in accounts of their learning.

The Discussion Document is organized around four themes, identified and elaborated by the IPC, each with an associated set of questions to be explored in ICMI Study 25:

1) Theoretical perspectives on studying mathematics teacher collaboration;
2) Contexts, forms and outcomes of mathematics teacher collaboration;
3) Roles, identities and interactions of various participants in mathematics teacher collaboration; and
4) Tools and resources used/designed for teacher collaboration and resulting from teacher collaboration.

These themes and questions provided the basis for inviting papers. Each submission was reviewed by at least two IPC members, and authors of accepted papers were invited to participate in the Study Conference. Accepted papers were then revised (when necessary) by the authors, before being published in this electronic conference proceedings.

The Conference Proceedings includes 80 papers from around the world. In addition, we invited four esteemed scholars to present plenary lectures related to each theme at the Study Conference, and four others to respond to their lectures. The lectures are also included in the Proceedings. (Papers representing both the lectures and the responses will be published in the Study Volume.) To ensure that teachers' voices were well-represented at the Study Conference, we invited four practitioners actively involved in very different collaborative experiences to participated in a plenary panel moderated by an eminent scholar whose career includes multiple collaborative projects with teachers. The panel reports and moderator's introduction and synthesis are also included in the Proceedings. The countries represented in the Proceedings include: Algeria, Argentina, Australia, Austria, Brazil, Canada, China, Colombia, Cyprus, Denmark, France, Germany, Greece, India, Iran, Ireland, Israel, Italy, Japan, Malawi, Malta, Mexico, Netherlands, New Zealand, Norway, Portugal, Slovakia, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Turkey, United Kingdom, United States, and the US Virgin Islands.

The ICMI Study Conferences are unique in that their primary focus is not formal presentations of participants' papers. Instead, almost all of the time is spent in Working Groups organized around the themes and led by IPC members. During these Working Groups brief presentations by the participants, based on their papers, serve as a springboard for in-depth exploration of the themes and associated questions; and these intense discussions are directed toward the preparation of a published volume. For this reason, a draft of the Conference Proceedings was disseminated prior to the conference so that participants would have time to read the papers for their theme's Working Group in advance.

Ms. Kelly Boles, the Study Project Manager, edited the Conference Proceedings after the Study Conference. We thank her for her tireless efforts to produce this volume.

The Study Conference was held after the beginning of the COVID-19 pandemic, before the world was aware of its nature and the rapidity with which it would spread. Several countries, including China, had already begun to implement travel restrictions. To enable participants' attendance, our conference hosts at the Instituto de Educação, Universidade de Lisboa were able to arrange their virtual participation. We are grateful for their efforts and note that this experience also presented the opportunity for our group to reflect on and learn from an additional form of collaboration.

Preparation of the ICMI Study 25 volume, begun during the Study Conference, is continuing in four virtual collaborative groups, one focused on each theme. We had set the ambitious goal of having the Study Volume ready to share at the ICME-14 conference in Shanghai, China in 2021. At the time we are writing, ICME-14 has been postponed to 2022. We hope to all meet again in person in Shanghai, and we look forward to sharing the Study Volume there.

Hilda Borko

Study Co-chairs: Hilda Borko (USA) Despina Potari (Greece)

Study Themes

ICMI STUDY 25

Theme A | Theoretical Perspectives on Studying Mathematics Teacher Collaboration

Theme Co-Chairs: João Pedro da Ponte, Portugal
Takeshi Miyakawa, Japan

Theme B | Contexts, Forms and Outcomes of Mathematics Teacher Collaboration

Theme Co-Chairs: Cristina Esteley, Argentina
Rongjin Huang, USA

Theme C | Roles, Identities and Interactions of Various Participants in Mathematics Teacher Collaboration

Theme Co-Chairs: Shelley Dole, Australia
Ronnie Karsenty, Israel

Theme D | Tools and Resources Used/Designed for Teacher Collaboration and Resulting from Teacher Collaboration

Theme Co-Chairs: Ornella Robutti, Italy
Luc Trouche, France

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ICMI STUDY 25

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Content-specific theory elements for explaining and enhancing teachers' professional growth in collaborative groups

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Theme B | Masami Isoda — Japan

Producing theories for mathematics education through collaboration: a historical development of Japanese lesson study

Reactor: Alf Coles, United Kingdom

Theme C | Konrad Krainer — Austria

Collaborative groups in mathematics teacher education: grasping the diversity of roles, identities and interactions

Reactor: Bettina Rösken-Winter, Germany

Theme D | Karin Brodie — South Africa

Resources for and from collaboration: a conceptual framework

Reactor: Kara Jackson, United States

ICMI Study 25
TEACHERS OF MATHEMATICS WORKING AND LEARNING IN COLLABORATIVE GROUPS
Plenary Lecture – Theme A
Lisbon, Portugal, 3-7 February 2020

CONTENT-SPECIFIC THEORY ELEMENTS FOR EXPLAINING AND ENHANCING TEACHERS' PROFESSIONAL GROWTH IN COLLABORATIVE GROUPS

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In order to explain and enhance teachers' professional growth in collaborative groups, sociocultural theoretical perspectives and the construct of community of inquiry have been established. This plenary paper builds upon these achievements and suggests eight kinds of theory elements that can refine these perspectives and allow theorizing in more content-specific ways, where content refers to both, classroom content and PD content. The paper suggests an adapted model for professional growth as the framework for content-specific theorizing.

As the insightful ICME survey (Robutti et al., 2016; Jaworski et al., 2017) has shown, collaborative groups form a widespread and promising environment for teachers' professional development (PD), and many different variants have already been explored in research and PD practice. However, the ICME survey also revealed that only 85 out of the 316 papers on teacher collaborative learning that were analyzed explicitly referred to theoretical perspectives on collaboration. This finding shows the need for more comprehensive and systematic theoretical foundations for explaining and enhancing PD in collaborative groups (PDCG). This plenary paper argues that in order to elaborate a theoretical foundation for teachers' professional growth in collaborative groups, further theory elements should be integrated at the classroom and the PD levels. In particular, it advocates including a theory of professional growth (Clarke & Hollingsworth, 2002) and content-specific theory elements capturing the PD learning content. In this way, the paper focuses on content-specific theorizing, i.e. of creating new theoretical contributions and the relation to the background theoretical perspectives the research builds upon (Mason & Waywood, 1996; Prediger, 2019a). To illustrate my meta-theoretical messages, I discuss a vignette and results from two PD research projects (Prediger, Fischer, Selter, & Schöber, 2019; Prediger, Kuhl, Büscher, & Buró, in press) where secondary mathematics and special needs teachers develop their practices for fostering slow learners and for differentiating in inclusive classrooms.

After a brief introduction of PDCG and the construct of practices in Section 1, an introductory vignette in Section 2 illustrates why we need more theorizing. Section 3 disentangles the kinds of theory elements required for a theoretical foundation of PDCG by exploiting the analogy of classroom and PD levels. On this meta-theoretical base, Section 4 specifies the relevant theory elements for the introduced vignette and the PD it stems from, which provides the concrete material for the meta-theoretical reflections in Section 5.

1. Existing background theories for professional development in collaborative groups: Communities of practice and inquiry

According to the ICME survey (Robutti et al., 2016), 80% of 85 papers analyzed that explicitly referred to teacher collaboration in their theory sections addressed the theoretical construct of community, often in the sense of community of practice (Wenger, 1998; Lave & Wenger, 1991) or community of inquiry (Jaworski, 2006). Whereas some articles only referred to community in order to name the PD setting for practical reasons, most articles also referred to underlying sociocultural theories of learning. Within these sociocultural theories, learning is conceptualized as being situated in communities of social practice, where novices are successively drawn into practices, first from a legitimate peripheral position then to the center of the experienced practitioners. When these theoretical perspectives are related to teachers' learning, teaching is thereby conceptualized as a set of social practices, and professional growth is described by increasing engagement and alignment.

Jaworski (2006) has enriched this theoretical perspective by emphasizing that PD itself is shaped by a set of social practices in which alignment should not be assimilation, but critical alignment. This offers the opportunity for collective professional growth within a community of practice rather than the stability of only enculturating novices. She is widely cited for the enriched construct of communities of inquiry in which "participants . . . align with aspects of practice while critically questioning roles and purposes as a part of their participation for ongoing regeneration of the practice" (p. 190). In both perspectives, practices form the key theoretical construct. Practices in mathematics classrooms can be defined as "ways of acting that have emerged . . . [that make] it possible to characterize mathematics as a complex human activity and in that it brings meaning to the fore by eschewing a focus on socially accepted ways of behaving" (Cobb, Stephan, McClain, & Gravemeijer, 2001, p. 120). To apply the analogy, *teaching practices* are ways of acting that have socially emerged to manage typical situational demands at the classroom level, while *practices of inquiry* refer to those at the PD level (see below).

Putnam and Borko (2000) had already advertised lifting sociocultural, situated theoretical perspectives from the classroom level to the PD level. In order to effectively exploit these perspectives, they underlined the need for further considerations on (1) where to situate teachers' learning experiences, (2) the nature of discourse communities for teacher learning, and (3) the importance of tools in teachers' professional learning experiences. In this plenary paper, I extend their call for further considerations into a call for further theorizing.

2. Why we need more theorizing: An introductory vignette from a community of inquiry

Vignette. The first phase our PD research project on inclusion involved observing communities of inquiry of mathematics teachers and special needs teachers who were attempting to collectively develop their teaching practices for a new pedagogical demand, namely, integrating students with special needs who were previously taught in segregated special needs schools. The vignette is from the first meeting in which the researcher facilitator started collaboration of 18 months with the teacher group. At the moment of this first meeting, the teachers had already spent 9 months on finding ways for differentiating their teaching material in order to adapt to students' diverse mathematical profiles. After nine months of intense collective work, they were proud to have substantially changed their teaching practices in order to adapt to all students' abilities, mainly in task-based individualized settings. When the researcher facilitator first met them, Paul, the mathematics teacher, reported about Suleika, one of their students with learning difficulties, and showed two of her products on multi-digit subtraction (in Figure 1):

Paul: Suleika can calculate the subtraction well, only the carries pose problems for her. But we can handle this successfully by differentiated tasks: I only give her subtractions without carries.

Although the community agreed on the success of their changes, Maria, the special needs teacher, took three months for convincing her colleagues to participate in a PD program, as she saw an additional problem to be tackled:

Maria: I tried to teach them subtraction with carries several times, but they always forget it.

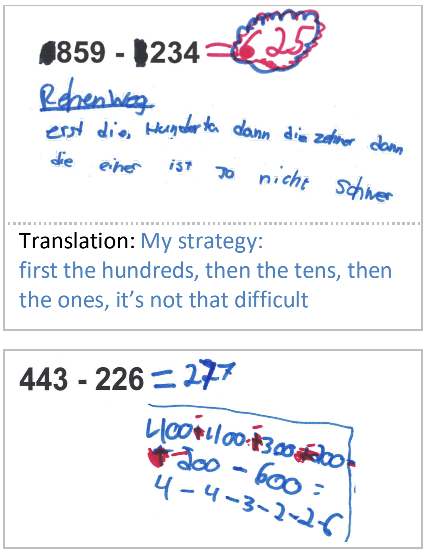

Figure 1: Suleika's written products

Analysis. Paul´s utterance was a characteristic expression of this community's adaptive differentiating practices: Students get individualized tasks that they can succeed in. Within the frame of the teachers' collective evaluation, the differentiation practice proved to be successful, as Suleika was able to complete her tasks. In spite of the intense engagement of these teachers' community of inquiry, they could not develop more productive practices for enhancing Suleika's learning. Although Suleika's second product reveals serious struggle with place value understanding (see Figure 1), this was not treated by the teachers. Maria's additional concerns that students "always forget" had also not yet entered the teachers' collectively shared space of discourse.

What the analysis leaves out. This brief analysis reveals the importance of shared orientations of what the teachers consider relevant in their community of practice. However, the language of analysis provided by the theoretical constructs from Section 1 is not yet well enough elaborated to identify the critical points in more detail. A rough analysis outside the given theoretical framework reveals: For them, differentiation meant adapting to students' abilities rather than really strengthening their learning, so their shared criterion for evaluation was reduced to task completion, not to learning growth. The reduction to this criterion also reduced the need for critical alignment with their own teaching practices. At the same time, the teacher community did not distinguish between procedural and conceptual knowledge, as they did not problematize whether teaching Suleika the algorithm with carries might miss the conceptual base. A theoretical foundation that can really inform facilitators' work in supporting the development of this community of inquiry will need to include these aspects. It will also need to find a language for explaining why Maria was not able to introduce her "students forgetting" criterion into the shared discourse.

Section 3 will introduce the meta-perspective on generic and content-specific theory elements for the first steps towards a content-specific sociocultural theory of professional growth. Section 4 then discusses the concrete elements that can explain the vignette and guide action in supporting a community's professional growth.

3. What kind of theory elements are required for a theoretical foundation for PDCG?

This chapter starts with structural meta-theoretical clarifications: What is a theory, and why do I only speak about theory elements? What kind of theory elements are required for a theoretical foundation of PDCG that provides a framework for explaining and enhancing teachers' professional growth? Theory elements vary in their *logical structure*, in *size* and in their *function* (Prediger, 2019a).

Niss (2007) characterizes a theory by its *logical structure* as an "organized network of concepts (including ideas, notions, distinctions, terms, etc.) and claims about . . . objects, processes, situations, and phenomena" (p. 1308). The claims can be basic hypotheses, statements logically derived from the fundamental claims, or empirically grounded propositions about connections and mechanisms.

Theories vary in *size*: some encompass a well-elaborated theoretical perspective with a complex network of constructs and propositions (such as sociocultural theory) and are sometimes reduced to single constructs or claims (such as the construct communities of practice). Rather than networking complete theoretical perspectives (Bikner-Ahsbahs & Prediger, 2014), this paper focuses on the local integration of several constructs and claims; this networking strategy is more suitable for fields that are not yet mature enough for big theories (as Jaworski, 2006, stated for the field of PDCG).

In order to decide which theory elements have to be integrated for a theoretical foundation for PDCG, a distinction according to their *function* in the design and research process us useful (Prediger, 2019a) and use the structural analogies between the classroom level and the PD level (see Figure 2). On the classroom level, theory elements (i.e., constructs, basic assumptions, and empirically grounded connections) for four main functions have been established (Mason & Waywood, 1996; Prediger, 2019a):

- C1 for specifying and structuring the mathematical learning content,
- C2 for explaining mechanisms of mathematics learning,
- C3 for explaining the nature and background of mathematics teaching, and
- C4 for designing and enacting learning environments.

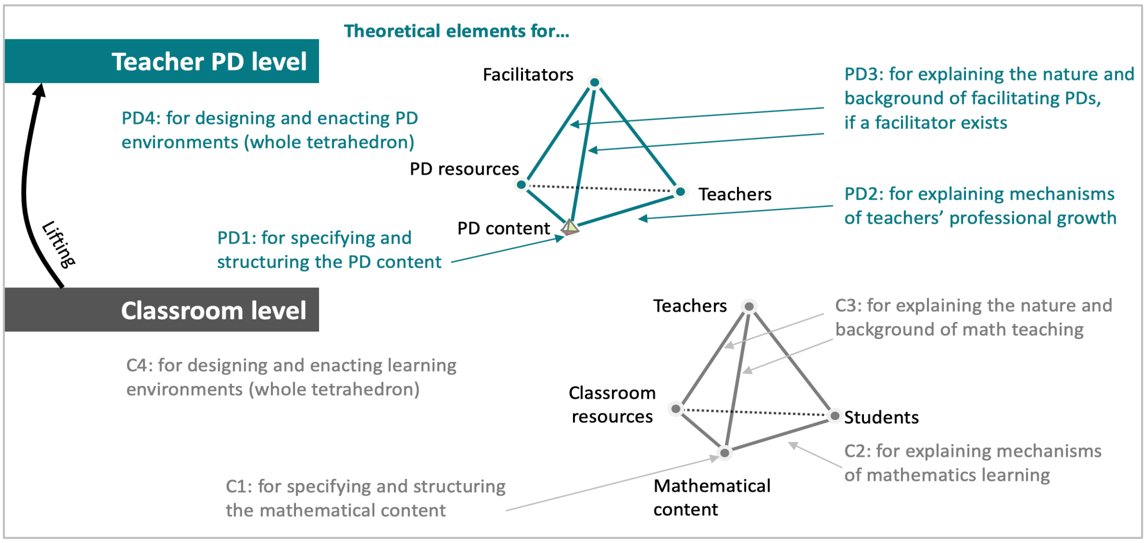

Figure 2: Lifting theory elements from the classroom to the PD tetrahedron

As Figure 2 visualizes, these functions refer to different parts of the didactical tetrahedron on the classroom level. Exploiting the structural analogy between the classroom didactical tetrahedron and the PD tetrahedron makes it possible to lift the elements (Prediger, Roesken-Winter, & Leuders, 2019), inferring the need for analogous theory elements on the PD level:

- PD1 for specifying and structuring the PD content;
- PD2 for explaining mechanisms of teachers' professional growth;
- PD3 for explaining the nature and background of facilitating PDs, if a facilitator exists; and
- PD4 for designing and enacting PD environments.

4. Generic and content-specific theory elements for explaining and enhancing PDCG

Coming back to the vignette from Section 2, the researcher facilitator intended to receive a profound understanding of the collaborative group's practices and challenges before changing the role from observer/analyzer to a facilitator of the PDCG and before supporting the group to develop their critical inquiry stance and their teaching practices.

These dual practical goals, the facilitator's understanding and then intervening, have a counterpart on the theorizing side: The repeated (and much more systematic) analysis of these kinds of vignettes can enhance the researchers' theoretical understanding. Systematically connecting the theoretical elements can generate a theoretical underpinning for typical facilitation practices and designs for PDCG. That is how we aim to find a theoretical foundation for enhancing teachers' professional growth. For this theorizing purpose, it proved to be highly relevant to unpack the theory elements on the classroom and PD levels, not only by means of generic theory elements, which apply to all classroom and PD content, but also content-specific theory elements, in this case, where the classroom content is understanding multi-digit subtractions and the PD content is differentiating in inclusive mathematics classrooms.

Although it is not possible to demonstrate the theorizing process with all its details here, this section intends to show the power of working with articulated theory elements unpacked down to the level of the mathematical content. I will successively introduce theory elements for all the different functions and use them for the analysis. The analysis starts at the classroom level to build the ground for analyzing the group's teaching practices and the group's processes of professional growth later at the PD level. It ends with a look at how the facilitator reacted and how this experience informed the PD design for future collaborative groups.

	Conceptual understanding	Procedural skills
Actual content (Grade 5)	Conceptual understanding of actual topic, in this case, regrouping units while subtraction	New procedures, in this case, multi-digit subtraction procedure with carry
Foundations from previous years (Grade 2)	Basic conceptual needs, in this case, place-value understanding (meaning of digits)	Basic skills, in this case, basic subtraction facts up to 10

Table 1: Theory elements of C1 for specifying the classroom content, with content trajectory for structuring it

Introducing theory elements for C1. On the classroom level, the theory elements C1 for specifying the classroom content in view are printed in Table 1. The table distinguishes conceptual understanding and procedural skills (Hiebert & Carpenter, 1992), but also the actual learning content and its foundations from previous years. The examples in Table 1 relate to the classroom content for multi-digit subtraction and its conceptual underpinnings (discussed in Hiebert

& Wearne, 1996). The arrow indicates the typical trajectory according to which the content can be structured (see C2): basic conceptual needs to underpin basic skills, and these are necessary for building further conceptual understanding, which then underpins the new procedures.

Analyzing the vignette with respect to C1. In Suleika's case (see Figure 1), she mastered the basic skills of subtraction facts up to10 and used them for multi-digit subtraction without carry. Subtraction with carry, however, is based on the conceptual understanding of decomposing numbers into digits. Suleika could not build on this due to the basic conceptual need of fundamental place-value understanding (which becomes visible in her decomposition of 443; see Figure 1).

Introducing theory elements for C2 and C4. The basic mechanism of learning C2 that helps to explain Suleika's challenge in remembering the procedure of subtraction is that sustainable learning always requires connecting to previous knowledge, best accomplished with settled understandings (Hiebert & Carpenter, 1992, p. 67). Hence, students who have no access to specific basic conceptual understandings cannot continue learning along the content trajectory, in other words, the actual conceptual understanding or the procedural skills building upon these basic needs are not accessible. This proposition about the structure of the generic content trajectory has been empirically proven in long-term assessment studies (e.g., by Moser Opitz, 2007) and resonates with early results on understanding multi-digit subtraction (Hiebert & Wearne, 1996). Consequently, our design research group has designed a learning environment that enables teachers to formatively assess and foster students' basic conceptual needs (Prediger, Fischer, Selter, & Schöber, 2019). The theory elements C4 underlying the learning environment include three design principles (for their empirical and theoretical justification, see Moser Opitz et al., 2017): DP1, focusing on conceptual understanding; DP2, ensuring adaptivity using diagnostic tasks; and DP3, promoting discourse. This learning environment was shown to be effective for giving students safe access to the basic conceptual needs, with significantly higher learning gains than the control group (Prediger et al., 2019). However, the learning gains varied largely and relied heavily on the teachers, so further PD research is required for optimizing support for all collaborative groups.

Analyzing the vignette with respect to C2 and C4. In the vignette, Suleika's learning pathway was not aligned to the trajectory C2 in Table 1, and the fact that the teachers tried to teach her later stages of the content trajectory without taking into account the earlier stages might explain why she always forgot the content. However, the teachers' practices for differentiating did not rely on the categories from C1 (Table 1), nor on the propositions C2 and design principles C4. Instead, they were based on the practices the community of inquiry collectively developed, independent from the design research team. Rather than blaming the teachers for not using this approach, theory elements C3 are required which allow the researcher facilitator to explain their practices and their forms of inquiry, acknowledging the enormous efforts they conducted in their community.

Introducing theory elements for C3. In line with Wenger (1998) and Jaworski (2006), teachers' professional practices have been defined as socially established ways of mastering recurrent situational demands in mathematics classrooms (such as differentiating and fostering low achievers). To explain these teaching practices in more depth, this paper refers to Bromme's (1992) situated construct of teacher professional expertise and adapts it to a situated sociocultural perspective. According to Bromme, practices are visibly characterized by shared pedagogical tools (e.g., tasks, teacher moves), but are explainable only by the underlying orientations (socially shared beliefs) and the activated categories for perceiving, thinking, and evaluating. Bromme's general framework gains its explanative power for content-related purposes when filled in content-specific ways (Prediger, 2019b): To explain teachers' practices, we identify the socially shared content-specific pedagogical tools, orientations, and filtering categories that underlie their utterances and visible behavior for mastering the self-posed situational demands.

Analyzing the vignette with respect to C3. With respect to our vignette, the practices to be analyzed by the theory elements C3 are Paul's and Maria's (and their colleagues') differentiating practices for their inclusive classroom and their fostering practices for her student Suleika. The teacher community was driven by the shared inclusive orientation <a good inclusive classroom is adaptive to students' abilities> (orientations are indicated by <…>), hence design principle DP2 here serves as the shared guiding orientation. In order to realize it, they use the pedagogical tools of differentiated tasks and activity settings of individualized learning. The major category by which they perceive the initiated processes and by which they evaluate and optimize their differentiating practice is ||task completion|| (categories are indicated by ||...||). Applying this category, the teachers evaluate the short-term success by assessing whether a student was able to complete the task with the given support and simplifications. And, indeed, simplifying the mathematical demands involved in subtractions without carries proved to be efficient for fulfilling their evaluation category ||task completion||. However, this category is guided by a <short-term> orientation, whereas <long-term> would refer to ||learning growth||.

Deriving theory elements for PD1. Brief analysis revealed first content-specific categories and orientations for the PD content "differentiating and fostering students in inclusive classrooms" that the observed collaborative group held. Careful analysis of many other vignettes revealed a specification of further content-specific categories and orientations that could bring the teachers' collective inquiry forward. The researchers who observed several groups finally structured them in Figure 3, which includes, as a nested core, the categories for specifying and structuring the classroom content. Figure 3 also later turned out to be a useful tool for communication with teachers (although it was not existent during the vignette itself). The C1 categories can help teachers to make decisions about learning goals and assess students' learning pathways along the content trajectory. However, the PD1 <short-term or long-term orientation> determines whether teachers focus on ||learning growth|| or on ||task completion|| as their major category of thinking, perceiving, and evaluating their practices.

Introducing theory elements for PD2. To explain the mechanisms of teachers' professional growth, I follow the ideas of Jaworski's (2006) communities of inquiry and exploit them content specifically by an adapted version of Clarke and Hollingsworth's (2002) model of professional growth.

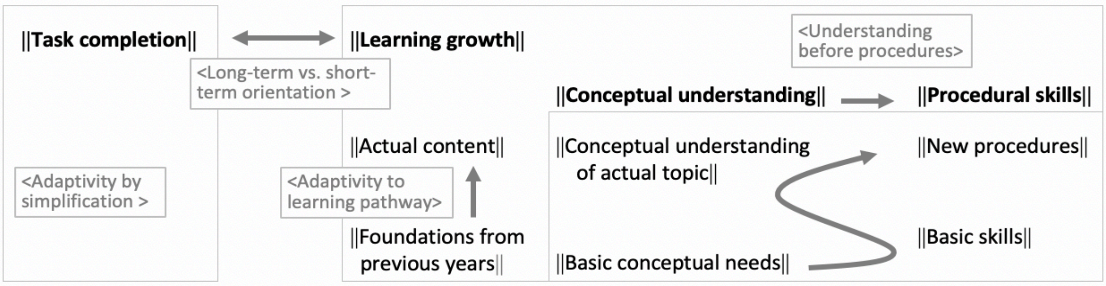

Figure 3: Specifying the PD content: Teachers' categories and orientations

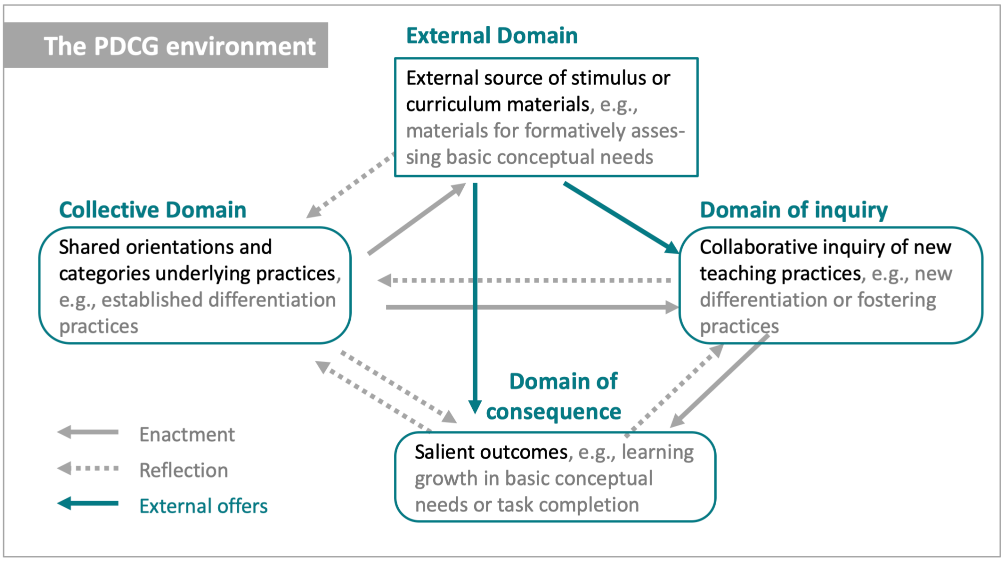

Figure 4: Adapted model for professional development in collaborative groups (PD2 & PD4)

Jaworski (2006) emphasizes the condition for community growth that communities do not only align with current practices but engage in inquiry for critical alignment of their practices (and of new input). For this, she describes the shared categories of evaluation as crucial. This idea can be combined with Clarke and Hollingsworth's (2002) model for professional growth, which has often been cited for designing and enacting PD environments (PD4) and for explaining teachers' professional growth (PD2), although mostly without explicit focus on collaborative groups. Clarke and Hollingsworth (2002) provided a model that relates the different domains of a change environment without bringing them into a naïve chain of implementation steps, from the *external domain* (with external sources of information, stimulus, or classroom resources) via changing the *personal domain* (teachers' knowledge or attitudes, here conceptualized from a sociocultural perspective as the *collective domain* of teachers' shared orientations and categories) to the *domain of practice* (here conceptualized as the *domain of inquiry for* new practices) and then to the *domain of consequence* (with salient outcomes such as students' learning gains). Rather than assuming such a simple unidirectional chain of effects, they emphasize the mutual interplay of different domains with different pathways of enactment and reflection. This model is suitable for our purposes as it builds upon Putnam and Borko's (2000) call for authentic situated learning opportunities, which is here realized by the emphasis on the domain of inquiry. As the model was originally articulated in generic terms and mainly for individual teachers, it is here adapted to the sociocultural perspective and content-specific explorations, as indicated for the vignette in Figure 4. Particularly, this adaptation also allows analysis of the connection between individual and collective concerns, which is crucial for the conditions of collaborative growth.

Analyzing the vignette with respect to PD2 and PD1. Until the facilitator came in and provided external sources, the vignette can be analyzed in the three lower domains: the collective domain, the domain of inquiry, and the domain of consequences. Based on the status quo in the *collective domain* (as presented in the analysis by C3), the community of inquiry was working hard on new differentiation practices (*domain of inquiry*). However, their current focus on the evaluation category ||task completion|| had substantial impact on the way they perceived their success in the *domain of consequence*. At the same time, outside the shared collective domain, Maria, the special needs teacher, puts a second evaluation category on the table, ||forgetting||. The category of ||forgetting|| is a remarkable one, since, while it refers to the category identified as crucial in PD1, ||learning growth||, it does not relate students' learning to teachers' practices, explaining unsuccessful learning growth solely within the students. In this way, it has not yet initiated a focused reflection in the *domain of inquiry,* but a call for external support. Indeed, Paul and other colleagues articulated that they were not interested in ||forgetting||, as this cannot guide their teaching and assumes too much responsibility by the students (Jackson et al., 2017). Here, incompatibility of teachers' orientations explain why the community of inquiry could not adopt Maria's concerns. The researcher facilitator first met the community of inquiry in this moment. With the theory elements of the specified PD content in mind (see Figure 3), she noticed that the teachers were not concerned about Suleika's place value understanding, although her written product in Figure 1 provides a strong evidence of its peculiarity. Thus, the teachers did not activate the category ||basic conceptual needs|| for assessing Suleika's work. The researcher facilitator assessed that the community of inquiry was neither driven by the orientation <understanding before procedures> nor by <long-term rather than short-term>, which would have led them to focus on her basic conceptual needs rather than trying to teach her multi-digit subtraction without any place value understanding. A first rough approximation of this analysis allowed the researcher facilitator to explain why the teachers' enormous efforts had not yet led to satisfying long-term results and how to enter the discourse with the collaborative group in order to turn Maria's concern into a collective and productive concern.

Consequences for PD3 and PD4 in the continued vignette. In order to draw consequences for PD3 and PD4, no further theory elements are required, as all relevant aspects can be articulated by the model in Figure 4. The facilitator researcher's practice for enhancing professional growth started with listening to the teachers and analyzing their collective efforts in all three domains by means of the theory elements PD1 and PD2 (Figures 3 and 4). Following the principle of building upon teachers' collective resources, she realized that <understanding before procedures> was not the ideal starting point to discuss in this community, as being too far from their actual collective concern. Instead, she chose <short-term versus long-term orientation> in order to relate Paul's and Maria's points and build upon Paul's intention to consider aspects that they can influence in order to establish a new shared orientation. The following utterances are synthesized from a longer conversation:

Facilitator: Paul says you can handle Suleika's difficulties successfully by giving her only subtractions without carries. However, Maria does not seem to be satisfied with the learning outcome. What is the problem with Suleika always forgetting the procedure, Maria?

. . .

Facilitator: You also seem to be interested in the long-term learning. Can we go back some steps and check what Suleika can master on her learning pathway towards multi-digit subtraction? I see how she decomposed the numbers (443 = 400 – 400 – 300); do you think this could have any impact on her ability to remember the procedure of managing carries?

After 30 minutes of discussion, Paul, Maria, and their colleagues collectively decided that they needed to go back in the content trajectory to stabilize Suleika's learning pathway in a more sustainable way. It took them much longer to realize that they needed to provide learning opportunities for the basic *conceptual* needs, and that this might be also the more productive practice of differentiation. Thus, the first external offer provided by the facilitator could strengthen Maria's implicit orientation <long-term instead of short term>, which also opened the teachers to other evaluation categories for their teaching success. However, to enact teaching practices towards the new evaluation category

||learning growth|| in ||basic conceptual needs||, they required further external offers, namely pedagogical tools for formatively assessing students' ||basic conceptual needs|| and teaching material for fostering them.

The externally offered curriculum materials for formative assessment and remediating sessions not only provided them with the required pedagogical tools, which they could now integrate into their practices, but also with access to the detailed pedagogical content knowledge on basic conceptual needs for other mathematical topics, such as place value understanding on the number line and meanings of multiplication and division (see Prediger et al., 2019). Once the teachers had incorporated these categories and orientations in their collective domain, their inquiries resulted in bigger changes of their practices and a closer approximation to the newly set goals: fostering all students' ||basic conceptual needs|| in order to assure <adaptivity to the individual learning pathways>. Based on these experiences, it took another year before the orientation <understanding before procedures> really started to guide their work. Interestingly, it entered their *collective domain* via the *domain of inquiry*, when experimenting with the curriculum materials for all students and experiencing "lovely aha-moments, when students say 'now, I really got it!'" In this way, the teachers' pathways of long-term collective professional growth reflected an interesting interplay between the four domains.

Tentative content-specific theorizing on PD2 and PD4. As these considerations illustrate, the adapted model for PDCG can serve as a theoretical framework for explaining teachers' professional growth (PD2) and for taking actions to enhance it (PD4). However, the general model only provides the framework for necessary content-specific theorizing. The analysis of this vignette and many further cases (e.g., Prediger et al., in press) resulted in the first tentative theorizing about teacher communities' learning pathways towards striving for differentiating practices (PD2) and the roles of external resources such as classroom material in supporting the process (PD4): When communities of inquiry work on innovative practices (in this case, on their differentiating practices), their evaluation categories in the domain of consequence might be the most crucial to develop.

Lessons learned. The various case studies from the PD projects in views are currently theorized. One central theoretical contribution is the *empirically grounded hypothesis* is that the teacher communities' learning of the PD content "inclusive mathematics teaching" might be characterized as progression of evaluation categories in four stages:

1. ||Work intensity|| "All students work eagerly." (no matter on what)
2. ||Task completion|| "All students complete the tasks."
3. ||Procedural learning growth|| "Students develop procedural skills with adaptive demands."
4. ||Conceptual learning growth|| "Students develop conceptual understanding, if necessary on basic conceptual needs."

Hence, initiating shifts in the evaluation categories might be the most crucial external input required to allow teacher communities to continue their independent inquiries. Additionally, providing curriculum materials for formative assessment and fostering sessions may not only provide a pedagogical tool for strengthening teaching practices, but it may also influence the collective domain, namely the shared pedagogical content focus on further basic conceptual needs in different mathematical content. These two hypotheses will require further systematic investigation before they can count as empirically grounded theory elements.

With respect to Putnam's and Borko's (2000) three areas of consideration, I conclude the following hypotheses for PDCG:

(1) It is worth situating teachers' learning experiences in communities of inquiry with emphasis on the domain of inquiry.
(2) Classroom materials can support the teachers' formative assessment and fostering practices and at the same time serve as a tool to extend the communities' shared PCK.
(3) The nature of the discourse in the PDCG is heavily influenced by shared orientations and categories, specifically by evaluation categories. Shifting these categories to the domain of consequence seems to be a crucial starting point for the external domain.

5. Meta-theoretical reflections on the necessary topic-specific theory elements

This paper intends to contribute to developing theoretical foundations for explaining and enhancing teachers' professional growth in collaborative groups. Building upon the general sociocultural perspective on teachers' practices (Wenger, 1998) and the construct of communities of inquiry (Jaworski, 2006), it used the exemplification in one vignette in the community of inquiry to show:

- Theory elements of PD content (PD1), teacher learning (PD2), facilitating (PD3), and PD settings (PD4) are all useful and necessary for explaining and enhancing professional growth.
- The structures of the big theoretical frameworks (communities of inquiry, models of professional growth) are helpful in understanding the complexities and intertwinement of different domains. However, they mainly provide a *generic search space*. Informing the concrete analysis and especially the concrete PD design and facilitation, they must be elaborated in content-specific ways for different PD content (Prediger et al., 2019). Whereas content on the classroom level refers to specific mathematics topics, content on the PD level refers to the specific teaching practices that the communities of inquiry have chosen to work on, in our case, differentiating and fostering the learning of students with difficulties.
- Although PD practices are always content specific, research papers and particularly theorizing processes tend to abstract from these contents; however, we should talk more about content-specific theorizing.
- The generic theory elements gain their explanative power when being filled in content-specific ways, and this also requires the nesting of corresponding theory elements (C1–C4) on the classroom level into the PD level. The more this nested structure is unpacked, the more we learn in content-specific ways about the PD content (ibid.).

Whereas the above arguments are not specific to the form of PD in view here, PDCG, the main result of the current study might be very characteristic for the collaborative setting: The shared evaluation categories seem to be the crucial point, more than the shared knowledge or orientations as a whole. As long as individual evaluation categories have not really entered the collective domain (as Maria's ||forgetting||), they cannot unfold their influence, and this can also hinder the professional growth of the individual within the collaborative group. This interplay of individual and collective learning in particular will require substantial further empirically grounded theorizing.

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PRODUCING THEORIES FOR MATHEMATICS EDUCATION THROUGH COLLABORATION: A HISTORICAL DEVELOPMENT OF JAPANESE LESSON STUDY

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This paper briefly sketches the Japanese theories for designing and reproducing better lessons to share and transfer the challenges and experiments of lesson study. Lesson study was initiated in 1873. The basic philosophy for developing children who learn mathematics by and for themselves and the basic manner of lesson study appeared in 1883. The major theories of mathematics education for designing and reproducing sciences were developed on the elaboration of theories proposed by various lesson study groups. Currently, it can be summarized as the theories for: clarify the objectives; distinguish teaching concept; establish the task sequence; and teaching approaches which includes assessments. This paper illustrates their historical development through lesson study.

Lesson study has been recognized as the collaborative activity of teachers and researchers on the lesson study processes to share better practice. In so many cases, it focused on the school based approach to establish the school as learning community. On the other hand, the Japanese lesson study was usually lead by the mathematics lesson study community which focused on specific content and lesson study theme. It produced the theories for mathematics education as for designing and reproducing better lessons to develop students who learn mathematics by and for themselves. And their achievements are embedded into national curriculum and textbooks. The objective of this paper is to illustrate how Japanese developed thier theories for designing and reproducing science in mathematics education.

Brief History of Japanese Lesson Study for Collaborations

An Origin of Lesson Study

The History of Japanese Lesson Study began with the establishment of the Elementary School at Normal School (Predecessor of Higher Normal School, and an origin of the University of Tsukuba) in Tokyo (1873) under the Code of School (1872) which intended to introduce French Style School System. From the beginning, observing other teachers' classes was an ordinal activity because it changed the teaching style from tutorial manner of Temple Schools for heterogeneous students to a whole class teaching system based on grades. Temple schools used a textbook up to Pythagorean Theorem and extraction of the square root using abacus. Additionally, they also alternated it to the adapted edition of imported textbooks. The first lesson study national workshop, a kind of training, set by the scholars of the School under the Ministry of Education was published as the book "Improvement of Teaching" (Wakabayashi and Shirai, 1883). The authors were Elementary School teachers at the time and it was the product of a 10 year experiment of lesson study. Some of them were scholars who taught at the normal school and made a survey of Pestalozzi method in USA. The book became a bible to guide the lesson study because it included the principles of teaching under the Pestalozzi method, dialectical questioning under Zen/Confucian style, and tasks for inquiry, objective-based lesson plan, and ways of critical discussion after observation of the class. Critical discussion points were the preferred teaching materials and methods, and the observed activities between teacher and students. These basic components had an influence through normal schools for practicum, and it became basic format of guidebooks published by teachers. Pestalozzi recognized the nature of students as active learners and Japanese teachers tried to develop students through the tasks for inquiry. These philosophy provided a ground for Japanese educational policy to develop students who learn the subject (mathematics) by and for themselves and with the objective to develop (mathematical) thinking and attitude. Tasks designing and the necessary components of lesson plans were the platform to develop the design science of lesson study. The guidebooks also included the model/sample protocols between teachers and students for every subject which was the origin for lesson study as a reproducible science.

Establishment of Journals and Societies for lesson study group

Elementary education became free in 1900. The Journal for Educational Research has published since 1904 by the Elementary School. These journals for teachers functioned for sharing the theories and themes of lesson study and supported lesson study groups nationwide. Various experimental/demonstration schools proposed their methods under their studies of German didactics and so on. One innovative approach by Jingo Shimizu (1924) was that students pose problems by and for themselves. Japan Society of Mathematical Education (JSME) was established for secondary (Junior High) school mathematics teachers (1919) and enhanced nationwide movements to introduce calculus and perspective-production geometry under the influence of Kline movements. Until Worldwide Financial Crisis (1929), lesson studies at elementary schools were oriented to develop and share the new methods of teaching under the exercise oriented national textbooks. Such lesson studies produced the necessary tasks and innovative task sequence for new national textbooks (1934-1939) which focused on problem solving to develop mathematical thinking. On the other hand, because various textbooks were preferable, secondary school lesson study was oriented to experiment the task sequence of the innovative unit for mathematics curriculum reform under Klein movement. The Secondary School textbooks category I and II (1943 and 1944) up to Calculus were the products which set the mathematization as for editing principle.

Emergent of Various Lesson Study Groups which Propose Curricula and Theories

After World War II, the government under the USA changed the national curriculum as a recommendation, so curriculum development became the role of every teacher and school. Compulsory education was extended until junior high schools, the secondary school became senior high schools and more teachers were hired.

Until World War II, lesson study to show practice for other school teachers used to be the custom of demonstration schools with support of scholars and inspectors. On the issues of democracy, Ministry of Education enhanced progressivism and the school board of education, under each prefecture and city, enhanced to demonstrate their challenges for other school teachers and parents. Teachers' union was established and every group began their lesson study against progressivism. At this era, lesson study became cultural activity of every school. Various researchers such as mathematicians began to collaborate. The Association of Mathematical Instruction (AMI) was established (1951) against the national curriculum under progressivism and the Soviet Union began to support it. JSME included the elementary school mathematics (1952) and the government began to support JSME financially. After the re-independence (1953), the national curriculum system came back and the withdraw group of AMI established new societies such as the Mathematics Education Society of Japan (MESJ, 1959). On their background, AMI enhanced specific theory based on materialism. MESJ was oriented to pure mathematics and the school curriculum up to World War II. From this era to the modernization (Japanese New Math), Japanese basic mathematics education theories for lesson study had been produced by various groups with support of math-educators and mathematicians.

Mathematicians provided various visions in relation to pure mathematics at the era of modernization. Some of them were supportive to introduce New Math with set and structure: Akitsugu Kawaguchi supported teachers' group in Sapporo for introducing dynamic geometry with transformation and proposed the experimental curriculum and textbook for their lesson study. Some of them did not agreed: Toyama group (AMI) infiltrated their curriculum and textbook based on their own deductive sequence and materialism theory by using their square-tile model, and so on. Toyama's textbook was never accepted by the Ministry but some commune used it as a textbook, illegally. Yasuo Akizuki, a notable mathematician, who was a chair of High School National Committee supported to put in mathematization and modeling process for making mathematical thinking process an aim, as well as put in set and structure for content. Odaka was a leading educator who came out from AMI and established the Study Group of Secondary School Mathematics under the Junior Secondary School at the University of Tsukuba. He published a number of books for teachers and students under the deep surveys of US and UK projects and lesson study experiments. His group proposed their original junior high school curriculum under their experiments which enhanced integration of different domain (subject) on the context of principles used before the occupation but had content that included New Math. He was a member of National Committee at second reform for modernization; however, he was discharged in the middle of his term because he proposed to put in other content instead of set and structure because of time restraints. Kiyoshi Yokochi, president of MESJ, also published a number of new math books with teachers. The above were only a part of the discussion, and in the era more than two hundred books of different teaching methods were published by various study groups to secure the modernizations and its implementations.

Various challenges and fruitful discussions were done amongst groups everywhere. Unfortunately, mass media participated in those discussion and teachers' union engaged in negative campaigning parents whom had no knowledge of Set and Structures. Set and Structures lost the position at the next reform. However there were fruitful practical theories being extended into textbooks. Their ideas got the position of major representations to teach number and operations in the textbooks under further national curriculum. Their ideas were elaborated as significant tools by teachers to develop students' mathematical thinking by enabling them to develop mathematics by and for themselves.

The institutes at every prefecture level renamed their teacher education centers to implement national training for every teacher since1985, which includes lesson study as a component. And lesson study became more oriented to the implementation of curriculum instead of curriculum development.

Theories for Lesson Study Produced by Lesson Study Groups

The theories for lesson study were developed by teachers, math-educators and mathematicians. In the world, theories are usually developed by researchers to contribute in Journals. On the other hand, in the Japanese tradition since the initiation of lesson study, educators also show their challenges at the classroom. Thus, lesson study researchers should be teachers as well as teachers should be researchers. In this context, the nature of Japanese lesson study theories are design and reproducible science.

Most of current known theories were established after World War II under the democracy. The Japanese theories as for the school subject specific theories (Herbst&Chazan, 2016) can be explained from four perspectives. The first perspective is the theories to explain why we teach it. It's clarify the aims and objectives of lesson and formative assesment. The national curriculum standards is an authorized document that explains the objectives. To clarify the objective of teaching, math educators have prepared related theories such as theories for mathematical thinking. The second perspective is the terminologies used to distinguish conceptual differences in teaching content. The third perspective is the theories used to establish the curriculum sequence and task sequence. The fourth perspective is the theories used to manage lessons and assessment for teaching such as a manner of questioning. These theories have been prepared by math educators and teachers through several lesson studies.

Theories to Explain Aims and Objectives

The Japanese aims of education have been described as three pillars: human character formation (such as values and attitudes), general thinking skills (such as mathematical thinking and ideas), and specific knowledge and skills (such as mathematical knowledge and skills) since 1956. Currently, these aims are special not only for Japan but also for other countries such as the Southeast Asian countries (Mangao et al., 2017). The first two aims are usually explained as higher-order thinking skills in Southeast Asia as the learning content for learning how to learn. According to the Japanese principle of the national curriculum, these aims are symbolized by a single concept: "Developing students who learn mathematics by and for themselves" (Shimizu, 1984). In Japanese mathematics education, this has been recognized in relation to mathematical activities as for reorganization of life in living (Ministry of Education, 1947). The activity has been re-explained as mathematical thinking and attitude (Ministry of Education, 1956) by Japanese math educators, who have tried to explain it further. Shigeo Katagiri (see Katagiri, Sakurai, and Takahasi, 1969 and Katagiri, Sakurai, Takahasi, and Oshima, 1971), who was a curriculum specialist in primary school mathematics in the Ministry of Education, established the framework for mathematical thinking with teachers (Isoda and Katagiri, 2012). Katagiri's Mathematical Thinking is categorized into three: Mathematical Ideas, Mathematical Ways of Thinking and Mathematical Attitude. Mathematical Ideas are the ideas for activities which are embedded in students' activities and later sophisticate. For example, the idea of set is generally embedded in various activities and later it represented by set as the technical term in the mathematics curriculum. Mathematical Ways of Thinking such as inductive thinking which uses various occasions, and Mathematical Attitude (mind set) as the representation of value. After re-independence, Japanese national curriculum standards became a brief as for the law. It included general aims and contents. Thus, theories to explain the objectives of the lesson were necessary: Theories of mathematical thinking provide the analytical terminologies to explain mathematical thinking has been used for the objective of lesson deepen. In Japanese lesson study, it is used for clarifying the curriculum, task sequence, teaching materials, and methods of teaching and to analyize teaching and learnig process. Acturally, Katagiri provided the list of questioning in the lesson to promote students' mathematical thinking. It is not only a list for problem solving strategies but is used for precise descriptions of objectives which explains why we should teach in the curriculum and how.

Terminology to Explain Sequences and Theories for Extension and Integration

The terminology to explain content distinguishes conceptual differences in curriculum. It is necessary to explain the process of reorganization of mathematical concepts in teaching sequence. The Japanese established most of it up to1960s (Japan Society of Mathematical Education, 2010). Japanese teachers need to learn the terminology for developing students who learn mathematics by and for themselves because the school curriculum sequence cannot exist as a system deduced from the set and axioms such as pure mathematics (see Freudenthal (1973)). The sequence in the Japanese curriculum standards has been explained by the principle of “extension and integration” since 1968, which is oriented toward enhancing mathematical activities and developing mathematical thinking. It corresponds to the principle of reinvention by Freudenthal (1973) who proposed the reorganization of mathematical experience as mathematization. Under this principle, the school mathematics curriculum can be seen as a set of partially ordered local mathematics theories, like a net.

Such inconsistencies through the extension and integration of local theories in relation to multiplication are explained by the theory (Isoda&Olfos, in printing) that is adaptation of the conceptual and procedural knowledge to meaning and procedure in Fig. 1.

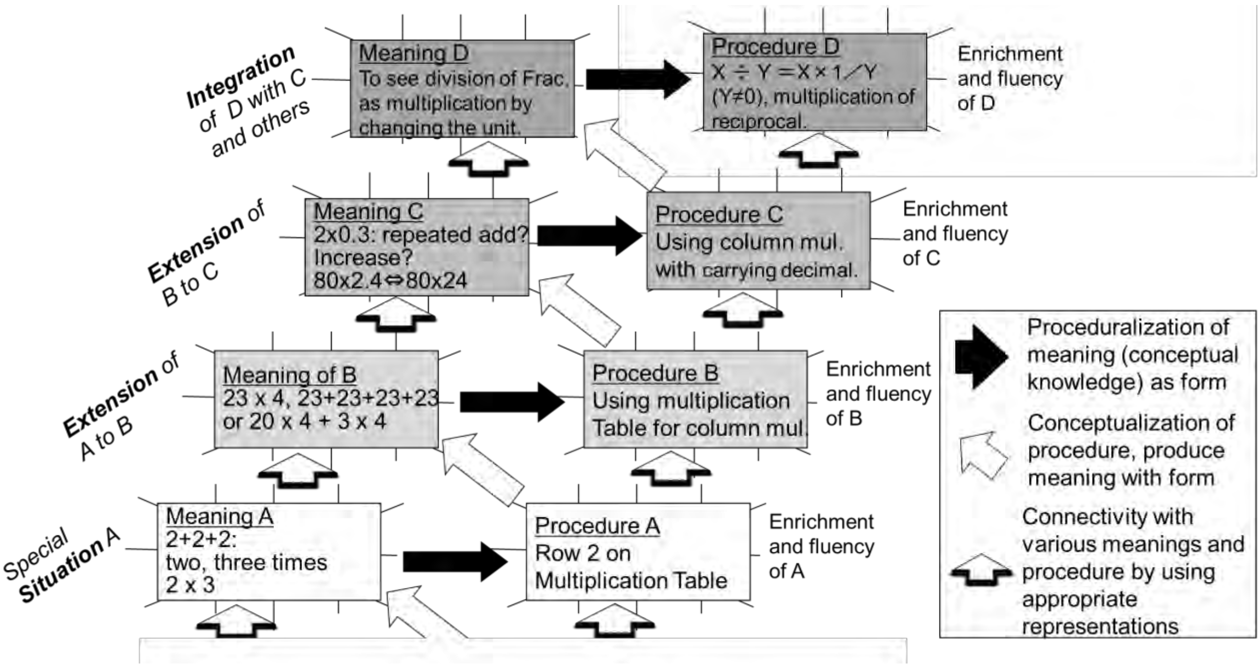

Figure 1: Extension and Integration Task Sequence for Multiplication

Conceptual and procedural knowledge (Hiebert, 1986) are used to explain the development of personal knowledge; however, in Fig. 1, Isoda (1992) used them to design and explain task sequences in the curriculum. In the curriculum sequence, as in textbooks, these are not discussed at the same time. Conceptual knowledge is usually taught to provide the meaning: it needs to use some known form of procedure. After introducing the meaning of multiplication as a binary operation (expression), the multiplication table is proceduralized from repeated addition; otherwise, students cannot distinguish it from addition as a new operation. In the process of extension and integration, inconsistencies usually appear. For example, for doing multidigit multiplication, students need to see the multidigit numbers under the base ten system for applying the multiplication table instead of just repeated addition. For the extension of multiplication to multidigit numbers with column methods, multiplication as repeated addition should be integrated with the base ten system. If we extend multiplication from whole numbers to decimals, the product of multiplication becomes small. It cannot be explained as repeated addition.

In the Japanese textbooks and Japanese teachers' lesson design these processes are discussed more precisely in relation to the task sequence. Conceptual and procedural knowledge are not seen as two different sides of the same coin in the textbooks. On the task sequence on extension and integration principle in the textbooks, it might be clearly distinguished. One of the reasons is that it is possible for students to learn the procedure without knowing when the procedure should be used. "If A, then B" is the format of the procedure. In the exercise in every chapter, students do only exercise B for solving given tasks. The condition, a part of A, is not necessary to consider in the exercise.

Before the extension of multiplication to decimal numbers, the product of multiplication only increases: "If it is multiplication of whole numbers, then the products become large." However, until extension of whole number to decimals, whole numbers meant numbers, so it looks correct to say, "If it is multiplication of numbers, then the products become large." This is possible learning content for students through the exercise in the textbook chapter. The necessity for all students to think about conditions in relation to "A" will be provided when students learn multiplication of decimals. Actually, at this stage, students do not know about decimals; they know only about whole numbers. Students are able to learn "A" when they encounter multiplication of decimal numbers. Another reason is related to the shortage of the capacity of working memory. If we limit working memory, some procedures are very convenient and faster for doing multiplication. Students do not need to consider the meaning of "A", because the numbers given in the exercise are not decimals. They have already established a convenient procedure that can be used without considering the original meaning of "A". After students attain fluency in the procedure, many students do not feel the necessity to go back to and interpret the original meaning of the situations. They may lose it because they do not need to think about the condition of "A" as long as they are applying it to learned situations. The opportunity for extension and integration is a chance to reorganize their mathematics by using their developed mathematical ideas. At the moment of extension and integration into the task sequence, students need to reconsider the real meaning through the overgeneralization of their ideas.

Problem-Solving Approach: Beyond a Teaching Method

If you have a chance to observe a lesson in a Japanese elementary school, the Japanese problem-solving approach looks the same as an open-ended approach, which involves posing an unknown task, solving it in various ways, comparing solutions with the whole class, and summarizing.

However, the Japanese problem-solving approach is prepared in the following way: aims and objectives for developing students who learn mathematics by and for themselves, terminologies to explain the content that students should learn, the curriculum and task sequence, and the teaching materials which embedded objective. On the other hand, an open-ended approach is characterized by an open-ended task. Consequently, the teaching materials used in the Japanese problem-solving approach are not the same as those in an independent task, topic, or content of mathematics because they exist under the aims, objectives, and task sequences under the curriculum. In the Japanese problem-solving approach, the task given by the teacher has the objective to produce the problematic which is necessary for students to recognize unknown. When teachers read a textbook without considering the theories, they cannot recognize the students difficulty because they know "A" already. On this condition, the key to the Japanese problem-solving approach is that the students reinvent the objective of the class set by their teacher as problematic, which should be solved because the problematic was planned by the teacher to encourage them to think mathematically. The contradiction in the planned sequence provides necessary dialectic-argumentation in this context. Given this limitation, the following exemplar (Fig. 2) on how the Japanese use the board in the lesson is meaningful. The Japanese approach means all those consequences and does not imply just a method of teaching like the scaffolding used to construct a building. Because the methods include the content of teaching. Every component of Fig 2 is explained by the theories for designing the classroom.

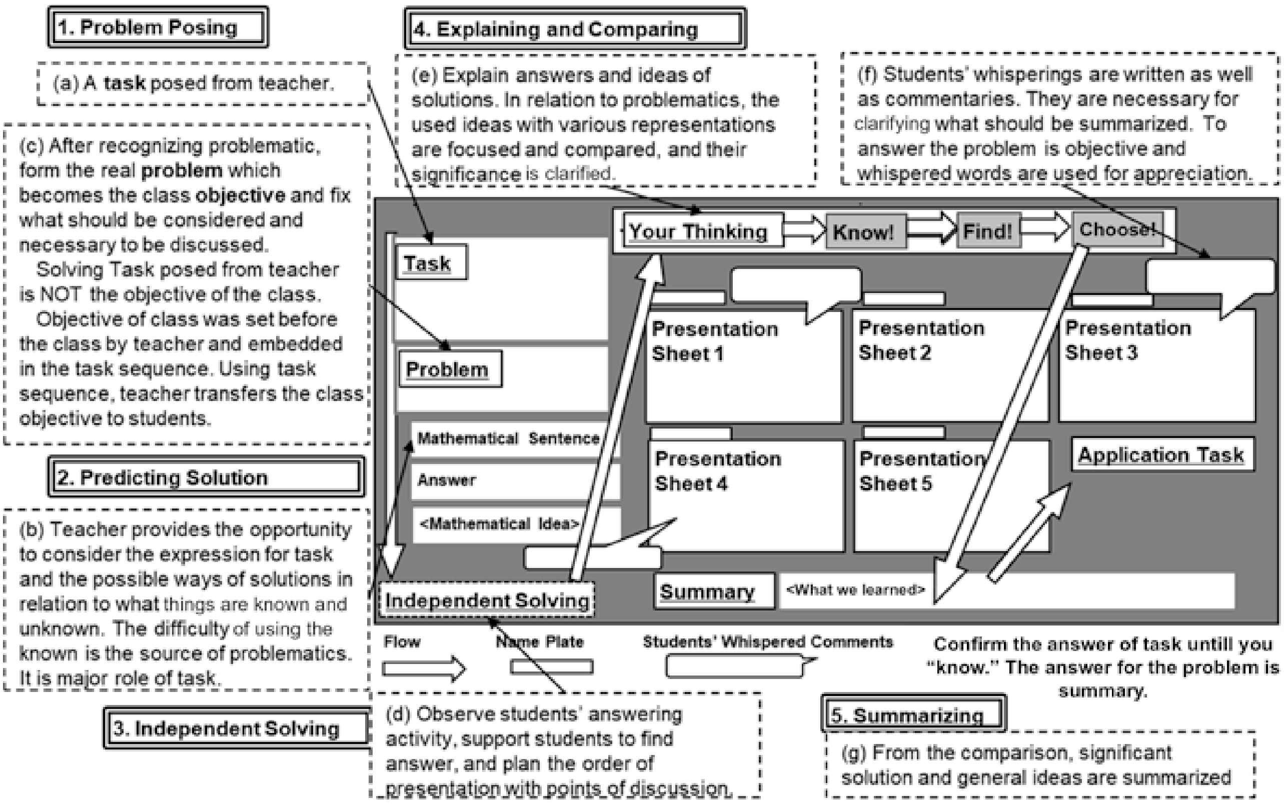

Figure 2: A Board Using Plan for Problem–Solving Approach (Isoda&Katagiri, 2012)

There are several other theories in Japan, and many of them have been proposed through critical discussions such as curriculum sequences. For example, the extension and integration principle provided provides task sequences that go against the general-to-specific principle proposed by Toyama group (AMI) since the 1950s which was called the water supply method (see Kobayasi, 1989). Toyama group criticized the National Curriculum and the certificated textbooks because they might produce misconceptions. They believed Toyama's theory never produce misconceptions because it avoided conceptual changes of Fig.1. Thus, the government side had to set the principle of extension and integration as the fallibilism nature of mathematics under modernization. Problem solving approach is the approach beyond misconceptions amongst task sequence. At that time, against the Toyama's critique, several counter theories were proposed to support extension and integration (on demands to develop mathematical thinking), such as Ito's theory (1961, 1962a, 1962b, 1962c, 1963a, 1963b, 1963c). Ito's theory to mediate ideas by models such as proportional number lines (Ito, 1968) was named "discovery methods" by Ito (published in English by Ito (1971)). He proposed the methods by preparing various representations and models for overcoming misconception by overgeneralization, and engaged lesson study with his lesson study groups. Odaka (1969, 1970, 1971, 1972, 1975, 1979, 1980, and 1982) established a schema theory with set tasks for the problem-solving approach, inspired by the idea of Piaget and the perspectives of mathematization in the 1943 textbook. Odaka produced a counter theory to explain an appropriate curriculum and task sequence—called the "exemplar approach"—against Toyama's sequence and schema theory based on materialism. Tadao Kaneko, written by Sakai and Hasegawa (1989), also theorized a task sequence for specific-to-general and general-to-special exercises (1987). Shigeru Shimada proposed the open-ended approach (1997; 1977 in Japan) based on his experience of the 1943 textbook under the mathematization principle, and Nobuhiko Nohda (1983) retheorized it as the open approach with perspective of German Didactics. There were discussions about embedding open-ended tasks into textbooks in the 1980s. Their theory for open-ended tasks itself did not indicate the task sequence for conceptual development directly. Odaka's, Kaneko's, and Isoda's theories were proposed for the ways to establish task sequence as for conceptual development on their theoretical bases. They published a number of books with several exemplar written by their lesson study groups. Their exemplar included the model protocols, not actual protocol itself, under the manner of design and reproducible science of lesson.

This paper illustrated how Japanese developed theories for designing and reproducing science.

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COLLABORATIVE GROUPS IN MATHEMATICS TEACHER EDUCATION: GRASPING THE DIVERSITY OF ROLES, IDENTITIES AND INTERACTIONS

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Initiatives with teachers of mathematics working and learning in collaborative groups show a huge diversity of roles, identities and interactions. This makes it difficult to get an insightful overview of this diversity, to compare initiatives or to grasp the specificity of individual initiatives. In this contribution we selected seven recent articles (covering all continents) and analyzed them along the following dimensions: relevant actors, relevant targets and relevant environments of the collaboration. We developed with RATE – Relevant Actors, Targets and Environments – a diagram which is used to highlight essential features of each initiative. Finally, we formulate some observations which could serve as starting points for future research on teacher collaboration.

Introduction

The discussion document “Teachers of mathematics working and learning in collaborative groups” (International Program Committee for ICMI-25 Study, 2019) identifies “Roles, identities and interactions of various participants in mathematics teacher education” as one of four major themes which need further elaboration. Regarding this theme C, the document indicates, among others that collaborative groups can include different “actors” (in various combinations). These can have a variety of roles which can shift over time. In collaborative interactions also the learning of all participants is important. The document indicates six “actors” in an exemplary way. It is easy to increase this list for example by stating: besides mathematics teachers themselves, other individuals like teachers of other subjects, lead teachers (or teacher leaders), department heads, principals, parents, teacher students, students, critical friends, facilitators, coaches, mentors, mediators, designers, multipliers, mathematics teacher educators, mathematicians, researchers, administrators, superintendents or policy makers can play an important role. And even this list is not complete. In addition also organizational entities like departments, schools, school boards, districts, committees, ministries or enterprises can be environments relevant to an initiative, for example, by influencing the goals, processes and results of collaborative activities – in projects, programs, teams, communities (of practice), networks, (study) groups, etc. (see e.g., Krainer, 2008).

This makes it difficult not only to get an insightful overview of this diversity, but also to compare initiatives or to grasp the specificity of individual initiatives. In preparation for this contribution we decided to look for selected articles and to analyze them along the following three dimensions:

- the relevant actors of the collaboration (e.g., teachers, teacher educators, researchers, etc.);
- the relevant targets (e.g., aims, goals, etc.) of the collaboration (e.g., improving the knowledge or beliefs of students and/or teachers in geometry, writing a new algebra curriculum, establishing or further developing a school’s emphasis on mathematics teaching, etc.);

- the relevant environments of the collaboration (e.g., a school or a district, a mathematical association, a university, policy makers, a new curriculum law, etc.).

Since we focus on **R**elevant **A**ctors, **T**argets and **E**nvironments, we call our tool RATE. In order to facilitate users' grasping of essential features of each initiative we visualize these features in a diagram, indicating the number of actors. In addition we highlight the key intervention (producing a relevant difference) of each initiative, its type and duration and the specificity of the collaboration. As a further means for describing the specificity of the collaboration, we indicate some key words (e.g., "collaboration" or "broker) which occur most often in a particular initiative.

In system theory (Willke, 1999, p. 12; see also Krainer, 2005) noticing of a relevant difference (observation) and producing a relevant difference (intervention) are important terms. In teacher education one kind of difference is of interest, namely the current (unsatisfied) status quo of teaching versus the desired target situation ("good" or "high quality" teaching, etc.). For example, if nobody (teachers themselves, researchers, educational policy, etc.) would see a need to produce a difference (improving teaching), reforms, professional development courses, etc. would rarely be initiated. But it is interesting to ask: What is the relevant difference that should be produced? Who defines this? By what means (professional development etc.) should the relevant difference be achieved? Who are the actors? Who has to learn, change, implement, etc.? Who supports, controls, etc.? Is collaboration between teachers fostered, are knowledgeable others involved (and in which role?)? Such reflections will be used to work out the specificity of collaboration in these initiatives and the type of intervention.

The RATE diagram will have the form of a double-triangle with the corners Teachers, Knowledgeable others, Authors and Relevant environments. The heading of the diagram indicates the main target and the type of the initiative (in parentheses and in bolt), (one-or bi-directional) arrows indicate interconnections between the corners (eventually characterized by a specific wording):

Main target (and type) of initiative

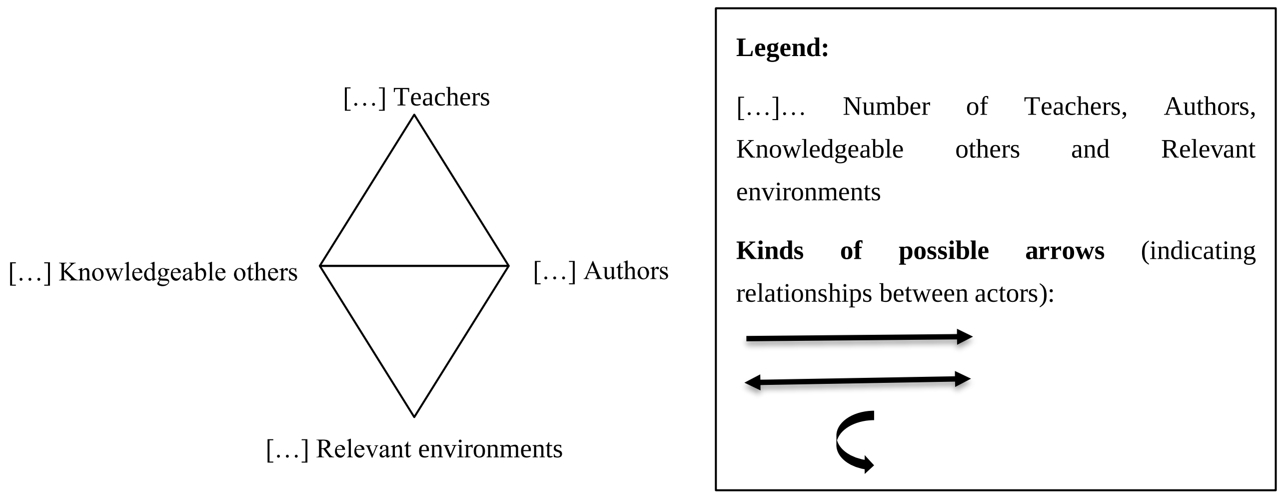

Diagram 1: RATE structure (Source: own illustration)

The selection of papers

The goal was to analyze one or two papers from each continent (Africa, Asia, Europe, North America, Oceania and South America). The selection of papers followed three criteria:

- Searching for papers with a clear focus on the topic (keywords were linked by logical operators: (mathematics) AND ((collaboration) OR (“teacher collaboration”) OR (“collaborative lesson research”) OR (“community of practice”) OR (“community of inquiry”) OR (“teacher interaction”) OR (cooperation));
- Searching for recently published papers (2018 and 2019);
- Searching in high quality journals (according to Williams & Leatham (2017) the focus was directed on the following two “very high” and five “high” quality journals: ESM & JRME and FLM, JMB, JMTE, MTL & ZDM); In order to fulfill the all-continent-goal a hand search for additional quality papers was connected (starting from literature in the discussion document).

When focusing only on titles and abstracts, the systematic search identified 20 papers. The reading of the full texts led to five papers (Asia 1, Europe 2, North America 1 and South America 1) focusing clearly on the topic. Therefore, a hand search was needed, focusing on Africa and Oceania. This led to eight further papers in journals and one article in an anthology. The selected African paper is cited as “in press” in the discussion document (and is published now), the Australian paper is published in a journal very close to “high” ranked (according to Williams & Leatham, 2017) and refers to authors cited several times in the discussion document.

In the following the seven selected papers are described and visualized using the RATE tool.

Description of the seven initiatives

Initiative 1 (Africa): A Case of Lesson Study in South Africa (Adler & Alshwaikh, 2019)

The relevant actors: 4 secondary mathematics teachers (“Lesson Study group”), 4 researchers (“project members”, “project team” from university, “we”) and 2 authors (part of project team).

The relevant targets: The targets refer to two target domains (teachers’ teaching and researchers’ co-learning), namely “improving the learning and teaching of mathematics in previously disadvantaged secondary schools” (p. 318), and “systematically research our co-learning” (p. 327). The key intervention (producing a relevant difference) is directed to teachers’ improvement of teaching.

The relevant environments: 1 university and 3 schools. Furthermore: one province in South Africa; provincial departments of education; school clusters; curriculum implementors; cluster leaders; further teachers at the school level; lesson study practice in Japan.

The type of initiative (duration): (Research-linked) professional development as Lesson study project (1 year). The key processes Planning-Teaching-Reflection were supported by using an analytic framework (Mathematical Discourse in Instruction & Mathematical Teaching Framework as a boundary object, moving between being a research tool and a tool for teaching).

Specificity of collaboration: Small collaborating groups of teachers from clusters of schools: “opportunities for teachers and researchers together to learn about teaching and how the tensions and dilemmas we [the researchers] faced were simultaneously opportunities for strengthening the coherence of the community” (p. 326). Although the initiative is regarded as a professional development, those carrying out this professional development do not name themselves as teacher educators or something similar (role) but describe themselves as researchers (identity); thus, the “actor system” shows with “teachers” and “researchers” a clear difference in identity and goals (learning in order to teach better; learning in order to generate new scientific insights), however, both having the role of “co-learners”. Compared to the six other initiatives, the statistics of key words in the text (excluding references) regarding this initiative shows the highest occurrence of “learning”, “reflection”, “group/s” and “lesson study”. In combination with the high frequency of use of “teacher/s” and “researcher/s”, this confirms the claim of co-learning.

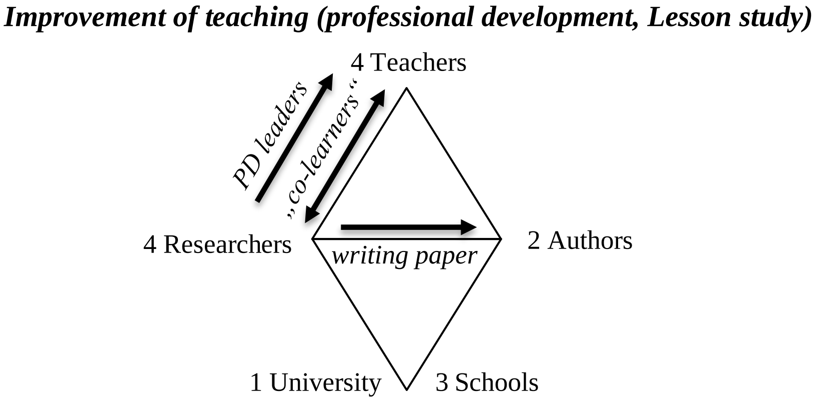

Diagram 2: RATE 1 – Africa (Source: own illustration)

Initiative 2 (Asia): Mathematicians and teachers sharing perspectives on teaching whole number arithmetic: boundary-crossing in professional development (Cooper, 2019)

The relevant actors: (“approximately”) 20 primary mathematics teachers (“teachers”), 2 mathematicians (“research mathematicians”), 1 “broker” (“participant-observer researcher”, “I”, “author”, “PhD in mathematics education”) from university and 1 author (broker).

The relevant targets: The quotation (referring to a setting in which) “the two communities could share their perspectives with each other, not only allowing the teachers to benefit from the mathematicians’ perspective, but also providing an opportunity for the mathematicians to attain the sensitive understanding” (p. 70) indicates a clear goal. The requirement by the Israeli Ministry of Education that primary-school teachers need “to enroll in mathematics PD courses in order to specialize in mathematics” (p. 71) expresses a second goal. The key intervention (producing a relevant difference) here seems to be to fostering communication and collaboration between all relevant stakeholders regarding mathematics teaching, in particular embedding mathematicians, thus overcoming “conflicts between the communities of mathematicians and mathematics educators” (p. 69).

The relevant environments: ICMI as a powerful international association (who encourages “a link between educational researchers, curriculum designers, educational policy makers, teachers of mathematics, mathematicians, mathematics educators and others interested” – see International Mathematical Union, 2019), Israeli Ministry of Education and 1 University;

The type of initiative (duration): professional development, co-taught by a PhD-student of mathematics and a Master’s student of computer science (1 academic year).

Specificity of collaboration: Two mathematicians provide a professional development course for primary teachers, whereby a mathematics educator serves as a broker between the two “communities” or “parties” (sociocultural difference): “I also highlight the role of a participant-observer researcher as a broker in this process, supporting events of boundary-crossing in which the parties came to explicate, and sometimes change, their own perspectives on teaching and learning mathematics with respect to the perspectives of others“ (p. 69). A main relevant difference refers to the goals which are mixed in a delicate way: teachers need to up-grade their mathematical knowledge, at the same time they are expected to be co-learners with mathematicians: „Together, these two lesson segments represent two types of PD activity – one “content based“ – designed and led by mathematicians and dealing with particular mathematical content – and the other “problem based“ – led by teachers and dealing with authentic issues of classroom teaching. In the first, episodes were selected and analyzed in detail to showcase opportunities for learning through boundary-crossing“ (p. 72). The frequency of words like “boundary/ies”, “sharing”, “mathematicians” and “community/ies” underline the descriptions above.

Improvement of teaching and of stakeholder-collaboration (professional development)

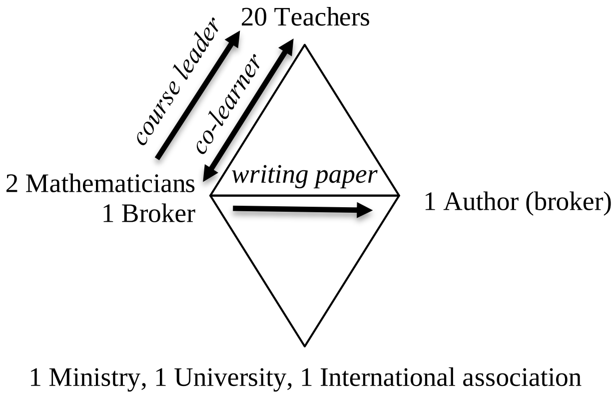

Diagram 3: RATE 2 – Asia (Source: own illustration)

Initiative 3 (Australia): Boundary crossing and brokering between disciplines in pre-service mathematics teacher education (Goos & Bennison, 2018)

The relevant actors: 6 project teams (“comprising at least one discipline academic and one education academic”, “mathematicians and mathematics educators”) from 6 universities, comprising 23 investigators (“the participants in the research were the mathematicians and mathematics educators who comprised the IMSITE project teams” (p. 261)) and 6 lead investigators.

The relevant targets: It is stressed that the project contributes to the “improvement in the quality of mathematics and science teachers” (p. 256). Further goals are “(1) fostering genuine, lasting collaboration between mathematicians, scientists, and mathematics and science educators who prepare future teachers and (2) identifying and institutionalizing new ways of integrating the content expertise of mathematicians and scientists […] with the pedagogical expertise of mathematics and science educators […]” (p. 256). This includes “[i]dentification of principles for fostering new forms of collaboration between discipline academics and education academics” (p. 258). The goals above indicate that the key intervention (producing a relevant difference) is a twofold one: through fostering collaboration between relevant stakeholders in mathematics and science education, the quality of mathematics and science teaching should be improved.

The relevant environments: 6 Australian universities (each working with a "Cascade university"); a project focusing on "teacher education strategies" (p. 257), funded by the Australian government.

The type of initiative (duration): Various (mainly pre-service, but also in-service) teacher education strategies (e.g., "Design courses" in order to improve recruitment and retention, or to initiate innovative curriculum arrangements; "Conduct a mathematics pre-service teacher education alumni conference to connect current students, graduates, teachers, teacher educators, and mathematicians in order to promote continuing professional learning" (p. 257) (3 years).

Specificity of collaboration: Initiating and sustaining interdisciplinary collaboration (mathematics and science as subjects, content and pedagogy/education as fields of expertise) and interuniversity collaboration (between universities and between universities and their cascade universities). This complexity is even amplified by integrated foci (primary and secondary schooling, prospective and practicing teachers). Besides the already mentioned goals, the project has also a strategic goal ("to promote strategic change in teaching and learning in the Australian higher education sector", p. 258) and ambitious research goals (e.g., investigating conditions that enable or hinder sustained interdisciplinary collaboration). This initiative has the highest occurrence of the words "collaboration" and "project", indicating the clear focus on bridge-building through a joint activity. In addition, also the word "mathematics" is used most in this initiative, mirroring the involvement of mathematicians in the project.

Improvement of mathematics and science teacher education and interdisciplinary collaboration (variety of teacher education strategies)

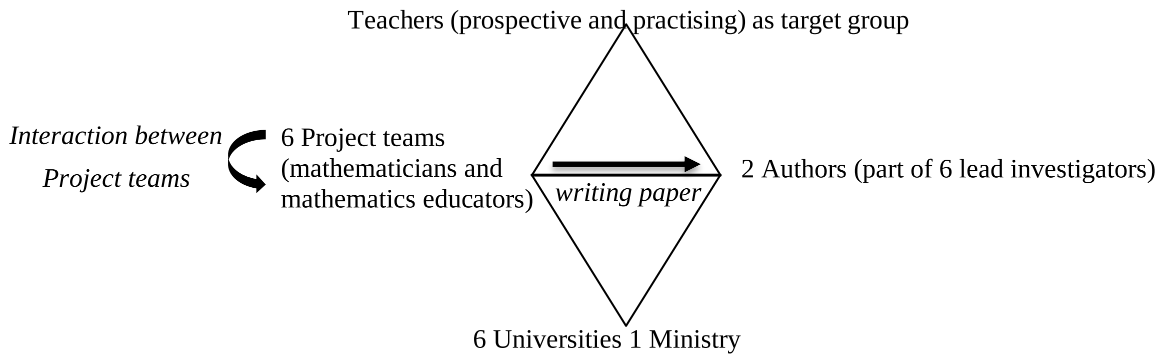

Diagram 4: RATE 3 – Australia (Source: own illustration)

Initiative 4 (Europe 1): Impact of professional development involving modelling on teachers and their teaching (Maass & Engeln, 2018)

The relevant actors: 326 secondary mathematics teachers (in the overall project, more than 1000 – mathematics and science, primary and secondary – teachers are involved); "course leaders" in 12 countries (teachers, pre-service educators, persons from CPD institutions) selected and educated by the "project partners" in each country (the "we" in the paper refers to the whole project, including all project partners); 2 authors (1 project coordinator and 1 researcher).

The relevant targets: The goal of the project is to achieve an impact "on teachers and their teaching" (p. 273) in direction of "implementing innovative teaching" (p. 273) through "inquiry-based learning" (p. 275). Regarding mathematics in secondary school, for example, the target is attaining "a significant change regarding the implementation of modelling in mathematics teaching" (p. 274). For improving teaching, the project also has the goal of implementing it at a large scale (double-goal). Thus, also the key intervention is doubled: the relevant difference aimed at is innovative teaching (as opposed to current teaching) and this should be implemented (scaled up) at many places (as opposed to some individual places).

The relevant environments: 14 universities ("project partners") from 12 countries, participating in a large EU-project; 7th Framework program of the EU which defines the context of the project.

The type of initiative (duration): continuous professional development (CPD) courses in 12 countries (running "within a timeframe of 2 years in each country", from "several weeks" to "the duration of 1 year", p. 277-278). With "about 100 teachers in each country" taking part, the project is relatively large. It is also complex, based on the doubled target group (mathematics and science teachers), the diverse cultural contexts of the 12 countries, the heterogeneity of course leaders' education and competencies, and their role as brokers between the goals of the project and the needs of the teachers.

Specificity of collaboration: In the courses 7 CPD principles (quality criteria) were implemented, including "stimulate cooperation between teachers so as to support teachers in the learning-on-job phases" (p. 277). In order to ensure quality across all 12 countries, the project partners "discussed the overall CDP principles and their implementation in the PD course at the biannual project meetings" (p. 278). The research questions reported in the paper refer to secondary level mathematics teachers participating in the project, and their students, focusing on the impact of a mathematics course on modelling. The collaboration among teachers, course leaders and project partners and between them is not a major focus of the paper. Given the size of the project and the various contexts in the participating countries, it is likely that the collaboration is manifold and takes very different forms. The size of the project and the large number of teachers dealt with in the research paper (macro perspective) seem to shift the view on concrete collaborations (micro perspective) to the background. It is not surprising that the occurrences of words like "collaboration" and "community/ies" is low, however, those of "teaching" and "student/s" relatively (to the other cases) high.

Implementing innovative (inquiry-based) teaching (CPD courses, scaled up through the "cascade model")

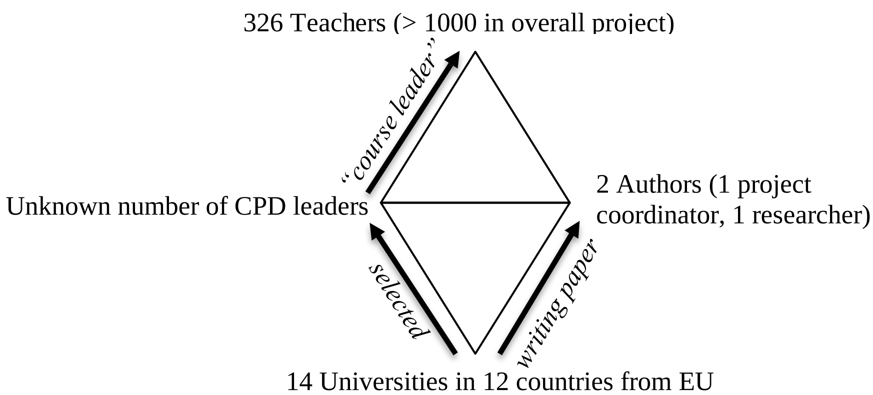

Diagram 5: RATE 4 – Europe (Source: own illustration)

Initiative 5 (Europe 2): Collaborative design of a reform-oriented mathematics curriculum: contradictions and boundaries across teaching, research, and policy (Potari, Psycharis, Sakonidis & Zachariades (2018)

The relevant actors: 34 members of a “design team” for a mathematics curriculum, comprising 11 “classroom” teachers (from kindergarten to secondary school), 15 “academic researchers” (2 mathematicians and 13 mathematics educators) and 8 “policy makers” (2 ministry and 6 school advisors).

The relevant targets: The goal is to develop “a reform-oriented national mathematics curriculum for compulsory education in Greece” (p. 2) in order to establish “the improvement of students’ learning as common goal” (p. 16). Reflecting on an interview, “the quality of students’ mathematical thinking and their future citizenship” (p. 14) is stressed. The key intervention is the shift from “traditional teaching” (resources) to “research-informed teaching” (resources), as highlighted in a paragraph heading of the paper on page 11. An important role plays the ministry who “initiated a curriculum reform through the New School act”, focusing on “active engagement of students, openness of the education to the society, …, and new roles for teachers as active agents of the curriculum” (p. 5). The commissioned “design team” amplifies this orientation on active engagement by giving key members a voice when generating interview data about the process.

The relevant environments: 1 Ministry of Education that commissions a design team, based on a new policy document (“New school act”, p. 5); a research team (from 2 universities).

The type of initiative (duration): A design team produces a final version of a curriculum (9 months). The paper reflects and analyses the design process, grounding it on a specific theoretical framework and on empirical data (e.g.,, based on interviews with 11 “key design team members”, p. 7).

Specificity of collaboration (here, not in a teacher education context): The ministry appoints a coordinator (researcher from a university) of a national mathematics curriculum and further members (design team) “based on the coordinator’s recommendations” (p. 6). During the curriculum design process “the coordinator acted as a broker between the educational policy activity and the designing activity” (p. 5). The social dimension of designing a curriculum and the use of an activity theory perspective is mirrored in a relatively high occurrence of words as “team/s”, “community/ies”, “member/s”, “colleague/s”, “broker/s” and “ministry/ies”. Also words like “teaching, “activity/ies”, “boundary/ies” and “mathematics” are used (relatively) often.

Designing a reform-oriented national mathematics curriculum (collaboration of a design team)

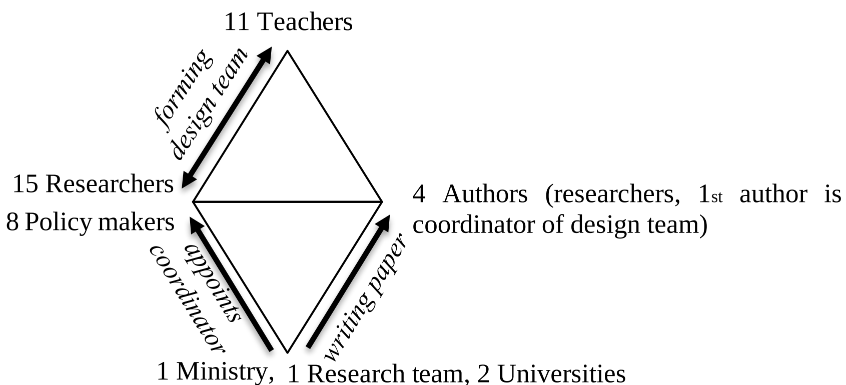

Diagram 6: RATE 5 – Europe (Source: own illustration)

Initiative 6 (North America): Supporting secondary rural teachers' development of noticing and pedagogical design capacity through video clubs (Wallin & Amador, 2018)

The relevant actors: 3 mathematics teachers "who comprised the entire mathematics department of one secondary school" (p. 1) and 1 researcher (first author of the paper) formed a "video club"; 1 additional researcher contributed to data analysis and writing of the paper (second author).

The relevant targets: The goal is a high level of teachers' capacity in "noticing" (of student thinking) and in "pedagogical design" (p. 1). This means that a key relevant difference (intervention) is teachers' improved competence. Among others the research focused on the question whether teachers' participation in the video club influenced "their view of collaboration" (p. 1). The quotation "Furthermore it is likely that without the video component of the collaboration process and the coparticipation among the teachers […] of these teachers […] would not have made the degree of growth they were able to accomplish due to their initial beliefs" (p. 20) shows that another relevant difference is seen between teacher collaboration and no teacher collaboration (through video club).

The relevant environments: 1 school (with its entire mathematics department; during the first phase "Introduction to school setting", also a "Meeting with administration" is mentioned) in a rural school district; 1 University (the teachers attended university PD courses etc. led by the 1st author).

The type of initiative (duration): The teachers attended 5 video club meetings (1 year). The paper reflects and analyses the design process, grounding it on a situated perspective and on empirical data, based on interviews with 11 "key design team members" (p. 7).

Specificity of collaboration: A major part of the collaboration was the reflection on lessons, based on lesson plans and videos. The video club aimed at fostering a "culture of supportive constructive feedback and discourse" (p. 7). "The researcher intentionally selected the video clips for the video clubs himself, as opposed to having the teachers select clips" (p. 9) in order to focus specifically on students' mathematical thinking. Recognizing "the value of coparticipation, these conversations were informal and mostly directed by the participants, but moderated by the researcher" (p. 9). The (compared to other cases) relative high occurrence of words like "participation", "participant/s", "colleague/s" and "school/s" indicates the collaborative nature of the video club. Often used words like (instructional) "decision making" and teachers' "beliefs" (regarding curriculum, mathematics, tasks, etc.) mirror the work on concrete instructional activities.

Supporting teachers' capacity development in noticing and in pedagogical design (PD project, video club)

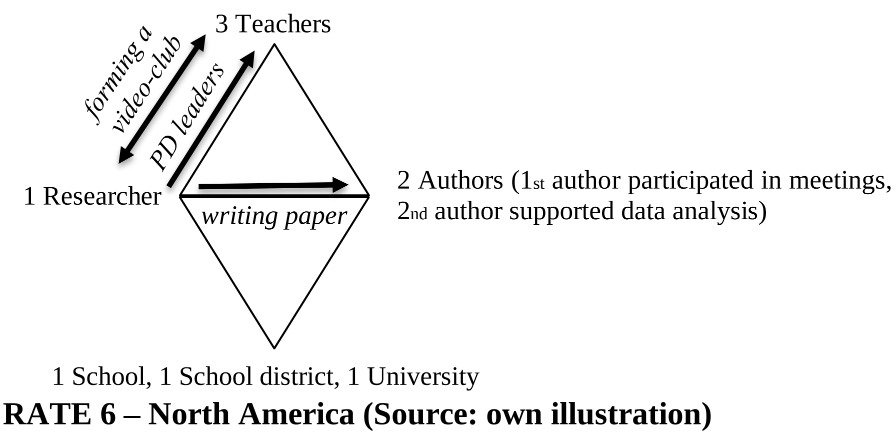

Diagram 7: RATE 6 – North America (Source: own illustration)

Initiative 7 (South America): How teachers learn to maintain the cognitive demand of tasks through Lesson Study (Estrella, Zakaryan, Olfos & Espinoza, 2019)

The relevant actors: 4 primary school teachers ("them"; teachers "with training in mathematics education and had more than 5 years of experience" (p. 5) and 3 researchers ("we"; "with experience in Lesson Study and teacher training" (p. 5)) worked together in a "Lesson Study group". There was also a research team (6 researchers – 3 of whom had worked in the Lesson Study group), who analyses the implementations.

The relevant targets: The overall goal is "the improvement of mathematics learning" (p. 1). The specific goal of this study was to investigate "how primary school teachers implement high-level cognitive demand tasks in a data analysis lesson in the context of Lesson Study" (p. 5). Implementing and maintaining a high level of cognitive demand is a special indicator for students' high quality. The key intervention is directed towards producing a relevant difference in students' learning, intending a shift from low level to high level of cognitive demand. Thereby, Lesson Study is seen as a teaching method "for transforming teaching" (p. 3), overcoming "the teacher-centered teaching model" which "remains dominant in most schools" (p. 5). The (rather general) conclusion of the study is that through "collaborative work among working teachers and researchers in the context of Lesson Study" it is "possible to design and implement tasks that maintain high cognitive demand in primary school" (p. 13). This indicates that "collaboration" (in the context of a Lesson study) is regarded as a key intervention leading to success.

The relevant environments: 4 teachers from 4 "Chilean public schools" (p. 5).

The type of initiative (duration): The Lesson Study group had eight 2-hours Lesson Study sessions (weekly). During these sessions "the group prepared the lesson plan and material and discussed the implementation of the lesson and how to improve it" (p. 5). The research team for the analysis of the implementation "met weekly for 2 h for 6 months" (p. 6).

Specificity of collaboration: One of researchers' most important work fields during the Lesson Study group was support: "With the support of the researchers, the teachers designed an open-ended task with consideration of the presentation of the data and the context and elements of high cognitive demand tasks for the grade, such as representing and arguing" (p. 5). Also one collaborative work field was the professional development of the teachers in the sessions – "The eighth session was dedicated to the teachers' self-evaluation and reflection on the experience of the Lesson Study cycle, the statistical knowledge acquired and the impact on their professional development" (p. 6). "[…] Lesson Study promotes the development of teachers' abilities to interact effectively with their students to promote their deep involvement in a task" (p. 12 f.). Not surprisingly this initiative (like initiative 1) has the highest occurrences of the words "lesson study" and "group/s". In this case, also "evaluation/s", "intervention/s" and "reflection/s" are more often used words, in contrast to other initiatives (with exception of "reflection" in initiative 1). This mirrors the teachers' active and self-critical stance as fostered by the researchers, for example: "The […] session was dedicated to the teachers' self-evaluation and reflection on the experience of the Lesson Study cycle" (p. 6).

Supporting teachers to implement high cognitive demand tasks (PD with Lesson Study)

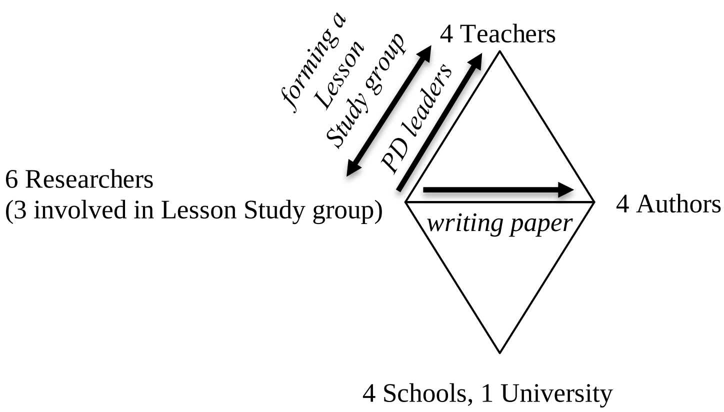

Diagram 8: RATE 7 – South America (Source: own illustration)

Comparing the cases

Comparing seven cases can only be a first step in grasping phenomena. However, it is possible to generate observations, possible hot issues and blind spots, and might create some ideas to develop a tool used in a larger study aiming at representativity.

Before we focus on communalities and differences between the seven cases, we check our seven cases with a survey of research in mathematics teacher education from 1999 to 2003 (Adler, Ball, Krainer, Lin, & Novotna, 2005). In this survey, two (of three claims) are relevant to our comparison. Claim 1, „Small-scale qualitative research predominates", can be confirmed: Since more than the half of our cases (4 out of 7) can be counted as small-scale (N<20), we have nearly the same picture as in the survey by Adler and colleagues (e.g., with 38 out of 65 JMTE papers based on small-scale research). Claim 2, "Most teacher education research is conducted by teacher educators studying the teachers with whom they are working", gets a very strong confirmation: all seven papers are written by people being involved into the activity (in six cases directly, in one case as a broker). A possible explanation is based on the fact that research on collaboration is a specific domain within the broad domain of research in teacher education in general (where a focus can e.g., be directed on teachers' beliefs, knowledge, etc., in a course etc. but not necessarily interconnected with an intervening activity besides collecting data). Of course, when dealing with research on "teacher collaboration", a research project just might investigate it without intervening. However, how can we deeply grasp the phenomenon of "collaboration", if we are not part of the collaboration? More general experts in system theory claim that in order to understand a social system (e.g., the ways, routines, patterns, hidden rules of collaboration etc.), we need to intervene in it (which happens automatically when we work with teachers and thus when we try to improve something). Interventions into a social system (a group, a course, a department, etc.) can cause a lot of reactions, towards those intervening (educators, facilitators, designers, etc.). The reactions can involve from open enthusiasm and collaboration (within the social system and with those intervening) to (open or hidden) resistance and tensions, with stark impact on the collaboration (internally and externally). We have many papers where successes (e.g., teachers' collaboration improved, the teacher-researcher collaboration was regarded as a powerful means) dominate, but no case which reports on activities which failed (at least partially). Also the seven cases reported here, representing very interesting research, don't report much on critical aspects of collaboration. However, collaboration would be an ideal topic to critically reflect interaction at different levels, for example: a) The teachers might explicitly be asked to share critical aspects of their collaboration (among teachers and with the educators); b) The teachers might explicitly be invited to comment on the results of the research (on collaboration); one could even think about inviting one or more teachers to be a co-author of a paper (eventually as an additional paper to a pure research paper); c) The authors (researchers) could – in addition to presenting their research results on collaboration – also integrate critical reflections on their collaboration as a team and on their activities of fostering teachers‘ collaboration.

In the following, we sketch some observations, using the RATE scheme, along relevant actors of the collaboration, relevant targets and relevant environments.

Relevant actors: The seven initiatives show – apart from mathematics (and science) teachers (in all 7 cases) – a variety of actors, including educators (6 initiatives), researchers (6), heads/principals (5), mathematicians (4), brokers (3), facilitators (3), heads (3), administrators (2), policy makers (2), multipliers (1) and teacher leaders (1). Besides the issue of being an actor, it should be mentioned that in all initiatives “students” appear, but no “parents”. As social entities we find (video) “clubs”, different “communities”, (lesson study) “groups”, (project) “partners” and (design and project) “teams”.

Relevant targets: In all initiatives one target is – although using different expressions and whether stating the goal explicitly or more implicitly – the learning of teachers and the improvement of teaching. The improvement of teaching is connected to quite different meanings like “innovative teaching”, overcoming “teacher-centred teaching”, “research-informed teaching” or supporting “inquiry-based learning”, hardly combined with a clear definition of the intended shift in teaching (marking a relevant difference between the status quo and the desired situation). In some cases, other adult learners like mathematicians and researchers are mentioned as co-learners.

Relevant environments: In all analysed papers occur universities (as working places of researchers), in many cases also schools (as working places of teachers, sometimes as places of intervention). Other environments concern ministries (2), a scientific association, a research team, a district, several countries or a political union. In the case of one initiative the text indicates that a requirement by a ministry established a distinct context for the initiative. It is assumed that also regarding other initiatives it would be interesting to read more about the context, in particular the goals and roles of stakeholders having an impact on the initiative.

Specificity of collaboration: The ways of collaboration are very diverse. Most initiatives describe extensively their particular approach. In many cases it would be interesting to read more about similar approaches and what the initiatives have in common or how they differ.

Summary

Finally, based on the analyses of this paper, we formulate some observations related to research on relevant actors, targets and environments of the collaboration.

Observation 1: Small-scale qualitative research predominates (see claim by Adler et al., 2005).

Observation 2: Most research is conducted by teacher educators studying the teachers with whom they are working (see claim by Adler et al., 2005).

Observation 3: Most research focuses on improvements and successes; (critical) reflections on researchers' (co-)learning – although focusing on collaboration – are rare.

Observation 4: Most initiatives focus on the learning of teachers and the improvement of teaching. However, the intended shifts in teaching (marking a relevant difference between the status quo and the desired situation = goal of intervention) is rarely defined in a clear way.

Observation 5: Only a few initiatives describe the context and relevant environments having a potential impact on the initiative.

Observation 6: Most initiatives describe extensively their particular approach. In many cases, it would be interesting to get comparisons to similar approaches (e.g., communalities and differences).

This paper doesn't allow the space to discuss about RATE as a tool for reflecting on research on teacher collaboration, for example, regarding its advantages, potentials, challenges and limitations. However, the conference could be a place, doing that. It is hoped that the description and the comparison of the seven cases and the formulated observations will serve as starting points for discussion and future research. In particular, the RATE tool might be further developed, for example by finding additional dimensions of analysis and/or for using it when analysing larger numbers of initiatives.

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RESOURCES FOR AND FROM COLLABORATION: A CONCEPTUAL FRAMEWORK

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Tools and resources are important for collaborative professional work and learning among teachers. They provide a means for the design of learning environments, for joint work in pursuit of learning goals, and can support links between teacher learning and teaching practice. In this paper, I will provide a framework for thinking about resources for teachers' joint work, in particular for systematic inquiry in professional learning communities. I elaborate four kinds of resources: knowledge; material/logisitical; affective; and human resources. Resources will function differently in different contexts and the absence of some resources may constrain certain forms of inquiry into teaching. I hope that this framework will be a useful resource going forward in our joint work in understanding the state of research in the field and how we might take it forward.

Introduction

Tools and resources are important for all joint work and learning, and particularly for collaborative professional work and learning among teachers. They provide a means for the design of learning environments, for joint work in pursuit of learning goals, and can support links between teacher learning and teaching practice. In this paper, I will provide a framework for thinking about resources as more or less tangible, drawing on the important point in the brief for Theme D that resources for and from collaboration are intertwined, and that developing and drawing on resources for collaboration can lead to further development of the resources as an outcome of collaboration, and as resources for practice. I will also address the question of missing resources.

I will develop the framework in relation to the existing literature and my own work in a four-year project with professional learning communities (PLCs), the Data Informed Practice Improvement Project (DIPIP), in Johannesburg, South Africa. DIPIP established and researched PLCs among high school mathematics teachers. The teachers engaged with a cycle of activities which focused on understanding and responding to the reasoning behind learners' errors in a range of teaching contexts: assessments, learner interviews, lesson planning, and interactions with learners in lessons. We supported teachers to see learners' errors as valid and valuable steps towards better mathematical thinking, and to become more responsive to learner errors as teaching opportunities. We researched the teachers' collaborations by analyzing their conversations in the communities and their teaching practices across the four years of the project. I will draw on various analyses and what we have learned to exemplify parts of the framework (Brodie, 2013b, 2019; Brodie, Marchant, Molefe, & Chimhande, 2018; Chauraya & Brodie, 2017).

The DIPIP project was located in Johannesburg, the major urban area in South Africa, a middle-income country, whose school system struggles more than those of many countries with lower GDPs. South Africa is the most unequal country in the world, as measured by the GINI coefficient, and our education system reflects this inequality, with the vast majority of learners in schools that struggle to support their learning (Motala, Dieltiens, & Sayed, 2012). Poor achievement in mathematics is widespread and strongly correlated with race and socio-economic status (Spaull & Kotze, 2015). The DIPIP project worked in schools that serve learners of mainly lower socio-economic status. While national policy requires PLCs in all schools (Department of Basic Education & Department of Higher Education and Training, 2011), there is little tradition of collaborative teacher work in South Africa. The teaching profession is not well respected, teachers are not well paid and there is substantial "teacher bashing" in the press. Teachers, particularly those in low socio-economic status schools, often have high teaching loads and teach large classes, and teacher morale is low. These schools are strongly managed by provincial departments of education, and there is very little trust among various levels of the system, with government, principals, teachers and parents often blaming and judging each other for the widespread low achievement of learners.

While my framework is informed by work in my context, I also draw on a range of work in other contexts. Sensitivity to context is an important element of any framework, however, there are a number of dimensions of the use of resources which can and do inform work across different contexts.

Conceptualising tools and resources for and from collaboration

The ICME-13 survey on teachers working and learning through collaboration defined collaboration as:

> co-working (working together) and can also imply co-learning (learning together). It involves teachers in joint activity, common purpose, critical dialogue and inquiry, and mutual support in addressing issues that challenge them professionally. (Robutti et al., 2016, p.652)

and teachers working together as:

> collaborating for some specific aims, which could be directed towards: improving students' learning; improving their professional role in the school; learning to use new resources (e.g., technological tools); creating a professional network within the school or region; and discussing institutional reforms and demands around the curriculum, the national evaluations system, etc (Robutti et al., 2016, p.653)

These definitions suggest that joint work produces learning and that collaborative learning requires the intent to work together and a purpose for this work. Collaboration can be local or more global, and can take place in face-to-face settings, or in virtual communities.

Wenger (1998) defines communities of practice as groups of people engaged with each other focused on a joint enterprise and creating a shared repertoire - a set of resources which support their enagement in relation to the joint enterprise. Communities learn together and are always sociohistorically situated in webs of social relations, as are the resources that support communities and their learning.

We have defined a PLC as a special kind of community of practice, with the distinguishing feature of professional learning (Brodie & Borko, 2016; Chauraya & Brodie, 2017), where professional learning entails becoming confident with and competent in the knowledge base of the profession and using it to make and justify professional decisions. Professional learning involves regular and sustained inquiry into various aspects of local practice, as they might relate to more global concerns (Jackson & Temperley, 2008). Louis and Marks (1998) argue that school-based PLCs allow teachers to "coalesce around a shared vision of what counts for high-quality teaching and learning and begin to take collective responsibility for the students they teach" (p.535). PLCs provide opportunities for systematic teacher collaboration and learning, usually facilitated. While collaborative work and learning can happen more or less formally, it is likely that thinking and practice develop more powerfully when worked on systematically. Stronger collaboration can become an outcome of PLCs, i.e. collaboration itself is both a means to and outcome of teacher learning.

The nature of tools and resources that support joint work and learning is an important theoretical concern for designing, researching and understanding teacher collaboration. It is important to distinguish between the tool/resource and the object of inquiry it mediates, and to be able to identify tool and object, and the relationships between them, in particular situations. Objects and tools do not exist outside of social relations and meaning-making. Objects of inquiry can become resources for further inquiry and new outcomes, through developing different saliencies for teachers. Tools created for collaboration can become resources from collaboration, i.e. an outcome of collaboration, if they change their meaning and use through the collaboration, i.e. the tools support teachers to see elements of their practice in new ways, and teachers come to see the tools and resources as helping them to work differently.

Vygotsky initially conceptualized tools as acting externally and/or internally, with a tool acting internally called a psychological tool or a sign (Vygotsky, 1978). For Vygotsky, material objects which mediate and support learning have a psychological function, i.e they change cognitive functioning. Tools that look similar can function differently, depending on how they are used. The most powerful tool, which acts both internally on our minds, and externally, on other people, is language. So for Vygotsky, tools are both material and go beyond the material, what is important is where and how they act. Perks and Prestage (2008) show how short phrases can act as tools which unlock reservoirs of meaning for pre-service teachers and support links between theory and practice.

Activity theory broadened the use of tools to include artifacts and resources which draw their meaning from and contribute to activity, mediating between subject and object in the context of a community (Engestrom, 1999). In activity theory, resources are both produced in and organize various aspects of activity systems and how different activity systems work together to produce learning (Akkerman & Bakker, 2011; Perks & Prestage, 2008). Wenger's (1998) theory of situated learning focuses on participation in communities of practice as they produce meaning, learning and identity, and introduces the notion of reification to denote the products of such participation, which inform future participation. Participation and reficiation continuously revivify each other. Reifications include knowledge, as well as material artifacts.

A view of professional learning as situated (Putnam & Borko, 2000) is informed by these theoretical perspectives on the use of tools and resources and how tools and resources might move and change across the different sites of professional learning - predominantly the community and the classroom (Akkerman & Bakker, 2011; Kazemi & Hubbard, 2008). While tools and artifacts may be thought of as material, the notion of resources extends these beyond the material. However, as I shall argue below, both tools and resources can refer to both the material and go beyond the material, and this is how I will use them.

A framework for tools and resources in collaboration

Drawing on the above, the following principles underlie my framework: tools and resources can be more or less tangible; they are socially and historically situated, they mediate the object of inquiry and therefore learning; their meaning and functionality depend on the contexts of their use; they emerge from and support activity systems and practice; and their influences will be more or less tangible or intangible. There are four kinds of tools and resources that support teacher collaboration: knowledge resources; material and logistical resources, affective resources; and human resources (people). Obviously, these four kinds of resources are related to each other. I will give a brief overview here and then elaborate them in the sections that follow.

Knowledge: The knowledge resources for mathematics teachers include mathematical knowledge and pedagogical content knowledge. How these are related to each other in the practices of teaching and professional development is the subject of much research and debate. Knowledge is developed in practice and activity, and informs future practice and activity. Developing professional knowledge requires ongoing, sustained inquiry, with a shared object of learning.

Material/logistical: These include the material resources used and developed in communities, including lesson plans, tasks and textbooks, videotapes of practice, and protocols for conversations. I include here "technological" resources, although I don't dicuss them in this paper. In addition to material resources, two important logistical resources are time for communities to work together and appropriate spaces for this work to happen, particularly in the service of long-term and sustained inquiry.

Affective: All learning involves emotions. For teachers to teach well, they need emotional sensitivity to themselves and their learners. For teachers to work together productively they need to be able to challenge each other's thinking and practices, in generative ways, and go beyond "contrived collegiality" (Hargreaves, 1991). Safety and trust are important to be able to learn with others, and emotions such as fear and bleame might work against collaborative learning.

Human: These include all the people involved in collaborative groups and supporting professional learning. People form an overarching category, since they make meaning of and draw together the other key resources. Since people in communities are the focus of Theme C, I will limit my discussion to the two key role-players in professional learning – teachers and learners.

Knowledge resources

Professional learning builds on, challenges and produces knowledge. This knowledge can be local, such as data or experiences from practice, and global, as in research findings and ideas for best practice. Successful professional learning should relate local and global knowledge (Jackson & Temperley, 2008; Katz, Earl, & Ben Jaafar, 2009). Mathematics teachers' knowledge is a key resource in collaborative learning - what teachers bring to their collaborations will inform the collaboration and its outcomes, and knowledge is obviously a key outcome of collaborative learning. Thus knowledge can be both tool and object, depending on the situation. Teachers' mathematical knowledge for teaching is best described as a combination of mathematical content knowledge (CK) and pedagogical content knowledge (PCK) (Ball, Thames, & Phelps, 2008).

Learning communities need a focus, or object of inquiry, that is both narrow and broad enough to allow for substantive discussion and sustained learning (Katz et al., 2009). The focus should allow for the development of both CK and PCK and should support teachers to draw on global resources to serve local understandings and vice versa. In the DIPIP project we chose the object as the reasoning underlying learners' mathematical errors. The assumption, based on the substantial errors and misconceptions research, was that systematic errors are built on partially valid mathematical reasoning and that making that reasoning explicit for teachers and learners can help both to value learners' current mathematical thinking and develop more robust mathematical thinking (Smith, DiSessa, & Roschelle, 1993). The focus on errors was a mechanism to access three important dimensions of teaching and learning mathematics: learners' thinking, which makes sense to them and can be worked with, even (and especially) when partially correct; teaching practice, which can work with learners' errors and thinking; and teachers' own knowledge, both content and pedagogical content knowledge. Errors can be seen as absences, as they often are by many teachers, or as presences, as a resource for future learning, and in this way they can become a knowledge resource for teachers.

Teachers were supported in a set of developmental activities to develop their knowledge of the reasoning behind learner errors in the various sites of teaching. They analysed tests, which provided an overview of strengths and weaknesses in learners' mathematical knowledge in a particular school, grade or class. Based on the test analysis, teachers chose learners who had made interesting errors that they wanted to understand more deeply and interviewed the learners. They then took the results of these two analyses and mapped them against the curriculum, working out when and how the key concepts were taught and what curricular issues might have contributed to the errors. Based on these three activities, teachers chose a leverage concept, which is a concept that underlies many of the errors that learners make in a topic, for example: the meaning and use of the equal sign. Once a concept was chosen, the DIPIP facilitator found literature on that concept, including learner errors in the concept. The community read and discussed these papers and drew on these discussions to plan lessons together. The lessons aimed to surface learner errors with the concept and to find ways to engage the errors, rather than avoid them. These lessons were taught and videotaped and the community then reflected on episodes in each teacher's lessons in order to understand their strengths and challenges in dealing with learner errors in class.

Our project design took a particular view of the development of teacher knowledge and practice. Our approach was that teachers tend to be most focused on what they do every day in their classrooms, i.e. their practice and their PCK. Therefore the best way to draw on and develop teachers' knowledge in an integrated way, is to start with their practice and PCK and to develop CK in relation to these (Brodie & Sanni, 2014). This is different from many PD programmes that start with CK and then move on to related PCK. One of our key principles was that in coming to understand learner needs, teachers can come to understand their own learning needs: what mathematics they need to learn and how to use this new knowledge to improve their practice (Brodie, 2013b). All teachers notice learner errors, however very few teachers see them as based on valid reasoning and as opportunities for deepening mathematical knowledge. We explicitly positioned teachers as both experts and learners. As experts, they contributed their knowledge of teaching mathematics, their contexts and their learners, and as learners they deepened this knowledge.

We have a range of evidence to suggest that the focus on errors was useful for some teachers and not useful for others (Brodie, 2019), and that we realized our aim to access both CK and PCK through a focus on PCK (Brodie et al., 2018). The first two quotes below show teachers who enjoyed the focus on errors and learned from it, the third quote is from a teacher who, while she enjoyed the focus, could not see how it benefited her practice.

> Being able to get the reasons behind learners' answers, I can now at least try to ask them…to keep on probing the learners until they realize their mistakes.
>
> We analysed question by question, concept by concept, that's where I saw that maybe, somehow, we are short-changing our learners. That was quite a rude awakening, ja, and I hope that I could use that even in the other subjects that I teach. Because I think it would have a far-reaching positive impact
>
> Where, what, how do I benefit from this ... it was nice arguing, identifying some of those errors made by learners, trying to think why they made these mistakes, how, why, you know, those different views from learners, people justifying those wrong answers, It was fun, it was fun, but you know, in as much as it was fun … I couldn't link what we were doing with what we are doing in the classrooms

The first two quotes suggest that the focus on errors supported teachers to see their learners differently and possibly become more responsive to them. The third quote suggests that for this teacher, a focus on more tangible, immediately applicable resources may have been more useful than our more long-term, less tangible knowledge resources.

We have some evidence that the focus on PCK supported conversations about both PCK and CK. In one community we found that 34% of conversations were on CK and 66% on PCK, depending on the particular activity (Brodie et al., 2018). Of the 58 CK conversations across the 17 meetings, 30 (52%) were triggered by PCK conversations. Of the 161 PCK conversations, 23 (14%) were triggered by CK conversations. Thus PCK conversations did lead to CK conversations, while the converse was less true (Brodie, 2014; Marchant & Brodie, 2016).

We also found that the knowledge content of the conversations in the PLCs varied in relation to the different activities and that taken together the different activities supported conversations on mathematics (CK), learner thinking, and teacher practice (PCK). For example, the lesson planning sessions supported conversations on mathematics content, as teachers tried out the tasks, and the videotaped reflections supported conversations on practice as teachers discussed responses to learners' errors (Chimhande & Brodie, 2016; Marchant & Brodie, 2016).

So knowledge is a key resource in teacher collaboration. Teachers come to their collaborations with knowledge and this knowledge is transformed during the collaboration to produce new knowledge. Professional learning activities can support teachers to talk about their knowledge with each other, to build explicitly on their own and others' knowledge, to develop ways of seeing differently and more deeply, and to develop different relationships between different forms of knowledge and between knowledge and practice.

Material and logistical resources

Much has been written on material resources, particularly the use of videotape in teacher development (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Karsenty & Sherrin, 2017). In our project, the teachers analysed tests, learners' responses to tests, interviews with learners and videotapes of their own lessons, and created tasks and lesson plans, which were also subsequently analysed and improved on. Joint work on tasks, texts and lesson plans are important to support teachers in using them well as resources in the classroom. We found that the teachers in our project struggled to anticipate the errors that learners might make as they planned tasks and lessons, even when they had analysed the errors in learners' tests, interviews and the literature.

In this paper I would like to focus on the less tangible resources of time and space, as these turned out to be crucial to the PLCs' functioning. Time and space are likely to function very differently in different contexts. When, and for how long teachers can explicitly do collaborative work and where they do this has consequences for the kind of work they can do, particularly the long-term sustained inquiry that is important for the success of PLCs. Time and space might be missing resources for teachers in lower socio-economic schools.

Time is probably the scarcest resource for busy professionals. All of the teachers we interviewed stated that time was a key challenge to participation: finding time to meet as a group and finding time to do the work of the project outside of meetings, for example the project readings or personal reflections on videotaped lessons (Brodie, 2019). None of the schools had time during the school day for teachers to meet and all of the communities met after school, which often clashed with teachers' personal commitments, such as picking up their own children from school. Some teachers spoke about making time for their collaborative work, for example:

> Because what I've also learnt is that here on earth there's no time, but one has to make time for anything after all
>
> But as for me I consider it as pressure that I can't do anything about, other than finding a better way to deal with … I must just find myself time

However, a number of teachers said that they could not find time and when there was a clash between school commitments and PLC commitments, they had to prioritise their school commitments. There were teachers in our project who had many, very big classes, leaving little time for other activities.

> You really find no time to give to this other project, because you know I have at least to give also my time to this because it's my contractual obligation … with the workload that we had, it's overwhelming, you know really, really it's overwhelming

Teachers at wealthier schools are more likely to have smaller and fewer classes, and therefore more time for joint work and professional development. At some wealthier schools, time can be found within the school timetable for collaborative work but in most schools in South Africa, teachers have to make use of their own time and personally prioritise their own development above other commitments. Chauraya (2016) showed that teachers' idea of collaboration before we introduced the PLCs, was informally asking for help from one other person. There were clear hierarchies as to who could help whom, in particular teachers teaching higher grades were assumed to have better knowledge and could help those teaching the lower grades, but the reverse was not always true. After working in the PLC for a year, teachers had a more robust sense of how all could contribute to each others' work and development. Less time for systematic collaboration might contribute further to inequalities across schools.

Space is also related to resource inequalities. Most government schools in South Africa have little space which can be dedicated to teacher collaboration. Classrooms, libraries, staffrooms, laboratories and small offices were variously used as spaces in our project, which meant that records of past meetings were difficult to display and all equipment had to be brought in for each meeting (equipment was also vulnerable to theft). So how material resources function in relation to space is an important area of further investigation. Not all collaborative groups need to meet face-to-face and online collaborations allow for teachers from different schools to meet and learn together. However, there are also inequalities between rich and poor in relation to access to the internet (data is very expensive in South Africa) and it is not yet known whether virtual collaboration can support the kind of systematic learning that PLCs do. Supporting ongoing sustained inquiry among teachers in the same school who can then make strong collective changes to their practices is central to PLC work and requires appropriate space in which to work.

Time and space are clearly resources for collaboration. It is less clear how time and space might be transformed into resources from collaboration. It may be that sufficient time for working together might support teachers to save time in other aspects of their work. If time spent in collaboration strengthens other resources, then more time may become available. Time and space might become objects of inquiry when teachers talk about how they sequence their teaching and organize their classrooms.

Affective resources

While it may seem strange to think of our emotions as resources, they are an important resource for learning and for collaborative learning. There is a strong history of research on emotions associated with learning mathematics (for example Black, Mendick, & Solomon, 2009; Hannula, 2012), but a definite absence of the role of emotions in mathematics teacher learning (Breen, 2009). There are no emotion words in the frequency cloud of words in the titles of the survey sources in the ICME survey on mathematics teachers' collaboration (Robutti et al., 2016, p.662), and a look through the volume: "Tools and processes in mathematics teacher education" (Tirosh & Wood, 2008) does not show a focus on emotions. So emotions are a missing resource in the research, and may be important when thinking about building equitable participation in collaborative learning.

A supportive environment for collaborative learning requires space for professional disagreement and conflict among ideas, so that there can be generative conversations and space for growth (Katz et al., 2009). At the same time, if conflict becomes personal, possibilities for collaboration and learning are reduced. So two key features for collaboration to be productive are safety and trust. Safety to challenge and be challenged, to agree and disagree, and trust that the process will support everyone's learning, and that contributions can both be given and received. Research suggests that where there are strong hierarchical relationships within schools and where teacher morale is low, it is difficult to sustain engagement in PLCs (Schechter, 2012; Wong, 2010). The following quotes show a range of trust that teachers experienced in their PLCS.

> We can talk to the other community members freely without, how can I say, stage fright. We are confident because we are talking with colleagues, knowing that no-one is judging you
>
> I don't know about videorising [sic]. I think we're coming from an era where people were critical about you, they were looking at all the bad things that you were doing ... Now, we understand you're videorising it so that we can see ourselves developing. But somehow, at the back of our minds, it's like, is it true? Are they not hiding something from us?
>
> Because at the end of the day, we would see it as a research which benefits someone and not us really

A real fear about working with others is being judged, explicitly, or without your knowledge. In some school systems, "poor" performance can affect job security, and so being judged can have material consequences for teachers. But it can also have emotional consequences, creating or reproducing doubts about not being good enough. Trusting that teachers and learners are the ultimate beneficiaries of the work may also be difficult for teachers working in schools and systems where their needs are not always taken seriously.

A the same time, communities that work well can support positive emotions associated with learning. A key role for faciliators of collaborative learning groups is to create and support safety and trust. In our project, we articulated the following facilitator moves for building community: validate all ideas as useful contributions to the conversation; articulate positive aspects of negative situations; identify with teachers' issues and concerns; and notice and counter disengagement and exclusion (Brodie, 2016).

One reason that we chose a focus on learner errors in our project is that we wanted to support teachers to see learners as mathematical thinkers, to hear and interact with their thinking. As teachers began to interpret and explain learners' errors and to see errors as reasoned and reasonable, they began to blame themselves, or other teachers, for learners' errors, finding the reasons for learners' errors in how they were taught previously. The PLC facilitators worked hard to counter this tendency to blame, but we saw it recurring in our data and believe that it is difficult to work against, given the widespread blame of everyone by everyone in our system, and the emotions associated with not suceeding in mathematics, which are made even more salient when focusing on what goes wrong: errors. As one teacher told us:

> Because now it was somehow it was a bit painful. If you find that learners who were doing grade eleven, and learners who were doing grade eight, they were given almost the same test. But now when you check the errors that were done by grade eleven learners, were the same as the errors that were done by grade eight.

Many of the teachers expressed similar pain when confronted by the fact that learners made the same errors as they progressed in their school careers. One teacher told us that when you see them making errors in grade 8, you can blame it on the primary school teachers, but when the errors persist when taught by herself and her colleagues, it becomes difficult to decide where to apportion blame.

For some teachers, this was a learning and growth point – they realized that there is no blame for errors, they are a normal part of learning mathematics and we can develop strategies for working with them to develop learners' mthematical thinking, but for other teachers, the explicit focus on learner errors was demoralizing and contributed to their feeling overwhelmed by their work.

So emotions have consequences for collaboration and learning, both positive and negative. Emotions are also clearly both tool for and object of inquiry, because an important part of teacher learning must be to focus on their own and their learners' emotions, particularly when dealing with errors. Emotions are clearly transformed through collaboration and learning – understanding why and how is an important task for researchers.

People

People are both actors and resources in teacher collaborative work, and as they bring together the three other sets of resources, they function at a different, overarching level. As Theme C notes, there can be a number of participants in collaborative teacher learning: teachers, facilitators, researchers and principals, among others, and they bring different strengths and resources to the collaboration. From the perspective of Theme D, how people revivify resources for learning is important, as well as how they may be or become key resources for collaborative learning.

Teachers bring their knowledge, material resources, time constaints and emotions to collaborations and draw on all of these to contribute to their own and others' joint work and learning. These contributions are made through language and also produce new language as part of the collaboration (Perks & Prestage, 2008). Dialogic inquiry supports participants to hear different perspectives and to co-ordinate these into new directions for practice. How to create and sustain the conditions for dialogic inquiry is an important role for facilitators (Brodie, 2016).

In order to understand how collaborative groups support teacher learning, we can look at the nature and extent of conversations in collaboration. As quoted above, we saw how conversations on PCK led to conversation on CK more often than the reverse (Brodie et al., 2018). We have also shown how conversations in the different activities focused on different elements of PCK and CK, with the activites taken as a whole focusing on both (Chimhande & Brodie, 2016; Marchant & Brodie, 2016). Shifting the conversations to different levels of depth was more difficult, and these tended to stay similar across activities and years.

How teachers feel about their collaborations is important in how they work and learn together. Chauraya (2016) showed how teachers' views of collaboration shifted over time in their community. As teachers begin to look to others, they can see themselves in new ways. We saw teachers asking for resources, for example in one community a teacher wanted to know why the better-resourced school down the road did not come to share their knowledge in the community. While this can be interpreted as a teacher assuming that better material resources means better knowledge resources, it can also be seen as understanding that extending the community beyond a school can be helpful. (Brodie, 2013a).

While much of the research focuses on teachers, and their use of resources, learners might be thought of as a missing resource in teachers' collaborative learning. Although learners are usually represented in teacher collaboration through their work, in videotaped lessons and other artifacts of practice, teachers often don't see learners' perspectives when thinking about learner work. Our project focused on learner errors, and as I have shown above, this supported some teachers to see learners differently, although with some pain and blame. But we did not find out about learners' views of their errors, or about emotions that are evoked among learners when teachers work with learners' errors.

There are some examples of work with learner perspectives in relation to teacher learning. Vogler and Prediger (2017) captured students' views of a teaching situation on videotape and then showed these to the teachers, showing how seeing students' perspectives helped the teachers to think about the consequences of their interactions with students and how they might interact differently in future. In work subsequent to the DIPIP project, we have shown that different ways of valuing and working with learner errors are important in learners' mathematical identities (Gardee, 2019). Also interesting is a study by Sherman and Catapano (2011), where students participated in a mathematics club and in-service and preservice-teachers participated as mentors in the club. From interacting with their own students in a different context, the teachers came to see them as productive mathematical thinkers. So introducing actual learners as real people, with thoughts and feelings about their learning may be an important missing resource for teacher collaboration and might support stronger collaborations and learning.

Conclusion

Teachers joint work can happen in a number of ways: from informal conversations to asking for help, to marking or planning together, through various forms of systematic learning in groups or PLCs. In each of these learning contexts, tools and resources will work differently and mediate the work and learning in different ways. In systematic learning, the role of tools is particularly important in that they provide ways of seeing and noticing key elements of practice in new ways, support teachers to see themselves, their learners and their work differently and can therefore contribute to shifting learning and practice.

Since tools and resources are socio-culturally situated, and interpreted and given meaning by people in communities, they will be used differently in different contexts and will support different kinds of learning. The absence of resources, including time and space, and less attention to emotions and learners, may constrain certain forms of inquiry into teaching.

I hope that this framework linking knowledge, material, affective and human resources will be a useful resource in our joint work in understanding the state of research in the field and how we might take it forward.

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Plenary Panel Reports

ICMI STUDY 25

Coordinator | Hilary Hollingsworth — Australia

Working and learning in collaborative groups: What's key to mathematics teachers? (Introduction and Synthesis)

Panelist | Yiyi Chen — China

China-England Mathematics Teacher Exchange Programme

Panelist | Christelle Fitamant — France

Institute for Research on Mathematics Teaching - Brest, France

Panelist | Lameck Dition Sandram — Malawi

The Improving Quality and Capacity of Mathematics Teacher Education in Malawi Project, A Norwegian and Malawian Collaboration

Panelist | Shelli Temple — USA

Professional Learning and Collaboration via Social Networks

WORKING AND LEARNING IN COLLABORATIVE GROUPS: WHAT'S KEY TO MATHEMATICS TEACHERS? INTRODUCTION

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This report examines the accounts of four mathematics teachers focused on improving their professional practice through collaborative group experiences. The teachers work in very different contexts and were involved in very different types of collaborative groups, yet their accounts reveal that there are key elements of their collaborative group experiences, and outcomes of those experiences, that resonate across locations and activities. The report is presented in three sections. First, in this Introduction, the aims of examining the collection of teacher accounts are identified and some observations related to design features of the four collaborative groups are presented. Second, the four teachers' stories are provided. And third, a synthesis of lessons learned from the experience of these teachers is presented, and questions are proposed to provoke consideration of how these might inform directions for mathematics teachers working and learning in collaborative groups in the future.

Research suggests that collaborative professional development has the potential for impactful effects on teaching practices and student achievement (Borko, 2004; Jensen, 2014; Opfer, 2016). Yet, the OECD's most recent Teaching and Learning International Survey (TALIS) data show a comparatively low percentage of teachers participating in collaborative professional learning activities (OECD, 2019).

This report examines the involvement of four mathematics teachers in collaborative groups, with the aim of (i) understanding the kinds of opportunities for learning that these groups provided and the impact that they had, and (ii) synthesizing insights from the teachers' participation in these groups to inform the ways that mathematics teachers might work and learn in collaborative groups in the future. Each teacher was asked to report on three areas related to their collaborative group work and learning, including: the context, purpose and design of the collaboration; outcomes of the collaboration; and, particular lessons learned (for example, in terms of: any factors that supported or limited the collaboration; any surprises or challenges encountered by participants; specific components of the collaboration that they believe provoked sustained changes in their own approach to teaching mathematics; or, ways they might reimagine the collaboration to make it more effective).

The four teachers who share accounts of their participation in collaborative groups work in different continents (Africa, Asia, Europe, North America), teach at different levels (primary, secondary, tertiary), and were involved in collaborative groups that have different forms (a pre-service lesson study group, an international teacher exchange program, a teacher/researcher network, an online teacher network). Table 1 provides an overview of the different collaborative groups, including their location, form, participants, and foci.

Table 1: Overview of the four teachers' collaborative groups

Location	Form	Participants and role of the teacher	Work and learning focus
Machinga Teacher Training College, Malawi, Africa	Pre-service lesson study - part of a national initiative aimed at improving mathematics teacher education	9 participants: - teacher educators from Machinga Teacher Training College - *role*: leader of the lesson study group	Planning, teaching and analyzing a lesson study research lesson for pre-service student teachers
Shanghai, China and England, United Kingdom (UK)	Teacher exchange program: China-England Mathematics Teacher Exchange Programme - funded by a UK federal grant - main activities: reciprocal school visits, collaborative workshops	- UK Year 1 to Year 9 teachers led by experts from the National Centre for Excellence in the Teaching of Mathematics (NCETM) - Shanghai teachers led by experts from the Shanghai Normal University - *role*: teacher in one of the Shanghai base schools visited by UK teachers	Collaborative planning, sharing and research in preparation for UK teacher visits to Shanghai; mathematics topics for lesson study research lessons purposefully selected
Brest, France	Teacher/researcher network - part of a network of Institutes for Research on Mathematics Teaching - monthly meetings - regular email communications	6 participants: - 3 secondary school mathematics teachers - 2 university mathematics teachers - 1 French teacher working both at a secondary school and a teacher training institute - *role*: secondary mathematics teacher	The teaching of logic in secondary school - specific related foci negotiated based on the needs of the group
Oklahoma, United States of America (USA)	Online Professional Learning Network (PLN): Math Twitter	Thousands of mathematics teachers from around the globe	Flexible content and foci developed by participating teachers

	Blog-o-Sphere - informal collaboration using social media hashtags and Facebook groups to connect subject-area teachers - organized efforts including: book studies, professional learning events, face-to-face conferences, websites, blogs	networking online - *role*: participant, virtual mentor, online partner	to connect with others

Some observations related to the design of the different collaborative groups follow.

Design feature 1: Overarching purpose

An overarching purpose for the work of the collaborative groups is consistently articulated across the teachers' accounts. Each group has a strong focus on the improvement of teaching practice with the subsequent outcome of improved student learning outcomes. The teacher from Malawi, for example, reports,

> … the overall aim was to improve the quality of mathematics teaching in Malawi. As such, lesson study was introduced as a way to achieving the aim. It was thought that lesson study will be a scaffold for student teachers and teacher educators learning in Malawi.

In the Malawi context, the collaborative group lesson study focused on improving the quality of teaching of mathematics educators working in the important area of initial teacher preparation.

The teacher from China reports that a strong focus of their teacher exchange program is on improving UK teachers' mathematics knowledge and pedagogy.

> The goal of the programme is to raise curriculum standards in the UK in mathematics by improving teachers' pedagogical and subject knowledge and to refine the curriculum to ensure that all pupils achieve their full potential in mathematics without anyone 'left behind'.

The teacher from France reports that their collaborative group is part of the network of Institutes for Research on Mathematics Teaching (IREM). These institutes provide collaboration between mathematics teachers from schools to university, focusing their work on issues related to improving mathematics teaching, disseminating outcomes, and conducting professional development for teachers.

The teacher from the USA reports that thousands of mathematics teachers from around the globe are engaging in an online Professional Learning Network (PLN), "actively working to promote quality mathematics instruction, mentorships for new teachers, and curriculum development". She notes that, although activities associated with the network have diversified over time,

> Throughout these efforts, the main goal has remained the same – a grassroots “for teachers, by teachers” professional learning network to improve the quality of mathematics instruction for our students.

The different groups’ focus on developing teaching quality is not surprising given that this is a priority consistently articulated in education policies, guidelines, and initiatives globally, and as such, it frames the profession’s improvement agenda. As suggested by Emeritus Professor Dylan Wiliam during a keynote presented in 2012, “every teacher needs to improve, not because they are not good enough, but because they can be even better” (Wiliam, 2012). The teachers’ accounts clearly show their commitment to continuous professional learning and improvement of their teaching practice.

Design feature 2: Participation in goal setting

A design feature common to the collaborative groups is the involvement of participants in determining the focus of, and explicit goals for, their work together. Each group used particular approaches to identify their foci and goals. In the context of the mathematics educators’ lesson study in Malawi, lesson study members worked with one another to set a long-term goal to increase the pass rate of mathematics student teachers by 2021. In the teacher exchange program between Shanghai and England, different core content topics were purposefully selected by group members for use in each round of lesson study, with guidance from an expert from Shanghai Normal University. The teacher-researcher network in France first collectively defined their main objectives, which were to focus on “developing classroom settings and tools,” related to logic and logical reasoning, and “disseminating these tools to teachers”. Then over a period of six months “each participant worked freely choosing his line of work” presenting their work, thoughts and questions at group meetings where they then identified agreed areas of work “based on the points of convergence and divergence”. And, the online teacher network in the USA formed different goals according to the particular work and learning tasks they engaged in, for example: virtual mentoring; book studies; sharing research articles; sharing of mathematics problems on websites; and inviting the public into classrooms virtually.

Potential benefits of the approaches used to set goals in these collaborative groups include authentic engagement of teachers and a sense of ownership associated with the process and outcomes of the work. In addition, as noted by the teachers from Malawi, France and the USA, participants were able to define and receive differentiated support specific to their individual needs.

Design feature 3: Focus on increasing technical capability

There is a consistent focus across all four collaborative groups on improving mathematics teachers’ technical teaching knowledge and skills. As mentioned earlier, in the case of the French teacher-researcher network the particular focus was on supporting increased capability related to the teaching of logic and logical reasoning, and this focus targeted not only secondary mathematics teachers but also a teacher of French language and university teachers. In Malawi, teacher educators had the opportunity to plan in great detail a research lesson, and then engage in observing, analyzing, and reflecting on the teaching of the lesson, as well as refining the lesson design. The teacher from Malawi emphasizes the importance of improving his technical practice through a fine-grained examination of teaching.

> Planning for a research lesson involves putting forward teachers' content knowledge of a particular topic and the best teaching practices that could be used. There is inclusion of critical thinking approaches, probing questions and challenging tasks. This has become my common practice when planning my lessons. More time is spent figuring out how students will think and learn a particular concept than the teaching itself.

Shanghai teachers involved in the China-UK teacher exchange were similarly very focused on increasing their technical ability when they explored how to teach selected topics in reform-oriented ways in order to share with UK teachers. And the teacher from the USA describes various ways that teachers collaborate in their online network to increase their technical capability.

This focus on increasing technical knowledge and skills suggests that the collaborative groups that the four teachers were involved in are, by design, examples of what Jensen (2014), drawing on the research of Rosenholtz (1989) and Clement and Vanddenberghe (2000), refers to as 'active collaboration'.

> Active collaboration, in which teachers learn from each other through team teaching, joint research projects and classroom observation and feedback has a positive impact on students. Collaboration that concentrates on administrative issues does not. (Jensen, 2014, p. 7)

Design feature 4: Access to specialist expertise

All four teachers express that they greatly value the way that their collaborative groups enabled them to access specialist expertise. This includes the expertise of other teachers, researchers, mathematics educators, and other education professionals. The mathematics teacher from France reports the tremendous opportunities provided to their teacher-researcher network by working with a specialist teacher of French language, as well as university teachers. The French teacher, for example, "helped the mathematics teachers at the secondary school to define words that did not have the same meaning in mathematics and French" and supported the analysis of language used in teaching sessions. The teacher from Shanghai involved in the teacher exchange program, reports that the importance of the collaborative partnership between Shanghai and UK teachers unfolded over time as the program developed, and teachers from both locations stated that they learned much from one another. The teacher in Malawi notes, "I have learned a lot of teaching techniques through observing fellow educators teach." And, the secondary mathematics teacher from the USA, strongly emphasizes the ways that the online teacher network she is a part of creates opportunities to share and learn with others. She reports,

> Due to the rural nature of Oklahoma, there are minimal opportunities for professional development as it relates to mathematics education.… when teaching a specialized curriculum such as AP [Advanced Placement] Statistics, the opportunities for traditional professional learning workshops are limited, often making it necessary to look for non-traditional methods of collaboration and networking… One of the major benefits of collaboration via social media is the 24/7 access to teachers around the globe. With a single Facebook post or Twitter tweet, you can easily receive responses from teachers with a variety of teaching experience, backgrounds, and geographic locations in a matter of minutes.

Reflection

Interestingly, despite the diverse collaborative group locations, forms, participants, and foci, there were several elements common to the design of the teachers' collaborative group experiences. There were also some common elements related to the outcomes of the teachers' collaborative group work and learning. A synthesis of lessons learned from the experience of the teachers is provided after the presentation of the teachers' stories that follow.

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PLENARY PANELIST REPORT

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Context, Purpose and Design of the Collaboration

The outstanding performance of Shanghai students in mathematics and science on the 2012 PISA assessment has attracted United Kingdom (UK) educators' attention to the Shanghai mathematics curriculum (called *Shanghai Maths*) as well as to mathematics teaching and learning in Shanghai (called *Teaching for Mastering*). Shanghai students achieved a mean score of 613 (119 points above the OECD average) on the PISA in mathematics with an excellence rate of 55.4%. Since 2012, education authorities in China and the UK have had frequent contact to explore mechanisms of collaboration. In February 2014 a delegation, including a representative from Ofsted (Office for Standards in Education) and other UK educational experts, visited Shanghai. During this visit, the Department of Education in the UK and the Education Commission in Shanghai agreed to launch a teacher exchange program, called the 'China-England Mathematics Teacher Exchange Programme (MTE)'. The goal of the programme is to raise curriculum standards in UK mathematics by improving teachers' pedagogical and subject knowledge and to refine the curriculum to ensure that all pupils achieve their full potential in mathematics without anyone 'left behind'. The programme is funded by a UK federal grant and is jointly led by the United Kingdom's Ministry of Education and the Shanghai Municipal Education Commission. Shanghai Normal University, UK National College for Teaching and Leadership (NCTL) and UK National Centre for Excellence in the Teaching of Mathematics (NCETM) are responsible for carrying out the programme.

By 2018, there have been three rounds of exchanges between Chinese and British mathematics teachers with more than 500 teachers from Shanghai and the UK directly participating. Additionally, approximately 12,000 British teachers have participated in *Teaching for Mastering* which disseminates what has been learned from the exchange programme. The programme will continue until 2023, with a goal of benefiting teachers from at least 9,300 primary and 1,700 secondary schools in the UK (Boylan et al., 2019). Major activities include reciprocal school visits and collaborative workshops. Throughout the programme, groups of UK teachers (Year 1-Year 9) led by experts from NCETM visit Shanghai schools. During these two-week visits they observe classroom teaching, participate in school-based teaching research activities, and attend workshops on developing lesson study. Similarly, groups of Shanghai teachers, led by Dr. Xingfeng Huang from Shanghai Normal University, visit England. During the two-week stay in England, the Shanghai teachers teach lessons in local schools to explore how Shanghai teaching methods could be adapted and implemented in England. The details of the implementation have been evolving according to the goals in each round of exchange. My school (Cao Guangbiao Primary School) is one of the programme base schools that has served as the platform for the exchange programme in Shanghai over the past four years.

Before the exchange visit, the Shanghai participating teachers are divided into collaboration groups to prepare for the UK teachers' visit, to learn about the UK education system and to update their knowledge about mathematics teaching in Shanghai. Some teachers who taught in the UK on their previous visit share lesson plans used, as well as their observations. When collaboratively planning lessons, teachers often simulate situations which may arise in the UK classes. In addition, a lesson study group with 10-12 teachers from programme base schools explores how to teach a purposefully selected topic in a reform-oriented way in order to share with the UK teachers. With the leadership of Dr. Huang from Shanghai Normal University, each lesson study focuses on a different core content topic such as equivalent fractions or fractions on a number line.

The collaboration not only takes place among teachers in Shanghai, but also between Chinese and British teachers. When the English teachers from the UK maths hub visit Shanghai, they are invited to teach a math lesson to the Shanghai pupils with the UK teacher and a partnering Shanghai teacher working together to develop the lesson plan. Similarly, when a Chinese teacher teaches in England, the partnering English teacher works with the Chinese teacher to develop the plan.

Overview of the Collaboration Outcomes

The programme has had a substantial impact on both British and Shanghai teachers and schools. In the UK, the Sheffield Institute of Education was commissioned by the Department of Education in December 2014 to undertake a longitudinal evaluation of the programme. Mixed methods have been used to analyze data collected over three academic years. Data from student testing has shown that in schools most directly involved in the exchange program, there has been an increase in pupils' KS1 mathematics attainment. (The UK national curriculum is organized into blocks of years called 'key stages' (KS). KS1 refers to pupils at ages 5-6, including Year 1 and Year 2). Moreover, survey and interview data has revealed that cohort 1 teachers (exchange participants in 2014/15) improved in their beliefs about mathematics teaching and commitment to learning from Chinese mastery teaching methods. Observing the mastery teaching in Shanghai classrooms was perceived to have been particularly impactful. Cohort 2 teachers (2016/17) particularly appreciated their visit to Shanghai, which deepened or challenged their previous understanding of Chinese mastery teaching methods. The visit by the Chinese teachers to England supported the UK teachers' implementation of teaching for mastering mathematics (Boylan et al., 2019).

To reflect on Shanghai teachers' observation and understanding of mathematics teaching in Britain, Professor Huang edited a book called *I Teach Mathematics in Britain*, which includes chapters by programme participants from Shanghai. The book includes four sections and highlights the differences between Chinese and English mathematics teaching as observed by the teacher authors. One salient point noticed is that homogeneous grouping is prevalent in English primary school which does not enable weaker students to progress as desired. Some teacher authors used vivid examples to describe differences in language, culture, and how best to address and learn from these. Additionally, some teacher authors highlighted their development as mathematics teachers, particularly in their use of hands-on activities.

Description of What Was Learned

Through participating in this project over the past three years, I have learned so much. I would like to highlight what I have learned: (1) learning to collaborate with UK teachers; (2) learning the cultural differences in defining mathematics concepts; (3) learning the differences in learning progressions; and (4) learning to use research-based teaching practices.

Learning to collaborate with UK teachers. At the beginning of the collaboration, we thought it would be easy for us to know the learning situation in the UK classrooms, simply by having the British teachers tell us what pupils had previously learned from the UK curriculum. But, we found it was often hard for us to design lessons based on just this information, and pupils having already been taught something does not mean that they have mastered it. Teachers from both countries realized that it is necessary to have deeper and more extensive discussions about students' readiness to have effective collaboration. Now teachers from both sides jointly select the teaching content and share more extensively about school culture and student learning. For example, my partner teacher, Mrs. Louis, gave me information about her school, Caroline Haslett Primary School, through videos and photos. These artifacts helped me to understand her classroom environment as well as student homework and exercises, so that I could understand their learning situation in advance.

Learning the cultural differences in defining mathematics concepts. Different cultural and educational backgrounds increase the difficulty of teaching in another county. Through collaboration, Chinese and British teachers are learning how to minimize the negative impact of these differences. For example, students recognize a rectangle and square at an early age, but when exploring the relationship between these shapes conceptually, the UK pupils struggled to realize that a square is a specific rectangle. I was surprised because this had never happened in the Shanghai classrooms where I taught. My British partner teacher shared that in British primary schools, attention is paid to the visual characteristics of the shapes of square and rectangle separately, without establishing conceptual connections between them. Furthermore, we checked the English National Curriculum which says that pupils should be taught to recognize rectangles (including squares) in Year 1, which is the age of 5. This means that students in the class which I will teach were taught the concept three years ago. Moreover, the curriculum does not give a clear definition of rectangle, which surprised the Shanghai teachers. After asking several teachers and pupils in the UK school, we found that the problem largely stemmed from ambiguous definitions. In the UK, most people think that the rectangle is a shape with four right angles and two pairs of opposite sides equal but different in length. In Shanghai textbooks, a rectangle is defined as a quadrilateral with four right angles and two pairs of opposite sides equal. To further understand how a rectangle is defined in British textbooks, we consulted the textbook *Maths No Problem* which is recommended by the government. It says that the opposite sides of a rectangle are always parallel and equal. Based on this, a rectangle should include square. Thus, we believe that the student learning difficulty could be solved by appropriate practice. After some deliberate practice, we were delighted to see that pupils could understand the concept very well. This makes me deeply feel that the difference between the pupils' mathematical achievement in the two countries is not due to pupils' different learning abilities, but to cultural differences. Reflecting on my own teaching, I now realize how important it is to focus on the knowledge that students have already learned and the context in which they learned it. This new idea leads me to think further about lesson planning.

Learning the differences in learning progressions. Through the China-UK collaboration we have discovered substantial differences in learning progressions in mathematics content between Shanghai and the UK. For example, when I was teaching addition and subtraction within 100, I found that the UK pupils had a weak foundation of addition and subtraction within 20, an issue which will likely result in them having difficulty learning addition and subtraction with larger numbers. This weak foundation may be caused by the school teaching plan which showed they have spent almost half of the term learning multiplication involving 2s, 5s and 10s, without any review of addition or subtraction. Nevertheless, British teachers insisted on continuing to teach this content. To seek a solution to this dilemma, I posed the following in the WeChat group of the programme team: How to design the lessons about addition and subtraction within 100 when pupils are not fluent with operations within 20. (WeChat is the most popular social media platform in China, and is routinely used by Shanghai teachers to share teaching resources and ideas and discuss problems in teaching.) Teachers in the group provided various suggestions. For example, some teachers suggested using songs to help pupils remember the number bonds of 10. These ideas prompted me to further study the teaching content of addition and subtraction within 100 and clarify the relevant content in the Shanghai textbook. After comparing the presentations in Shanghai and British textbooks, it was found that the initial operation relies on the fluency of decomposition of number. As a result, I added a warm-up activity of reciting *Make 10* songs in the teaching design. While students were excited and interested, they also naturally make sense of the decomposition. Based on this design, my teaching in the UK went well.

Learning to use research-based teaching practices. Teachers in Shanghai conduct self-evaluations and reflect on the reform of mathematics teaching in China-UK workshops. I will illustrate with the example of an ongoing lesson study of fractions on the number line which occurs in two stages. First the learning trajectory of the content (fractions on number line) across grades in the Shanghai textbook was examined and the ways of presenting the content in different textbooks was compared. To understand student readiness, we gave a pretest. To broaden our understanding of the presentation of the topic, we also consulted textbooks from other counties. Finally, learning goals were set: (1) to find the position of the proper fraction, (2) to use the number line as a learning tool to compare the size of the fraction, (3) to have a preliminary experience in the integration of numerical and pictorial representations. The second stage includes the cycles of design, implementation, reflection and revision. After teaching the first class, teachers shared their thoughts about this lesson one by one, such as the large capacity of the whole lesson and insufficient teaching time. For language sentence patterns, such as 2 one-thirds is two-thirds, the meaning of fraction can be emphasized. Drawing on the feedback from other teachers and my self-reflection on the lesson, I changed my lesson plan to mainly focus on understanding how to locate fractions on a number line while using comparison of fractions to help students build links between fractions and integers on the number line. The lesson was taught a second time using the revised lesson plan. Pupils had more time for discussion and communication, and naturally established the relationship between the previous knowledge of fractions and number lines. For example, I asked pupils to find more fractions on the number line after they had found the fractions with denominators 2, 3 and 4. Students discussed in groups and then shared their group's ideas in the whole class. They realized that all fractions can be found on the number line because they could go on forever. Furthermore, with the help of language sentence patterns like four-thirds means 4 one-thirds, they can locate improper fractions on number line.

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PLENARY PANELIST REPORT

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Collaborative work within an IREM

IREM of Brest

The IREM of Brest is part of the network of Institutes for Research on Mathematics Teaching. These institutes provide a collaborative organization between mathematics teachers from schools to university. They can work together on issues of mathematics teaching, disseminate their outcomes and conduct professional development for teachers. The meetings take place on the premises of the IREM at Brest University.

Birth of the group

In 2009, new mathematics curricula was set up. During the annual conference planned by IREM of Brest, there were informal discussions about the teaching of logic in secondary school. These discussions led to an observation: On the one hand, the students receive little teaching of logic, on the other hand, the teachers feel a lack of resources to develop a practical teaching of logic as described in the curricula. Subsequently, a working group was formed to focus on this subject in September 2010.

The group consisted of six people: two mathematics teachers from Brest University, three mathematics teachers from secondary school and a French teacher, working both at a secondary school and at a teacher training university institute. In a rather traditional way, the role of university teachers is to bring theoretical content and analyze the activities proposed to the students. Teachers at the secondary school design class sessions, test them in their classrooms, and analyze them. The French teacher designs class sessions for French courses and points out the different language elements used in mathematics and in French.

The purpose and design of the collaboration

During the first meeting, we collectively defined our main objectives: at first, developing classroom settings and tools (like exercises, test, mind mapping …) wherein students can practice logic and logical reasoning without theoretical courses, and second disseminating these tools to teachers.

One of the university teachers proposed to lead the group. We decided to meet once a month, Friday afternoon, at Brest University and we communicated regularly by e-mail. One of the participants (often the same) took care of the minutes of the meeting and reminded the work to be done by each participant with an e-mail just after the meeting and just before the next meeting.

During the first six months each participant worked freely choosing his line of work. At each meeting, the participants presented their work, their thoughts and their questions. This allowed the group's leader to identify areas of work based on the points of convergence and divergence on which we agreed to work.

For the secondary teachers, the goal is not to teach theoretical logic, but the University teachers use theoretical logic with their students from the first year. So we needed to design practice activities for secondary school that prepare students for the academic logic of the university and design additional practice activities for the university's students that connect with the secondary school.

One of the objectives of our collaboration was to create class activities that can be used in mathematics courses, French courses in secondary school and University. Together we created the same written tests for each of these levels. We asked, in our school, teachers who did not participate in the group to take the test to their students (fewer university teachers were involved in this phase).

With the data from the results of the test, we were able to assess the needs of our students. In order to prepare students for the teaching of logic at the university, the secondary teachers suggested that theoretical logic professional development be set up for all mathematics teachers by the University teachers.

During our meetings, we found that the new mathematics curricula and schoolbooks did not provide any progression in logic learning. Then, we had to plan a teaching of logic from the first year to the last year of secondary school. We have designed practice activities fitted to the level of our students. These activities are sequenced to progress from simple to more complex logic concepts over time. The French teacher helped the mathematics teachers at the secondary school to define words that did not have the same meaning in mathematics and French, and the secondary mathematics teachers recorded their course sessions so the whole group could analyze the sessions.

Collaboration outcomes

Written tests were conducted in all educational levels from first year of high school to first year of University. The activities for secondary school students were tested by secondary teachers and analyzed by the whole group. The statistical results from the test and the analysis of the class sessions were published in the "logique au fil de l'eau" brochure.

The needs of secondary teachers regarding the notions of logic were identified and gave rise to a professional development. It was set up with the help of the IREM of Brest Director during the annual conference. This professional development lasted one afternoon and could not address all of the secondary teachers' questions. Some logic concepts (the most significant for secondary teachers) were reviewed and a list of resources was given to the participating secondary teachers in order to supplement their learning from the day.

Our work dissemination was made thanks to the "logique au fil de l'eau " brochure published by the French APMEP (Association des professeurs de mathématiques de l'enseignement public). In this brochure, the mathematics secondary school teachers described the sessions and their analysis. A University teacher wrote a logic course for the secondary school mathematics teachers. The French teacher and university teacher wrote a text on the links between logic and language.

Moreover, secondary mathematics teachers conducted logic workshops at conferences organized either by several IREM or the APMEP.

The Inter IREM Committee of secondary schools is a commission which includes secondary teachers and University teachers from several IREM. This commission has meetings in Paris five times per year. Several IREMs have also worked on logic teaching, so a group named "logic" was set up within the commission. A broader collaborative work has been put in place with several IREMs. Our group chose a representative, a secondary teacher, who goes to Paris and has exchanges with the representatives of other IREMs about logic teaching during Inter IREM Commission meetings in Paris. Currently, members of the commission are writing a brochure summarizing work on logic teaching from several IREMs.

What was learned

Factors that supported or limited the collaboration

The existence of IREM was the key element in setting up this collaborative work. The IREM is a place of discussions about mathematics teaching, known to all the teachers from primary school to the university and easily accessible. The annual conference provides an opportunity to discuss without the institutional hierarchy. Additionally, IREM has a library of teaching resources to which all teachers have access; so when teachers want to improve their practice or have a question on mathematics teaching, it is natural to look to an IREM for collaborative work. Furthermore, the institution recognizes IREM so the schedule of teachers who work at IREM can be fitted so that they can attend the meetings.

An issue was the organization of remote work: often we want to present a perfect document to the group but it takes a lot of times to prepare, and even then, our document was not perfect for the group. It is difficult to present an unfinished work to the group but the discussions are more open. Furthermore, collaborative tools have not been fully explored. Some of us did not use these tools and the others used different tools (Google Drive, Dropbox, email…). This situation has limited our ability to exchange our work between the meetings. One of the lessons learned was that the group members should take time to choose and learn how to use the same digital tools.

Challenges encountered by the participants

Another secondary teacher and I were to lead professional development for teachers for the first time. We were apprehensive about facilitating this experience for other teachers. The teachers who attended the professional development did not have a lot of knowledge about teaching of logic but they did teach the same level we do in mathematics. Some participants did not want to teach logic without theoretical logic. We needed to convince them that our practical activities have a good impact on students' progress and prepare secondary school students for the academic logic of University. All the participants of our group prepared this first professional development. We chose to prepare a debate and presentation of our work rather than a course of logic. During the session, one participant of our group was present with the teachers attending the professional development to feed the debate between the attending teachers and the leaders of the professional development, if necessary. Finally, teachers who participated in the professional development worked well during the sessions with us, but it is difficult to determine how they use this training in their classroom. After this first experience, we continue to lead additional trainings with less apprehension.

During our work, we discussed with other IREM groups outside of ours at Brest. I was appointed to be the reporter of our work during the Inter-IREM Commission meetings in Paris. On this occasion, I met specialists in mathematical logic who work together in a committee with secondary teachers. They needed the point of view of several secondary teachers, so I accepted to join this committee.

Since then, I continue to meet them regularly, and now I plan the committee meetings with another secondary teacher regarding mathematics learning in secondary school.

Changes in my approach to teaching mathematics

This collaborative work provoked some changes in my mathematics teaching practice. It forced me to have more perspectives on my teaching. In a collaborative work, there are different points of view of the participants, sometimes contradictory. Each participant has to argue, defend his point of view and be able to make it evolve through the others. For example, during our work, I had to explain to the other participants why I chose some activities. I learned how to analyze the activities to be convincing, and now I continue to analyze the activities that I propose to my students. We also recorded course sessions; we listened to the recordings together to analyze the students' reactions. Since the study of these recordings, I believe I am more attentive to the reactions of my students.

Teachers are alone in class in front of their students; collaborative work is an interesting way to improve teaching practice.

PLENARY PANELIST REPORT

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Context, Purpose and Design of the Collaboration

Collaboration in mathematics takes different forms and is practised in different contexts. This paper discusses experiences of collaboration in lesson study. This is a case of Malawi, a Southern African Country of an area of 118 484 square kilometres and with a population of about 18.7 million people. The collaboration is actually taking place at Machinga Teacher Training College, one of the eight public teacher training colleges for primary school teachers in Malawi.

It all started at national level as both a professional development program and a network of teacher educators from three Teacher Training Colleges. The program was under the Improving Quality and Capacity of Mathematics Teacher Education in Malawi Project with funding from the Norwegian Program for Capacity Building in Higher Education and Research for Development (NORHED). The project was a collaboration between University of Malawi and University of Stavanger, and the overall aim was to improve the quality of mathematics teaching in Malawi. As such, lesson study was introduced as a way to achieving the aim. It was thought that lesson study will be a scaffold for student teachers and teacher educators learning in Malawi. Forty-six participants were involved in the program.

From the national level, lesson study activities trickled down to college level. At Machinga Teacher Training College, nine mathematics teacher educators were involved. A lesson study group was formulated, and in the group, there were five male educators and four female educators. I took the leading role of the group and with the other educators as members, a goal was set. A long-term goal was to increase students' achievement by improving the pass rate from the current 40 percent to 95 percent by 2021. Generally, students' performance in mathematics was not impressive and as a mathematics section of the department of mathematics and science at the college, we thought of putting in place strategies that could assist to rectify the problem, so we welcomed lesson study as one strategy. Eight of us mathematics educators collaborated in the planning of the research lesson by developing a lesson plan, identifying teaching and learning resources and observation tools to be used during the teaching of the research lesson. One of the eight mathematics educators taught the research lesson while the other seven observed and collected data. Thereafter a reflection process was initiated and the teacher educators discussed the research lesson and shared their experiences. The experiences were also shared at national level during another professional development workshop with teacher educators from two other colleges and experts from the project.

Overview of the Collaboration Outcomes

The first outcome of the collaboration is change of attitude. As indicated in one of the limitations to collaboration, the attitude of some educators who were not ready to have their lessons observed by fellow educators has greatly changed. Educators are increasing their flexibility, accommodating presence of observers and taking part in the sharing of teaching experiences.

The second outcome is on improvement in instruction. Lewis and Hurd (2011) argue that, "If you want to improve instruction, what could be more obvious than collaborating with fellow teachers to plan instruction and examine its impact on students?" (p. 3). Indeed through collaboration, educators are able to develop lessons rich in critical and problem solving strategies. This is helpful to both students and teachers. Students are challenged with activities that keep them active throughout the lesson. For instance in one of the research lessons, students were asked to model the addition of 45 and 16. The students came up with different ideas like using place value boxes and tins, using an abacus, and counters. Of interest was the use of stones in the place value box instead of sticks. At the same time, student teachers are also developing teaching skills and chances for them to use critical thinking approaches. CORD (1999) argues that many teachers tend to interpret the learning environment according to their own experience as students – that is, they teach the way they have been taught. So, the likelihood that student teachers will use ideas of lesson study in their teaching, using critical thinking and problem solving approaches to be specific, after having experienced it themselves, is very likely.

Description of What Was Learned

There were some factors that supported collaboration in the teaching of mathematics. The first one was environmental in nature. That is the context in which the collaboration was taking place. The college had everything the teacher educators needed to carry out lesson study. Rooms and curriculum materials were available, and students were also in college. The college administration gave the group a go ahead and made teaching resources available.

Culturally teachers are lifelong learners. It is in their tradition to seek knowledge. They would always want to learn to update their knowledge base, and when such a chance unveils itself, they go for it. Any initiative that proves to be productive in improving achievement of students, is often taken seriously by teachers. Lesson study came at a time when it was needed most. There was a need to understand a reviewed teacher curriculum. This called for a collective effort of teacher educators to understand its contents. Lesson study was the timely solution and motivator to that cause.

On limitations, the size of the class involved was big. There were about forty student teachers involved during the research lesson. That affected mobility of the teacher educator as well as the students during the lesson because of limited space in the classroom. It also became difficult for the teacher educator to reach every student and give individual assistance. Resources were also inadequate for every student to be in contact with them. For an effective follow up on each and every student during a lesson, it could be good to have students not more than twenty in one class. A lesson study lesson requires full understanding of how instructions are influencing learning in each and every learner, and this is only possible where the size of the class is small.

Practically, most schools in Malawi have large classes and that will take some years to be solved. I see this as the greatest challenge, and it cannot be overlooked when planning for lesson study. However, there are a number of aspects with research lessons that can be accomplished and improve learning other than focusing on the learning of individuals. For example; team planning, collective reflection and use of critical thinking approaches can enhance learning. Hence, I feel modifying some areas of the lesson study process to suit the Malawian context can help teachers to carry out lesson study in highly populated classrooms.

Another limiting factor, which came as a surprise, was the unwillingness of some teacher educators to participate in some stages of the lesson study. They could neither make themselves available during planning nor accept the role of teaching the research lesson. The thinking that they are the best teacher educators kept some participants away from the collaboration process. The solution to this has been geared towards attitude change. This is being coupled by allocating two educators to one class so that they can plan and teach as a pair. By working in pairs people will see the relevance of sharing ideas and working as a team.

It was also not all that easy for the leader of the collaboration group to lead through a model which was new to all of them. This demanded more time for the leader to study and search for more information so that a right track could be followed. This helped the leader to become more knowledgeable about the lesson study.

Components of the collaboration that provoked sustainable change

The following paragraphs describe three components of lesson study that provoked sustainable change in my approach to the teaching of mathematics. The first component is planning. Lesson planning is a daily activity that teachers do as they prepare for their lessons. It is in its natural context to see a teacher planning for lessons. Success and failure of a lesson depends heavily on its planning. However, planning a research lesson collaboratively becomes more rewarding than planning it individually. Planning for a research lesson involves putting forward teachers' content knowledge of a particular topic and the best teaching practices that could be used. There is inclusion of critical thinking approaches, probing questions and challenging tasks. This has become my common practice when planning my lessons. More time is spent figuring out how students will think and learn a particular concept than the teaching itself.

Furthermore, I have learned a lot of teaching techniques through observing fellow educators teach. The way they approach their lessons and taking it through developmental steps, is an important practice. For example, starting a lesson by asking students a challenging question. Then building the lesson on students' responses until the objectives of the lesson are met. This was also the case with other educators who were involved in the collaboration.

The third component is about conducting a research lesson. In this stage, one member of the lesson study group teaches a research lesson while the rest of the group members observe and collect data. A data collection tool is used where experts observe the lesson and collect relevant data. This is the data that inform instruction and bring improvement. When one teaches a lesson individually without colleagues monitoring the proceedings, very little data is obtained. However, the practice of collecting data when teaching is what is very important. I now treat my lessons as sources for data collection for my learning about my students' learning.

I am able to identify gaps in my teaching and learning of students. For instance, I was teaching about subtraction of mixed numbers, $7\frac{1}{3} - 3\frac{2}{5}$. I asked students to explain how they can solve the problem. One student explained, "First subtract 3 from 7 and get 4, then subtract $\frac{2}{5}$ from $\frac{1}{3}$." The student proceeded up to this stage: $4\frac{5-6}{15}$. And then the student said, "We take 1 from 4, the whole number, and add to 5 (minuend) to make 15 and then subtract 6 from the 15 which means we have now $3\frac{15-6}{15}$. This assisted me very much because I was able to understand the student's thinking on the problem. The gap was identified and ways of handling the problem were shared with other educators.

Reflection forms an integral component of the lesson study circle. This is the stage where the teacher and members of the lesson study group share data from a research lesson. Members share what they feel are the successes and the challenges of the lesson they observed, as well as what they learn about the students‘ learning. Collectively, they once again plan the lesson, fusing in new ideas and approaches and eradicating elements of the lesson that are not significant in realising the objectives of the lesson. There is power in reflection and every time a lesson is being reflected upon, new insights are realised. No wonder reflection has become part of the Teacher Education Philosophy of the reviewed Initial Primary Teacher Education which states, "to produce a reflective, autonomous, lifelong learning teacher, able to display moral values and embrace learners' diversity" (Malawi Institute of Education, 2017), and is being implemented now.

It is my wish that one day in-service primary school teachers be introduced to lesson study as a form of collaboration. This will greatly assist to improve instruction and the performance of learners in mathematics. That might take a long time, but it will be a good undertaking. The challenge I anticipate is a lack of research skills in the primary school teachers. Lesson study lessons are research lessons and research skills are very crucial to the lesson study process.

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PLENARY PANELIST REPORT

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Context, Purpose and Design of the Collaboration

Jenks Public Schools is a suburban school district serving approximately 12,000 students in grades Pre-K to 12 in Northeast Oklahoma. Oklahoma is a mostly rural state, located in the Central Plains of the United States, with two main metropolitan areas, Tulsa and Oklahoma City. Jenks is a southwestern suburb of Tulsa, and the school boundaries cover the city of Jenks as well as a section of the southern city limits of Tulsa. Jenks High School, serving grades 10-12, has a graduating class of approximately 750 students, with a mathematics department of 15 regular education and 5 special education teachers, teaching classes ranging from Algebra 1 to Calculus 3. During my 20-year tenure at Jenks High School, I have taught a variety of classes, with a current teaching assignment of Advanced Placement® (AP) Statistics, Geometry, and Forensic Science and Data Analysis. In addition, I serve as our site Professional Development Coordinator as well as on our Leadership Team.

Due to the rural nature of Oklahoma, there are minimal opportunities for professional development as it relates to mathematics education. In many districts, there may only be one or two mathematics teachers, so Jenks is fortunate to have a large department of educators. However, when teaching a specialized curriculum such as AP Statistics, the opportunities for traditional professional learning workshops are limited, often making it necessary to look for non-traditional methods of collaboration and networking.

In the late 1990s, teacher message boards and email listservs were vital elements to online teacher collaboration, but in the mid-2000s, online teacher journals, called blogs, started to become more popular, followed soon by the use of social media such as Twitter, Facebook, and Instagram as a way to connect these teachers together and create real-time collaborative conversations revolving around lesson ideas and pedagogy. There are now thousands of mathematics teachers around the globe that are active participants in an online Professional Learning Network (PLN) called the #MTBoS, or the Math Twitter Blog-o-Sphere. Through these online connections and social networks, the members of the #MTBoS are actively working to promote quality mathematics instruction, mentorships for new teachers, and curriculum development.

In general, the collaborative nature of the #MTBoS is fairly informal, using social media hashtags and Facebook groups to connect subject-area teachers. However, there have been organized efforts regarding book studies, outreach at national professional learning events, and even a face-to-face multi-day math teacher conference, called Twitter Math Camp (TMC), during the summers of 2012 through 2018. Throughout these efforts, the main goal has remained the same – a grassroots “for teachers, by teachers” professional learning network to improve the quality of mathematics instruction for our students.

Overview of the Collaboration Outcomes

The nature of social media as a medium for collaboration lends itself to opportunities for discussions with a wide reach, both geographically and longitudinally. A single tweet can create a multi-hour or even multi-day discussion with contributors around the globe, all sharing their input and guidance on an activity, lesson plan, or classroom management advice. Collaborations via the #MTBoS have resulted in pedagogical books being written, open-source software and curriculum, free sharing of lessons and Desmos activities, and even public outreach programs such as “Math on a Stick” at the Minnesota State Fair. The impact of the online teacher collaboration is, in many ways difficult to measure, but the effects are far-reaching. One example of this can be seen with the success of Twitter Math Camp (TMC), drawing both presenters and participants from the greater #MTBoS community, as well as from the local hosting region. In 2012, the original TMC hosted approximately 40 teachers from a variety of teaching experiences and backgrounds for a 3.5 day workshop. In 2014, Jenks High School hosted TMC and the workshop had grown to close to 150 teachers. At its end, in 2018, TMC had impacted close to 600 teachers and classrooms through in-person attendance plus an additional unknown number through virtual interactions.

While the physical TMC conference lasted 3.5 days each year, the virtual portion of the conference lasted year round. In the weeks and months preceding each TMC, the conference presenters were hard at work preparing for their sessions. Since many of these presenters were not in geographic proximity, they organized their presentations using online collaboration tools such as Google Docs and Skype calls to hash out the details. During the actual conference, the whole-group sessions, such as the keynote speakers and the ‘My Favorites’ portions were videoed and shared via the YouTube channel, plus participants were encouraged to “live-tweet” from each session using a social media hashtag so people not in attendance could follow along. In the days and weeks following the conference, the conversations continued as teachers shared their learning experience through blog posts and Twitter discussions.

Description of What Was Learned

One of the major benefits of collaboration via social media is the 24/7 access to teachers around the globe. With a single Facebook post or Twitter tweet, you can easily receive responses from teachers with a variety of teaching experience, backgrounds, and geographic locations in a matter of minutes. The exposure to teachers from different cultures and teaching environments enriches the personal professional learning experience, which can lead to richer experiences for students, from both a pedagogical and social justice aspect. With traditional professional learning opportunities, teachers tend to be limited due to geographic proximity and as a result, the participants generally come from very similar backgrounds and teaching experiences. In contrast, developing a PLN via social media allows for a diversity of perspective, which in turn creates a robust and responsive professional learning experience as classrooms and social environments evolve.

Within the AP Statistics community, one limiting factor of traditional collaboration is isolation, with most AP Statistics teachers being the only person in their district and surrounding area that teaches the course. Through the power of social media, these teachers, including myself, are no longer alone. By reaching out through a Facebook post or via Twitter, new AP Statistics teachers have ready access to experienced teachers to help guide and mentor them through the course and how to best teach challenging content. Around fifteen years ago, a young teacher from Hattiesburg, Mississippi reached out on a then-active teacher message board looking for another AP Statistics teacher to discuss course content and share teaching ideas. My response to that post and the virtual mentorship that resulted is a key reason why I am so invested in the power of social media for teacher collaboration. This commitment to helping new AP Statistics teachers has continued throughout the years, including the development of a Facebook group in 2015 called the "AP Stat Teachers Support Group" and through the online AP Statistics community on Twitter. While every teaching context is unique, these online partnerships are very empowering to teachers as they seek to best prepare their students for the standardized AP exam given each year in May – a test that can earn students college credit for specific scores.

Throughout the history of the #MTBoS, educational trends can be seen, often before they show up in traditional professional learning opportunities. One of the most powerful movements that I've been involved in was in the area of student assessment. Approximately ten years ago, several prominent teacher bloggers started implementing Standards Based Grading (SBG) in the mathematics classroom, based on works by Robert Marzano, Dylan Wiliam, Ken O'Connor, and others. During this same time frame, I had become disillusioned with traditional grading methods and the inability of the grading system to clearly communicate what my students knew. The desire to read the works of these authors and discuss thoughts with my virtual colleagues led to the creation of an online book club via Twitter, with weekly group chats to support the use of formative and summative assessment in the classroom using hashtags of #sbarbook and #eduread for easy curation. The change from traditional grading systems and appropriate use of formative assessment tools is one that has been slow to take off in mainstream educational circles, but is quite common within the online teacher community.

A more recent collaboration of the #MTBoS is the use of instructional strategies that truly inform and transform student learning. Through the use of rich mathematical tasks, teachers and students alike are growing as mathematical learners and thinkers. The online teacher community regularly shares these "low floor – high ceiling" or "open middle" problems with each other, presenting them freely for feedback and use by teachers around the globe. Several websites have been developed and crowd-sourced by the #MTBoS, including Visual Patterns (www.visualpatterns.org), Which One Doesn't Belong (https://wodb.ca), and Open Middle (www.openmiddle.com). Within my own classroom, these rich tasks have been vital in helping students see themselves as mathematical knowers and doers. In the past, there has been a disconnect between the mathematics classroom and what mathematicians actually do – look for patterns, explore curiosities, and enjoy challenging problems. By utilizing these tasks, students are able to showcase their thinking and reasoning skills and truly see the joy and beauty of mathematics.

While lengthy conversations with distant colleagues and websites full of tasks can definitely have a positive impact on the classroom and student learning, another powerful influence can be found through the collaborative efforts of the "180 blog". In the United States, an average school year consists of 180 days, so several years ago, a few teachers decided to use social media platforms as a way to invite the public into their classrooms virtually to observe the day-to-day learning that takes place. Originally, the "180 blog" utilized online blogging platforms, such as WordPress or Blogger, to journal these daily activities, but over time, this idea has morphed to the micro-blogging platforms of Twitter and Instagram. By using the social media hashtag of #teach180, teachers are able to easily share photos each day of student work and learning activities with their parents and local stakeholders, as well as with the greater #MTBoS community. This initiative, whether through a traditional blog or through Twitter or Instagram, is an excellent way for teachers to receive a daily dose of inspiration and to spark new ideas for the classroom.

All of the above initiatives are important to the improvement of mathematical instruction, but by far, the most powerful outcome of the #MTBoS are the relationships formed by teachers that would otherwise not know each other. The exposure to teachers from a variety of teaching environments, with diverse student populations, the ability to get teaching and learning advice from experienced educators, and the real-time feedback for lesson development are the most valuable aspects of the #MTBoS community. By forming friendships across time zones and geographic boundaries, teachers are no longer limited by the size of their physical mathematics department within their district or surrounding area, they now have infinite opportunities for learning and collaboration within the virtual world.

WORKING AND LEARNING IN COLLABORATIVE GROUPS: WHAT'S KEY TO MATHEMATICS TEACHERS? A SYNTHESIS OF LESSONS LEARNED

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This section presents a synthesis of lessons learned from the experience of the four teachers' working and learning in their collaborative groups, and proposes questions to provoke consideration of how these might inform directions for mathematics teachers working and learning in collaborative groups in the future.

As noted in the Introduction, and evidenced in the teachers' accounts, the four collaborative groups were very different from one another, and there is much that can be learned about the nature, design and implementation of collaborative groups from the teachers' experiences in these groups. Following is a synthesis of some observations related to lessons learned from the teachers' experiences, together with questions to provoke consideration of how these might inform mathematics teachers' participation in collaborative groups in the future.

Factors supporting collaboration

The four teachers identified and described a number of factors that supported the work and learning that took place in their collaborative groups. These included cultural, social, environmental, and physical factors. Some of the key supporting factors were:

A culture of learning. The teacher from Malawi proposed that there is a strong tradition among teachers to see themselves as lifelong learners and this positively influences their participation in professional learning.

> Culturally teachers are lifelong learners. It is in their tradition to seek knowledge. They would always want to learn to update their knowledge base, and when such a chance unveils itself, they go for it. Any initiative that proves to be productive in improving achievement of students, is often taken seriously by teachers.

In the French example, the connection to an existing organization, the Institutes for Research on Mathematics Teaching (IREM), was considered to be a key element in setting up their collaborative group. The IREM is familiar and accessible to teachers, respected by them, well-resourced, and "when teachers want to improve their practice or have a question on mathematics teaching, it is natural to look to an IREM for collaborative work". Participation of teachers in the collaborative group was encouraged and the scheduling of their attendance supported. Implicit support of this kind can be invaluable to the success of collaborative activities.

Motivation and timing. For participants in the Malawi lesson study collaborative group, there was high motivation to be involved as the mathematics educators needed to understand a reviewed curriculum.

> Lesson study came at a time when it was needed most. There was a need to understand a reviewed teacher curriculum. This called for a collective effort of teacher educators to understand its contents. Lesson study was the timely solution and motivator to that cause.

The teacher from the USA reported that her motivation for collaborating with others online was sparked by a young teacher reaching out for support.

> Around fifteen years ago, a young teacher from Hattiesburg, Mississippi reached out on a then-active teacher message board looking for another AP Statistics teacher to discuss course content and share teaching ideas. My response to that post and the virtual mentorship that resulted is a key reason why I am so invested in the power of social media for teacher collaboration. This commitment to helping new AP Statistics teachers has continued throughout the years…

She also described how at various points across her teaching career she has had opportunities to source information and discuss ideas with virtual colleagues and these occasions have motivated her to reflect on and transform different aspects of her teaching.

Available resources. The availability of needed resources, including physical space and materials, student-participants' time, and approval from administrators, was seen as key to implementing the collaborative group lesson study in Malawi.

> The first one was environmental in nature. That is the context in which the collaboration was taking place. The college had everything the teacher educators needed to carry out lesson study. Rooms and curriculum materials were available, and students were also in college. The college administration gave the group a go ahead and made teaching resources available.

Partnership. The teacher involved in the China-UK exchange highlighted the importance of a genuine partnership in their collaborative group. She noted that as their exchange program unfolded the teachers from both locations realized they needed to work closely together to ensure deeper understandings about teaching and learning in the two countries, and she reported that now "both sides jointly select the teaching content and share more extensively about school culture and student learning."

Connection to others. The teacher from the USA reported that a positive aspect of online teacher collaboration is that it connects teachers together to "create real-time collaborative conversations revolving around lesson ideas and pedagogy". She noted,

> The nature of social media as a medium for collaboration lends itself to opportunities for discussions with a wide reach, both geographically and longitudinally. A single tweet can create a multi-hour or even multi-day discussion with contributors around the globe, all sharing their input…

She described benefits of the exposure to teachers from different cultures and teaching environments through online platforms, contrasting these with traditional professional learning opportunities.

> With traditional professional learning opportunities, teachers tend to be limited due to geographic proximity and as a result, the participants generally come from very similar backgrounds and teaching experiences. In contrast, developing a PLN [Professional Learning Network] via social media allows for a diversity of perspective, which in turn creates a robust and responsive professional learning experience as classrooms and social environments evolve.

She also highlighted the important role that online collaboration can play for mathematics teachers who specialize in less common courses, or who may be working in less-populous areas.

> … one limiting factor of traditional collaboration is isolation, with most AP [Advanced Placement] Statistics teachers being the only person in their district and surrounding area that teaches the course. Through the power of social media, these teachers, including myself, are no longer alone.

Factors limiting collaboration

The teachers also identified and described factors that placed limitations on their collaborative group outcomes. These included:

Participation avoidance. The teacher from Malawi reported that some mathematics educators were initially unwilling to participate in components of the lesson study process, requiring the group to implement strategies to provoke attitude change. A practical strategy that they applied involved pairing educators to work together, so that those who were reluctant would see the relevance of sharing ideas and working as a team.

Resource constraints. In Malawi, when teaching the lesson study research lessons, student class sizes were not conducive to the particular instructional approaches they were trying to implement, and lesson materials for students were limited. The teacher from Malawi has signaled that there may need to be some modification to lesson study approaches for the Malawi context, because most schools in Malawi have large classes.

Leading 'new' ideas and approaches. Leading change in areas and approaches that are new is challenging. The teacher in Malawi noted that it was not easy for him "to lead through a model that was new to all of them", and he needed additional time to study and research relevant information to support the group. Similarly, the French teacher reported that when she was to lead professional development for teachers together with a colleague for the first time, they were apprehensive. She noted the pressure she felt when working with peers, and the need "to convince them that our practical activities have a good impact on students' progress and prepare secondary school students for the academic logic of university." She also reported, however, that she continues to provide training with less apprehension, and additionally she contributes to a committee working with mathematics specialists.

Communication protocols and tools. The French teacher reported that one issue arising in their context related to the ways that their group members organized and shared their work remotely. She noted that they were reluctant to share documents that were not "perfect" with one another, and the preparation of such documents requires time. She also noted that there was a lack of consistency in members use of collaborative tools, limiting the ability of the group to exchange work between meetings. A lesson learned, she suggested, is that "group members should take time to choose and learn how to use the same digital tools."

Provoking and sustaining personal professional learning and growth

The teacher from Malawi reported three particular components of lesson study that "provoked sustainable change" in his approach to the teaching of mathematics. The first involves collaborative planning. He noted that "planning a research lesson collaboratively becomes more rewarding than planning it individually" because you need to contribute and scrutinize knowledge and ideas, and you learn much from what others contribute. The second involves conducting the research lesson. He suggested that because colleagues monitor the lesson proceedings, there is an opportunity for collecting data about one's teaching.

> When one teaches a lesson individually without colleagues monitoring the proceedings, very little data is obtained. However, the practice of collecting data when teaching is what is very important. I now treat my lessons as sources for data collection for my learning about my students' learning.

The third component involves reflecting on the teaching of the research lesson and refining the lesson. The teacher noted, "There is power in reflection and every time a lesson is being reflected upon, new insights are realized."

The teacher from Shanghai noted the following key aspects influenced her professional learning over time: close collaboration with exchange partners; the detailed examination of mathematics curriculum and teaching approaches; thorough planning of lesson study research lessons; and, close examination of lessons taught (including receiving feedback about the lesson, reflecting on the lesson, and refining the lesson). The exchange program that she is involved in has been in place for several years, and as she noted, "the details of the implementation have been evolving according to the goals in each round of exchange." The program duration and its evolving nature (facilitating program relevance and currency) appear to have contributed to her sustained learning over time.

The teacher from France suggested that her involvement in the collaborative group provoked some lasting change in her mathematics teaching practice. She believes it forced her to consider more perspectives about teaching.

> In a collaborative work, there are different points of view of the participants, sometimes contradictory. Each participant has to argue, defend his point of view and be able to make it evolve through the others. For example, during our work, I had to explain to the other participants why I chose some activities. I learned how to analyze the activities to be convincing, and now I continue to analyze the activities that I propose to my students.

She also noted the impact that analyzing recorded teaching sessions had on her.

> We also recorded course sessions; we listened to the recordings together to analyze the students' reactions. Since the study of these recordings, I believe I am more attentive to the reactions of my students.

The teacher from the USA reported a variety of ways that different online opportunities have stimulated her collaborative activity and improved her mathematics instruction. However, she suggested that the most powerful outcome of her involvement in her collaborative online network are the relationships formed with other teachers. It is these, she suggests, that provoke and sustain her commitment to ongoing learning and improved teaching practice.

> The exposure to teachers from a variety of teaching environments, with diverse student populations, the ability to get teaching and learning advice from experienced educators, and the real-time feedback for lesson development are the most valuable aspects of the #MTBoS community. By forming friendships across time zones and geographic boundaries, teachers are no longer limited by the size of their physical mathematics department within their district or surrounding area, they now have infinite opportunities for learning and collaboration within the virtual world.

Reflection and questions

The stories of the four teachers included in this report provide evidence of considerable work and learning in different collaborative contexts around the globe. Their experiences provide some insights into what's key to mathematics teachers working and learning in collaborative groups, and also provoke questions about this. Such questions include:

- Which support elements are absolutely critical to effective collaborative groups?
- What kinds of processes facilitate authentic partnership roles in collaborative groups?
- How might flexibility and responsiveness be effectively incorporated in collaborative group work and learning?
- How might competing professional learning needs of collaborative group members be effectively managed?
- How might the processes and products of collaborative group work be effectively shared?
- How might effective collaborative group activities and outcomes ‘reach’ more mathematics teachers?
- How might cross-cultural insights related to mathematics teachers working and learning in collaborative groups be effectively shared?

It is anticipated that consideration of questions such as these might usefully inform directions for mathematics teachers working and learning in collaborative groups in the future.

Theme A

Theme Chairs: João Pedro da Ponte & Takeshi Miyakawa

Theoretical Perspectives on Studying Mathematics Teacher Collaboration

COLLABORATION BETWEEN TEACHERS AND RESEARCHERS: A THEORETICAL FRAMEWORK BASED ON META-DIDACTICAL TRANSPOSITION

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The meta-didactical transposition framework is an important reference for studying existing relationships in collaborative research between teachers and researchers. Born in a training context, it is important to extend it to describe and analyze these same relationships in the context of research. It is by finely analyzing the objects on which the protagonists focus that we can identify significant elements of interaction description leading to a better understanding and improvement both from a theoretical and methodological point of view of collaborative research.

Introduction

The framework of meta-didactical transposition, born in Turin in a teacher training context (Aldon & al., 2013, Arzarello & al., 2014), has been taken up, expanded and enriched to adapt to the analysis of interactions in so-called collaborative research, involving actors from different institutions, researchers, teachers, computer scientists, etc. The purpose of this paper is to show how the theorizing of relationships between actors can help to better understand and act on the improvement of collaborative research. The fundamental hypotheses that support this work are developed in order both to justify the scientificity and the usefulness of collaborative research in general and particularly in mathematics education. This construction is being tested in a case study coming from the work done with teachers in the European FaSMEd project[1]. The work presented in this paper is based on the work done in the EducTice team of the French Institute of Education (IFÉ – ENS de Lyon), in particular in the FoRCE project (Formation et Recherche Collaborative en Éducation), a Franco-Canadian project involving IFÉ, the Lyon's Local Education Authority and the Faculty of Pedagogy of the Université de Sherbrooke (Monod-Ansaldi & al., 2019, Nizet & al., 2019).

An attempt to theorize the relationships between researchers and teachers

Fundamental hypotheses

As anthropologist, Marcel Mauss (1923) pointed out, education is a total fact, "a fact that sets in motion the entire society and its institutions" (p. 102). It is therefore a complex phenomenon, which,

[1] Formative Assessment for Science and Mathematics Education, The research leading to these results has received funding from the European Community's Seventh Framework Programme fp7/2007-2013 under grant agreement No[612337].

as the philosopher Edgar Morin points out, can only be understood through complex thinking (Morin, 1990):

> It can be said that what is complex is on the one hand empirical, uncertain, unable to be certain of everything, to formulate a law, to design an absolute order. On the other hand, it is also a matter of logic, that is, the inability to avoid contradictions. (chap. 27/57) [2]

Educational research thus has the need to take this complexity into account and integrate it into its research paradigms:

> We are in an uncertain battle and we do not yet know who will win. But it can already be said, however, that if simplifying thinking is based on the domination of two types of logical operations: separation and reduction, both of which are brutalizing and mutilating, then the principles of complex thinking will necessarily be principles of distinction, conjunction and implication. (Id. chap. 33/57)[3]

Asking questions about the type of research and the position of actors in research raises at least three types of questions, of an epistemological, methodological and ethical nature. Epistemological questions relate to the positions of the research regarding the actors and the aims of this research. They make it possible, in a way, to position research in the tensions between antagonistic poles: academic research to produce new knowledge or research to train actors. And from another point of view, research related to the construction of teaching materials or research to understand a practice. What are the researcher-practitioner relationships? Is it a question of considering the teacher in the research as a parameter, i. e., as an element of the system to be studied, or as a variable, i. e., as an element of the system to be characterized, or finally as an object of research? (Roditi, 2015). What are the targeted productions and what are the conditions for their scientificity?

From a methodological point of view, the organization of collaborative work is not self-evident and must be thought of in relation to the theoretical frameworks at the same time as the goals that the actors set for themselves. The construction of tools to provide collaborative working conditions is essential.

Finally, ethical issues are essential in any work with actors from different institutions. Everyone has different issues in the common work that need to be considered and addressed in the research. Issues relating to institutional recognition and the capitalization of work are at the heart of the "contract" that must be concluded between the actors of collaborative research. This contract seems to me to have two very distinct facets: the first concerns the setting in stone of the conditions and objectives expected by the research actors: what is expected of each party and how it will be possible to achieve these objectives, but also the expected results and the methods for disseminating these results (training, text, actual achievements, etc.). The second facet concerns the object of the research, which

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[2] Translated by us from: "On peut dire que ce qui est complexe relève d'une part du monde empirique, de l'incertitude, de l'incapacité d'être certain de tout, de formuler une loi, de concevoir un ordre absolu. Il relève d'autre part de quelque chose de logique, c'est-à-dire de l'incapacité d'éviter des contradictions."

[3] Translated by us from: "Nous sommes dans une bataille incertaine et nous ne savons pas encore qui l'emportera. Mais l'on peut dire, d'ores et déjà, que si la pensée simplifiante se fonde sur la domination de deux types d'opération logiques : disjonction et réduction, qui sont l'une et l'autre brutalisantes et mutilantes, alors les principes de la pensée complexe seront nécessairement des principes de distinction, de conjonction et d'implication."

can only be defined *a priori* through questions or hypotheses about an object whose boundaries have not yet been precisely constructed. The contract is then similar to the didactic contract of the Theory of Didactic Situations (Brousseau, 1990), i.e. it cannot be fully explained. The contract is negotiated through a devolution of research, just as learning is the result of a devolution of the didactic situation negotiated between teachers and students. Ethical issues are at the root of the conceptualization of collaborative research and cannot be neglected under penalty of breach of contract and therefore failure.

These reflections naturally lead to hypotheses on which we base our research and which tend to consider all actors as partners in research. It is therefore a question of working *with* teachers rather than *on* teachers, considering the tensions already encountered.

What answer can be given by research to formalize collaborative research between researchers and teachers?

The Meta-Didactical Transposition

The meta-didactical transposition is based on Chevallard's anthropological approach of didactics (1989, 1992). It describes a dynamic of relationships between teachers, researchers, trainers and any actor involved in collaborative research. In the research team "EducTice" where I was working, we were seduced by this approach (Sanchez & Monod-Ansaldi, 2015, Aldon et al., 2013, Aldon & Panero, 2017, Monod-Ansaldi et al., 2019) and we tried to extend this theoretical model to describe and analyze collaborative research. This model describes a dynamic of relations between researchers and teachers based on five pillars:

1. Double dialectic: didactical dialectic between the thee vertices of the didactical triangle knowledge, teaching and learning and meta-didactical dialectic between didactical dialectic and pragmatical or theoretical justification of this dialectic.
2. Meta-didactical praxeologies: according to Chevallard (1989), any human activity can be described as a praxeology modeled by a quadruplet (T, τ, θ, Θ): in front of a task t, one (or several) technique (s) τ allows the task to be solved in the same way that all tasks of the same type T can be solved. This technique, which depends very closely on the institution in which the type of task is proposed, is justified in the institution by a discourse (θ, technology: discourse on techniques) and a theory Θ. The double dialectic then leads us to consider two types of praxeologies: a didactical praxeology in which the type of task is built with reference to the knowledge to be taught, the technique being recognized in an institution and the technological and theoretical justifications depending both on institutional habits and learned knowledge; and a meta-didactical praxeology in which the type of task is didactical praxeology: for example in the European project FaSMEd which dealt with formative assessment in science and mathematics, the praxeology of a lesson on fractions, where writing a fraction is based on a technique justified by calculation rules and more theoretically by algebra, is the basis for a formative assessment lesson, itself justified by empiricism and theories of formative assessment.
3. Institutional aspects: for each of the actors, the praxeologies are directed by the fact that the actors belong to an institution; in the case of a type of task, the technique used and the justifications for this technique depend functionally on the actors' institution.

4. Internalization: this phenomenon describes the exchanges between actors from different institutions and is at the heart of the collaborative work that could not exist if the actors did not take advantage of the interactions to modify their knowledge system. Internalization is linked to the knowledge at stake, whether practical or theoretical. Internalization can be considered as a modification of a praxeology at a didactical or meta-didactical level.
5. Brokering: for dialogue to take place in a fruitful way, actions are needed to make the ideas at stake explicit (brokering) in order to facilitate dialogue by clarifying each other's positions. The broker is essential in the phenomenon of internalization to make the link between the points of view expressed and to reformulate in the language of each of the institutions involved the concepts and knowledge at stake.

Figure 1 shows an initial schematization of meta-didactical transposition that describes the dynamics that can be created in collaborative research: through joint work, with the help of brokering actions, teachers and researchers build a shared praxeology by internalizing *a priori* external components to one or the other community (Fig. 1). At the meta-didactical level, i.e. at the level of the discussion of didactical practices, teachers and researchers have developed praxeologies in their own institutions: they have techniques as well as justifications for these techniques. The transposition phenomenon must thus lead to a shared praxeology at the same time as the external components are internalized in the practices.

This schematization shows the dynamics that can be created, but it also raises questions: how can this dynamic be set in motion? How and why is it maintained? What is the meaning of the term "shared" when describing a shared praxeology?

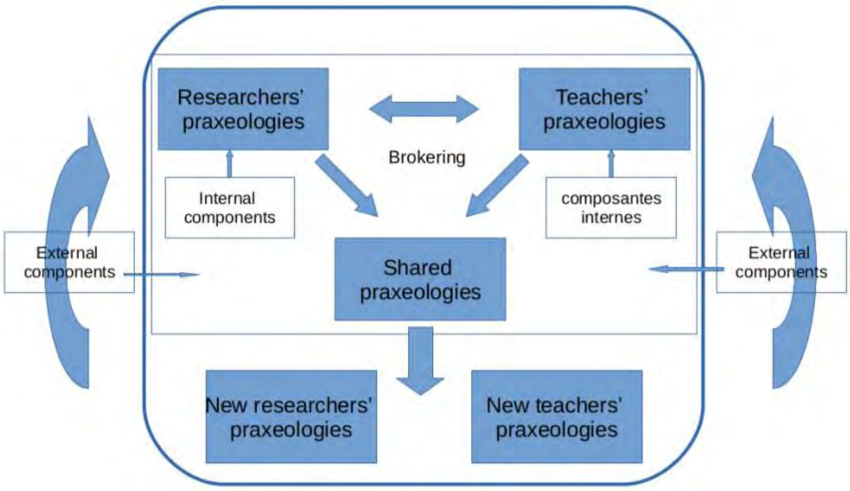

Fig. 1 : Schematization of the Meta-didactical Transposition (from Aldon & al. 2013)

Boundary objects

To go a little further in the concept is to precisely define this work object. By maintaining the conceptualization of meta-didactical transposition, it is possible to provide some answers to these questions by formalizing the nature of the work object (or object of interactions) that can be seen as a boundary object in the sense given by Star and Griesemer (1989) (Robutti & al., 2019). This idea of a boundary object, resulting from an anthropological approach, is particularly interesting for describing interactions between communities and developing the concept of shared praxeology:

> Boundary objects are objects which are both plastic enough to adapt to local needs and constraints of the several parties employing them, yet robust enough to maintain a common identity across sites. (Star & Griesemer, 1989, p.393)

In addition to this interpretative flexibility, the structure of the boundary object is also of great importance and the "information needs" trigger the actions that can be carried out on the boundary object at stake. The metaphor of the object as an object in the object-oriented programming paradigm can help to better understand this structure and the scale issue highlighted by Star (2010):

> The two other aspects of boundary objects, much more rarely cited or used, are (1) the material/organizational structure of different types of boundary objects and (2) the question of scale/granularity. Boundary objects are a sort of arrangement that allow different groups to work together without consensus. However, the forms this may take are not arbitrary. They are essentially organic infrastructures that have arisen due to what Jim Griesemer and I called "information needs" in 1989 (Star 2010, p. 602)

But what is particularly interesting in this quotation is the fact that the boundary object exists if and only if the actors act on it. It is through interactions that this frontier can be widened, enlightened by the collective activity resulting from all the individual actions of the protagonists. Following Carlile (2004), we distinguish three types of activity on the components of a border object:

> […] we scale the relative complexity of the circumstances at a boundary using Shannon and Weaver's (1949) three levels of communication complexity: syntactic, semantic, and pragmatic. (p. 557)

At a syntactic level, the "transfer" action proposes a clarification of the knowledge of the object; it corresponds to a situation where a common vocabulary is established or is *a priori* constituted. These actions keep the frontier in the initial state but allow all actors to explore together by agreeing on the terms used, by highlighting different components of the object but without disrupting the relationships between the objects.

At a semantic level, the "translation" action moves the boundaries of the frontier or affects the relationships between the components of the boundary object. The search for common sense concerns the relations between the components, the constituents of the boundary object as well as a common construction of the meaning of one of its components. The translation activity leads the protagonists to build a sufficient compromise to agree on the subject of study within the specific framework of their discussion.

Finally, at a pragmatic level, "transformation" actions lead to a sharing of knowledge with a view to its use in the rest of the work; we affirm the pragmatic nature of the intervention because there is an awareness of a modification of a technique with regard to a type of task. This "negotiation" (Monod-Ansaldi & al. 2019) can be accepted as an evolution of the work perspectives with the object in a dimension of training and professional development that we can relate to the phenomenon of internalization of meta-didactical transposition.

Transfer and translation activities highlight the mutual understanding of the manipulated components by acting both at a syntactic level of mutual understanding of the manipulated components and at a semantic level of the meaning that can be given and shared to these components. Transformation actions modify the relationships to objects by including them in all the tools that can be used for action at a didactical level. To make the link with Meta-Didactic Transposition, transfer leads to a shared understanding of the type of tasks, translation leads to a modification of technologies (in the Chevallard sense) and theories when the transformation leads to a modification of techniques related to the types of tasks related to the components of the boundary object at stake allowing a practical investment of the studied component (Fig. 2).

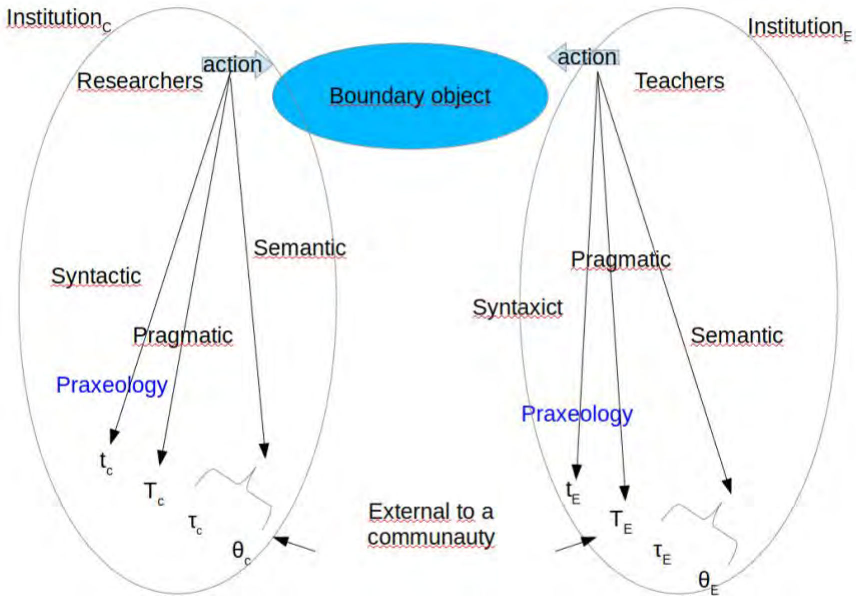

Fig.2 Boundary objects and MDT. Illustration

The announced methodology of the project was built on the paradigm of Design Based Research (Swan 2014). In this project, the general discussion on formative assessment at the meta-didactical level, using in particular the results of the research, promotes the construction of classroom activities and their implementation in the classrooms. At the same time, these classroom implementations provide feedback that, through observations and analyses, evolves the model and participates in the professional development of actors, teachers and researchers. During the experimental phases of FaSMEd (task design, *a priori* analyses, *a posteriori* reflections, etc.), researchers share research results while teachers mainly offer their professional knowledge and pragmatic justifications of their practices. Thus, during the meta-didactical transposition process, the researchers' praxeologies come up against those of the teachers, and it may happen that components of praxeologies that were external to a certain community gradually become internal, within a praxeology "shared" by the two communities. The concepts of "formative assessment" and "use of technology", for example, are the subject of internalization phenomena that have been highlighted and studied in the conduct of FaSMEd. The awareness of the phenomena of internalization appear clearly in the discussions between researchers and teachers; the dialogues clearly show the two didactical and meta-didactical levels, in particular in lines 8-10 and 13-15 where the comparison of class observation with analyses of the constructed situation calls into question the didactic positioning of teachers in a violent way (line 15):

> 8 H1 And then after that, that's how we expect from... they're in check because maybe what we wanted to do is not what... it's not that in fact...
>
> 9 H2 yes yes yes... yes and then it's...

> 10 H1 ...and, and it's interesting to have it... this exchange there and I think it's great yeah it's great yeah it's great interesting there's that... really if we can continue at the school level it's good eh: really because there's a wealth and an exchange like that and it's... not people like that as you see it in hindsight...
>
> […]
>
> 15 H1 because sometimes we are actually in our thing aah ! fuck and shit (blow on the table) what the fuck they don't understand and in fact it's because yes in fact it's us we didn't propose, put the problem of, of, of, of, and it's really true it's....

But there is also an internalization of the reality of the class that must fit into the model (lines 16-23) and which illustrates this double movement of internalization necessary for a real sharing of praxeologies.

A second meeting illustrates the actions on the boundary object. In the dialogue, the boundary object, as a container, is indeed the formative assessment, which is however never mentioned. On the other hand, interactions focus on Quiz, open questions and technology. First, the question of the relationship between Quiz and technology objects arises and is quickly reduced to discussing the type of question that can be asked, thus integrating the "open question" object as a component of the boundary object. Later on, the protagonists distinguish the role of the use of technology: media-method relationships exist and it is these relationships that need to be studied (line 10-11) from both the student and teacher's point of view; the dialogue then leads to a point of widening the object's boundary by linking "evaluation modalities" to the differentiation strategies specific to formative assessment.

In terms of action on boundary objects, the transfer appears to agree on the relationship that may exist between the assessment and the means of assessment; thus, lines 9 and 10, the distinction to be made between means and end is highlighted:

> 9 C2: yes, it's not due to technology
>
> 10 CP1: no, it's something else

This is followed by a translation action:

> 11 E1 : this is the evaluation method

In the "Quiz" component of the boundary object, the questioning mode is questioned and linked to the assessment methods, which is then repeated to specify the relationships that may exist between the way of questioning.

Conclusion

Using the example of design-based research from the European FaSMEd project, I presented theoretical and methodological frameworks for analyzing the interactions between teachers and researchers in collaborative research. The essential elements of the Meta-Didactical Transposition are not all developed but remain as a watermark of this text. In particular, the importance of the institutional dimension of transposition, which is not well developed here, remains a fundamental framework for studies. It is clear that analyses of praxeologies but also of the components of a boundary object only make sense in a given institution and that the dialogue between the actors takes into account the institutional positions of each (Fig. 2). Similarly, the dialogues that have been more finely analysed are only proposed here as illustrations of the theory and should be more broadly detailed. Finally, actions on the components of a boundary object can only lead to internalization to the extent that they are constructed in a sequence, not necessarily linear, of transfer, translation and transformation. The drivers of these actions therefore necessarily involve brokering acts that can be highlighted in the analysis of interactions.

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THEORISING A WAY OF WORKING WITH TEACHERS OF MATHEMATICS IN COLLABORATIVE GROUPS

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Working in collaborative groups is a motif of all of the mathematics teacher education courses at the University of Bristol, including our teacher education course and our Master's course in mathematics education. We set out some of the history of working in collaborative groups in the UK and the influences on our practice. We have theorised how we observe change and learning taking place in these groups. We offer examples of working in collaborative groups, with our prospective mathematics teachers and also from our Master's course. These examples are analysed in relation to our theorising about learning and also to point out some of the roles of the facilitator. We conclude by considering the affordances and constraints of the way of working. One affordance is the energising effect on teachers when the form of interaction is defined, whereas the content is led by their interests. We suspect it is necessary to experience the way of working, in order to facilitate it.

Introduction

In this manuscript, we aim to set out a particular form of mathematics teacher collaboration that has developed at the University of Bristol in the UK. We set out the features and goals of the way of working and point to some of the affordances and constraints, in particular, focusing on the learning of the mathematics teachers involved. The way of working has a history.

In October 1979, aged 27, Laurinda, a mathematics teacher, joined one of the educators at the University of Bristol, Joan Yates, and two other teachers to examine their practice "critically and systematically" (Yates, 1983, p. 35). The group was influenced by Stenhouse's (1975) book *An introduction to curriculum research and development*, which was written instead of a text-book "in the hope that it will serve until an adequate text-book can be written" (p. vii). The practice of this collaborative group was firstly to identify an "interesting intellectual phenomenon"; then each teacher collected instances of the phenomenon; reflecting on their lists led to focusing down on a specific area of mathematics; and agreeing a methodology of: i) writing down the intended approach before lessons, ii) writing an account of what happened, highlighting the interesting phenomenon, and iii) writing an account of conversations with some pupils about their experience of the phenomenon. The group shared writings and reflections with each other considering samenesses and differences:

> Finally we can certainly say that "samenesses" were identified … but probing beneath the surface revealed substantial differences. It is the implications of this that are of prime importance in our teaching. (Yates, 1983, p. 39)

In exploring samenesses and differences in their observations the members of the group were also developing a common language that was closely linked to practice. A number of action research and curriculum development centres thrived from the mid-70s onwards. For instance, John Elliott, who had worked with Stenhouse on the Humanities Curriculum Project (1967-72), founded the collaborative action research network (CARN), where, at the University of East Anglia, Bridget Somekh joined him. Journals followed with John Elliott being a founding editor of the *Educational*

Action Research Journal (1993) linked to CARN. In Bristol, one of the Nuffield-funded Resources for Learning Development Units (RLDU) began life. In the late 1980s, having been a member of various curriculum development groups at the RLDU, Laurinda became mathematics editor full-time for a year on secondment from her school, followed by two years as a job-share. The role of the mathematics editor was to work with groups of teachers "on either developing materials related to government initiatives or from perceived needs of teachers themselves" (Llinares, Krainer, & Brown, 2014, p. 439). The research behind the RLDUs was by Philip Waterhouse (Dickinson, 2001) and, unlike the Stenhouse group she has been in, Laurinda was to learn that now the group was to be less than 10. What seemed an important question to ask from these experiences was "How many teachers or prospective teachers to place in a collaborative group?" for varying purposes. The explicit focus for the teachers in an RLDU group was on the development and then production of materials that had been tried out in all their classrooms. Where discussions were rich across a range of schools, the lesson idea would move to production. The "implicit focus of the role of mathematics editor [and the other subject editors in the team] was on the professional development of those teachers in the groups" (Llinares, Krainer, & Brown, 2014, p. 439). The RLDU had a design studio of graphic designers and a printworks on site, so the production of materials was important (some resources are now freely available, see https://www.stem.org.uk/resources/elibrary/resource/31296/addendum-cockcroft).

Having arrived at the University of Bristol as a full-time lecturer in 1990, steeped in experiences of working in and facilitating collaborative groups, there was no outlet for Laurinda's skills on either the one-year course for prospective teachers, nor as part of the Masters in Mathematics Education programme. In 1993, the text book imagined by Stenhouse was published, *Teachers investigate their work* (Altrichter, Posch, & Somekh, 1993), which was the impetus she needed to develop a Master's module, using action research, designed for local teachers. In the academic year 1993-94, she was successful in obtaining a University of Bristol Continuing Education Grant to develop a Master's double unit *Professional development through working in a collaborative group on an issue in mathematics education*. This unit moved away from the traditional model of taught sessions, to supporting a collaborative group of part-time students, local teachers researching in their own classrooms. The grant allowed Laurinda to buy a set of the recently published course text-book when the module became part of the mathematics education Master's offer. (For more background and a case study of one student's journey with facilitated action research, see Brown (with Dobson), 1996).

These biographical experiences are of necessity UK-centred, but the movement was and is international. To name only a few examples from other countries, Japanese lesson study is spreading and adapting; in France there is the IREM network (www.univ-irem.fr/); Barbara Jaworski and colleagues' (Jaworski et al., 2007) work in Norway where "the team decided to replace 'researchers and practitioners' with 'teachers and educators' ('both of whom are researchers')" (Krainer, Chapman, & Zaslavsky, p. 433); and work in Austria collating a key-word searchable database (http://imst.ac.at) of papers "written by teachers for teachers" (ibid.).

In 1997, Alf, as a mathematics teacher, started the Masters in Mathematics Education course at the University of Bristol. His first module was the collaborative group with Laurinda, as tutor, facilitating the group. Having moved into mathematics teacher education in 2010, the collaborative way of working as university tutors has passed down through generations of people working at the School of Education, forming a learning community with the teachers in school.

Other influences on practice

Over the last 20 years, we have continued to develop our practice in running collaborative groups and also our theorising about what takes place within them. Later in this paper, we will be offering transcripts from meetings, which we will analyse in terms of learning. We also learn from running the groups and reflecting on experiences. We have looked into a range of theories to help us make sense of what we notice. We have been influenced by writing from the psycho-analytic tradition and notions of inter-subjectivity (Tahta, 1993; Ogden, 1994; Brown & Coles, 2007); we engaged in the ideas that emerged at the turn of the century about embodiment (Lakoff & Johnson, 1999) and complexity (Davis & Simmt, 2003). The enduring theoretical influence on us has been from the perspective of enactivism (Varela, Thompson, & Rosch, 1991). In the next section, we briefly set out some of this thinking as it relates to change and learning. It is important also to note that mathematics is more than simply a context for our work with teachers. We will frequently take opportunities to do some mathematics together with teachers. We view mathematics as the study of relationships and see a parallel between the way mathematics learning can proceed through the development of awareness, and the way in which we view learning to teach mathematics in terms of the development of awareness (see next section).

Theorising change

Our thinking about teacher learning starts from relationship, including how and why teachers might become interested and affectively involved in re-thinking their practice of teaching. A central term for Gattegno was that of awareness, as that which enables action (Mason, 2018) and we view the starting point for teacher development as a shift in awareness. Teacher learning often seems to begin with a recognition of something not working – a habit that no longer fits a situation, or a moment of surprise. We draw on the enactivist theorist Rosch, in conceptualising how humans categorise the world as fitting into three broad levels, of increasing abstraction: a detail, or sub-ordinate level; a basic-level (our most common categories); and an abstract or super-ordinate level (Varela, Thompson, & Rosch, 1991, p. 177). What characterises the basic-level is that these are the categories that are the most abstract ones where the same kinds of action are used for all members of that category. To take a non-mathematical example, we generally perform the same kinds of actions with "chairs" (i.e., sitting on them). But, with the more abstract category of "furniture", (unless you are a furniture remover) you would not perform similar actions with members of this category and so the word is not at the basic-level. What is at the basic-level changes over time for each individual. But it was one of Rosch's findings that most of the time we speak in basic-level categories. Indeed, to function effectively as a teacher there is a *need* to interpret classroom events at the basic-level, since these are the categories that are linked to immediate action. As a teacher, I need categories such as "the class is confused" to be linked to actions, with little need for deliberation about what to do next.

The way in which basic-level categories are linked to action offers one interpretation for why it can be hard to learn from experience. If our experience comes to us (as it were) in relation to basic-level categories, it is hard to see how we might use that experience to question or un-pack these categories. But, in that case, the possibilities for us developing new actions as responses to situations seem limited. One way we have theorised that learning can take place (Coles, 2013) is through a descent into the detail of experience, into Rosch's sub-ordinate level. If we can access the detail of our experience, there is potential for us to notice new distinctions. Within the enactivist world-view, the making of distinctions is seen as fundamental to cognition (Spencer-Brown, 1994). In other words, making discriminations (noticing similarities/differences) is a mechanism underlying cognition.

From new distinctions there is the possibility of new labels which, over time, can develop into new basic-level categories. In terms of a model of how learning takes place, this is a phase of symbolising in which the teacher may provide, or highlight, a label for something that has been noticed. In learning to teach mathematics, the symbolising takes the form of a labelling of new distinctions that have the potential to inform future planning. But new distinctions and labels and symbols will only become effective if there is the opportunity for them to be used and to become habitual so that they become basic-level categories and attention can move on to something else. A teacher or teacher-educator might then disrupt habitual expectations and provoke a return to the detail of events, allowing the potential for further development. The process can be conceptualised as a cycle (see Figure 1).

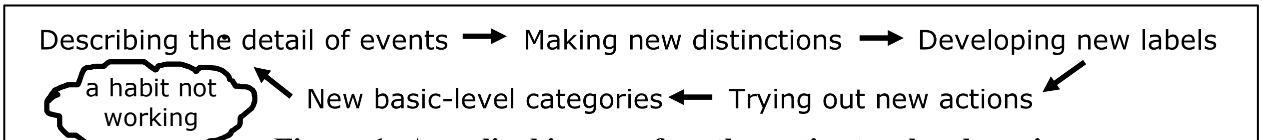

Figure 1: A cyclical image of mathematics teacher learning

In the next two sections, we offer two illustrations of how we operate with collaborative groups at the University of Bristol. We use our theorising about learning in order to analyse the way of working and to point to affordances and constraints. We first describe and analyse a mode of working with prospective teachers of mathematics, in a collaborative group and then do the same, in the context of working with in-service teachers on a Master's course.

Case 1: Working in a collaborative group of prospective teachers of mathematics

A practice that was established by Laurinda in the 1990s, on the teacher education course at the University of Bristol and that persists to this day, is the use of a particular mechanism for running collaborative group sessions, following early experiences that prospective teachers have of being in schools. The teacher education course is structured so that students' time at university is interspersed with times in schools, teaching lessons. Once a week, for the first five weeks of the course, the prospective teachers of mathematics will meet in a group of around 10, with a university tutor, in order to de-brief on their experiences in schools. The way in which we de-brief on experience takes a particular form. A related way of working operates at other times, with the whole group of 30, when prospective teachers discuss issues in groups of three before moving to a whole group discussion.

We take the view that the early experience of prospective teachers on a teacher education course are vital to orienting them towards productive ways of learning from experience and, as such, we are protective of introducing anything that will get in the way of their learning. For this reason, we have consistently taken the position that it would not be ethical to engage these prospective teachers in discussion relating to getting informed consent to record early meetings. There we have never taken audio or video recordings of group sessions. We do, however, have experience of running sessions for around 10 years (for Alf) and around 30 years (for Laurinda). It is from these experiences that we have chosen to fictionalise (Hannula, 2003) an account of a proto-typical collaborative group session. This account was created via a role play enactment, with Alf acting as the "teachers". So, although we are analysing this account in relation to our own framework, the account was not constructed to illustrate the fraemework but from the accumulation of our experiences in such groups. The account is somewhat condensed, we then analyse it, drawing on Figure 1's image of a process of learning.

1. Laurinda:	So, I want you all to imagine being back in school and find an incident or a moment that is still with you either because it makes you feel comfortable or because it feels uncomfortable, a moment that you remember and that stands out somehow. I want you to try to describe this moment in a brief but vivid manner.
2. Teacher 1:	So I felt really uncomfortable in one class I am going to take over. I think the teacher had really confused the class while getting them to do linear equations with an x on both sides.
3. Laurinda:	Can you get down to story, something that happened, don't justify, try to get to the detail of what happened.
4. Teacher 2:	One of the students put their hand up and looked at me. Every time the teacher looked as though they was going to go over, the student put their hand down; I eventually went to help. The student said 'you'll tell me the answer won't you, the teacher just confuses me'.
5. Laurinda:	That is a story. Can someone offer another story that is sparked off by that one? This one might not now be the one you were thinking about originally.
6. Teacher 3:	Well I was working with a group of students and they were also stuck on linear equations and I tried to help them and explain how to answer the question and one student said to me 'don't worry sir, I'll ask our teacher'.
7. Laurinda:	Anybody got a story related to that one? Another story, similar or different to these two?
8. Teacher 4:	I was called over by a child who said that they didn't know what to do next and I didn't know what to say.
9. Laurinda:	Okay enough stories, what are we talking about here?
10. Teacher 5:	These are all about helping individuals.
11. Teacher 6:	It's got to be their own work.
12. Teacher 7:	Taking on the role of being a teacher in the room.
13. Laurinda:	Yes, I see an issue around the relationship between the teacher and student, you all getting into the school culture and knowing what to do; where are the children placing you in terms of your expectations on them and on yourselves? So what are some strategies for what to do, if you don't know what to say to a student?
14. Teacher 4:	I could have done the next line for them.
15. Teacher 2:	I feel uncomfortable about that, my teacher never gave us answers.
16. Teacher 8:	Ask the student how they did a question they got right.
17. Teacher 3:	We could see if someone sitting next to the student could help them.

Learning in the collaborative group

In an actual meeting there would then be an invitation for another teacher to start, from one of their original stories. Stories could be mathematical in focus or, like this one, more general. We observe, in line 1, Laurinda prompting teachers to re-enter their experience in schools. There is a role here for emotion, in terms of identifying moments or incidents that are still somehow unresolved or where something did not go smoothly (a habit, perhaps, that did not work). For the way of working to be effective there must be a move away from talking in basic-level categories. Laurinda notices that in line 2, the talk has not got into enough detail and makes this explicit in line 3 and again in line 5, when the story (line 4) moved away from basic-level categories. After hearing a number of stories (there might have been more here), Laurinda moves the group (line 9) to considering similarities and differences, with the aim of provoking new distinctions. The label "what to do, if you don't know what to say to a student" provokes a sharing of a range of strategies (lines 14-17). These are all things that the teachers might try out in the classroom and, if the response indicates the strategy was effective for them, may lead to developing new basic-level categories, linked to engaging students in dialogue.

Case 2: Working in a collaborative group of in-teachers of mathematics on a Master's course

The following data was inspired by audio recordings of two years' worth of Master's meetings, at the University of Bristol, on the course *Professional development through working in a collaborative group on an issue in mathematics education*. Over two years, 18 teachers took this unit. The teaching sessions were always twilight, allowing in-service teachers to attend. The meetings ran fortnightly. As with the section above, we have fictionalised the transcript, but based it on audio recordings, in order to give a condensed sense of the progress of the first meeting. The first meeting was chosen since this is a vital one in terms of setting up expectations for ways of working. Teacher names are pseudonyms. Teachers had done pre-reading from a text on action research (Altrichter, Posch, & Somekh, 1993).

1. Alf: Welcome everyone. We will be working as a collaborative group and so what that means is each of us supporting the learning and the journey of each other. How the meetings will generally run is that each of you will have time in the meeting to talk about or to get feedback on what you are doing. When somebody's got *time* the role of the rest of the group is then to try and support that person, so it may be lots of things get triggered off in your mind; jot down on paper and try and stay with what the person is saying, try to explore, to tease out, question if it makes sense. We'll see and Laurinda and I will police that as well. Okay any thoughts anybody about action research?

2. Claire: Picking something relevant to yourself and finding what you want to find out first of all.

3. Georgie: I see it is as cyclic process, taking in an idea and building on it, to form new ideas.

4. Sally: And not having too many pre-conceived ideas, do something to impact straight away

5. Adam: I want to be building on strengths and overcoming weaknesses, not letting myself petrify.

6. Laurinda: I'm interested in what Adam meant by petrify? Did you mean frightened?

7. Adam: No, I meant something that goes hard, something that stays the same and doesn't change

8. Laurinda: Not letting your practice petrify, can you say a bit more

9. Adam: Well I guess people do things in a certain way and don't change. I don't want to get to the stage where I think, this is how I teach ratio and it is fixed forever. I want my practice to get better and I don't want to stop wanting for it to change.

[After further discussion, all the teachers then have time to talk through their initial ideas or starting points for their own action research, with the others supporting. We exemplify this with Sally.]

10. Sally: I work with a lot of low ability pupils in maths and trying to encourage them to engage, remember things and stuff. So, a lot of low ability groups and there's just a real frustration about the resources that are out there, that they're not necessarily enough or they're not doing what I want them to do. So part of the project or the issue I'd like to come at is looking at how do you get those pupils to make progress that's sustainable.

11. Alf: You said the students are not doing what I want them to do, can you say a bit more?

12. Sally: Did I?

13. Laurinda: You said the resources.

14. Sally: The resources, yeah, are not doing what I want them to do. Having something that's just useful to my teaching and also to their progress because there's lots of different ideas out

there but actually just trying to make them work. I feel that a lot of the time I try one thing, try another, try this, try that, hope for the best, pray a little sometimes and just keep going. But to research that and come up with something, this is what I've done to help these pupils make progress and this is what works and this is what is sustainable, is what I'd like to, is what I think about every day actually.

15. Claire: So is your key aim to boost their achievement or engagement, or is it interlinked between?

16. Laurinda: [pause] Good question

17. Sally: Umm, really, really good. I think it's both, I think if they're engaged, and sometimes they are, sometimes they're not, I think if they are engaged their achievement will improve

18. Laurinda: That's an example of exactly what you can get out of a question. If you haven't got an immediate answer that's good. That's why I said it was a good question.

Learning in the collaborative group

In setting up this first meeting, Alf comments (line 1) on the group task being "to explore, to tease out, question if it makes sense". This practice is illustrated by Laurinda in line 6, where she flags up a word that is ambiguous. Without more detail, we can only guess what Adam means by "petrify". Laurinda is attuned to when she is placed in the position of having to guess meaning. In line 11, Alf mirrors Laurinda's move of questioning a phrase, picking up on Sally's comment (line 10) "they're not doing what I want them to do". However, Alf interpreted the comment as being about Sally's students, which gets a puzzled response (line 12); Laurinda follows up suggesting the comment was about resources, which gets an energetic reply (line 14) judged by its length. A significant moment in the group is line 15, where another teacher takes on the role of questioning and teasing out meaning. Laurinda comments to the group about the process (lines 16,18) to highlight that this is precisely what we want the group doing for each other. The practice being established here is one where we stay with the detail of experience and prompt and provoke new distinctions arising from this detail.

Discussion

A similarity across the cases above is the way in which discussion is focused into the detail of events. This either happens through the mechanism of getting teachers to share "stories", or through a discussion norm, that is enacted by the facilitators, of questioning ambiguity and descriptions that are too abstract in terms of the categories being used. In both cases, we (as facilitators) take responsibility for the manner in which discussion will take place, but not the content. In other words, so long as a "story" (in the first case) is offered with sufficient detail, it can be about any element of a teacher's experience in school. Similarly, the action research focus of teachers (in the second case) can be on any element of their mathematics teaching, but they will be constrained to describe it in ways that allow the possibility for new thinking and new distinctions. We take opportunities to do mathematics together as a group, when this arises from stories or issues. Getting into the detail of how we each do mathematics, allowing a re-seeing of mathematical concepts, is a key part of the process of learning.

In both contexts, teachers' learning is supported by structures around meetings that guide the work that takes place within them. In the first example, teachers will be back in school on the next working day, with opportunities to explore new actions and ways of being with their students and with mathematics. In the second case, there is a structure to the action research that teachers are constrained to follow. Later meetings may have a focus on, e.g., collecting data, and then on analysing data. We are aware we have mainly illustrated the early stages of the cycle of learning (Figure 1). Partly this is because the processes take time and we only know in retrospect if a new distinction has proved useful.

One of the affordances of the way of working that we observe consistently over time, is that teachers adapt quickly to intended discussion norms, something not often reported in working with teachers. We also observe teachers becoming energised by re-looking at their practice; our own "story" for this, is that the re-looking comes from being constrained not to speak in the basic-level categories that are used to be effective in the classroom. However we suspect that, to learn to facilitate, there is a need to experience a group working effectively; we worked together with a group (Case 2) partly to support Alf's learning as a facilitator. It is not obvious to spot when a teacher is talking in basic-level categories and yet that is vital, to direct discussion to the detail of events. Having conceptualised a learning process for teachers (e.g., Figure 1) seems key to our conviction as facilitators. We suspect such conviction is significant and is supported by the history of mathematics education courses at Bristol, meaning new staff and new teachers have enough faith to take the risks entailed in defining the form, but not content, of meetings and in being vulnerable to a continual re-seeing of practices.

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ZONES OF ENACTMENT IN TEACHER COLLABORATION

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This paper reports on zones of enactment (see Spillane, 1999) as a theoretical framework used to study teacher collaboration in a continuing professional development (CPD) programme. Learning to teach mathematics through inquiry (LTMI) is a 14-month CPD programme designed to support secondary school teachers of mathematics in Malta towards inquiry teaching. LTMI, which offers participation in summer workshops followed by ongoing on-the-job meetings, is based upon research-informed principles of effective CPD, including long-term engagement in a community of practice. Due to their personal resources (prior knowledge, beliefs and practices) and contextual conditions, teachers work within different zones of enactment. This paper shows that the interplay between teachers' personal resources in enacting inquiry and external factors (pupils, policy, public, private and professional sectors) can be investigated through zones of enactment. When teachers' zones of enactment become social, and involve ongoing negotiation, feedback and support from external factors, teachers become better informed to make changes to their practices.

Introduction

Research shows that continuing professional development (CPD) is effective when it is practice-based, ongoing, involves collaborative and reflective practice (Loucks-Horsley et al., 2010; Stoll, Harris, & Handscomb, 2012). Collaborative opportunities for teachers are generally embedded within the design of CPD programmes (Guskey, 2002). In other words, rather than isolated off-site training workshops disconnected from practice, CPD is conceptualised as including ongoing collaborative on-site experiences of practice-oriented learning. Undertaking a community of learners persepective to CPD requires the need for schools to minimise teacher isolation and instead instil a collaborative learning culture (Loucks-Horsley et al., 2010).

Sociocultural theories of learning originated in Vygotsky's (1978) work on child development, particularly following his work on the concept of zone of proximal development (ZPD). ZPD gravitates around the notion of what the learner can achieve alone and with the help of a more knowledgeable other. Zones of enactment (Spillane, 1999) support Vygotsky's (1978) emphasis on the social dimension of learning and the extent to which a teacher changes in practice, provided that there is support to do this. Similar to other research carried out by Jaworski (2006) and, more recently, Potari (2013) and Goos (2013), *zones of enactment* theory focuses on the role of community and the context of teacher learning within education settings. This paper shows how zones of enactment is used as a theoretical perspective to study mathematics teacher collaboration and to better understand teacher change through their engagement in a collaborative CPD. In this study, learning to teach mathematics through inquiry (LTMI) is a CPD programme specifically designed to provide supportive structures with access to a range of online resources (see www.iblmaths.com) and ongoing opportunities for teachers to collaborate, interact and reflect on their classroom practices. Through a range of supportive and collaborative structures, LTMI was intended to offer more social zones of enactment for teachers.

Zones of Enactment to Study Teacher Collaboration

Teachers enact reform practices differently (Martinie, Kim, & Abernathy, 2016) for different reasons: personal beliefs about subject-matter, teaching and learning, their knowledge, dispositions, the school environment and conditions that they work in. However, the influences of teachers' enactment extend beyond the individual school. Spillane (1999) argues that the extent to which teachers reform their practices depends on the characteristics of their *zones of enactment*.

Zones of enactment are defined as the space in which teachers "make sense of, and operationalize for their own practice, the ideas advanced by reformers" (Spillane, 1999, p. 159). According to Spillane (1999, p. 144), *zones of enactment* delineate zones in which "teachers notice, construe, construct and operationalize the instructional ideas advocated by reformers". Ideas about reform in the curriculum are usually communicated to teachers in different ways: by the policy, professional (working context), private (textbook and curriculum publishers) and public (parent and community) sectors. Opportunities for teachers to learn about and to reconstruct their practice are, according to Spillane (1999), mobilised by these sectors. Moreover, consistency in conveying ideas about reform practice among these sectors would influence teacher enactment.

Zones of Enactment Model

Spillane (1999) proposes a model to account for the way teachers respond to and enact mathematics reform. This model (see Figure 1) positions the *personal* resources that teachers have for learning about practice – their existing knowledge, beliefs and dispositions towards mathematics – as central to the learning process. Teachers notice opportunities for learning about teaching or stimuli for change based on their *personal* resources. Such noticing (see Mason, 2002) is, of course, not automatic – some teachers may notice many opportunities for learning while others may only notice a few. But learning opportunities also arise from within their environment – *pupils*, *professional*, *policy, private* and *public* sectors.

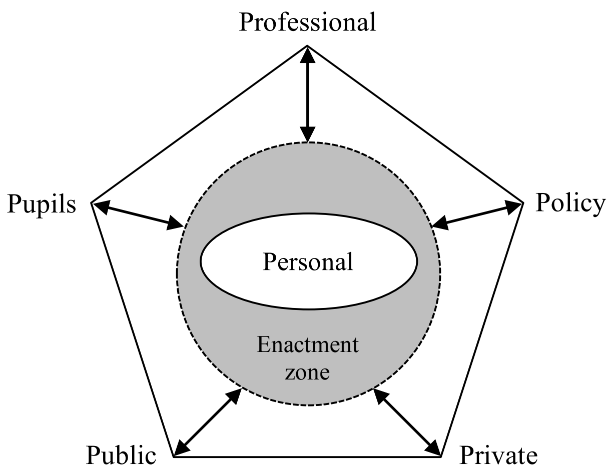

Figure 1: Zones of enactment model adapted from Spillane (1999)

Teachers encounter new ideas about practice from within the environment they work in and process these ideas through *personal* resources. In this model, the *personal* is at the centre as influences from *pupils*, *professional*, *policy, private* and *public* are mediated through teachers' personal resources. Hence, for example, the way in which the *policy* sector influences teachers' learning about practice depends, to a large extent, on what teachers notice in relation to their existing beliefs, knowledge and dispositions. The two-way arrows linking the *personal* to the *pupils*, *professional*, *policy, private* and *public* represent the influence that the environment has on the *personal* resources of teachers in enacting reform practices, but also on what teachers notice and their influence of practice as mediated through their beliefs, knowledge and dispositions.

The model depicts the array of external factors that impinge on local enactment. The five outer factors of the pentagon represent those organisations, associations and individuals, and the opportunities and incentives they mobilise from which teachers might learn about enacting reform practices. One external factor refers to the *policy* sector which includes government and school policies. This includes both formal policies and informal policy talk. Another represents the *professional* sector, including both formal associations and informal contacts among educators. Professional interactions may occur with colleagues at and within other schools. Usually, for Maltese teachers, this happens as a result of subject departmental meetings, staff development sessions, informal discussions in staffrooms and participation in in-service courses. This also comprises conversations with other professionals, for example, education officials when visiting to observe and discuss lessons with teachers. Another factor is *pupils*, and the influence that their responses to schooling has on teachers. Teachers' perceptions of students have an important influence on teachers' practice and their response to implementing inquiry (Hunter, 2010). The fourth factor denotes the *public*, which involves the concerns of parents and community towards the practices promoted by the reform. The fifth factor denotes the *private* sector, particularly textbook and curriculum publishers.

These external factors bear on the enactment of reform, and might therefore support or hinder teacher opportunities and incentives to learn about and change their practice. Spillane (1999) argues that the influence of external incentives and opportunities for learning in changing practice impinges, to a large extent, on teachers' *zones of enactment*.

Enactment Theory as a Social Construct

The quality of teachers' enactment zones is enhanced by both personal and social resources. Spillane (1999, p. 170) states that:

> The extent to which teachers' enactment zones extend beyond their individual classrooms to include rich deliberations about the reforms and practising the reform ideas with fellow teachers and other experts, the more likely teachers are to change the core of their practice.

Assisting teachers to change their practice depends, in a significant way, on an environment that supports ongoing collaborative inquiry about improving practice (see Lotter, Yow, & Peters, 2014). Enactment zones that are social promote changes in both the personal resources of teachers and in the way teachers enact their learning in practice. This means that CPD has the potential to create enactment zones for teachers that extend beyond individual classrooms to include collaborative deliberations between teachers that eventually support learning and change (Spillane, 1999).

According to Spillane (1999), enactment zones that have strong social connections are an indispensable condition for profound teacher change. Hence, besides considering changes in the individual teacher, the influence of the wider social contexts and resources available for learning (including CPD opportunities) play a significant part within teachers' enactment zones. For Spillane (1999), teachers' zones of enactment have three key characteristics. First, enactment zones have a social dimension built on ongoing deliberations with colleagues and experts. Second, deliberations are supported through discussion and, finally, deliberations are enabled by expert teachers, university academics and material resources. In other words, social resources are critical in mediating between policy incentives and teachers' personal resources. To enable teacher change, it "is not so much a matter of telling teachers to reflect nor simply of providing access to a wide variety of mathematical experiences, but rather one of nurturing teachers' rich participation in a variety of settings" (Hodgen & Johnson, 2003, p. 231). In facilitating change, a crucial influence is the existence of a genuinely collaborative environment and dialogue between professionals. School contexts matter. Hence, to enable teacher change, there is a need for the creation of structures, both within and across schools, that provide spaces and tools that facilitate constructive and productive dialogue to emerge.

The CPD Programme: Learning to Teach Mathematics through Inquiry (LTMI)

LTMI is a CPD programme designed to provide a blended approach of off-the-job summer workshops and on-the-job meetings for secondary school teachers of mathematics to collaboratively immerse themselves in and learn about inquiry. The off-the-job component of LTMI offers teachers four summer workshops focusing on four inquiry-based learning (IBL) features: mathematical tasks for inquiry, collaborative learning, purposeful questioning, and student agency and responsibility. Summer workshops followed a consistent pattern of activities – teachers first worked collaboratively to solve an inquiry task, then discussed their experience working on the task and later watched a video from a local classroom demonstrating a teacher using the same task with students. Discussions alternated between pair, small-group and whole-class. Such discussions were intended as additional opportunities for teachers to further investigate teaching approaches, clarify concepts and problematize issues related to inquiry teaching. At the end of each workshop, teachers were encouraged to collaboratively plan a lesson using the activities presented and the ideas generated. The CPD materials are available online (see www.iblmaths.com).

Follow-up meetings were then intended to provide collaborative ongoing support for teachers to discuss, evaluate and develop practice-based learning. These meetings, held from October 2015 until May 2016, followed a structured set of activities led by a facilitator. The opening activity prompted participants to reflect on their inquiry practices. Reflections included personal strategies for using IBL, challenging situations encountered and classroom incidents. This was followed by reporting back and sharing of IBL lessons and tasks. Finally, participants discussed and agreed upon an agenda for the following meeting.

Teacher Collaboration within this Study

Situative theorists posit that learning occurs when knowledge is distributed within communities of practice (Wenger, 1998). When diverse groups of teachers with different types of knowledge, experience and expertise come together, community members can draw upon and incorporate each other's expertise to create rich conversations and new insights into teaching and learning. Effective

CPD programmes value distributed cognition, and hence that learning develops from and between participants, because each participant can contribute knowledge, beliefs and practices to a learning community. Through social interactions, LTMI offered teachers ongoing opportunities to exchange knowledge about their students, the particular settings in which they teach, their teaching practices and their beliefs. In the process of engaging with such deliberations, teachers may develop both their knowledge about practice and experience change in their beliefs.

Researchers interested in teacher professional communities have drawn on the community of practice (CoP) perspective (see Jaworski, 2006; Wenger, 1998) – also referred to as professional learning communities – to explain the social processes shaping teacher learning. In this study, communities of practice are viewed as dynamic learning spaces where participants have opportunities to engage in learning through "actions whose meaning they negotiate with one another" (Wenger, 1998, p. 73). Identities, of community participants, become shaped as they collaborate, share ideas, discuss and reflect upon their practices with others. This situated perspective of learning has been applied widely to teachers' learning to teach (e.g.: Coburn & Stein, 2006; Jaworski, 2006).

In this study, LTMI was an intervention programme designed to provide ongoing opportunities for teachers to meet and share knowledge of inquiry practices within a learning community. LTMI was intended to cultivate a professional learning community to address the debilitating effects of teacher isolation. As reported by Attard Tonna and Shanks (2018), a competitive culture is still dominant within Maltese schools. As a result, teachers tend to work in isolation and opportunities for collaboration are rather limited. Using the zones of enactment theory, inquiry teaching is investigated within both the school and the LTMI community. Carried out over a period of one scholastic year (2015-2016), this study involved a range of data collection methods including pre- and post-LTMI questionnaires, interviews held before, mid-way and at the end of LTMI, video recordings of teacher CoP meetings and a focus group discussion. Data analysis was informed by the zone of enactment theory to study how teachers enact inquiry practices in their classrooms. More importantly, zones of enactment theory was used to study the extent to which teacher collaboration shaped teacher learning, enactment and change.

Understanding Teacher Collaboration and Learning through Enactment Zones

Prior to LTMI

Prior to LTMI, teachers adopted transmission practices generally led by their transmission beliefs. Teachers' enactment zones were individualistic, where deliberations with other teachers to create resources for teaching and developing practices were missing. These teachers perceived challenges emanating from the *private*, *public*, and *policy* sectors as a result of an exam-oriented system (see Table 1). Their willingness to participate in CPD, to learn to enact IBL, was driven by their personal motivations and perceived needs to improve students' learning of mathematics.

The *professional*, *public* and *policy* sectors, reinforcing a competitive exam-oriented education system appeared common to all teachers and viewed by them as restrictive towards enacting IBL. While non-state schools afforded Chris and Greta some authority over the content to teach, exams and choice of textbooks, Sarah had to abide by instructional materials that were imposed on her. Moreover, albeit school leadership teams appeared supportive for these three teachers participating and employing IBL practices, a *lone-fighter* culture (Krainer, 2001) was dominant within schools.

Table 1: Enactment zones of teachers prior to LTMI

Enactment Zones	Teachers		
	Chris	*Greta*	*Sarah*
Personal	Transmission beliefs* and transmission practices	Connectionist beliefs* and transmission practices	Transmission beliefs* and transmission practices
Policy	Summative assessment practices; content-based syllabi		
Public	Parents demanding notes, homework and success in examinations		
Private	Syllabus, exams and textbooks are decided between teachers		Syllabus and textbooks are imposed on teachers
Professional	Supportive SLT towards CPD and IBL; well-resourced school but no culture of collaboration between colleagues on teaching and learning		
Pupils	High-attaining, well behaved and have traditional experiences learning mathematics	Mixed-attaining, well behaved and have traditional experiences learning mathematics	Low-attaining, lack motivation and have traditional experiences of learning mathematics

* *Teachers held a mixture of transmission, discovery and connectionist beliefs with one being more predominant*

During LTMI

Enactment zones capture how teachers' willingness to learn, and to develop knowdelge, beliefs and practices is influenced by the learning opportunities offered by external factors. There were sociocultural factors influencing teachers' understanding and enactment of IBL. For these teachers, the school system restricted their enactment zones. Collaborative supporting structures and a culture of sharing and negotiation were still lacking within schools. Teachers found this support in the CoP created within LTMI. Learning to teach within the CoP contrasted with that of school life, with qualities of collegiality emergent from the sharing of practices. Collegiality and collaborative practices within the CoP contrasted the 'modus operandi' within schools.

Within the professional sector, teachers' isolated practice often saw them retreating into their own classrooms and keeping professional exchanges with school colleagues to a minimum. These teachers reported teaching in rather traditional ways before joining the CPD programme, and eventually made important changes to their practice – both in terms of their new ways of thinking about teaching and learning, and also in integrating IBL. To understand these changes, it was necessary to examine their zones of enactment that extended beyond their individual classrooms. Their efforts to enact IBL were supported and sustained within a social dimension provided by LTMI. Ongoing deliberations with colleagues from different schools, supported by a CPD facilitator, were central to their efforts. Their enactment zones were grounded within discussions and considerations of inquiry teaching ideas and their simultaneous implementation of IBL in their classrooms. An important characteristic was that their deliberations were enabled by the online materials provided in the CPD and the resources they created. While CPD materials were important for teachers to teach through IBL, they enabled conversations about not just what to use but how to use these effectively. Ongoing follow-up meetings served as opportunities for teachers to feed each other with knowledge about practice-based experiences in enacting IBL. Essentially, in their enactment zones, teachers engaged in a norm of collaboration and deliberation that contrasted sharply with the norm of privacy that still dominates Maltese schools (Attard Tonna & Shanks, 2017). For these teachers, translating IBL into practice was mediated by enactment zones that were more social, offering support and feedback over time.

Conclusion

Enactment zones refer to how CPD initiatives and the learning opportunities they offer are encountered, interpreted, constructed and operationalised by teachers. Enactment zones are context dependent and, as Spillane (1999) argues, vary on a continuum – from individualistic to social. Enactment zones that extend beyond teachers' individual classrooms, as offered by LTMI through a CoP, are more likely to support teachers both in developing a deep understanding of reform and in enacting it. Contextual constraints of teachers' work, incentives and learning opportunities were stimulated by three particular sectors (see Figure 2). The two-way bold arrows represent the interplay between the personal resources of teachers (now represented by a bigger area) and the *pupils*, *professional* and *policy* sectors (now more closely linked to personal resources). This interplay is created by interaction, negotiation and support. This broadens teachers' enactment zones where reform meets practice. On the other hand, the one-way arrows show the absence of collaborative learning opportunities for teachers' enactment of reform practices with the public and private sectors.

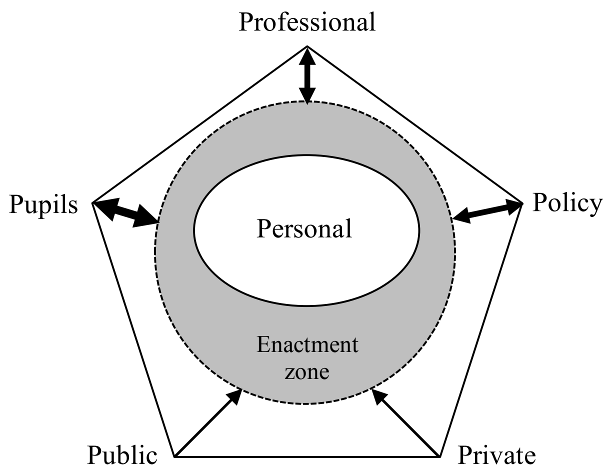

Figure 2: Enactment zones influencing teacher collaboration and learning

Teachers in this study valued enactment zones that offered means for feedback, collaboration, support and negotiations. The CoP meetings offered teachers the opportunity so that their personalised enactment zones become shared with others. They also attributed their persistence with inquiry teaching to students' increased engagement and positive response to inquiry tasks – *pupils* sector. Students' developed capability to discuss mathematics encouraged teachers to pursue with their efforts to promote discussion, thinking and reasoning. It appears that, as teachers recognised their own limitations of influencing the *private* and *public* sectors (see Spillane, 1999), they were more strongly motivated to implement changes within their classrooms driven by the collaborative learning opportunities created within LTMI. These two sectors were perceived by teachers as limiting and hindering their implementation of IBL because they imposed non-negotiable curricular practices.

To promote teachers' capacity to learn and to act as agents of innovation, CPD providers, school leaders and policy makers must look critically into zones of enactment. In the absence of dialogue, interaction and negotiation, external sectors are likely to hinder teachers from implementing the ideas promoted by reform. Any planned intervention must take into account that it is teachers who need to own the reform and, in the process, control the destiny of their change (Chapman & Heater, 2010). Teacher agency is thus a crucial factor. Rather than imposing reform, teachers should learn about it through communication with external sectors. Through social zones of enactment, teachers may negotiate their understandings of reform, resolve conflicts related to its relevance and eventually make changes that facilitate its access into their classrooms.

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NETWORKING OF THEORIES FOR A MULTIFACETED UNDERSTANDING ON LESSON STUDY IN THE ITALIAN CONTEXT

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This paper explores the possibilities offered by combining three theoretical frameworks to observe some experiments in different contexts, in the field of mathematics teachers' professional development, based on the Lesson Study methodology and framed into Cultural Transposition, through the Networking of Theories lens. The three researches share a broader goal: studying what might happen when a "foreign object", such as Lesson Study, is introduced into teachers' practices. The specific goal is to explore how dissimilar theoretical frameworks can highlight different aspects related to the cultural transposition of Lesson Study into the Italian context, and how this cultural transposition can improve, modify or strengthen teachers' practices. Does the confrontation with a foreign culture shed light on which teachers' practices and beliefs are more stable, and which are more malleable and subject to change? Findings provide a positive answer to the applicability of LS in the Italian cultural and didactic context.

Lesson Study, in the last years, has been gaining increased attention in the teachers' professional development research field (Bartolini Bussi & Ramploud, 2018), although in Italy it is not very widespread. This research is part of three research projects in Mathematics Education, two at the Department of Mathematics of the University of Turin and one at the University of Salerno. These studies are rooted in a consolidated Italian culture for a meaningful, long-life teachers' professional development, attentive to the cultural and institutional context. The Cultural Transposition framework (Mellone et al., 2019) stresses the need for a careful approach to the confrontation between practices situated in different cultural contexts. This encounter can fuel a reflection on the reasons behind teaching practices, thus fostering the growth of teachers' professionalism. In this paper we take on this challenge, reflecting on three experiments each with its own peculiarities: they have been conducted with prospective and practicing teachers (both in primary and secondary schools) and they assume three theoretical frameworks (Semiosphere and Semiotic of Cultures for primary school teachers, Semiotic Mediation for high school mathematics teachers, Boundary Objects for prospective mathematics teachers). We will show how the Networking of these theories enrich the discussion on the seemingly common findings.

What is Lesson Study?

Lesson Study (LS) is a collaborative methodology for teachers' professional development rooted in the Confucian Heritage Culture. LS is a three-steps cycle: establishment of long-term learning goals and lesson planning, implementation and observation of a research lesson, discussion on the lesson. These steps can be repeated, like a life cycle in which each lesson is the foundation for new growth. In a LS, a group of at least three practicing teachers and in case some university experts and prospective teachers, collaborate to the detailed planning of a one-hour lesson, which will be taught by one of the practicing teachers in his/her classrooms observed by the others, and discussed by the group. On the one side, LS is a culturally situated methodology and it may not be invariant by translation (in the mathematical sense) between cultural contexts. On the other side, Cultural Transposition proposes "the decentralization of the didactic [and, in our case, teachers' professional development] practice of a specific cultural context through contact with the didactic practices of different cultural contexts" as a way to bring forward in teachers and researchers the implicit assumptions in which practices are rooted, eventually revisiting them through an enriched point of view (Mellone et al., 2019).

Italian institutional context and Lesson Study

In the Italian context, teachers' professional development is defined as compulsory, permanent and strategic by the Ministry of Education (law 107/2015), and it is recognised as an opportunity for effective professional growth. The widespread feeling of professional isolation on the part of the teaching community, whose work is becoming increasingly complex from the scientific, humanistic and social points of view, is the main cause for the law to highlight the promotion of collaboration between teachers as a key principle, and encourages professional development in collaboration at the level of the individual school, and at territorial, national and international level. The preparation of quality teachers, as a key mediator of student performances, is not exclusive to the Italian context. It has gained increasing attention in recent years at the international level (OECD, 2009, 2012); in Europe, it is the cornerstone of the Europe 2020 development strategy. Italy already has many teams in which the culture for a meaningful, long-life collaboration in teachers' professional development is deeply rooted: for mathematics, a never-ending tradition started in the 60s with the *Nuclei di Ricerca Didattica* (Arzarello & Bartolini Bussi, 1998), grew with the *Matematica per il Cittadino* project (MIUR, 2001; 2003; 2004) and currently goes on with many local and national projects coordinated by the Ministry of Education (e.g. m@t.abel, Piano nazionale Lauree Scientifiche). LS can be a further support in the struggle to respond to the demands of the institutions, and another support to the research community in the ongoing development of a culture for collaboration between mathematics teachers.

The three experiments

In the following, we will describe the three experiments designed to understand how to promote, design, and assess relevant collaborative professional development practices for mathematics teachers, each with teachers of different school levels and with different theoretical frameworks.

The first LS experiment is set in a primary school in Piossasco, near Turin. The working group consists in a retired teacher-researcher, four teachers of four different classes, and a researcher. Three complete LS cycles are carried out by three teachers in their 1st-grade classes. The theme of the lesson is the introduction of the 'plus' sign and its institutionalization. The goal on children is to understand the concept of addition as the sum of two quantities in its epistemological meaning of putting things together, and relate it to the sign of mathematical language. Later on, another cycle is carried out by the fourth teacher in the 3rd-grade class, in which the activity designed for this LS is part of the educational path that includes the knowledge of weight measurements and the study of state transitions, through experiments with water. The aim is to accompany students in reinvesting their mathematical knowledge and argumentation skills to the transversality of the disciplines. Each teacher implemented the lesson in his or her class. The experiment is observed through a semiotic lens, adapted from the Semiosphere (Lotman, 1990), looking at a space in which we can observe the dynamics that develop among teachers in the design, implementation of the lesson and in the *a posteriori* discussion. A semiotic lens is used to look for "a truth already present [in the teachers, and] that [only] waited to be recognized" (Sedda, 2006). According to Greimas (Ricoeur & Greimas, 2000), the semiotic point of view provides the scientific knowledge that enables us to investigate that spectrum of knowledge "always known" (and in contrast to the spectrum of "I never thought about it"), but not explicit. In particular, through the introduction of a "foreign object" (the LS methodology) in the usual practices and beliefs of Italian teachers, a process of *deconstruction* is carried out (Bosch & Gascon, 2006, p. 53; Mellone et al., 2019) which "exudes" from the noosphere (Chevallard, 2002, p. 9), influencing the levels of didactic co-determination (Chevallard, 2002, p. 10) and the reflection by teachers. The Semiosphere is in itself a research lens based on collaboration, interaction, in fact it is alleged that no semiotic system can culturally function in isolation.

The second experiment describes a LS trial conducted in Salerno, which involves four teachers from a scientific-oriented high school in Avellino and three researchers from the Department of Mathematics of the University of Salerno. The trial is connected to the well-established tradition of designing Learning Units and carrying out activities within the project Liceo Matematico (Capone et al., 2017): groups of teachers are systematically in contact through meetings with university researchers, to implement collective planning of *ex ante* educational activities and *ex post* analysis of processes. For this LS experiment, the theme "tessellations" is chosen for the learning unit "the art of geometry", connecting with natural sciences and art. Semiotic Mediation (Bartolini Bussi & Mariotti, 2008) is the framework that characterizes the experiment. Two artefacts are used. The first one, used in the Engage phase in Inquiry mode, uses as traces: a sentence, two images and a technological tool (smartphone). The situated texts produced by the students are transcribed on appropriately-made observation sheets. In the final part of the sheets, students are asked to formalize their observations in a mathematical text: each group will therefore provide its own "definition of tessellation". The second artefact is used to solve the real problem: in this case, the traces are cardboard polygons, while no technological tools are used. Once again, the students will write their texts on an observation sheet. The mathematical knowledge is expressed through oral communication. All the implemented activities are socio-semiotic, both because they arise from the sharing between teachers and because they are designed taking into account the Vygotsky perspective of knowledge as a shared experience.

The third experiment involves 29 prospective teachers at the Department of Mathematics of the University of Turin. The aim is to find out the reproducible components useful to implement LS with practicing teachers in the Italian context. The prospective teachers have no previous teaching experience. They worked in small groups: each group is required to create a different activity on continued fractions, from which to draw up a Lesson Plan for a 20-minutes lesson. The lesson is to be performed in front of the researchers and the other prospective teachers, and subsequently discussed within the group. We can stress some differences with usual LS contexts: first, LS is usually performed inside schools and participants have some teaching experience; second, lessons usually last one curricular hour; last, LS is a non-evaluative methodology. As LS is a new methodology for both prospective teachers and researchers, the Boundary Object and Boundary Crossing framework is used to analyse how the two communities act to cope with the novelty, and how LS (the Boundary Object) evolves as a consequence (Star, 2010; Akkerman & Bakker, 2011).

Theoretical Framework

We use part of the Networking of Theories framework (Prediger et al., 2008) as an appropriate (meta)language that makes possible the reference to new conceptual entities connecting our frameworks. In particular, we refer to Radford (2008). He describes a theory as a way of producing understanding and ways of acting based on: *a system P* of basic principles (not a set, for which there is a strong relationship between many of its elements), which includes implicit views and explicit statements outlining the frontier of what will be the universe of the discourse and the research perspective adopted; *a methodology M*, which includes techniques for data collection and data interpretation supported by P, that is, both a theoretical characterization and the very functioning of the methodology (Bernstein, 2000) [the minimum requirements for M are operability and consistency with respect to P.]; *a set Q* of paradigmatic research questions. Table 1 describes the Principles, Methodologies and Research Questions for our frameworks:

Table 1: the three theoretical frameworks

	Semiosphere	Semiotic Mediation	Boundary Objects
Principles	Semiosphere: a multi-cultural dynamic space, interwoven with flows of text, processes of elaboration and understanding of meanings generated by individuals as they interact and know each other. Outside of it there can be neither communication nor language. It is the result and the condition for the development of culture (Lotman, 1990). Its characteristic elements are: heterogeneity and dynamism (it is linked to natural or human aspects of social relations, homogeneity is an anomalous instance); boundary as one of the main mechanisms of semiotic individuation, a porous membrane that marks the passage between "me and the other"; translation as the	At the centre of semiotic mediation is an artefact that embeds mathematical meanings, but is not transparent to embedded meanings. Students, interacting with the mediator, will leave "traces" of their activities (through situated signs). These traces constitute the Semiotic Bundle (Arzarello et al., 2009), that is the dynamic system of signs of various nature (e.g. gestures and words) and of their relationships (e.g. the contemporaneity of a gesture and a word) produced by one or more subjects who interact during the execution of a task. Learning as a teacher mediated social activity. Roots in the vygotskian cultural approach favouring social knowledge.	Boundary as a sociocultural difference leading to discontinuity in action or interaction between communities. Continuity and discontinuity, in the sense that within discontinuity two or more sites are relevant to one another in a particular way. When different communities share a goal, they negotiate a platform at the boundary that allows permeation of practices and preservation of the identity of each community. This crossing might generate tension, which might be the harbinger of new knowledge (Akkerman & Bakker, 2011). Boundary Objects are dynamic objects residing at the boundary, ill-structured with the potential of creating a bridge

	primary mechanism of dialogue and knowledge due to the generating asymmetry, characteristic of the space of the Semiosphere.		between the different communities, robust enough to maintain their identity when they become tailored to local use (Star, 2010).
Methodology	Comparison and analysis of texts, considering all the productions, both of the teachers and of the students, as texts. Context analysis. A posteriori analysis of texts, both verbal and written, produced collectively or individually (transcriptions of audio and video, protocols).	Context analysis. A priori analysis of the semiotic potential of the artefact. A posteriori analysis of texts, both verbal and written, produced collectively or individually (transcriptions of audio and video, protocols).	Introduction of the Boundary Object in a community. Analysis of group dynamics and documentation to investigate how they evolve in the interaction with the Boundary Object. Analysis of the evolution of the Boundary Object when communities act on it.
Research Questions	1. How does Cultural Transposition interact with teachers' beliefs and educational practices? 2. Which specific methodological elements, encountered in the experiments in the light of Cultural Transposition, are highlighted by the different theories and their Networking? 3. Which methodological components of LS are relevant to question 2, with respect to the evolution analysed in question 1?		

A networking of different theories can be seen as a set of connections involving at least two theories. A connection depends on at least two parameters: the structure of the theories involved in the connection; the purpose of the connection. In the framework of Prediger, Bikner and Arzarello (2008) the panorama of networking, seen as a dialogue between theory and cultures in multi-theoretical research (Bikner-Ahsbahs & Vohns, 2019), is painted by strategies. Since our intention is not to unify theories, but rather to make them communicate with each other, we focus on the goals of the networking strategies: in 'comparing', it is to discover similarities and differences; in 'contrasting' to highlight differences; in 'coordinating', elements from different theories are chosen and put together to investigate a given research problem. In 'combining', the elements chosen do not necessarily show the coherence observed in coordinating theories (it is rather a 'juxtaposition' of theories - Radford, 2008). In our three researches, with different theoretical frameworks and therefore with principles and methodologies that are not entirely congruent, we answer the same research questions. For this reason, to achieve our goal we will use the strategy of combining theories.

Findings

Because of space constraints, data supporting these findings will be presented in a future paper. Each theory has its own specificity. Combining three different points of view on similar dynamics, therefore, has the same effect that panting a complex object from three different positions of view can have: it highlights aspects and details, relations between components, dynamics that do not belong to the visual cone of a single observer. With our theoretical frameworks, we focus our attention on a common space: the way in which Cultural Transposition allows us to contextualize and deconstruct (Bosh & Gascon, 2006) teachers' beliefs and practices. The three frameworks conceive, in different ways, LS as an element that interacts with the Italian context and its components.

As a *Boundary Object*, LS is a dynamic object, moving on the boundary between communities of practice. The analysis focuses on the meeting at the boundary between the community of prospective teachers and the one of researchers, which also metaphorically represents the practicing teachers. On the one hand, the two communities collaborated on the reflection on LS, which allowed the researchers to highlight some components of the methodological object that had remained implicit in the brokering of LS from the Japanese cultural community to the Italian one. On the other hand, the researchers were able to observe how LS helped the *boundary crossing* of the prospective teachers towards the practicing teacher's community, making sense of a different perspective. Moreover, it was possible to observe how the encounter with the robust components of LS (Star, 2010) developed a reflection of the prospective teachers on their own meta-didactic praxeologies (Arzarello et al., 2014), possibly transformed into a hybrid between the long-term approach, typical of the Italian context, and the fine analysis used in LS. The analysis of the evolution of both communities of practice and of the Boundary Object itself, allowed the researchers to highlight the potential of LS as a Boundary Object to trigger some dialogic mechanisms for professional growth indicated by Akkerman & Bakker (2011). The community of researchers developed new praxeologies for the introduction of LS in the Italian institutional context.

Immersing ourselves in the visual cone of the *Semiosphere*, we see how the LS methodology, becoming part of the mathematics class' semiosphere, allows the deconstruction of practices and beliefs, so producing a new awareness. In particular, it allowed to look at the collaboration between teachers, and thus at the elaboration, exchange, and archiving of mathematical knowledge and professional development, as mutually inclusive continuous texts. The texts are written (e.g. Lesson Plan), graphic (e.g. drawings of students or graphic representations of teachers), oral (e.g. dialogues in the various phases of the LS), technologically mediated (e.g. worksheets, machines), embodied (e.g. interaction in the classroom), institutional (e.g. curricula), local (e.g. specific epistemological, didactic and pedagogical needs), and others. Through their heterogeneity in mutual continuity, the semiospherical texts allow to keep connected aspects that would seem distant. Distance (understood here in a cultural and sfemiospheric sense - Lotman, 1990) would generate a loss of meaning.

Semiotic Mediation allows us to reflect on the importance of the teacher's role in the appropriate choice of artefact linked to its semiotic potential, and on the importance of the teacher's role in the management of discussion and sharing of individual signs; the teacher also seems more aware that better time management involves better class management. In the teaching practice, LS, shared with the whole teaching community, seems to contribute not only to the professional growth of the experimenter teacher, but transfers to the whole community the refined skills and the acquired awareness of their role in guiding the discussions in the classroom. The identification shared by the experimenter teachers with the semiotic potential of the proposed artefact was the necessary background to its use in the classroom. The careful planning of the didactic intervention, of the possible tasks and the didactic organization foreseen into LS allowed to assume the right semiotic perspective to focus on the production of signs and on the process of transformation of these signs. The teacher, after just one LS cycle, becomes more aware of the choice and use of the artefact to make it functional to semiotic mediation.

The combination of the three theoretical lenses allows us to go beyond the single point of view. If the frameworks of Semiosphere and Boundary Object carry out a meta-analysis of LS in relation to the context in which it operates, identifying the points and tools for intervention in a complementary way, Semiotic Mediation provides us with the tools to observe LS in its operational practice, reading and interpreting the didactic action of the teacher, and therefore promoting an analysis of the effectiveness of LS itself as a professional development practice. In this sense, in absolute consonance with the Semiotic Mediation, the lens of the Semiosphere allows us to focus on the patterns of reasoning that the students use, and on the essential components of socialization of reasoning in building meaning. Starting from the three experiments and through the juxtaposition of the three lenses, we were able to highlight some of the specific teaching practices within the Italian context. From an institutional point of view, our teachers have expressed great difficulties in organizing the time in which to do their work and, at the same time, a need for flexibility with respect to the management of time in the classroom. Moreover, in particular due to the cultural and institutional context but also to emotional aspects, teachers feel the need to adequately respond individual students' needs, something possible only by giving the right importance to design, planning, and assessment of teachers' actions. Finally, from a content point of view, we found deeply rooted fears about the management of mathematical misconceptions, alongside the underestimation of the possibilities offered, in this sense, by research in Mathematics Education.

The LS methodology has contributed to this study, thanks to the new perspective with respect to teachers' meta-didactical and collaborative praxeologies (Arzarello et al., 2014), providing them a tool for microanalysis of the phases of the lesson in a context accustomed, for historical and institutional reasons, to the design and analysis of long-term development strategies. The encounter with other people's practices within LS is an opportunity to observe and reflect on one's own different praxeologies. The apparent contrast between a fine lesson planning and the attention to the needs of the individual student accentuated the careful design of the didactic intervention. The detailed and collaborative design of possible tasks and the didactic organization provided by LS have allowed to take the right semiotic perspective to focus on the production of signs and on the process of transformation of these signs.

Conclusion

In this paper, we have used part of the Networking of Theories framework (Prediger et al., 2008) as a (meta-)language that made possible the connection and harmonization between our three theoretical frameworks (Semiosphere, Semiotic Mediation and Boundary Objects). LS methodology in the Italian context allowed us to closely study the practices of prospective and practicing teachers in collaborative contexts. By combining the three theoretical lenses we have tried to extend the experimentation of LS to different contexts (practicing teachers in primary and secondary school and prospective teachers at university level). The results highlight the collaborative dimension in teaching/learning practices as a possible key for a real reform of teaching, seeking and creating connections between teaching practices of different school segments; the collaboration between school and academia can be an added value towards more conscious teaching practices in the light of the research results. This was possible thanks to a careful cultural transposition of LS into our educational context, which provided a solid methodology for teachers' collaboration within the institutions: the data of the three different experiments on different school levels from different points of view provided a novel understanding on how to promote, design, and assess relevant professional development practices for mathematics teachers. The combined findings seem to have provided further support to the applicability of LS in our didactic praxologies. Overall, LS seems to be replicable as an effective teachers' professional development practice, suggesting the potential of a not only horizontal collaboration. A training that flourishes from below can be of support to the communities of practicing teachers and a stimulus for prospective teachers. Observing one's own work through the practices of others allows a more conscious reflection on one's own practices, laying the foundations of a modern teachers' professional development.

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A COMPARATIVE ANALYSIS OF DIFFERENT MODELS OF MATHEMATICS TEACHER COLLABORATION AND LEARNING

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In this paper, we employ an analytical framework to examine the design features, teachers' learning processes, and outcomes expected in three selected models of teachers collaboration and learning, namely the Action-Education model, the Learning Study model, and a Community-Centered model for teacher learning. Based on our analysis, we outline the affordances and limitations of the selected models from the three perspectives: analytical vs. holistic ways of thinking of the relationship between research and practice in teacher's collaboration and learning; cooperation vs. collaboration in interactions between researchers, knowledgeable others and teachers; and the tacit nature of knowledge for teaching mathematics. Through our analysis we suggest a flexible approach to the use of such models, reconsidering them as tools for which it is essential to propose sets of practical principles that can inform teachers of the choice of models as tools for particular purposes, as well as more generally informing the design, evaluation and research of professional learning.

ICMI Study 25 seeks to better understand and address the challenges in the relationship between mathematics teachers' collaboration and learning (MTC&L). Across the world, various forms of teacher collaboration have been developed to support and study teachers learning, including Lesson Study, Action Research, and Design Research (Chen & Zhang, 2019). To date, however, issues that have remained unclear include the boundaries of these for supporting MTC&L at a scientific level (e.g., Wood, 2017; Ding et al., 2019) and the relationship between theory and practice in these various models (e.g., Huang & Shimizu, 2016; Kempe, 2019; Morris & Hiebert, 2011). Collaborative work across professional communities is highly valued to develop a deeper understanding of the interface of theoretical and practical principles of research and practice that is the key to generating knowledge to improve teaching. In this paper, we aim to contribute to theme A (Theoretical perspectives on studying mathematics teacher collaboration) by focusing on the following two questions:

- What is illuminated by the different perspectives and methodologies and what needs further investigation?
- What are promising research designs and data collection and analysis methods to study teacher collaboration?

From a literature review of studies of MTC&L across professional communities, we selected three models that have been conducted in different cultural contexts for a more in-depth analysis. The three models, each designed for supporting (and studying) inservice teacher collaboration and learning, are: (1) The Action-Education (AE) Model (Gu & Gu, 2016), a combination of *Keli study* (exemplary lesson development) practiced by researchers and teachers in schools in China and action research; (2) Learning Study (LS) (Lo & Marton, 2012), a combination of Lesson Study and design study originally conducted in Hong Kong; (3) The Community-Centered (CC) model for teacher learning (Borko et al., 2005), a university-based summer institute program for supporting MTC&L in the U.S..

We recognize the different notions of the framework/model/form in the literature. In this paper we adopt the terminology used by the authors of the models according to the analytical framework by Boylan *et al*. (2018). In what follows, we summarise the selected studies and use the analytical framework to analyse each one. Finally, we discuss the findings of our analysis of the selected studies and propose further issues to be tackled in our future work.

Research design of the different models

Gu and Gu (2016) examine how the AE model works to improve teaching through, in particular, the nature of the work of the *knowledgeable other* (in their study, mathematics teaching research specialists, TRS) in mentoring teachers' practice during post-lesson debriefs in *Keli study* in China. Influential in China as a school-based form of Teacher Professional Development (TPD) that is widely applied (Ding et al., 2019), it aims to update teachers' theoretical ideas of teaching and learning, to support them to design new learning activities, and to improve classroom practice through *Keli study* in the context of the ongoing national curriculum and pedagogy reforms in the country.

Lo and Marton's (2012) LS model focuses on using variation theory as a source of pedagogical principles within teachers' practice. First carried out in Hong Kong in 1999, and subsequently applied in other parts of the world, including Sweden, Brunei, and the UK, the original conception of LS, as explained by Lo and Marton (2012), was to allow the research team to learn the potential value of variation theory. They consider the LS an appropriate model as it allows researchers and teachers to see how the objects of learning are dealt with in the classroom.

Borko et al. (2005) develop the CC model of guiding the design of a university-based TPD programme focusing on cultivating in-service middle-grade teachers' understanding of mathematics teaching, and learning. This model explicitly brought together constructivist and situated perspectives on teachers' learning that Cobb et al. (2017) see as an example of university-based TPD design studies in the U.S. that situate teachers' activity with respect to the TPD learning environment.

Table 1: An analytical framework for analysing models of professional learning

Categories	Questions to be focused in the analysis
Components & relationships	To what extent do the components of the model map onto the components of the focus PD programme or activity? Are there important aspects of the PD programme or activity that are not easily accounted for by the model? What are the change processes that underlie the PD programme or activity? Do these accord with the model?
Scope	Is the programme focused on the micro, meso or macro scale? What outcomes are the foci of the development programme or activity? Is the focus on discrete PD episodes or broader than that? What is the context of the PD? Does it require a systemic perspective?
Theory of learning	What theory of learning is espoused by the programme or activity, or is expected to be relevant? How far is the model congruent with this?
Location of agency	How is agency conceived within the programme - is it focussed on individual teacher agency, or does it include broader conceptions?

Analytical framework for analyzing the models of professional learning

Given the questions that we address in this paper, we employ the analytical framework by Boylan et al. (2018), which is based on a critical analysis of a number of models of theorising the nature and process of teachers' professional learning. Their framework focuses on categories of model components, purposes, scope, and explicit and implicit theories of learning and change processes, etc. (see Table 1). We first analyse the three selected models according to the categories and questions in

Table 1, and then we accordingly reflect on, and discuss, the different perspectives and methodologies of the models. Note that the category of *philosophical paradigms* included in Boylan et al. (2018) is not used in our analysis because the selected models do not address this aspect in detail.

Analyzing the three models of teachers' collaboration and learning

Given our focus, in this section we report our analysis of the features of the selected models, the teachers' learning that occurred in the processes of the collaboration, and their learning outcome.

Model 1. The Action-Education (AE) model

Components and relationships. The AE model includes three phases of teaching action and two reflections between the teaching actions on developing a *Keli study*. The three teaching actions are: (1) Existing action, focusing on a teacher's previous personal teaching experience; (2) New design, focusing on the new design of the *Keli*; (3) New action, focusing on the new classroom practice. A fundamental feature of the AE model is *Keli* - one or more cycles of planning, delivering, debriefing, revising and re-teaching of the exemplary lesson in a school setting. The *Keli* group is a professional learning community consisting of teachers and teaching research specialists (TRS) who usually work in various layers of teaching research system in China and have considerable experience and expertise in teaching mathematics and working with mathematics teachers (Gu & Gu, 2016).

In Gu and Gu (2016), two dimensions of TRS' mentoring are closely studied: one is of *mentoring content* (namely the types of teachers' knowledge such as mathematical knowledge, pedagogical knowledge, and practical knowledge) and the other is the *mentoring interaction* between the TRS and practicing teachers. The research team identified a four-core component model for conceptualizing practical knowledge (comprising goal analysis, task design, formative assessment, and behavior improvement) and provide an explanation of the relationships among these four components.

Four types of mentoring strategies were identified to account for the nature of conversations between TRS and teachers: general comments, comments on anticipated problems, responses to teachers' questions, and dialogues with teachers. Gu and Gu (2016) found that in their study, Chinese TRS mainly focused on discussing *practical knowledge* when mentoring practicing teachers during post-lesson debriefing. Interestingly, their study indicates a close yet complicated relationship of the three types of professional knowledge for teaching mathematics; that is, Chinese TRS did not tend to discuss theories and knowledge at a general and abstract level. They tried to help teachers to understand mathematical knowledge and pedagogical knowledge through analyzing concrete instructional cases that embrace mathematical and pedagogical ideas (which are regarded as parts of practical knowledge). In terms of the four-core component model, their study shows that Chinese TRS pay greater attention to task design and implementation by focusing on teaching behavior improvement and less attention to goal analysis and formative assessment.

Scope. The study is, to an extent, a mixture of micro (teachers' moment-to-moment learning experience), meso (teachers in the context of their school-based professional development activities), and macro (researchers with motivation for a national TPD programme in the wider context of curriculum and pedagogy reforms). The initial study on mentoring activities within the AE model took place in an elementary school in Zhejiang province. Two teachers were selected to develop the lesson subtraction with two-digit numbers through a typical three cycles of a Keli study. Four TRS mentored the entire cycle of the Keli study. Pre- and post-tests were given to students immediately before and after each lesson, and all the lessons and debriefs were videotaped. After each lesson, interviews with the teachers and selected students were audio recorded. In the later stages of the study, the research team organized over 20 TRS (including those who mentored the practicing teachers) to watch the videotaped mentoring meetings and to try to explain the mentoring activities in terms of their purposes, actions, intentions, and effects. The discussions about the nature of the videotaped debriefing meetings were also videotaped. The nature and the model of practical knowledge characterized in Gu and Gu (2016) is being continuously refined as the AE model has become a form for the school-based teaching research activities and TRS usually works in different layers of teaching research system in the country (Chen & Zhang, 2019; Ding et al., 2019; Huang & Shimizu, 2016).

Theory of learning. The AE model refers to two Chinese classic theoretical ideas of human learning: one is of the *wisdom of action*, which refers to the practical knowledge that integrates subject knowledge with pedagogical knowledge in the context of purposefully improving action; and the other the *unity of knowing and acting*, which is rooted in the Ancient Chinese philosopher's (Wang Shouren, 1472-1529) epistemological theory of learning. Their study largely examined the features of practical knowledge, which is considered by the researchers as a combination of *knowledge-in-practice and knowledge-of- practice*) (for details of the literature references see Gu & Gu 2016). Their study suggests that TRS' practical knowledge is closely related to PCK, but it is built on content and pedagogical knowledge beyond a combination of them.

Location of agency. The study examines the features of practical knowledge and its relationship with PCK and the expertise of mathematics teacher educators (in the Chinese context, TRS) that are considered as a key to support individual teachers' effective learning and improving their teaching mathematics in their classrooms. Their study illustrated that the TRS tended to comment on lessons in general and address anticipated problems based on their previous experience, and pay less attention to address issues raised by the teachers or to engage in dynamic dialogue with them. This finding is different from our early studies to which the AE model was largely referred (e.g., Ding et al., 2014, 2015). This shows a complicated feature of the *Keli* group collaboration in which knowledgeable others collaborate or cooperate with teachers. We return to this issue in the discussion section.

Model 2. The Learning Study (LS) model

Components and relationships. Broadly speaking, the LS model (Lo & Marton, 2012) adopts the Japanese *Lesson Study* model (Stigler and Hiebert, 1999) that involves teachers (with or without researchers) working together through one or more cycles of planning a lesson, and then teaching, observing, evaluating, and modifying the lesson by the team. Noticeably, however, the researchers also tried to reformulate the Japanese lesson study model as a form of 'design experiment'. That is, the research lessons in LS are based on a specific theoretical framework of learning, that of Variation Theory, and the research team wish to learn how well the theory can work.

Lo and Marton (2012) used two lesson episodes extracted from their LS to show how variation theory serves as a guiding principle of pedagogical design. The first case examines the relationship of the following components of the LS model: the last episode of the research lesson (topic is Cantonese Opera) of both cycle 1 and cycle 2, and pre- and post-tests of students' learning outcomes between the two cycles. One of the key components of teachers' collaboration and learning in the team is that in the course of a LS, teachers practiced the pedagogical principle implied by variation theory in their classrooms, and were supported to develop a deeper understanding of the pedagogical principle and its application. In this case, teachers learned that developing a lesson plan with a variation pattern design was not sufficient. Enactment of the lesson must allow the variation pattern to be experienced by the students. The teaching strategy was thus needed to enable the intended pattern of variation to be experienced by the students. The second case shows the relationship of a research lesson plan and its implementation (topic is of the electrochemical series at secondary school), and students' interview of learning outcomes. In this case, the researchers identified an important aspect of teachers' teaching activity that was not easily accounted by the theory of variation. That is, when the object of learning is complex and more than one critical feature must be discerned simultaneously, it is not always clear how to act out the patterns of variation in the lesson to bring about the desired effect. The 'ingenuity' of people in teaching practice leads the researchers to think alternatively about "The science of the art of teaching?" in the debates about teaching as a science or as an art.

Scope. The LS is also a mixture of micro, meso, and macro (regional scope). According to Lo and Marton (2012), over 300 learning studies have been developed through various projects of the Hong Kong Institute of Education, and many schools have developed learning studies on their own. Learning study has been found to improve student learning, reduce the gap between the high and low achievers, and contribute to teachers' professional development and the learning of researchers.

Theory of learning. Lo and Marton (2012) suggest that variation theory serves as a guiding principle of pedagogical design, and could be applied as an importance research approach of developing a strong theoretical mode of professional interactions that build teachers' learning and commitment to future inquiry and maintain their focus on student learning. Variation theory brings the focus of the LS sharply on the object of learning and provides a theoretical grounding to understand some of the necessary conditions of learning.

Location of agency. Lo and Marton (2012) shows researchers and teachers' efforts to address the links between practical knowledge of teaching strategies shared in teachers collaboration and learning in Japanese lesson study and the implicit and unclear nature of the pedagogical theories that might underpin such knowledge building in LS in a different context.

Model 3. The Community-Centered (CC) model for teacher learning

Components and relationships. Borko et al. (2005) designed a two-week long university-based summer institute for MTC&L that comprised 60 contact hours of meeting time structured around four major types of activities: solving mathematical problems; examining children's thinking; reading and discussing current literature; and reflecting on one's own learning. Borko et al. (2005) shared their efforts to develop (and research) the community-centered (CC) model for enhancing teachers' knowledge of algebra. The CC model connects two constructs that are central components of their TPD program. One construct—teacher learning communities—recognizes the impact of sociocultural factors upon teacher learning. The other construct—knowledge for teaching (teachers' mathematical and pedagogical knowledge)—focuses on teacher change. These researchers consider that a unique strength of their model is the emphasis on the symbiotic relationship between the two primary goals—community and mathematics understanding.

Borko et al. (2005) considered four features of classroom life that are fundamental to establishing and maintaining a successful learning community: safe environments, rich tasks, students' explanations and justifications, and shared processing of ideas. The first vignette given in their article, derived from an activity that occurred on the first day of the institute, illustrated the goals of the instructor of establishing a safe environment and creating a culture to support the sharing of ideas. The second vignette, occurred on the fourth day of the summer institute, depicted an activity in which the instructors deliberately guided teachers to develop algebraic knowledge and reasoning. Nevertheless, these researchers raised up several key questions about the challenges of the improvement and expansion of the CC model. For instance, how might the model be brought to some larger scale? How dependent is this model on the skills and temperaments of the instructors? What kinds of supports must be in place for teachers over time, enabling them to build upon the growth they experienced in this professional development program?

Scope. This study is meso in scope. Sixteen teachers from three different school districts participated in the summer institute. Thirteen of them were teaching at the middle school level, and three were elementary school teachers. The institute was taught collaboratively by two mathematics educators, who were university lecturers and members of the research project.

Theory of learning. The TPD programme and its research are firmly rooted in a situative perspective on teacher learning (i.e., it recognizes the contextual influences on knowledge construction for TPD).

Location of agency. The teachers worked collaboratively with their colleagues throughout the institute. While they addressed a wide range of algebra problems (often from contemporary curricular programs), the professional developers selected these problems with the focus on the goals of the program. Borko et al. (2005) developed the vignettes and the analysis that focused primarily on the ways in which the instructors created a professional learning community with the teachers, and how this community contributed to the development of teachers' knowledge of algebra. Borko et al. (2005) further showed that analyses of pre- and post-institute algebra content tests and interviews, teachers' daily reflections, and their final papers provide initial evidence that the summer institute had an impact on participating teachers.

Discussion and Conclusion

To discuss the different perspectives illuminated by the comparison of the three selected models, we summarize, in Table 2, our analysis of the features of the selected models.

We discuss three perspectives of the selected models and the value of the comparative perspectives:

- Analytical vs. holistic ways of thinking of the relationship between research and practice in MTC&L;
- Cooperation vs. collaboration in interactions between researchers, knowledgeable others and teachers;
- The tacit nature of knowledge for teaching mathematics.

First, both AE and LS models emphasize the cycles of the Lesson Study approach that aims to generate knowledge to improve teaching. As recognized by Morris and Hiebert (2011, p. 8), the outcome of Lesson Study, "an instructional product", has the potential to "guide actions towards helping students to achieve the learning goals". Moreover, as shown in Table 2, both AE and LS models address simultaneously lesson plan design and implementation as a whole professional learning and knowledge-generating process. Huang and Shimizu (2016) further argue that both *Lesson Study* and *Learning Study* provide evidence about how theory can be used to guide teaching and how teaching experiments can further refine theory. Note that the western TPD design studies share this analytical way of thinking of the relationship between theory and practice. As explained in

Cobb et al. (2017), pragmatically, TPD design studies involve supporting teachers in improving specific aspects of their instructional practice. Theoretically, TPD design studies involve developing, testing, and revising conjectures about both the process by which teachers develop increasingly sophisticated instructional practices and the means of supporting that development.

Table 2: Goals, learning processes and outcomes of the models

	AE	LS	CC
Goals	Update teachers' theoretical ideas and action on curriculum reform-based mathematics teaching and learning.	How well the VT theory has worked in teachers' practice of learning objects.	Cultivate teachers' understanding of algebraic thinking, teaching, and learning.
Learning processes	Three phases of teaching actions and two reflections on one or more cycles of *Keli* study.	Cycles of lesson plan, teaching and observation, evaluation, and modification guided by VT.	Four types of teaching activities: problem solving; understanding children's thinking; reading literature; and reflection.
Learning outcomes	Core elements of practical knowledge & its relationship such as task design and lesson implementation.	Lesson design and implementation of necessary conditions for learning according to VT.	Content knowledge, mathematics-specific pedagogical knowledge, and recognition of the importance of learning community.
Contexts	School-based teacher research activities system in China.	Projects of the HK Institute of Education in collaboration with schools in Hong Kong.	Pull-out sessions in a university in the U.S.

Here, we should draw researchers and practitioners' attention to the holistic way of thinking of the relationship between theory and practice that has yet remained to refine and understand in the AE model. That is, as noted by Gu and Gu (2016), Chinese TRS tend to help teachers to understand mathematical knowledge and pedagogical knowledge through analyzing concrete instructional cases that would otherwise be difficult for teachers to understand if given the theories and knowledge at a general and an abstract level. We believe that this professional interactions in which theory is interpreted and understood in teachers' teaching practice and practice is deliberately guided and controlled by theory should be a main focus in our future study of MTC&L. In so doing, it is likely to help teachers to overcome the gap between theory and practice and develop teachers' knowledge and reflection skills for making their own teaching theories (Ding et al., 2019; Kempe, 2019).

Secondly, findings from the analysis of the three models lead to further questions that need to be investigated in future work: Whose knowledge is the focus in the TPD models? What is expected to be learnt or improved through the models, and how does this occur? Kempe (2019) highlights the issue of teacher-researcher collaboration through different forms of practice and research. For instance, design research is mainly university-driven (e.g., the CC model analysed above), while lesson study can be teacher-driven. Kempe (2019) explains that the LS model emphasizes a teacher-researcher collaboration because both have a common object of research. That is, it is research *with* teachers, rather than *on* teachers and focuses on constructing knowledge concerning objects of learning as well as teaching-learning relationships. The AE model shares the same strengths of LS model. It is recognised that closing the research-practice 'gap' can actively involve teachers in a genuine process of collaboration where there are shared and common object of research (e.g., Ding et al., 2019; Kempe, 2019). Nevertheless, in line with Borko et al. (2005) and Huang and Shimizu

(2016), strong leadership by *knowledgeable others* is evident in all the three models, and the obstacle that needs to be overcome is to do with the question of what variation may be necessary to enable the further development of these models so as to be accessed by less experienced, or new, teachers (Ding et al., 2019). This leads to the final point we wish to make.

It is important to develop new research designs to collect data to enable researchers to overcome the challenges to understand and characterize the tacit nature of professional practical knowledge for teaching mathematics and the difficulty to disseminate it by the existing models (Kempe, 2019; Morris & Hiebert, 2011; Stigler & Hiebert, 2016). For example, Marton's variation theory is a general theory for learning, and in the work of Gu and Gu (2016) there remain a number of questions about the relationship between mentoring content and mentoring model. Such things mean that there is a problem to understand the types of knowledge, mathematical, pedagogical or practical knowledge. In our future work, we consider that it is necessary to understand not only the tacit nature of practical knowledge for teaching, but also people's attitudes and thinking of such knowledge for teaching mathematics rooted in their philosophical underpinnings (Boylan et al., 2018). We support Boylan et al.'s (2018) suggestion for a flexible approach to the use of models, reconsidering them as tools, and it is essential to propose a set of practical principles that can inform teachers of the choice of models as tools for particular purposes, as well as more generally to inform the design, evaluation and research of professional learning.

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A POSSIBLE PATH FROM TEACHERS' COLLABORATION TOWARDS TEACHERS' CHANGE IN PRACTICE

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This research follows a professional-development that encourages the participating teachers to integrate technology in their lessons and reflect upon their practice. The research aimed to find out whether taking part in such a reflection-encouraging PD may demonstrate detectable changes in the participating teachers' discourse regarding their practice. The commognitive framework enabled us to identify and track the changes in one participating teacher's professional-identity. Hence, we hypothesize that providing an opportunity for reflection in a professionally supportive environment may lead towards the improvement in a teacher's professional identity.

Introduction

It is by now within the consensus of the mathematics education research community that teachers play a major and critical role in the learning processes of students. This acknowledgement yielded the mapping of teachers' knowledge (Shulman, 1986; Ball & Cohen, 1999). Since then, the establishment of teacher-education programs based on progressive principles drawn from research is evolving. In spite of this, facilitating in-service teachers towards initiating a sustainable improvement in their practice is a tough nut to crack. The components that compose a successful professional development are challenging and not always easy to address (Borko, 2004). Present approaches in developing PDs for in-service mathematics teachers minimize "deficiency talk", and instead, provide participating teachers with opportunities to adjust their practice, building on their own expertise and on collaboration with peers (Dadds, 1997; Horn & Kane 2015). The research depicted ahead followed the teachers' learning in one PD with purposefully designed characteristics, as we describe henceforth.

The research presented in this paper involves looking closely at an in-school professional development program, aimed at encouraging teachers to use technology in class. The PD was designed based on guiding principles that emerged from both authors' extensive experience in leading PDs : (1) The teacher is an adult-learner; therefore, the PD should be planned to be relevant to the teacher's authentic professional needs; (2) Outcomes emerging from those needs may lead to long-term changes in her practice of teaching; (3) At present, in the Israeli high school math education system the teacher is autonomous to decide which technology to integrate in her class and in what manner; (4) Integrating technology into teaching is a process that should encourage the teacher to reflect upon her practice. The uniqueness of the PD that stems from these principles, is that it gives teachers an opportunity to define their pedagogical needs, and it also provides them with resources of time and professional support to find technological solutions that meet those needs. The fact that the topic of this PD was the integration of technology is important, because the participating teachers were never told, not even implicitly, that their pedagogy should be altered according to some externally decided principles. On the contrary, the facilitator, who is the first author and an experienced mathematics teacher herself, regarded the participating teachers as professional experts.

The purpose of the PD was to allow the teachers to spend some "paid-for" time collaborating, exercising their expertise and reflecting upon their practice, discovering their "need to improve" issues and implementing technology in their mathematics classes to broaden their teaching repertoire. The research aimed to discover whether taking part in such a reflection-encouraging PD may demonstrate detectable changes in the participating teachers' discourse regarding their practice.

Theoretical Framework

The commognitive theory of learning (Sfard, 2008) is our overarching framework. It defines the basic concepts with which we intend to establish our claims. The commognitive paradigm adopts the Vygotskian tenet saying that every human skill is a product of a process of individualization of collective activities. In particular, thinking is a unique human skill emerging from the collective activity of communication and thus we refer to thinking as communicating with oneself. The term commognition is a constant reminder of the identification of thinking with communication. According to Sfard (2008), a *discourse* is a well-defined form of communication characterized by its keywords, visual mediators, routines and endorsed narratives. In addition, learning is the process of becoming a proficient interlocutor in an established discourse, and the detection of learning is the detection of changes in one's participation in that discourse. The commognitive theory offers a complementary perspective on learning through the concept of *identity* of a person, which is operationally defined as the collection of all reified, meaningful and endorsable stories that are told about that person (Sfard & Prusak, 2005). To be reified, a story should tell us something about what this person *is*, and not only about what he is doing. A meaningful story is one whose teller thinks is depicting critical characteristics of its subject. The endorsability is achieved through supporting the story with empirical evidence. Those three characteristics are responsible for the difficulty to alter identity and for the differentiation between just any story about a person and stories about a person that constitute her identity. Identity can be either *actual*, as the stories are told about a person at present, or it can be *designated*, as the stories are told about the expected future of that person. Such stories, told by either first, second, or a third person, are according to Sfard and Prusak, the missing link that enables us to reveal the mechanism through which a community instigates the learning of its members. Therefore, we may look at learning as narrowing the gap between designated and actual identities.

Characterizing discourse, as conceived by the Sfardian paradigm, has methodological implications; we should distinguish between *mathematising* - talking about mathematical objects - and *subjectifying* – talking about people that participate in the relevant discourse. The stories that comprise the identity of a person are naturally retrieved from the subjectifying-talk.

The discourse this study sets out to follow is the participating teachers' talk about their practice during the PD. We follow Heyd-Metzuyanim and Shabtay, and signify the discourse about the practice of teaching mathematics as *pedagogical-discourse*. Also, in-line with Heyd-Metzuyanim and Shabtay, we suggest the concept *professional-identity* of a mathematics teacher as the collection of all reified, meaningful and endorsable stories told about the practice of that teacher (Heyd-Metzuyanim & Shabtay 2019; Heyd-Metzuyanim, 2019).

Our last theoretical claim regards the *process of reflection*. This concept has few references in the context of mathematics teachers' professional development, where it is characterized as a tool that helps teachers turn their experience into knowledge (McAlpine & Weston, 2002; Clarke, 2000), and may or may not have implications for the teachers' practice (Ricks, 2011). Accepting the claim that teaching in general is a reflective practice, we may learn from the relevant literature that although becoming a reflective practitioner is highly advocated and often taken as self-evident, it is difficult both to teach and to maintain. Yet, attempts to operationalize the actions that constitute reflection, are not unequivocal (Finlay, 2008). For this paper, we draw from Schön's seminal work (Schön, 1983, p. 68) three basic stages of reflection, to help us detect the reflective component in teachers' pedagogical-discourse and identity-talk. The first stage that should trigger the process is a sense of discomfort or puzzlement; the second stage is thinking, or telling a story about the action that gave rise to the sense of unease; and the third stage is an attempt to generate a new understanding and suggest an alternative action to solve the problem. Looking at reflection through the commognitive lens - as discursive process – implies that in order to follow its three stages, we need to carefully analyze teachers' pedagogical-discourse, that is, the way teachers communicate about their practice.

It should be noticed that although a colloquial use of the word reflection usually signifies any recollection and communication of past actions or thoughts, in this paper the phrases *reflection*, or *reflective-process* are reserved only for processes that begin with some sense of discomfort or discontentment. Practitioners who communicate self-contentment, as thoughtful as it may be, will not be considered in this paper reflective.

Having delineated the theoretical framework, we may now translate our research goal to a research question: *How can the opportunity to participate in a reflective-process, lead to changes in the participating-teachers' discourse? If a change in discourse is indeed detected, is it in the pedagogical-discourse, in the identity-talk, or in both?*

Methodology

The research follows a PD program that was conducted with teachers from several school mathematics departments during the school-years of 2016-2019. In this report we focus on one group of teachers that participated in this program's first year. Seven teachers from the same school took part in the PD, most of them teach mathematics in both middle-school (grades 7-9) and high-school (grades 10-12). Most of the participants were second-career teachers who had made extensive use of technology in their first career. As a result, they were more inclined to make use of technology in their classes, and had acquired extensive experience in this practice. The PD involved 10 meetings of 90 minutes each, and they were two weeks apart to allow teachers to experience the integration of technology in their lessons between PD sessions. The main idea of the PD, as reported in the introduction, was to offer the participating teachers resources of time and professional support to allow them to collaborate with their peers and come-up with technological solutions to their authentic pedagogical needs.

Each PD meeting except the opening and closing sessions was conducted according to the same pattern: At the beginning participants reported (individually or in small groups) what they had enacted since the previous meeting and what are their working plans for the present meeting. The main part of the meeting was dedicated to teamwork on planning the next lesson. In the final part of the meeting, again, each participant or team reported shortly to the group what did they achieve in this meeting. To receive credit for the PD, teachers were required to attend all the meetings, enact three lessons with technology, and submit a written report (following a prescribed template) regarding each enacted lesson. Let us emphasize that although the PD was not explicitly directed to initiate a reflective process, some of its components, such as making plans for technology integration, writing reports and talking within a group and to the assembly about an enacted lesson, did encourage the participating teachers to reflect upon their practice, in accordance with the stages stated above.

In order to answer the research questions, the following data was collected: The facilitator of the PD managed a journal documenting both the planning and the execution of each PD meeting and her thoughts that followed. All the meetings were audio-recorded. All the written products of the participating teachers were collected including lesson plans, lesson-reports and computer files. Two of the teachers' lessons were video-taped, and four of the participating teachers were interviewed a few weeks after the PD ended by an interviewer who was not familiar with the particulars of the PD.

The data analysis comprised two phases: The first phase was to use the facilitator's journal and the participating-teachers' written reports to locate one teacher who showed the most salient inclination towards changes in discourse. The second phase of analysis focused on that particular teacher - Yosef. For a deeper analysis of his discourse, we thoroughly reviewed all the audio recordings that documented Yosef's participation in the PD meetings, all Yosef's written products and his interview. The analysis identified Yosef's pedagogical discourse and in particular his subjectifying talk that may be considered as identity-talk or stories that can be interpreted in a way that uncovers implicit identity-talk. The findings, reported henceforth, are the evidence we selected in order to support our story regarding the change we detected in Yosef's discourse[1].

Findings

This section presents Yosef's story, from the perspective of the researchers, throughout the PD. Yosef is in his fourth year of teaching math. He became a teacher after working in the high-tech industry for many years. He is extremely competent in using technology and his students are low-achieving students in both middle and high-school grades. In this section we unfold Yosef's change of discourse as it appeared to us, supported by evidence from the beginning, middle and the end of the PD.

Beginning

At the beginning of the PD Yosef had several opportunities to share with the assembly his point of view regarding the practice of teaching. We hereby analyze two of the episodes, which we believe clearly demonstrate the stories that Yosef told in the early stages of the PD. The following is taken from Yosef's oral report to the assembly at the very beginning of the second meeting (0:00:24-0:01:30), which was the first meeting Yosef attended:

> I'm teaching the so-called non-calculus this year, "three units" (the lowest level of high-school mathematics). When I teach arithmetic sequences $a_{n+1} = a_n + d$, the students, many of them, have a problem of comprehension, what is this n? They have no idea. The fact that I'm saying [they have] a problem, that does not mean I'm not explaining ... fine, even after two lessons of thorough explanation

[1] Different aspects of the research, presented in this paper, with similar data but different analysis, were published in the following conference proceedings: Elbaum-Cohen, A., & Tabach, M. Reflection as a mechanism to explain changes in teachers' identity: The case of Yosef. *Proceedings of the 5th ERME Topic Conference MEDA 2018*. Copenhagen, Denmark. https://www.math.ku.dk/english/research/conferences/2018/meda/proceedings/MEDA_2018_Proceedings.pdf

> to small groups of students, there is still a problem of comprehension. I'm trying to think, if we use computerised tools, what can be done in this group? It is clear to me that it should be something different, more exhaustive, maybe.

In the above quoted episode Yosef is talking about his experience as a teacher and so this is a part of his pedagogical discourse. He is describing the problematic situation of his students. Not only do they have difficulties understanding mathematics, this difficulty is persistent even when Yosef is explaining the difficult mathematics with deep intention. Let us emphasize that it was Yosef's choice to share this experience with his peers and instructor at the PD, therefore we may consider his story significant. In addition, Yosef is describing his students' characteristics as math learners and his talk is a third person identity talk describing his students. We suggest that the way Yosef portrayed his students, as totally incompetent, implicitly reflects his sense of impotence as their teacher. Therefore, we believe this story is a representative story from the collection of stories that form Yosef's professional-identity. Yosef is accepting the invitation to reflect, and takes the first step to express his discomfort or even frustration from the present state of affairs in one of the classes he teaches.

In what follows, Yosef continues to describe the poor mathematical disposition of his students and his doubts regarding the possibility of improvement using technology. The facilitator asks Yosef: "What in your students, do you think you can rely on? On what can you build their understanding?". Yosef was reluctant to answer this question and the conversation ended with the facilitator's advice, that a good start would be to try a small scale technological demonstration and to take it from there.

Yosef's first written report presented a different angle on what we believe is the same phenomena. In the first lesson of integrating technology, Yosef decided to let his students experience an inquiry-based activity, in which they were instructed to infer the significance of each parameter in the linear-function $f(x) = a \cdot x + b$, by using sliders in DESMOS that continuously change a and b. We assume that he decided to implement an inquiry-based task, because most of the participating teachers did so. Yosef reported that his students failed to accomplish the task, and in the "reflection" part of his report, he wrote: "It is a critical problem regarding the aptness of inquiry-based activities to this specific population". Yosef chose to refer to his students as "population", and their inability to perform as expected was due to the characteristics of this "population". We interpret Yosef's choice of vocabulary as signifying the irrelevance of this "population" being his students and the irrelevance of any possible action he may take as their teacher, as if it was predestined, and there is nothing he can do to improve their learning. He continues his report: "The attitude in this setting (inquiry-based task) is totally different and it requires higher-level competencies. The question is whether this is to the benefit of the students, or not". Yosef presents the question whether inquiry-based activities are appropriate for low-achieving students, as a theoretical issue that may have an absolute answer regardless of Yosef's actions. We may infer that although he is their teacher, he does not feel that he has, or may ever have, a positive impact on these students' learning.

The above analysis of Yosef's pedagogical discourse uncovers an implicit identity-talk component. These implicit identity stories present him as a teacher who does not think that he has any true responsibility for his students' learning, nor is he able to change their poor qualities as learners. In spite of that fact, we must state that he is the one that chose to join the PD and in addition, he chose to let his students experience an inquiry-based activity. We believe that those choices are a sign that

Yosef was willing to try and be guided into a new route that will enable him to tell new stories that may re-define his professional-identity.

Middle

In the second technology enactment Yosef planned three consecutive lessons (far more than the participating teachers were required to do). In the first two, he decided to use technology (DESMOS) from the teacher's post in order to demonstrate graphic representations of parabolas and let his students participate by referring to this demonstration. At the end of the second lesson, Yosef showed a drawing of a "face" constructed from parabolas and instructed the students to reconstruct the "face" at home using DESMOS on their cell-phones. According to the report, the third lesson was disappointing for Yosef, since the attendance was low and most of the students failed to complete their homework. Nevertheless, Yosef reported this whole experience as more successful than the previous one. He said that now he understands that the inquiry-based approach that let the students read written instructions and independently figure out mathematical ideas as they used the computers independently, was too demanding for his students. We notice that Yosef chose to accept the facilitator's advice, and used a small demonstration. He did so only after his attempt to conduct an inquiry-based task (as reported in his first report), failed.

This second report presents a new and different story that replaces the one that was told at the beginning of the PD. Now, it is not the students who are incompetent at learning while participating in an inquiry-based activity, it is the activity which is unsuitable for the students. This seemingly minor change can open the door for bigger changes to come, because it shows signs of Yosef shifting the responsibility for the favourable outcome of the lessons from the students to himself. Looking deeply into Yosef's second written report, we found some utterances that support this claim. For example, when he describes one of the moves he conducted during the first of the three lessons, he said: "Maybe here, I made a slight mistake". On the one hand, we may say that this utterance can be interpreted as a critical view of Yosef towards himself. But, when we broaden our perspective, a story that may make sense of it is that Yosef sees himself as someone who can make mistakes, therefore he can also improve his performance, mend those mistakes and do good. Again, the impotency that characterized Yosef's view of himself in the beginning, is replaced, after his second report, by a more optimistic view, as Yosef considers himself a significant figure to his students' learning.

We assume that Yosef's change of practice from inquiry-based to demonstration stems from his decision to accept the facilitator's advice, given in the second meeting, to use a simple demonstration, and give it a try. We could not find any evidence for him trying to analyze his former actions or trying out a solution he thought about. Therefore, we can only say that the attempt to solve the unease Yosef shared with his peers and facilitator, was not a completely independent reflective process.

Towards the end of the PD

Yosef decided to enact the third technology-implemented lesson in his class of low-achieving 7th graders. He followed his peers who made extensive use of an interactive game played using cell-phones (kahoot), especially in low achieving classes. The subject was addition and subtraction of signed-numbers. In his oral report to the assembly, Yosef explained his decision to use kahoot: "I needed it in order to break the…course of the lesson… so it won't be too long. Letting them know that we are going to play kahoot at the end, it gives them some kind of motivation to work". The game is used here to achieve a pedagogical necessity of keeping the students motivated and positively active for the whole lesson. Looking at all the data, this is the first time Yosef referred to the integration of technology as addressing a pedagogical necessity. This may further support the story Yosef began telling previously: The students have difficulties and it is the teacher who helps them to overcome those difficulties. Furthermore, Yosef's pedagogical decision may be interpreted as the completion of a full reflective-process. The unease and analysis stages are implicit, while the solution is presented via the use of kahoot in the lesson.

As before, Yosef is critical regarding his decisions. In the third written report he says:

> Although I tried giving very simple exercises in the kahoot, repeating the principles in their simplest form, even good students, relative to this group, made mistakes due to their attempts to give quick answers. It seems that the attempt to give quick answers made them overlook the procedure.

After identifying the problem, Yosef suggest a solution: "Questions about the procedure should be added to the kahoot", and he elaborated several possible questions. In general, it seems that Yosef was looking both on the advantages and disadvantages of his pedagogical choices, and was willing to learn from his experience and suggest solutions to the new problems that arose. Another sign of the change in Yosef's stories is his referring to some of his students as "good students". Therefore, we may claim that of the same students, who in the beginning had persistent difficulties, at least some of them are now described as good students who may make mistakes. In this last report we claim that a full reflective process has taken place. Yosef detected the problem, he analyzed it, and found a tentative solution.

A new dimension that was not a part of Yosef's directly told stories appeared towards the end of the PD as he started taking an active and meaningful part in the teachers' discourse in the PD. His oral reports evoked the other teachers to ask questions and to suggest pedagogical ideas to answer Yosef's queries. In the last PD meeting Yosef presented a new self-made Geogebra applet that visually defines the trigonometric relations in a right-triangle. His applet and presentation received positive and encouraging responses from his fellow teachers. We believe that although it is only implied, we may add to the stories that constitute Yosef's professional-identity a story that describes him as a teacher who contributes to his community of peers, and whose community contributes to his practice.

Discussion

The goal of this study was to enquire whether participation in a reflection-encouraging PD may demonstrate detectable changes in participants' pedagogical-discourse and professional-identity. The findings depicted above show that participation in this specific PD for the integration of technology indeed resulted, in the case analyzed, in significant changes in the stories that constitute one teacher's professional-identity. Interpreting Yosef's stories, we may claim that his professional-identity took a meaningful turn from an impotent teacher who has no impact on his students' learning, to a teacher who is sensitive to his students' needs and is able to attend to those needs with pedagogical and technological means. In addition, Yosef's participation in the PD enabled him to perform some complete reflective-processes. One caveat should be declared with regard to the interpretative nature of depicting Yosef's professional-identity. It is known that adults are reluctant when it comes to sharing identity-talk. Hence, it is much more difficult to find reified stories that are voluntarily told by adults. Some of our detected stories were interpretations of observed communication.

Looking at the investigated PD, we cannot say that specific pedagogical or technological tools were taught, nevertheless, we assume it is the resources of time and support and the unique design that let the participants work in collaboration, communicate safely about their practice and form a supportive community. This community of collaborating teachers yielded and nurtured a significant change in a teacher's professional-identity and in parallel, allowed him to perform some complete reflective-processes.

After looking at the changes that can evolve in a teacher's professional-identity it seems inevitable to ask, how can we explain that while participating in the same PD, Yosef underwent such impressive progress but looking at other participating teachers we detected a less impressive change, or no change at all. We believe that this query can be answered if we further dwell on the stories that form all the teachers' designated-identities. As stated in the theoretical introduction, learning may be looked at, as narrowing the gap between the actual and designated identities. If there is no such gap, we would expect no learning. Therefore, the attempt to explain the differences between the performance of different teachers will entail looking at their designated identities, which is beyond the scope of this paper.

The theoretical contribution of this paper sheds additional light on the productivity of discourse analysis within the commognitive theoretical framework. The research settings produced an authentic pedagogical-discourse that revolves around a concrete pedagogical action (teaching mathematics with technology) and the meticulous interpretative endeavor using commognitive language yielded a coherent story about the change in identity and professional development of one teacher.

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INTERACTIONIST PERSPECTIVES ON MATHEMATICS TEACHERS' COLLABORATIVE LEARNING: IMPLICATIONS FOR INTERVENTION DESIGN

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Interactionist analyses of teachers' professional conversations respond to open questions about collaborative mathematics teacher learning in ways that are proximal and relevant to their lived experiences and everyday work. Drawing on situative theories of learning, we analyze partitioned conversational records for evidence of learning. Key findings from our prior studies point to four design considerations for interventions that seek to leverage the potential of mathematics teacher collaboration: (1) deeper collaboration is relatively novel and rare for teachers; (2) development of a shared vision for teaching is essential and deliberate work; (3) adequate representations of teaching are necessary for supporting intersubjectivity about core instructional ideas; and (4) frames are an important site for reconceptualization of key ideas about teaching. Examples from our current projects show the application and broader utility of these findings for interventions that use collaboration to support mathematics teacher learning.

When researchers find large groups of students performing above what demographics and prior achievement would predict, they consistently find teachers collaborating with a sense of collective responsibility for student learning. This "beating the odds" phenomenon is a robust finding in the sociology of education; groups of teachers taking collective responsibility for student learning leads to better educational outcomes for students (Gutiérrez, 1996; Langer, 2000; Lee & Smith, 1996; McLaughlin & Talbert, 2001). Researchers identify several reasons for this. First, when teachers work together with a shared commitment to students, they garner emotional support in what can often be an isolating profession. Second, teachers working together can reduce individual workloads, delegating redundant tasks like designing lessons or assessments. Finally, in sharing teaching decisions with like-minded colleagues, teachers can delve into problems of practice and improve their instruction, supporting transformative professional learning.

This last conjecture is the starting point for our work (Bannister, 2018). As mathematics educators, we see teacher collaboration as a promising mechanism for instructional improvement. At the same time, as learning scientists, we seek empirical specificity about the forms of teacher collaboration that might support transformative professional learning. In this paper, we argue that interactionist perspectives offer a promising research design for studying collaborative learning. We substantiate our argument by sharing 4 design principles that have emerged from our empirical work in this vein, illustrating how they can inform interventions seeking to build mathematics teacher collaborations.

Theoretical Approach: Situative Teacher Learning and Collective Concept Development

To study mathematics teachers' collaborative learning in general, we draw on situative theories of learning (Putnam & Borko, 2000; Greeno, 1998). Unlike traditional cognitive theories that look at changes in *individuals*, situative theories look at *individuals-in-contexts.* That is, we attend not only to individual teachers but also to the social world of their teacher communities. In particular, we have taken up situative perspectives to investigate *collective concept development* (Horn & Kane, 2015), the way teachers develop understandings about core ideas about mathematics instruction, ambitious and otherwise, such as *assessment, lesson launches, proportional reasoning*, or *struggling students* in the complex context of schools. Over time, as teachers share dilemmas, exemplars of practice, language for teaching, activities, or critical classroom incidents, they develop taken-as-shared meanings (Horn & Kane, 2019), informing and guiding individual and collective pedagogical decisions.

Interactionist perspectives on mathematics teachers' collaborative learning

Opportunities to Learn. To investigate mathematics teachers' collaborative learning, we delve into the details of their conversations. As groups' collective concepts change, we take this as evidence of learning. Of course, not all teacher groups are equal in their support of concept development. To understand variations in learning opportunities across teacher groups, we look for how conversations (a) provide teachers with conceptual resources for understanding instruction, and (b) mobilize teachers for future work (Hall & Horn, 2012; Horn & Kane, 2015). For instance, many teacher groups never delve into the meanings of ideas like *quiz* or *low student* — conversations that would aid with concept development. Instead, most teacher groups we have observed focus on pressing logistical issues of planning, developing materials, and responding to administrative demands — conversations that focus on mobilization for future work.

Epistemic stances. Even when teacher groups have deeper conversations about their shared understandings, not all "concepts" are equally aligned with visions of ambitious instruction put forth in documents like the U.S. National Council of Teachers of Mathematics' *Principles and Standards* (2000). To look at the quality of the ideas under discussion, we attend to teachers' *epistemic stances* on core issues of teaching, learning, and mathematics. These are perspectives on what can be known, how to know it, and why it is of value. Analytically, teachers' conversations reveal epistemic stances on different timescales. Most obviously, bald declarations in single turns of talk manifest these stances (e.g., "what matters here is getting the kids to stay motivated"), what we call *epistemic claims*. Sometimes, a question provides an interpretive frame that implies an epistemic stance (Bannister, 2015; Horn & Kane, 2015). For instance, the question, "The kids aren't getting it. Should we do some review on homework or warm ups?" implies a stance that *reviewing a topic through more problems helps student understanding*.

Over longer time periods, epistemic stances surface throughout workgroups' activities, as interactional emphases reveal commitments to what can be known and how to know it, while providing teachers with substantially different interpretative resources. In our framework, different epistemic claims signal more traditional or ambitious ways of thinking about teaching. For instance, we would locate the epistemic stance underlying the question about reviewing within a traditional teaching epistemology, since the emphasis is on the teachers' actions and not on analyzing and cultivating students' understanding.

Methodological Implications:
Analyzing Conversational Records for Evidence of Learning

Our situative lens on mathematics teachers' collaborative learning invites an ethnomethodological approach to research (Goffman, 1959), foregrounding participants' sensemaking. To understand how conversations contribute to that sensemaking, we also draw on sociolinguistics (Hymes, 1994), which emphasizes social and cultural aspects of communication. Our participant-centered perspective works with a variety of research designs: we listen to and analyze teachers' conversations from a variety of participant observer positions. At times, we have skewed largely on the observer end (e.g., Horn et al., 2017). At other times, we have studied teachers' talk in response to an intervention (e.g., Bannister, 2015) or where we ourselves actively participated in the conversations *as* an intervention (e.g., Horn, in press). Across these studies, our data center rich records of teachers' talk, through various combinations of detailed fieldnotes and audio or video recordings, which we supplement with artifacts of practice, classroom observations, and interviews with teacher participants. These studies are all grounded in initial ethnographic work that allows us to see the local meaning systems at work in any collaborative group (Jordan & Henderson, 1995).

Using an interactionist perspective, we prioritize the details of teachers' talk in our analysis of learning. An important part of the analytic work includes looking closely at what teachers say as they discuss instruction to make claims about their understandings. When additional conversational details are available, we use them to develop richer accounts of these understandings-in-context. For example, we can look at *pacing* to understand issues of deliberateness, press, and intensity; *voice intonation* to gain insight into emotionality; *overlapping talk* to indicate conflict, excitement, or involvement; *seating arrangements* and *body positioning* to understand social alignments; and so on. These additional details help uncover the social side of the knowledge production in teacher collaborative groups: who people are matters for how their ideas get taken up and how they take up the ideas of others. A respected department chair's feedback may carry more weight for a novice teacher than another colleague's (Horn, 2007), and such details make this available for analysis.

Parsing conversational records to track concept development

To look for teachers' opportunities to learn in conversational records, Horn (2005) developed a unit of analysis called *episodes of pedagogical reasoning* (EPRs), the segments of talk where teachers describe issues or raise questions about teaching practice, accompanied by elaborated reasons, explanations, or justifications. Sometimes, these are single turns of talk ("I am doing a notice-and-wonder for this because my kids *love* it!"). More interestingly, they can be multiparty co-constructions. When these center a core idea about teaching and learning mathematics, these are rich sites of collective concept development, particularly when teachers delve into multiple explanations for something that already has occurred (diagnostic exploration) or consider alternative approaches to something they are planning (prognostic explanation). Thematically related EPRs can be examined over time across meetings to understand a group's concept development (e.g., Bannister, 2015).

Seeing teachers mobilize for future work

Opportunities to learn require not only concept development but mobilization for future work. As noted, most mathematics teacher meetings focus solely on mobilizing for future work, such as setting up pacing calendars, developing common assessments, or planning lessons. Thick concept development, as we have described, happen less often, but it is a hallmark of teacher communities who work together to teach ambitiously (e.g., Horn & Kane, 2015; Nasir et al., 2014). In fact, when teachers connect the concepts they develop to future work, those are the richest learning environments (Horn et al., 2017), as they link teachers' actions to a shared analysis of key ideas.

Key Findings about Mathematics Teachers' Collaborative Learning

Our combined research produced hundreds of hours of observation and analysis of mathematics teacher groups. In what follows, we summarize key findings from our larger body of scholarship.

First, conversations that sustain teachers' attention to pedagogical reasoning and, in turn, support concept development are relatively rare (Horn et al., 2017). Instead, pressing logistical and administrative concerns typically take precedent when teachers gather. An implication of this is that simply putting mathematics teachers in groups does not guarantee the kinds of learning policymakers and administrators hope to find (Bannister, 2018). Second, even in conversations richer with learning opportunities, teachers' warrants for decisions frequently make moral appeals rather than empirical ones (Hall & Horn, 2012). That is, teachers justify decisions based on "helping students understand" or "aligning with the standards"—both pointing to their responsibility as teachers—rather than a sense of evidence-based practice. We note this observation not out of a sense of judgment but instead because calls for "evidence-based practice" (e.g., Davies, 1999) do not align much with teacher sensemaking. Because moral appeals to a shared sense of professional responsibility anchor collective concept development, collaboration works better among teachers who share a vision of teaching.

Third, in richer collaborative conversations, teachers interweave talk about teaching with specific details of practice (Horn & Little, 2010; Horn, 2010). These representations include reports of classroom events (*replays;* Horn, 2005), anticipated events (*rehearsals;* Horn, ibid.), artifacts like student work, or instructional materials. From a learning perspective, we know that, for collective sensemaking, having a common object can support learners' intersubjectivity (Vygotsky, 1936/1984), a sense of shared meaning. Representations are especially crucial to teachers learning about their practice, since, unlike other professions, discussions about the work are almost always asynchronous from instruction itself. As concepts are being developed, teachers' conversations move between the specificity of the representation and their own epistemic claims about what matters, what is knowable, and what is doable (Horn, Kane & Wilson, 2015), making adequate representations crucial for supporting teachers' collaborative learning.

Fourth, as teachers' collaborative conversations align more with ambitious mathematics teaching, their collective framings and habits of mind about underperforming students shift to become more actionable. For instance, their framings about struggling students might move from stereotypical "kid blaming" and generic advice-giving to specific accounts of obstacles to student success, coupled with localized descriptions of "evidence-based" instructional interventions (Bannister, 2015). These frame shifts signal teachers' increasingly productive uptake of core ideas in ambitious teaching. In addition to shifting to more agentic frames, teachers' frames also become more complex as they consider problems of practice ecologically; that is, they do not just frame problems as relating to just *mathematics* or *students* or *instruction* but instead think about the interconnections among these facets (Horn & Kane, 2015). From a data analysis perspective, such framing shifts provide empirical evidence of within-group learning about ambitious teaching in a mathematics teacher community over time (Bannister, 2015, 2018), which underscores the important role that frames play in documenting, analyzing, and making space for teacher learning.

To illustrate how our interactionist approach surfaces moments of teacher learning, we share the following excerpt. It comes from a meeting of middle school mathematics teachers who were looking at student work together to diagnose what it revealed about their students' understanding of *unit rate*. Deanna, a teacher, and Lindsay, an instructional coach, were looking with their colleagues at Tommy's work. He made a place value error, writing \$3.50 instead of \$35 as his solution. After considering the source of his error (e.g., a possible calculator error), they had the following exchange, developing the concept of what it means for a *student to have the concept of unit rate.*

Deanna:	I mean, I think for the most part, he has the *concept*, but
Lindsay:	Well, but it sounds like he has the concept of, or he has the *procedure* for how to find—
Deanna:	—the *procedure*
Lindsay:	—unit rate.
Deanna:	Yeah
Lindsay:	I'm not sure.

As Deanna negotiates the distinction between *concept* and *procedure* with Lindsay, she has the opportunity to reconsider this distinction and its implications for addressing Tommy's (and others') ideas in her lessons. Over time, we see these distinctions get taken up by teacher groups, with specific examples anchoring their shared understandings as they make sense of their practice.

Together, our key findings point to design considerations for interventions that seek to leverage the potential of mathematics teacher collaboration, particularly when instructional improvement is a goal. Designers need to (1) consider that deeper collaboration is relatively novel for teachers; (2) work deliberately to develop a shared vision for teaching; (3) provide adequate representations to support teachers' intersubjectivity about core instructional ideas; and (4) attend to frames as a site for reconceptualization of key ideas about teaching.

Designing Interventions for Productive Collaboration

Building off of the previous section, we now illustrate two interventions that sought to leverage the potential of mathematics teacher collaboration in support of teacher learning.

Example 1: Designing Video Based Coaching as Co-Inquiry into Practice

In Horn's current research, *Project SIGMa*, she has used principles about teacher collaborative inquiry to design a video formative feedback (VFF) activity, developed in partnership between her research team and a professional development organization, Math for America Los Angeles (M*f*A LA) that works with experienced secondary mathematics teachers in Los Angeles to support the development of ambitious mathematics instruction. The VFF sought to fill a gap in the teachers' professional development experiences — namely, that they wanted more feedback on their actual instruction.

Through a few development and refinement cycles, the VFF process unfolded as follows:

(1) *Setting up the inquiry:* A team coach talks to the focal teacher about what they are working on in their teaching and how we can look for evidence of how they are doing in their classroom.

(2) *Documenting the class session:* Researchers use two cameras to document the lesson from the focal teacher's point-of-view and the whole class.

(3) *Research team review of video records:* The research team reviews the video, looking for interactions that support the teacher's inquiry.

(4) *Teacher, partner teacher(s), and researcher collaboratively debrief video clips*: During the same week as the recorded lesson, the partners meet to discuss moments as a site for collective inquiry into their practice. These conversations are the target for teacher learning.

Considering the novelty of deep collaboration. As a research team, we outlined the VFF process to prospective collaborating teachers. M*f*A LA teachers work in school-based teams, so we sought teams who were interested in working with us. This "go with the goers" approach has its trade-offs. On the one hand, teachers were willing participants, not coerced into participation. On the other hand, teachers who might need the most support were able to opt out. Because of the novelty of the process, we hoped that, as teachers made positive reports to their colleagues about the intervention, a diffusion of innovation would happen, prompting more risk-averse teachers to join in.

Working with a shared vision of teaching. By recruiting teachers from M*f*A LA, we could make some assumptions about the teachers' professional commitments to ambitious mathematics instruction. In addition, setting up the VFFs around teachers' questions (step 1) ensured that our work was relevant to their current concerns. As our work with them unfolded over time, some of the most powerful learning came from teachers working to align their professed commitments with what was captured in the video records. For example, Julie, a teacher committed to teaching mathematics for understanding, had to reconcile this commitment with the pace of her lessons that rushed students through some activities.

Providing adequate representations of practice. The rich video record gave teachers unprecedented access to details of their instruction. To continue the example of Julie, she saw not only what students did while she was present during their groupwork time, but also what they did after she walked away. In the project's exit interview, she explained how the debrief discussions shifted her understanding of her practice. "[Now] when I come up to a group, just really try to ask them what they were thinking [...] whereas before I might have assumed I *knew* what they were thinking." Julie's slowing of her instructional pace to listen closely to students' thinking aligned with her goal of teaching mathematics for understanding.

Attending to teachers' framing as a site for reconceptualization. As facilitators, our team members attended to teachers' framing. Initially, this meant negotiating their inquiry questions developed in step 1. For instance, if a teacher wanted to know how "low" students were engaging during the lesson, team members would ask them to elaborate on what they meant by "low." During the debrief conversations, we pressed teachers to connect epistemic claims about teaching, students, and mathematics to each other to support the ecological reasoning we identified with teachers who successfully use ambitious instruction (Buenrostro, Garner, Marshall & Horn, in preparation).

Example 2: Designing Video Based Working Groups as Responsive Professional Development

In Bannister's current research, she has used principles about video-based collaborative teacher learning to design a responsive professional development model (RPD), which was developed by her research team in partnership with a mathematics teacher community comprised of middle and high school teachers from four geographically proximal rural school districts. Following the success of a multi-year professional development model that involved monthly PD meetings during the school year and weeklong summer workshops that were organized around a content-based theme, the RPD model was designed to respond to participating teachers' advancing learning needs in grade-level collaborative contexts. While we kept structural aspects of the whole-group program that worked well for the group (e.g., monthly PD meetings, weeklong summer institutes), the RPD model supported small groups of teachers to use video clips of their instruction to collectively examine and improve their teaching with increasing autonomy.

After facilitating the formation of small groups based on grade-level, content, and individual learning goals and interests, the process that we used for the RPD model is as follows:

(1) *Organizing for inquiry*: Toward the end of each monthly PD session, each group chose a "spotlight teacher" for the next session, who shares what they are working on and how it connects to student experience. The group helps the spotlight teacher choose a target mathematical practice that they would all try prior to the next session. While each group selects their own spotlight teachers, the facilitators established that every person should be the spotlight teacher at least once and encouraged all to take a similar number of turns as spotlight teacher (i.e., about 3 turns).

(2) *Documenting practice*: Given the large geographic spread that separates researchers, teachers, and schools, each teacher was given a camera kit (e.g., digital video camera, tripod, high capacity SD card, extension cord) to capture their own lessons. The spotlight teacher set up a camera in their

classroom and recorded the lesson where they tried the target mathematical practice. They then reviewed the video and selected a brief clip (<5 minutes) to show to their group. In addition, they prepared for the meeting by reflecting on the target practice and the potential impact on student experience. Their group members also made brief notes about the attempted practice and potential impact on students prior to the next sessions, although they are not required to film their lessons.

(3) *Spotlight on practice*: After a brief small group check-in, the spotlight teacher told the group about the lesson, explaining the learning goal(s) and the attempted target practice. The spotlight teacher shared the video clip with the group, along with student work samples if helpful. Teachers took turns sharing their observations about student thinking, with the spotlight teacher sharing last.

(4) *Group goal setting*: The group concluded their conversation by identifying common student contributions and bottlenecks across their lessons, and conjecturing ways to use mathematical teaching practices to respond to them. Using these ideas, they thought about mathematical teaching practices their students needed them to work on and any challenges they have experienced with this practice previously. This conversation naturally fed into the selection of the next spotlight teacher and target practice, which ignites the next RPD cycle.

Considering the novelty of deep collaboration. Our research team reviewed the RPD process with prospective participants from our partner school districts. While participation was incentivized through camera kits and stipends, it remained optional. Despite the long distances that some teachers had to drive to attend the monthly meetings (45-60 minutes), participation rates were consistently high following implementation of the RPD model (~ 20 teachers). Moreover, while the mere thought of filming their classrooms brought some participants to tears, all participants filmed a lesson and shared a video clip of their instruction with their group, including teachers who had attended multiple years of prior PD and never expressed readiness for recording. The cultivated safety of their small working group supported teachers to risk deprivatizing their practices and speaking frankly with trusted colleagues about their classrooms.

Working with a shared vision of teaching. Our team opened participation to any mathematics teacher from our partner rural districts in the spirit of meeting teachers where they are. Constraints related to driving distance, outside commitments, and grant funding resulted in consistent participation for about 20-25 teachers. While every teacher demonstrated eagerness to participate fully, we did not observe a shared vision of teaching relative to ambitious instruction across all participants. That being said, teachers became increasingly comfortable speaking frankly about what was going on in their classrooms and accepting where they were in the process over time. For example, one self-described "old school" veteran teacher continued collaborative inquiry and publicly experimented with her engagement with student thinking until her retirement a year later. Given the range of starting places within our community, the small group context gave teachers significantly more resources and opportunities to grow toward a shared vision of teaching.

Providing adequate representations of practice. We organized the RPD model around activities that were likely to support teachers to make sense of the target practices with actual student experiences, with the emphasis on video artifacts and student work as representations of teaching. The protocols we used made the "before and after" public to the spotlight teacher's working group, which provided them with a supportive accountability structures when learning to do this work.

Conclusion

We presented key findings about our approach to understanding the possibilities of mathematics teacher learning through collaboration, offering them as design principles for other efforts to support mathematics teachers' collaborative learning (Collins et al., 2004). Our extended examples applying these ideas offer a proof of concept, showing how two different interventions take up these findings to design for teacher learning. As we mine the teacher learning in these interventions, we hope to refine our understanding of how mathematics teachers learn in collaboration with their colleagues.

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IDENTIFICATION PROCESSES IN TEACHER COLLABORATION DURING PROFESSIONAL DEVELOPMENT: DOCUMENTARY GENESIS IN A CONTEXT OF SCHOOL MATHEMATICS CURRICULUM REFORM

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It is reported here a case study on the professional development of a group of 14 in-service middle school math teachers, who worked collaboratively on the design of new lessons, in the context of the Mexican middle school mathematics curriculum reform that focused on the development of competencies. In particular, tasks and context that made to successfully enact teacher collaboration are described. It is important to note the blending of intellectual perspectives utilized, in particular, the model of professional growth by Clark & Hollingsworth (2002) was useful to understand the role of the context of mathematics school curriculum reform as an important part of the change environment, and the documentary genesis approach by Trouche et al (2010) alone with the concept of personal philosophies or images of mathematics from Ernest (2007, 2012) were the analytical tools that finally allowed for explaining teacher grow achieved during the two phases of the professional development case presented here, mainly through the analysis of teacher productions.

This paper is based on data and results that are part of a larger longitudinal study that sought to build a Mexican model for the professional development of in-service middle school math teachers. It accounts of information on the general characteristics of the official professional development programs –implemented by the Ministry of Education in Mexico at the epoch. The empirical work in this study started by identifying teachers' personal philosophies or images of mathematics (Ernest, 2007, 2012), in the particular case of the in-service math teachers who participated in the study. Therefore, the authors' hypothesis from this case is that the professional development model that was finally obtained (and that will be shown in next pages) serves to work on teacher professional development based on identification processes that underlie teacher collaboration on the design of new lessons.

Antecedents

Different points of attention have been stressed in the study of the professional development of in-service teachers. For example, according to Azcarate (1998), professional problems of teachers mostly turn around the design and/or materialization of the curricular proposals. On another hand, the quality of teacher practice has been also a persistent problem, as noted by Hiebert et al. (2003), who suggested that progress can be made by designing programs that could influence the nature and quality of this practice. In addition, Hiebert and colleagues noted that teacher training programs have an expiration period, which makes necessary to continuously review their design and implementation. And, according to Marcelo (2002), initial training provides the teacher with baggage of knowledge that must be complemented throughout teacher active professional life.

All of these combined with educational contexts marked by the implementation of large-scale school mathematics curriculum reforms makes imperative to offer professional development programs for in-service teachers (Montecinos, 2003). In this regard, Mexico is particularly not an exception given a constant series of reforms to the school mathematics curriculum (SEP, 1993; 2004; 2006; 2011; 2018). The main objective in the case study that is presented here was to build a cyclic model of professional development for in-service teachers focusing on the learning of new pedagogical practices (those induced by the introduction of new contents in the curriculum reform) and based on their reflection on what constitutes their profession and their practice into the classroom. Finally, it is also important to point out the need, emphasized by several authors as Peñas & Flores (2008), Cobb (2005), and Llinares (2005), to approach or to establish links between the theoretical knowledge on teaching and teacher's practices into the classroom.

General Characteristics of the Official Teacher Development Programs in Mexico

In the successive changes that have been implemented through the different school mathematics curriculum reforms in Mexico, teachers have been considered always as protagonists of the educational transformation (SEP., 1993; 2004; 2006; 2011). The central features of the 2011 integral curriculum reform, which concentrate the guidelines and developments of the 2006 reform, in relation to the professional development of teachers, are based on the recognition that reflection and educational practice in the school are key to strengthening the continuous training of teachers and the additional academic staff, and to promote collaborative school management processes. In particular, the different approaches of curriculum reforms have appealed to the commitment and professional development of teachers to consolidate them (SEP., 1993; 2004; 2006; 2011). In this way, teacher professional development has been considered a fundamental axis in the reform process that has been carried out in Mexican middle schools, since it has been highlighted as the possibility of generating substantive transformations in pedagogical practices.

This fact has even been fully recognized by the Ministry of Public Education, underlining that: promoting professional development is the best tool to improve teacher performance in front of the group (Ortega et al., 2005). But, unfortunately, all of these considerations, recognitions, calls, and underlining of promoting the professional development of mathematics teachers have mainly resulted to be only rhetorical.

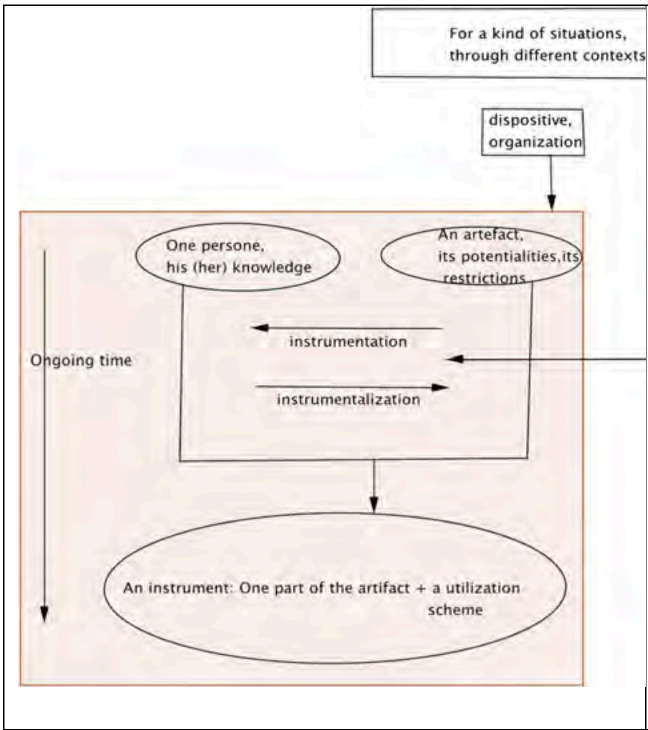
For example, according to Sandoval (2001), one of the characteristics of the official programs of professional development for in-service teachers in Mexico, has been the scarce teacher’s participation. These programs of professional development, according to Martínez (2005), end up being framed in a course, which only acquires meaning for its recipients if it awards points for a promotion on the official teaching career. On the other hand, the additional academic figures, whose formal function is to guide teachers in their work, in reality scarcely attend schools and when they do it, their work is carried out in a purely administrative format. That is why they do not constitute an important reference in teaching practice and even less they constitute educational support within the framework of teacher professional development (Sandoval, 2001). In this context, the professional development of in-service math teachers, in reality, has become a model where trainer’s activities during the course focus on carefully developing their own new materials and the organization of the new courses to disseminate the proposals of the reforms. It is to say that the training model of the official courses for teacher professional development is focused on the trainer and based on the implementation of homogeneous courses.

Finally, another important disrupter in this scenario, highlighted by León (2005), has been the meritocracy, constituted by the graduates from a good number of graduate programs, which with honorable and few exceptions, turn out to be proposals for improper training, decontextualized and whose operativeness has almost never been evaluated.

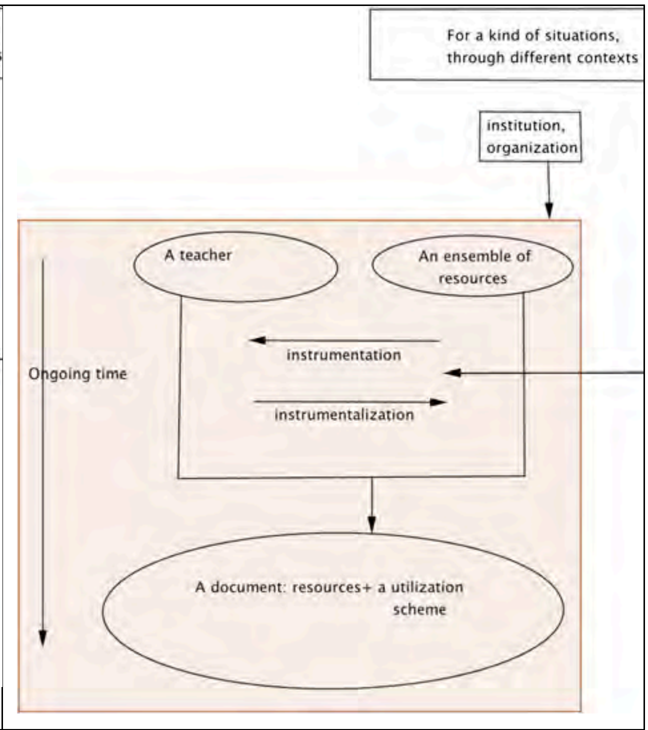
Theoretical Underpinnings

Teachers' Collaborative Work During Professional Development Programs

Based on the documentary genesis by mathematics teachers and in their construction of collaborative design using digital resources, Gueudet and Trouche (2009) have extended the concept of instrumental genesis proposed by Verillon and Rabardel (1995) to the one of documentary genesis. In this respect, it is illustrative the following schematic representation of both concepts (see Hoyos 2012). Therein, one can notice that the concept of document connects teacher practice with their images of mathematics (Ernest, 2012), really through managing the resources they have at hand, as mathematics curriculum prescriptions, textbooks, digital technology, etc. But, "whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (Thom, 1973, p.204. Cited in Ernest, 2012, p.9).



Schematic representation of *instrumental genesis* (Trouche 2010. Cited in Hoyos 2012).



Schematic representation of *documentary genesis* (Trouche 2010. Cited in Hoyos 2012).

In words of Gueudet & Trouche (2009), and Sabra (2010), the documentary approach provided tools for the analytical study of the processes that underlie the professional development of math teachers, both individually and collectively.

Personal Philosophies or Images of Mathematics

Ernest (1994) argued that differences in mathematics teachers' practices cannot be explained sufficiently attending only to mathematics knowledge. Such differences may be attributable to particular belief systems about what is mathematics and on its teaching and learning, which in particular constitutes the rudiments of certain personal philosophies or images of mathematics that teachers maintain, although often these personal philosophies are non-articulated in a coherent manner. Personal philosophies of mathematics provide a general epistemological and ethical framework, under which the conceptions about the teaching and learning of mathematics are considered, and they are subjected to limitations and opportunities of social context. Ernest (2012) provides a detailed characterization of the concept of personal philosophy or image of mathematics: "Concept images represent a deep level of meaning, partly implicit, and may influence their holder's disposition and actions. Similarly, images of mathematics can include a wide range of representations and associations from sources including philosophy and accounts of the nature of mathematics, but also including representations from the media, classroom presentations and parent, peer and other narratives about mathematics. Personal images of mathematics can utilize mental pictures, including visual, verbal, and narrative representations, originated from past experiences, social talk, etc., and include cognitive, affective and behavioural dimensions, including beliefs". (Ernest, 2012, p. 9)

Identification Processes

The work of Cerulo (1997) provides an antithesis to traditional identity studies, and mainly the works cited there refocus scholarly attentions from the individual to the collective. According to this author, many of her reviewed studies have approached identity as a source of mobilization rather than a product of it, and particularly in relation to identification processes, attention to collectives (p.394) has re-energized scholarly interest in the identification process itself. In this way, "a growing literature explores the mechanics by which collectives create distinctions, establish hierarchies, and negotiate rules of inclusion" (Cerulo 1997, p. 394). In Cerulo's work there is a section where one can find a variety of identification processes currently under study. (p.395). Related with the case study we are presenting here, this author highlights that the study of objects also proves key to recent research on identification. For example, Cerulo claims that several works noted the ways in which individuals and groups used [art] objects (Martorella 1989. Cited in Cerulo 1997), specifically to articulate and project identities.

Emergence of Teacher New Collaborative and Pedagogical Practices from Teacher Professional Development

Although teacher collaboration wasn't considered during the activity developed by participant teachers in the professional development program implemented by Hoyos et alt. (see Hoyos 2012, 2016), this work was an important antecedent for the case study presented here, because it highlighted the potential of starting teacher activity during professional development from materializing the knowledge of the teacher about teaching, to move towards another level in the development of new collaborative and pedagogical practices.

Methodology for Construction and Obtention of Data

The case study that we are presenting here was developed in two phases. The first, was exploratory and constructive, and the second, comparative and inferential. First phase was specifically developed through enacting teacher identification processes to know of collective teacher images of mathematics and about their teaching (see below the specification of the implemented tasks). The second phase turned around teacher design of new lessons or activities (to be implemented in classroom) on new curricular contents included in the 2006 school mathematics curriculum reform, namely the generalization of patterns, for the learning of school algebra. The participants in this research were 21 in- service math teachers belonging to public middle schools in Mexico City. All of them were experienced teachers in math middle school. Teacher participation in this case study took place in a framework of meetings carried out face-to-face, in an official center for teacher professional development or specific workshops, during the months of October 2007 and November 2008 (in the first phase of the study), and from January to June 2009 (during the second phase of the study), all of these was set up in the context of the review of the 2006 middle school mathematics curriculum reform. Teachers were always grouped in teams of three, or four participants, and the general objective of the meetings was to analyze the new 2006 official teaching approach for the development of the mathematical contents in the classroom.

The first general task. For the start of professional development activities there was a general task. It consisted in analyzing all the new mathematics contents proposed in the reform alone with the fundamental purpose of highlighting the basic contents for each middle school grade[i], specifically those that could be taught using the half of the time normally assigned for teaching them in regular classes. In our opinion, this particular task provided the framework to explore teacher production generated from two types of teacher resolution: on one hand, there were resolutions that probed to have a baggage of basic mathematical knowledge for teaching in middle school; and, on the other hand, there were resolutions that allowed to explore on teacher knowledge by teacher materialization of particular teaching for specific contents in the classroom. Specially these second type of answers constituted the suitable space for the design and testing of the type of tasks for professional development that corresponds to the core of this research, i.e. the co-construction of a cyclic model for the professional development of in-service middle school mathematics teachers.

At the start of the investigation, we worked with the productions of the 21 teachers, however once the attention was focused (for the second phase) on the approach for teaching the contents of generalization of patterns, only rested 14 participant teachers, those that particularly expressed their urgency to address this topic.

Task 1. As it was said before, the data in this case study were obtained from teacher collaborative teams during collective meetings, and teacher productions were elaborated during team collaborative resolution of the required professional tasks. Some of the principal teacher productions consisted in the elaboration of a schematic drawing or diagram that specifically showed their personal philosophies or images of mathematics, those that they used to implement during their practice.

> *Task 1. "Setup in a diagram or schematic drawing the important pedagogical elements you display to address mathematical issues with pupils".*

Some of the complete teacher's collaboratively produced diagrams are showed next. They were here titled as Diagram 1, Diagram 2. Each of these diagrams were reached by teacher negociation within the team to draw a single schema. In fact, each diagram reflected a teacher team product, as a result of teacher collaboration and negociation within respective teams. Finally, is important to note that after each team had finalized their drawing, it was implemented a discussion on the meaning of their production. These discussions were managed and registered (by taking notes) by the second author of this paper (R. Garza), who really played the role of teacher educator, or more precisely, as a teacher team tutor or person in charge of whole implementation of teacher activities during this investigation.

Analysis and Discussion of Results

On Teacher Productions of Diagrams 1 & 2

Absolutist philosophies, subject to constraints and opportunities afforded by social context. The first interpretation extracted from Diagram1, resides on the existence of teacher images of mathematics that are subject to constraint and opportunities afforded by social context (Ernest 2012), impositions that forced teachers to propose strategic conformity. In other words, it shows teaching from an absolutist teacher position, it alludes to consider the teacher as the empowered element in classroom, providing an explanation to the student. Through this teacher production, teaching was characterized according to the assumption that learning is a process directed mainly from the teacher to the student, centering the action on the teacher, and where students adhere to explanations and validations directed by the teacher. The problem posing statement expressed by the teacher that is showed in the diagram, and the students oral communications talking about different solution strategies, are clearly perceived as the communication of doubts that in this case should be clarified by the teacher.

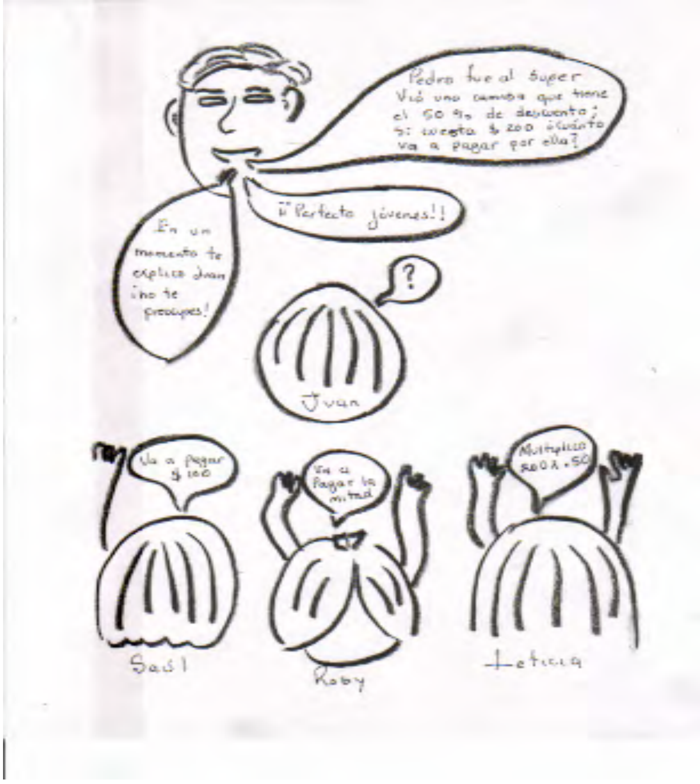


Diagram 1

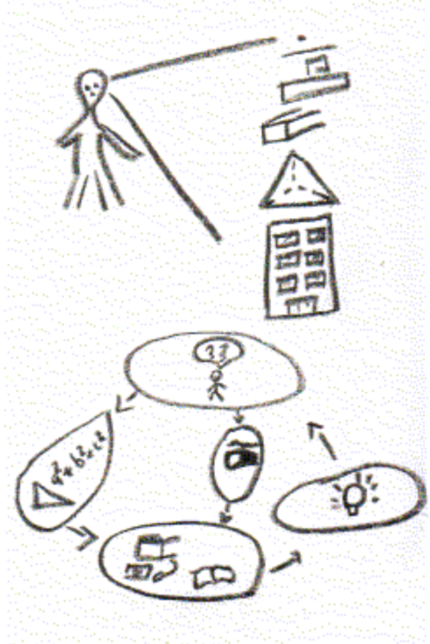


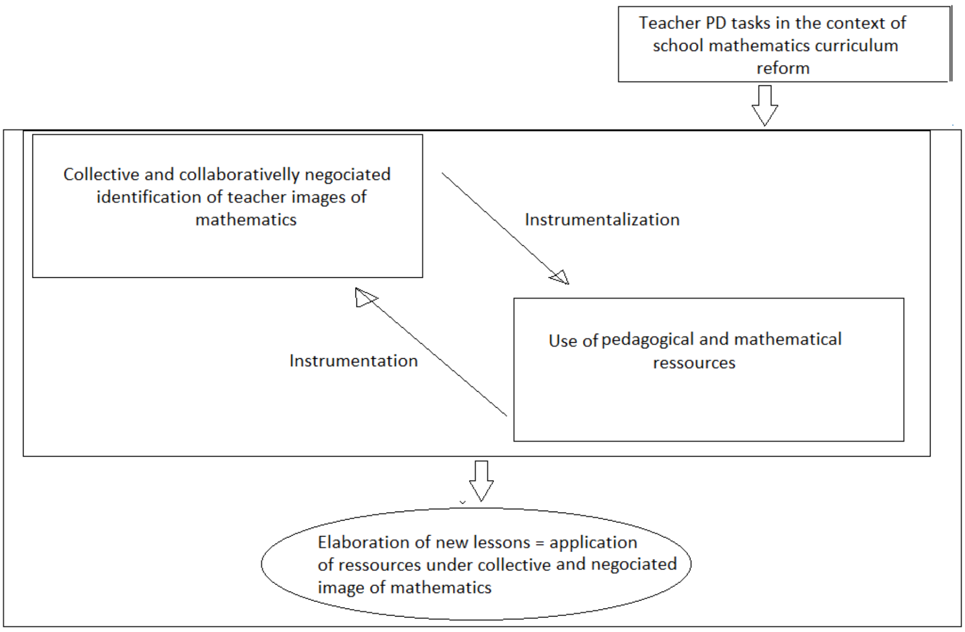
Diagram 2

For this team, the learning of mathematics has been associated, according to the corresponding teachers, to <<... a problem for solving that counted with the presence of different paths of solution ...>>[ii]. In fact, the teachers have linked their diagram to social and symbolical forms, and to the social context of the students. Although the diagram could appertain to a fallibilist perspective: <<Pedro went to the supermarket and saw a shirt that had a 50% discount. If it costs $ 35 USD, how much did he should to pay for it?>>", because it has been projected an image of mathematics that has a character, intuitive, active, collaborative and creative, the last teacher clarification on their meaning of the diagram has to do with absolutist issues of transmission and validation of a path that is completely directed by the teacher: <<... the teacher has to intervene to explain, the students who did not understand are the object of our practice at that time ...>>.

It is important to highlight that the collective identification process accomplished here, evidenced by teacher production, integrated different levels of elaboration, abstraction and generality, as well as different forms of representation, those sustained and negociated by all teachers in the team. However, in the particular team case that produced the Diagram 1, it could be observed a preeminence of the path of absolutism, where the teacher is who lead the class, posing problems to solve or situations to explore, and the students are trying to express conjectures that await for teacher validation. Moreover, in the second phase of this study it was important too to note how participant teacher teams used of pedagogical and mathematical ressources to collectively create or design new lessons, in order to articulate and project their teacher identities (Cerulo 1997).

But in order to adjust the extension of this paper to the writing limit of 8 pages, the analysis and discusion of the other teacher's productions will be given in additional pages that will be distributed at the time of the conference, in Portugal. But we can't leave without at least showing a diagram of the model of the cyclic professional development, the main result of this case study.

Cyclic Model for the Professional Development of In-Service Mathematics Teachers



Cyclic model for the professional development of in-service mathematics teacher

From the diagram for the cyclic model of professional development of in-service mathematics teacher, based on the empirical data of this investigation, it could be seen that it echoes the schematic representation of documentary genesis by Trouche (2010) –see the section of theoretical underpinnings in this paper. The place of a document –or the result from a documentary genesis in Trouche's schematic representation, here (in the schematic ciclyc model for teacher PD) is occupied by collective teacher design or elaboration of new lessons. A detailed analysis and discussion on the other diagrams and new lessons produced by participant teachers, could be given during the ICMI Study 25 conference, and/or in a next extended version of this paper.

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[i] Middle school in Mexico is constituted by three years that correspond, more or less, to 7th, 8th, and 9th grades in the USA.

[ii] It is important to note that to correctly follow the analysis of teacher productions, the protocol descriptions of the subsequent teacher discussions were made using the graphic signs << …>>, to quote teacher statements, those obtained and registered by the second author of this paper, after teacher had finished their production.

TEACHERS' COLLABORATION IN MIXED GROUPS FOR LEARNING MATHEMATICAL INQUIRY: A THEORETICAL PERSPECTIVE

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Drawing on the Anthropological theory of the didactic, we propose a condition for successful collaboration between elementary and secondary teachers in solving mathematical problems. Using excerpts from on-line teachers' teamwork within a graduate course, we observe teachers' progress through lenses of various dialectics (idionomy and synnomy, conjecture and proof, black boxes and clear boxes). These tools allow us to examine various aspects of teachers' collaborative learning through the construction of mutually accepted solutions to given problems.

Introduction

Problem solving is among the processes emphasized by contemporary mathematics curricula (see e.g. NCTM (2000)). More generally, the inquiry-based pedagogy in which "students are invited to work in ways similar to how mathematicians and scientists work" (Artigue & Blomhoj, 2013) is elaborated in several educational frameworks. However "the question of how to support practicing teachers in implementing the reforms [the theory] envisions looms ever larger" (Farmer, Gerretson & Lassak, 2003). As suggested in Ball and Cohen (1999), teachers' professional development and education programs should engage students in practices they wish to encourage. Thus, in order to improve the state of affairs with inquiry properly conducted in the classrooms, teachers should themselves be situated in problem solving settings and explore mathematical teaching and learning through reflecting on their own mathematical experience of producing solutions at the grade levels appropriate for their teaching. As well, placing teachers in working groups should provide them with ideas of how to organize and support similar setting with their students. Examples of such practices in the case of elementary teachers are given in Farmer (Farmer et al, 2003).

Several studies (Olsen & Kirtley, 2005; Cooper & Karsenty, 2018; Kondratieva, 2013a) report that interaction between participants possessing different expertise and perspectives on mathematics, such as elementary teachers, secondary school teachers, and mathematicians could be enriching and sometimes critical for gaining new insights for all participants. In this paper we consider a case of elementary and secondary teachers' teamwork within a teacher education graduate on-line course that allowed the participants to advance their experience in mathematical problem solving. Our goal here is to provide a theoretical approach helpful to conceive a mechanism and nuances of teachers' collaboration in mixed groups and examine why such teamwork could be problematic or successful.

Theoretical framework

We draw our theoretical framework from the Anthropological Theory of the Didactic (ATD), in particular the notions of praxeology, institutionalization and several dialectics associated with the processes of inquiry (Bosch, Chevallard, García & Monaghan, 2019). Here, we briefly describe some notions and ideas from the Glossary section of the above-mentioned book. Defining *inquiry* as the action taken to provide an answer to a question, ATD views the practice of inquiry as a source of possible upcoming critical changes in the traditional curriculum. The changes comprise the emergence of a new study paradigm of *questioning the world*, where the curriculum is formulated in terms of questions rather than in terms of existing works (i.e. intentional products of human activity) that students need to study. The study of existing works in the form of finalized answers is the main focus of current paradigm of *visiting works*. The problem is that often students are not sufficiently motivated to study these works and their learning then acquires a formal character, while resulting knowledge becomes neither deep nor long lasting.

ATD views the works in terms of *praxeologies* that consist of *praxis* and *logos* blocks. Further, the praxis block consists of types of *tasks* and *techniques* to solve them, while logos consists of *rationales* explaining and justifying the techniques used in praxis and *theories* within which these rationales exist. Praxeoloxy is a basic notion to describe human acts in particular situations. For example, praxeologies may be identified in solving mathematical problems, in teaching classes, and in many other human performances of given tasks. Historically, praxeologies have been developed in human activities and resulted from the questions that people posed to deal with real or hypothetical situations occurring in their lives. However this is not the case in the current educational system: the links between works to be studied and the reasons for their existence are often unclear not only for students but also for their teachers. The new study paradigm of questioning the world addresses this issue and envisions that works to be studied by groups and individuals should be those that are needed in order to answer certain questions (known as generating questions of study).

ATD proposes that praxeologies undergo a transposition as they occur in different social *institutions* such as research communities, practice communities, or learning communities e.g. schools. Within different institutions different conditions are present that support or restrict individuals in the process of familiarization with objects of their study. *Constraints* are defined as conditions that are beyond control of the individuals subjected to them. *Dialectics,* being a base for discourse*,* is viewed by ATD as a praxeology that "enables one to overcome two opposed types of *constraints* by turning them into a new kind of *conditions* that supersede them" (Bosch et al, 2019).

ATD suggests that learning is more productive in groups of students who together develop their individual and shared knowledge. Dialectics of *idionomy and synnomy* (*the individual and the group*) produces a group answer to a question from the answers given by individual participants of the group. The study paradigm defines the way these answers are formed. In the traditional paradigm of visiting works the answers to all considered questions are known by the teacher ahead of time. And even if students' answers are solicited in a classroom, they tend to be overridden by their teacher's answers, and thus validated or rejected by the authority. In the questioning the world paradigm, a *milieu* is formed from students' answers as well as from answers existing prior to students' inquiry and other available tools (e.g. technological environments). Then a collective answer results from the milieu in the process of negotiation, comparing, testing, defending and criticizing various approaches. Navigating between individual and group answers, participants make their contributions to the collective outcome, for which everyone in the group is responsible.

Dialectics of *conjecture and proof* (also known as dialectics of *media and milieu*) is vital for searching and identifying the truth. It orients learners towards critical revision of the information obtained from media (e.g. from a book or the Internet, a teacher, a peer), and treatment of this information as a conjecture (i.e. as a statement based on incomplete evidence) unless it is explained/proved by means provided by the constructed milieu. Dialectics of *black boxes and clear boxes* helps the learner to determine the depth and level of details required in a study of each existing work in order to address the generating question. The learner may consider the functioning of certain elements of milieu as black boxes, some elements -- as completely transparent for their understanding, and for most elements it may be a state in transition between the two. These three types of dialectics identified in ATD will suffice for the purpose of our discussion.

Data collection

The data had been collected in a graduate course offered at Memorial University (Canada) on-line for in-service K-12 teachers. The course focuses on mathematical thinking (Mason, Burton & Stacey, 1982) and includes such activities as individual problem solving, group discussions, and journal writing. One major assessment is based on five *interconnecting* problems, defined in (Kondratieva, 2011) as problems that (1) allow a simple formulation; (2) allow various solutions at both elementary and advanced levels; (3) may be solved by various mathematical tools from different mathematical branches, which leads to finding multiple solutions, and (4) can be used in different grades and courses and understood in various contexts. The following are examples of interconnecting problems used in the course and served as generating questions for study.

Problem 1: *In a square ABCD with E the mid-point of CD, join B to E and drop a perpendicular from A to BE at F. Prove that the segments DF and AB are of equal length* (Totten, 2007).

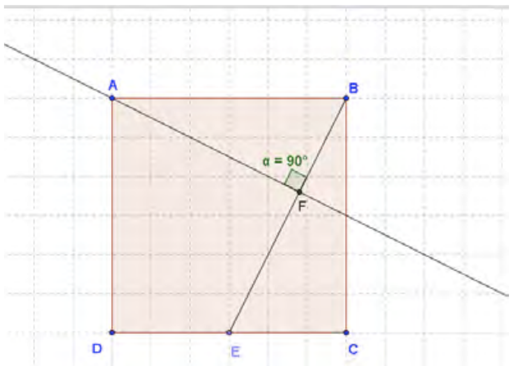

Figure 1: prove that segment DF and side of the square are of equal length

Problem 2: *Fred runs half the way and walks the other half. Frank runs for half the time and walks for the other half. They both run or walk at the same speed. Who finishes first? Explain your answer* (Mason, Burton & Stacey, 1982).

Students discussed the given interconnecting problems in randomly formed groups (normally) composed of 2 elementary and 2 secondary school teachers registered for the course. There were no restrictions on using any resources or technology. Each group required constructing at least 3 different solutions at various levels of sophistication. Each problem was discussed by students on-line during one week. After the groups' reports were submitted the instructor produced a summary of all solutions and a whole class discussion followed aiming at re-connection of different areas of mathematics in terms of the problem at hand. Then the next problem was considered and so on.

The data collection took place between 2007 and 2015 in the semesters when the author was teaching the course. The class size varied from 12 to 18 students. Participants' on-line discussions, group reports, and individual journals reflecting student's experiences with problem solving in the course have been analyzed. A range of possible solutions including that presented by teachers in their group reports was analyzed in Kondratieva (2013a, 2018) with focus on mathematical connections between the solutions. It was also observed that the mixed group discussions allowed the participants to reflect on each other's praxeologies and sometimes alter their beliefs (Kondratieva, 2013b). For example, statements from personal teachers' journals such as "I never thought before of a possibility to prove the same claim in multiple ways" and "This is how problem solving should happen in our classes – in collaboration not in isolation" witness certain shifts in students' beliefs related to inquiry-based pedagogy. In this paper we further analyze the effects of collaboration of teachers possessing different expertise by looking at 2 episodes from the on-line group discussion. The research questions are: (1) Why is productive mathematical communication between elementary and secondary teachers possible? and (2) How can the process of teachers' collaboration in mixed groups be observed through lenses of different dialectics?

Analysis of the results

From the ATD viewpoint, elementary, secondary and post-secondary schools are distinct institutions with their own specific praxeologies. However, as noted in Kondratieva (2018), a continuity of praxeological development may be observed in the sense that the logos present at a lower level contribute to praxis at the subsequent levels. It was proposed that this continuity is essential for meaningful learning of mathematics as students gradually build their cognitive universes, that is, the sets of objects, concepts and statements that they can mentally operate. Specifically, in the solutions considered in Kondratieva (2013a, 2018) we detect the following elements of praxeologies at different levels. We present them here from the student perspective.

Praxis at the elementary level consists of physical actions such as direct measurement or folding paper as well as of working with concrete numbers and pictures. However even at this introductory level students can develop the sense of generic structure and view concrete examples as members of some larger families of situations or classes of objects, which constitutes important elements of logos (basis of explanation) for them.

Logos at the elementary level gives rise to the praxis presumably common at the secondary level, which includes generic arithmetic and pictures, "a specialization which nonetheless speaks for generality" (Mason & Pimm, 1984). This praxis is supported by the secondary level logos consisting of use of algebraic symbolism, rules, equations, including analytic geometry of circles and lines. The study of meaningful use of symbols is the main direction of students' development at this level. Students may learn to manipulate with symbols formally, however, meaning and interpretation in terms of concrete examples should be accessible to them in order to maintain control over the calculations they are conducting.

At the post-secondary level working with algebraic equations become a part of the routine and therefore constitutes the praxis. The logos consist of theorems proven within a chosen axiomatic system. At this level the development of formalism is in the main focus of study of mathematics.

The above analysis of praxeologies found in a set of possible solutions to given interconnecting problems 1 and 2 leads us to the following answer to our first research question. The theoretical overlap between praxeologies (i.e. the overlap that was identified by the researcher, but may or may not be accessible to the teachers participating in the activity) makes the communication between teachers working at different school levels possible and even fruitful. We suggest that exactly due to this overlap of praxeologies mutual learning in mixed groups may occur. However, this does not happen in an instance (or even does not happen at all!) because in real settings teachers meet two contradictory curricular constraints: “Elementary teachers *can not* use algebra in their classrooms” and “Secondary teachers *must* use algebra in their classrooms.” These constrains give rise to a tension that calls for dialectics to overcome it by producing a new kind of condition, in our case – the conceptualization by teachers of “generic structure” that on one hand generalizes concrete calculations and on the other hand exemplifies algebraic expressions.

Dialectics comprises three phases of development: formation of a thesis, an antithesis, and the synthesis through which the differences between the thesis and antithesis are being resolved. Now we will illustrate this process with two exchanges of elementary teachers (ET) and secondary teachers (ST). In the first episode depicting an on-line discussion of Problem 1, the proposition ‘to measure the distances on the given diagram’ constitutes the initial thesis (line 1.1, 1.14) while the proposition ‘to use equations of circle and lines letting the size of the diagram be a parameter’ is an antithesis (line 1.9, 1.11). The synthesis leads to the idea ‘to assign a specific size to the diagram in terms of units and proceed with analytic geometry’ (lines 1.12, 1.17 - 1.19). Since the unit is not specified, this accounts for generality.

1.1. ET1: Point A is 6 units from D, as seen in the diagram provided. And point C is too 6 units form D. So we need to show that between D and F are 6 units. Aha, so D is the center of the circle and A,C,F must be on the circle! This we can check with a compass.

1.2. ST1: But the size of the square is not given in the problem. This is just an illustration with 6 by 6 square. We can’t assume the size.

1.3. ST2: And also how do you know that F is exactly on the circumference?

1.4. ET2: Isn’t the diagram a part of the problem? So we can assume that 6 units is the size! And it could be any 6 units, right? Like unit could be cm or mile or else. And then we just measure the side and the segment DF. I mean, with compass.

1.5. ET1: If you don’t trust a compass you can do in Geogerba.

1.6. ST2: Yes, Geogebra confirms that the circle passes through F. This is curious. But still it is not a proof, not for our level.

1.7. ST1: Right. No, only if we knew the coordinates of the point F then we would know for sure.

1.8. ET1: How is that?

1.9. ST1: The equation of circle of radius R is x^2+y^2=R^2.

1.10. ET2: But here R=6, right?

1.11. ST1: No, no. R is a number. This is the catch. You don’t know the size but the point should be always there. This is what I can’t see how to prove it.

1.12. ET2: Can you explain it for R=6? Like, ok, the circle is x^2+y^2=36, right?

1.13. ST2: See, for point A, x=0, y=6. You plug the numbers in the equation and it works, 0+36=36. So then you know that point A is exactly on this circle. This is why I like your idea of drawing the circle. But we do not know the coordinates of F to plug them in.

1.14. ET1: Can't we just measure them? On the diagram. In Geogebra. Look, x=4.8, y=3.6, so we can check the equation.

1.15. ST1: No! This would be the same as measuring DF….

1.16. ST2: But hold on, we can find F from the lines intersection. If we knew the equations of the lines.

1.17. ET1: In the diagram line EB has slope 2 and it passes via E(3,0). So we can find y=2x-6, correct?

1.18. ST1: And the line perpendicular to it must have slope -1/2. So it is y=6-x/2.

1.19. ET2: We can verify x=4.8, y=3.6 for the lines. 3.6=9.6-6, works! 3.6=6-2.4, works! So Geogebra was correct. So, now plug them in the circle equation … works! This is the proof!

1.20. ST1: I am still uncertain if we can stop here. But hold on… I know what to do. We should do everything we did, but in terms of R, not 6, you understand?

1.21. ET1: Sort of… this is getting a bit too complicated, guys!

In the second episode showing an on-line discussion of Problem 2, the proposition 'to use specific values for distance, time and speed' constitutes the initial thesis (lines 2.1, 2.3), while the proposition 'to introduce variables and solve inequalities algebraically' is an antithesis (lines 2.4, 2.6). A germ for emergence of a synthesis, a structural explanation is seen in line 2.5.

2.1. ET1: Let the speed of running be 4 miles/h and speed of walking be 2 miles/h. If Frank runs for 2h and walks for 2h, he will cover 8+4=12 miles. But then Fred runs 6 miles and walks 6 miles. So his time will be 6/4+6/2=4.5h. Thus, Frank comes first.

2.2. ST1: True. But how do we know it will always be so?

2.3. ET2: Let's try another case. If Frank runs for 1h and walks for 1h, … Same conclusion!

2.4. ST1: Great! but to be sure we should use some algebra, pick variables, set equations. This is how we teach them to proceed.

2.5 ET2: You do not understand! See, Frank always runs more than half of the distance and Fred exactly half. So, because running is faster Frank will always come first. This is my answer.

2.6. ST1: I think I see what you are doing. Ok, but in a different way. Let v and w be speed of running and walking. Then Frank going for 2T hours covers (v+w)T miles. And now we find the time for Fred and compare … how can we handle this?

Let us now turn to the second research question and review the above excerpts through lenses of different dialectics aiming to observe what each participant contributed to and acquired from this collective experience. Dialectics of idionomy and synnomy was employed when one member of the group proposed their solution to the question and other members of the group could either accept or reject, or more importantly, modify this proposition. In both Problems 1 and 2 elementary teachers started with examples (see lines 1.1 and 2.1) and found their (tentative) answers to the generating questions. These concrete examples had a generic structure (lines 1.4 and 2.5), which secondary teachers tried to detect and negotiate in view of analytic geometry (lines 1.7, 1.9) and algebra (lines 2.4, 2.6). Even though elementary teachers do not teach algebra in their classes it is important for them to understand simple algebraic calculations. Being present in pre-service teacher education programs, algebraic symbolism may be forgotten or felt as irrelevant by some elementary teachers. The moments of productive collaboration observed here suggest that algebraic techniques enriched by links to concrete examples could get a new and refreshed meaning for both parties (line 1.19 - 1.20). When algebraic expressions become a bit too cumbersome, the students as a group may develop the need for more “human”, that is, more structural or pictorial explanation. The latter was available to elementary teachers to a greater extend (line 2.5) than to the secondary teachers who were more accustomed to the algebraic formalism (as members of the secondary school institution, many of them believed that this was the *only* right approach, see line 2.4). Finally, the group developed an answer that was understood by every member. But the route of negotiation they walked together towards this solution taught them a lot about each other’s praxeologies.

Dialectics of conjecture and proof was evident when concrete examples suggested certain answer and therefore a conjecture was formed. Very often individual students jump right away to a general conclusion. However within the group, a question of validity of such conclusions was raised and called for an explanation (lines 1.2 and 2.2). It was important that all students contributed their ideas to the milieu formed by the group. This milieu relied on previous works and ideas (explained by ST in line 1.13) and the use of technology (line 1.6, 1.14). Students reflected on others’ proposals, asking clarifying questions (lines 1.8, 1.10, 1.12, 1.14) and developing meaning of the solution found within the group. We can observe how concrete calculations (for R=6) performed by secondary and elementary teachers (lines 1.12, 1.17, 1.18) helped elementary teacher to develop a generic proof in Problem 1 (line 1.19). This generic proof was a starting point for a meaningful algebraic generalization in line 1.20.

Dialectics of black boxes and clear boxes allowed elementary teachers to use algebra only to the extent they needed to understand secondary teachers’ explanations (line 1.12, 1.21). It also allowed secondary teachers to pay attention to some of the explicit but at the same time generic viewpoints specific in primary education (line 1.19, 2.5). In other words, teachers were experts in the domain of their own teaching but they familiarized themselves with the approaches used in the adjacent level, which caused them to rethink their own ideas, and therefore constituted for them moments of professional learning. Teachers learned not only about solutions new to them but also about different ways to present and explain the same idea. This is an important skill for teachers to develop because they need to disseminate mathematical knowledge to various groups of students. Therefore the ability to adjust the level of presentation according to students’ capacity is essential in their profession.

Conclusion

Olsen & Kirtley (2005) discuss how a secondary teacher enriched her understanding of a routine mathematical procedure by working with an elementary teacher. Here we attempted to describe a mechanism of teachers’ teamwork within a problem-solving context. Using theretical constructs of ATD, we reflected on data collected from mixed groups of in-service teachers. We identified an overlap of elementary and secondary school praxeologies consisting of generic exemplification, which belongs to logos in the former case and to praxis in the latter. We observed that theoretically this overlap provides grounds for fruitful elementary and secondary teacher collaboration. However, in real settings, we observed a tension between two contradictory constrains: ‘arithmetic way of doing’ in elementary school and ‘algebraic way of doing’ in secondary schools. The tension could be resolved during (re)construction by teachers of the missing overlap of their praxeologies. This development was analyzed through lenses of various dialectics (idionomy and synnomy, conjecture and proof, black boxes and clear boxes) capturing its complexity and shifting emphasis on different processes occurring within groups. The use of interconnecting problems allowed employment of both arithmetic and algebraic models, which supported a productive exchange of ideas between elementary and secondary school teachers. As a result, the participants experienced many elements essential for learning to enquire and to think mathematically (Mason et al, 1982). For example, "specializing" was largely promoted by elementary teachers and "generalizing" emerged from their collaboration with secondary teachers within dialectic of conjecture and proof. The processes of "convincing a friend" and "convincing an enemy" could be observed in idionomy-synnomy dialectics when teachers negotiated their answer within the mixed groups. In order to achieve a group answer, they needed to communicate their ideas effectively and be attentive to others' proposals. Of course, teachers defended their identities as members of their institutions (elementary or secondary school), but they broadened their expertise through participation in joint discussions. The dialectics of clear and black boxes required secondary teachers to 'descend' towards the arithmetic level and elementary teachers to 'climb' towards the algebraic level, helping each other to clarify certain ideas and approaches previously alien to them. Consequently, these experiences supported participants in revising and expanding their mathematical knowledge for teaching and made them aware of various scenarios that could happen in their own inquiry-based classrooms.

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Theoretical Preparations for Studying Lesson Study: Within the Framework of the Anthropological Theory of the Didactic

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In the anthropological theory of the didactic, we propose two theoretical resources of different types for studying lesson study. The first is the six dialectics of lesson study: of stakes and gestures, of period and study program, of milieu and infrastructure, of the predidactic and the postdidactic, of school and noosphere, and of the designer and the analyzer. These are tools for clarifying the nature of lesson study. The second is the scale of levels of paradidactic determinacy, which helps to identify conditions and constraints on different implementations of lesson study.

Introduction

Since the end of 1990s, Japanese lesson study has been studied by researchers in the United States (e.g., Lewis & Tsuchida, 1997), and the success of a well-known book "*The teaching gap*" (Stigler & Hiebert, 1999) increased the interest of researchers to study the impact of lesson study internationally. A recent direction of reseach on lesson study is the theorization of it. For instance, the first part of the latest scholarly book of lesson study "*Theory and practice of lesson study*" (Huang, Takahashi & da Ponte [Eds.], 2019) is devoted to different theoretical perspectives, where different models of lesson study are presented through seven chapters. In this paper, we contribute to this research trend by introducing some new conceptual tools within the framework of the ATD, i.e. the *anthropological theory of the didactic* (cf. Chevallard, 2019a). The study of lesson study (and of other teachers' collaborative work) probably need stronger theoretical constructs than the case of research of didactic situations at school. This is an opinion based on a fact that many didacticians more or less occupy two different positions: the *researcher* and the *teacher-trainer*. As far as we are unconscious of the presence of such schizophrenic status, the boundary between our research of the teachers' work and our intervention on it can be easily undefined. This means that the research lies side-by-side with a risk that it becomes spontaneous and non-scientific. In our view, we have to construct and elaborate scientific theories for being awake to such difficulty inherent in the theme of this ICMI-study.

The six dialectics of lesson study as a type of inquiry

On the notion of reference model

Didactic research confronts with an inherent obstacle on research, which is probably shared by many domains of social science—we "know" about the object of study before *scientific* research. This fact brings about the *transparency illusion* on social reality, which means that we tend to take social entities for granted (cf. Chevallard, 1992). Under this illusion, we are apt to apply unconditionally didactic worldviews or *theories*—which should be understood in broader meaning—of institutions in which objects of study are involved, especially institutions of schoolteachers and of curriculum-developers. This kind of research attitude hinders the growing of didactics as a scientific domain, because such theories are not for didactic research. For emancipating us from the taken-forgrantedness of didactic reality, we need scientific "yardsticks", that is, *reference models* (cf. Bosch & Gascón, 2006), which are different research assumptions about the object of study. For example, Chevallard and other researchers create a reference model of school algebra as tools for mathematization related to every domain of mathematics, while keeping a distance from the dominant school model of algebra as a mathematical domain (cf. Bosch, 2015). This fact that we need reference models is not a problem in itself. However, if didactic researchers default to construct their own reference models and embrace unquestioningly other models produced without scientific aims of didactics (e.g., mathematicians' epistemological models and curriculum-developers' pedagogical models), then didactics lapse into "spontaneous didactics" which only pedantically exaggerates common sense.

Let us emphasize here three points for avoiding the misunderstanding of the idea of reference model. The first point is that the challenge of constructing reference models is a never-ending endeavor. Any reference model is provisional and tentative, and can be modified with new theoretical constructs and newly gathered data. The second point is that every reference model does not have a normative and prescriptive nature for intervening in didactic reality. It is only a possible alternative which supports didactic researchers to understand didactic phenomena. The third point is that the reference model is not only created for knowledge to be taught but also for study processes. The reference model of knowledge is called the *reference epistemological model*, and the reference model of study processes is called the *reference didactic model* (cf. Gascón & Nicolás, 2019). In this paper, the notion of reference model will be applied to the teachers' work of lesson study, introducing the notion of *reference paradidactic model*. We will explain this notion in the following section.

Lesson study as a paradidactic type of inquiry

In this paper, we propose a reference model of lesson study. However, before that, let us introduce an assumption and a notion which are essential parts of the basis of our research. The starting point of our reference model is the following assumption—*lesson study is a special type of inquiry*. According to the ATD, a reference model of inquiry is the *Herbartian schema*: $S(X, Y, Q) \to A^{\heartsuit}$, which means that "students X produce their own answer $A^{\heartsuit}$ to a question Q with the help of teachers Y" (cf. Bosch, 2019; Chevallard, 2019a). As with the other notions from the ATD, the Herbartian schema has wider analytic scope. It is not a specific model for describing the inquiry in school classroom, but is a general model for any inquiry by humans which also include lesson study, since the notions of student and teacher have broader meaning in the ATD. In this paper, as a first approximation of lesson study, we regard it as follows—lesson study could start with the initial question "*How to teach a certain content during one lesson?*", and end with the final answer of "a report on a teaching implementation". This is a type of teachers' inquiry outside their own lessons. This characterization of lesson study leads us to a notion of the ATD—the *paradidactic* activities (Winsløw, 2012), which means teachers' activities outside their lessons. Generally speaking, there are three types of teachers' paradidactic activities: preparation, observation, and reflection. In this perspective, lesson study is a paradidactic type of inquiry, in which X can be a group of some teachers and Y is a group of advisers.

Let us add here that every study of lesson study consciously or unconsciously assumes some reference paradidactic model of lesson study. Several research results conceptualizes the process of lesson study as cyclic model. For example, Lewis's model (2016) expresses lesson study cycle in terms of four phases: 1. Studying curriculum and teaching materials, considering long term goals for students; 2. Planning the research lesson, anticipating students' thinking; 3. Doing research lesson, collecting data; and 4. Reflecting on lesson data and implications for teaching-learning. What we propose in this paper is another possibility of understanding of lesson study. Any reference model is constructed according to some research question and theoretical framework of a given research.

Reference dialectics of lesson study

For making a specific reference model of lesson study, we rely on the notion of *dialectic of inquiry*, which represent different bipolar pairs. In fact, the ATD uses the assembly of the *seven dialectics of inquiry* as another reference model of inquiry: for instance, the *dialectic of conjecture and proof* (Chevallard, 2002). According to the terminology of the ATD, the term of dialectic indicates a pendular movement between bipolar acts in the inquiry. The Herbartian schema represents the process of inquiry going from an initial question to a final answer. On the other hand, the dialectics of inquiry describes fundamental actions, which promotes progress of such process.

The seven dialectics of inquiry proposed by Chevallard are general notions related to any inquiry that includes lesson study. By contrast, in the following, let us propose *six new specific dialectics of lesson study*: the dialectics *of stakes and gestures*, *of period and study program*, *of milieu and infrastructure*, *of the predidactic and the postdidactic*, *of school and noosphere*, and *of the designer and the analyzer*. There are specialized points of view for studying lesson study.

The first dialectic of lesson study is the *dialectic of stakes and gestures*, which is related to the notions of *didactic stake* and *didactic gesture* (cf. Chevallard, 2019a). The didactic stake is the thing studied by students with the aid of teachers, typically, some pieces of knowledge to be taught. The didactic gesture is an act for teaching something. In the case of lesson study of mathematics, teachers think about some given piece of mathematical knowledge to be taught and/or generic competence on the one hand. On the other hand, teachers think about the way of teaching of it.

The second dialectic is the *dialectic of period and study program*, which is related to the planning and reflecting on a lesson and a sequence of lessons, that is, the scheduling of *didactic time* (cf. Chevallard & Ladage, 2008). Our word usage for describing scales of segments of didactic time—which are closely relatied to sizes or granularities of didactic stakes—follows to Bosch & Winsløw (2018). As a rule, a period correspond to one lesson, and a study program means a whole curricular project in a given school level.

The third dialectic of lesson study is the *dialectic of milieu and infrastructure*. The notion of *didactic milieu* means the set of resources of inquiry, which consists of *ready-made answers*; a variety of *works* necessary for constructing some final answer; new *derived questions*; and gathered *data collections* (cf. Chevallard, 2019a). In lesson study, teachers design and analyze students' milieus in given lessons. This focus is usually explicit for teachers. By contrast, the didactic infrastructure for lessons could be more implicit or transparent for teachers' perspective. Within the ATD, the infrastructure is a basis of acts, which makes them *viable* (cf. ibid.). In other words, the acts disappear or "die" without a relevant infrastructure. For instance, getting to Portugal by airplane cannot be realized without some aero-infrastructure with airplanes, airports, pilots, and so on. In the case of ordinary didactic situations, the classroom equipment, the school equipment, the amount of available time for lessons, the school textbook, and the teacher can be crucial parts of the didactic infrastructure.

The fourth dialectic of lesson study is the *dialectic of the predidactic and the postdidactic*. In lesson study, a teacher or a group of some teachers can repeatedly implement some lessons about a certain topic. In some cases, this could be done in different classes during one school year, and in the other cases, several years could be devoted to such repetition of lessons of a given kind. There could exist two kinds of *paradidactic time* around a lesson or a sequence of lessons at a given segment of didactic time : before and after. Winsløw (2012) introduced the adjectives *predidactic* and *postdidactic* for describing such asynchronous types of paradidactic time.

The fifth is the *dialectic of school and noosphere*. In lesson study, teachers could consider about not only national and/or international demand on education but also conditions of a given school such as average of students' achievements, and some specific mission or educational motto of the school. Within the framework of the ATD, the term *noosphere* means people thinking about the teaching (cf. Chevallard, 1992). A main role of the entity of noosphere mediates between society and its school system. A national committee for curriculum development is a representative subset of a given national noosphere. In the next secsion, we will return to this notion.

The sixth dialectic of lesson study has already been appeared implicitly throughout this paper, especially in the description of the definition of the adjective *paradidactic*. It is the *dialectic of the designer and the analyzer*, which are different roles observed in any didactic engineering. Teachers plan their lessons and analyze them. Let us emphasize here that the term *analysis* has broader meaning. It is not restricted to teachers' reflections after lessons, but includes expectation before lessons. In other words, the analysis means not only the *a posteriori* analysis but also the *a priori* analysis. In the same way, the design should be understood in wider meaning over activities in predidactic phases. Teachers more or less *re-design* their lessons in the postdidactic context.

A brief example of analysis with the six dialectics of lesson study

Let us give here a "fictive" example of lesson study in an imaginary one-day workshop, for illustrating a way of using of the six dialectics. In a workshop held in a Japanese teacher-educational institution, the preservice teachers are introduced to some variations of cyclic models of lesson study, and experience the designing of lesson plans. Under this organization, on the one hand, the preservice teachers could more or less deal with the dialectics of stakes and gestures, and of the designer and the analyzer—the qualities of their handling of them do not matter here. On the other hand, realizations of the other four dialectics—of period and study program, of milieu and infrastructure, of the predidactic and the post didactic, and of school and noosphere—could be restricted. Some reasons are as follows. About the dialectic of period and study program, the preservice teachers are not responsible for a whole study program. About the dialectic of milieu and infrastructure, they cannot consider about a given infrastructural elements other than very common and superficial matters, since they do not belong to any school yet. About the dialectic of the predidactic and the postdidactic, the workshop organization do not include postdidactic activities. About the dialectic of school and noosphere, they cannot assume different school conditions, because they do not have real students, and consequently do not have positional responsibility for them in a given school institution. In this imaginary example, we have identified some malfunctioning of using lesson study as a method for training preservice teachers. This kind of analysis is related to conditions and constraints of lesson study. Within the ATD, such an analysis is called *ecological analysis*. In the next section, we will introduce a tool for ecological analysis of lesson study and other kind of teachers' work.

Paradidactic ecology and the scale of levels of paradidactic determinacy

On ecological analysis

A main act of didacticians is *didactic analysis*, which is an analysis of different phenomena involved in the diffusion of knowledge. Chevallard (2019b) identifies two types of didactic analysis: *forward didactic analysis* and *backward didactic analysis*. Forward analysis aims to find out new conditions for realizing given (desirable) phenomena (e.g., the overcoming of a misconception). This popular analysis focuses on the changing of the didactic reality. On the other hand, a backward analysis tries to understand the existing didactic reality without research intervention on it. This kind of research attitude is not so prevalent in contemporaly didactics, but could be crucial for elucidating the mechanism of dissemination of knowledge in our societies. Within backward didactic analysis, the ATD emphasizes ecological analysis, which is led by the following crucial and *problematic* question—*why does (or does not) a given phenomenon exist there?* In the ATD, the notion of *ecology* means a set of conditions and constraints on a given didactic phenomenon.

Humankind
↓↑
Civilizations
↓↑
Societies
↓↑
Schools
↓↑
Pedagogies
↓↑
Didactic systems

Figure 1: The scale of levels of didactic co-determinacy

A central idea of ecological analysis is that any lesson is surrounded with various conditions of different levels from classroom to social environment. The *scale of levels of didactic co-determinacy* (Figure 1) highlights such multilayered nature of a bundle of conditions on a given phenomenon (cf. Chevallard, 2019a). The model represents that a didactic phenomenon occurs in a didactic system (e.g., a lesson), which is regulated by pedagogy (e.g., the problem-solving approach), at school (e.g., a junior secondary school) in society (e.g., Japan), civilization (e.g., OECD), even humankind (i.e. *Homo sapiens*). In short, the interest of the ATD lies on every kind of data necessary for the didactic research, without hesitation in going outside classroom.

Ecological analysis is a core of the gestures of ATD-researchers, and so is in the research of teachers' paradidactic work. A first approach to the paradidactic ecology has been done in terms of the notion of *paradidactic infrastructure*, which is a set of conditions for encouraging paradidactic activities (Winsløw, 2012). For instance, lesson study in Japan is supported by an infrastructure of the *open lesson*, which, roughly speaking is a kind of teachers' conferences for lesson study (cf. Miyakawa & Winsløw, 2013). In our view, the creation of this notion maks the discrepancy between the habitat and niche of the *didactic system* of lesson and them of the *paradidactic system* of teachers' collaboration such as lesson study more explicit. In other words, the notion of paradidactic infrastructure highlights that these systems live in different bundles of conditions, i.e. different ecologies. This means that we need some specialized scale for studying the ecology of paradidactic systems. In the following, we show a newborn tool for ecological analysis of paradidactic activities.

The scale of levels of paradidactic determinacy

Before introducing a new paradidactic scale, let us explain models of didactic systems and paradidactic systems for describing and clarifying relationship between them. In the ATD, any didactic system can be denoted by $S(X, Y, ♥)$ consisting of students X, teacher(s) Y, and a *didactic stake* ♥ which is something taught in the system. The Herbartian schema described in the previous section is an application of this model. Based on the triplet of didactic systems, we can denote the paradidactic system by $\mathfrak{S}(\mathfrak{X}, \mathfrak{Y}, S(X, Y, \heartsuit))$, where $\mathfrak{X}$ is a set of teachers, $\mathfrak{Y}$ is a set of supervisors, and a *paradidactic stake* $S(X, Y, \heartsuit)$ is a didactic system designed and analyzed in a given paradidactic system. These different notations implies two circumstances: first, the didactic system and the paradidactic system are interrelated in a nested way, and second, these are different systems which have respectively their own autonomies and dynamics.

The paradidactic system is an *open system* as with the didactic system, that is to say, such a system depends on different conditions out of the system itself. Let us introduce here the *scale of levels of paradidactic determinacy* (Otaki, Asami-Johansson, & Bahn, 2019) (Figure. 2), which is optimized for paradidactic research, based on the scale of levels of didactic co-determinacy. This scale represents all kinds of conditions and constraints on paradidactic activities, setting the presence of *paradidactic systems* as the first and lowest level. The second one is the level of *professions*, which indicates possible occupations related to the paradidactic activities such as schoolteacher, governmental official, teacher educator, specialist of a given discipline such as mathematician and didactician. Each profession affects the functioning of paradidactic systems based on its own occupational equipment. This level emphasizes a professional diversity of persons possibly involved in paradidactic systems. On the other hand, different professions can also possess similarities—e.g., epistemological, ethical, political and in philosophy of education—, which can be explained by the next third level of *noospheres*. Let us note that every school system bears its own noosphere. For example, the noosphere of elementary school is different from the noosphere of university. In the scale of paradidactic determinacy levels, the term of noosphere means a noosphere around a didactic system, which is a paradidactic stake of a given paradidactic system. Thus, in the cases of analysis of lesson study involved in Japanese elementary school, its noosphere is of the Japanese elementary school. This analysis naturally leads us to inclusion of the levels of *societies*, *civilizations*, and *humankind*.

Humankind
↓↑
Civilizations
↓↑
Societies
↓↑
Noospheres
↓↑
Professions
↓↑
Paradidactic systems

Figure 2: The scale of levels of paradidactic determinacy

A brief example of analysis with the paradidactic scale

Let us exemplify ecological analysis of lesson study within the new scale, based on Otaki et al. (2019). The paper reports a paradidactic phenomenon and its ecology, which is especially related to the dialectics of stake and gesture and of period and study program. On the one hand, in the post-lesson discussion, the teachers focused on small topics taught in a lesson or a sequence of some lessons without looking at larger bodies of knowledge, which include the topics. In other words, they took for granted the curricular articulation of mathematical knowledge, and did not take into account the possibility of reconstruction of it. This phenomenon is called the *thematic confinement* (cf. Barbé, Bosch, Espinoza, & Gascón, 2005). On the other hand, the teachers talked about general educational aims and teaching method without explicitly considering the knowledge to be taught. Such phenomenon is called the *pedagogical generalism* (cf. Florensa, Bosch, Cuadros, & Gascón, 2018). These two phenomena are usually observed simultaneously in lesson study, and thus we synthesized them into a phenomenon: the *paradidactic bipolarization*.

Using our new model, we made an ecological analysis of the paradidactic bipolarization, and identified two conditions on it. The first condition is the *schoolteachers' developmental byplay* at the levels of professions and noospheres. Schoolteacher are usually responsible for planning each lesson. In other words, their professional work does not include curriculum development, that is, the designing of relatively large bodies of knowledge. As a result, they focus on specific topics and generic pedagogy. The second condition is the lack of didactic notions like the *praxeology* at the levels of noospheres and societies. In general, it is difficult for us to consider about large bodies of knowledge over small pieces of knowledge. Indeed, we unconsciously tend to accept the organization of mathematical domains forced by national curricula even in our research. Didactic notions are useful for avoiding such naturalization of knowledge and talking about different possibilities of it. However, the diffusion of didactics into teachers' communities are not achieved well.

Final remarks

Huang and Shimizu (2016) indicate two types of theorization of lesson study: *of its entire process* and *of its core components*. The first type aims to model the whole (cyclic) process of lesson study. The second type focuses on teachers' acts and feats in each part of lesson study. In the second section, we introduced the six dialectics of lesson study, which are related to both of the entire process and the core components. It is because they are based on the Herbartian schema as a generic model of inquiry, and indicate specific foci in the inquiry of lesson study. By contrast, the scale of levels of paradidactic determinacy is more global, that is to say, it is not limited to research on lesson study. In our view, this is a useful model for different kinds of research of teachers' collaborative work when there are any ecological interests. A possible contribution of these two theoretical instruments is that they call to our attention to the necessity of advancing theorization of lesson study. In our opinion, we have to become conscious of the fact that we explicitly and implicitly rely on a "language" of the Japanese teaching profession whenever we investigate lesson study and lessons studied: e.g., the words *neriage*, *bansho*, *kyōzai kenkyū*, and *jugyō kenkyū*. In short, we have frequently used the theory that schoolteachers $\mathfrak{Y}$ in paradidactic systems ($\mathfrak{X}$, $\mathfrak{Y}$, $S(X, Y, ♥)$) construct, for studying the systems. Epistemological inadequacy of such a research condition should be recognized. Let us highlight here that the abovementioned theoretical framework subsumed under the ATD is provisional and open to argument. We already notice that there are some questions to be asked: for instance, "does there exist some differences between lesson study and other kinds of schoolteachers' paradidactic work from the perspective of the dialectics?" and "why do we need such a specialized framework for studying lesson study, although the ATD is general enough—or *anthropological*—for including lesson study as an object of study in the first place?". In our view, these questions are crucial not only for our framework but also for the identity and autonomy of the research domain of lesson study and paradidactic work. We believe that producing these questions is another merit of the framework.

Before closing this paper, let us add some comments about the roles of theories of lesson study. In the first section, we highlighted the importance of *scientific* theories of lesson study. However, the theorization is also crucial for actualization of lesson study. Winsløw, Bahn, & Rasmussen (2018) discuss the necessity of theoretical progress in research of lesson study not only from a scientific point of view but also by a practical reason. In their view, for the implementations of lesson study, we need something more than instructional manuals and descriptions of procedures, because there are probably many transparent facts taken for granted for the insiders of lesson study. As a result, it is difficult for outsiders of lesson study to understand and reproduce the ecology and economy of lesson study. The theorization of lesson study could be also essential for disseminating it.

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STUDYING TEACHER COLLABORATION WITH THE DOCUMENTATIONAL APPROACH: FROM SHARED RESOURCE TO COMMON SCHEMES?

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In this theoretical contribution we discuss in which ways the Documentational Approach to Didactics, more particularly 'schemes', contribute to our understandings of mathematics teacher collaboration when they interact with (digital) curriculum resources, and of the outcomes of such interactions. We analyze three studies of mathematics teacher collaboration and interaction with resources through the lens of 'scheme/s'. We contend that whilst theoretically we anticipated the existence of collective schemes, from our three studies we could not detect any regularity in the mobility of the same operational invariants. However, we can claim that the move from teachers' individual to agreed shared schemes had, in turn, influenced and modified their individual schemes, and hence enhanced their professional learning.

The contribution we present here concerns Theme A of the ICMI study: "Theoretical perspectives on studying mathematics teacher collaboration". We focus on the Documentational Approach to Didactics (DAD; Trouche, Gueudet & Pepin 2019), more particularly the notion of *scheme/s* (Vergnaud 1998), which has been used in several studies of mathematics teachers' (and students') collective work. The question we study here is linked with the two first questions proposed under Theme A (i.e. How do the different theoretical perspectives enhance understanding of the processes of teacher collaboration?; How do they enhance understanding of the outcomes of teacher collaboration?), and is formulated as: "How does DAD, in particular the notion of *scheme*, enhance our understanding of teacher collaboration and its outcomes?" We claim that within DAD, the concept of *scheme* is central for such understanding.

In what follows, we first recall the main elements of DAD, in particular the distinction between *resource* and *document*, the two interrelated processes of *instrumentation* and *instrumentalization* (Rabardel 2002), and we present in detail the perspective on teacher activity and teacher learning provided by *schemes* (Vergnaud 1998). Second, we draw on selected previous studies to evidence what we have learnt about teacher collective work and its outcomes, by drawing on DAD and schemes. In particular, we are interested in identifying individual and collective documents stemming from this work. Third, we discuss our theoretical results and their implications for further research perspectives, in terms of theory and of empirical studies.

Teacher collective work: the DAD theoretical perspective

DAD focuses on the interactions between teachers and resources, and on the consequences of these interactions. The definition of *resources* within DAD comes from the work of Adler (2000), considering as a resource anything likely to "re-source the teacher's practice". While Adler emphasizes the importance of "human resources", for this study we propose to study resources (outside the teacher) shared within various groups involving teachers (Pepin & Gueudet 2018). Drawing on the instrumental approach (Rabardel 2002) which proposes a distinction between artefact and instrument, DAD makes a distinction between *resource* and *document*. It considers that a given subject, engaged in a goal-directed activity, develops a document from a set of resources, associating recombined resources and a scheme of use (Vergnaud 1998) for these resources.

Document = *Resource* + *Scheme* (for a particular goal)

The process is called documentational genesis. Since we consider the interactions between teachers and resources within collectives, we claim that these geneses, or at least parts of these geneses, can be collective. We argue that this introduces a specific perspective on teachers' collective work, and we explain and illustrate this by leaning on the instrumental approach (Rabardel 2002) and, very importantly, by focusing on the concept of *scheme* (Vergnaud 1998).

Instrumentation/instrumentalization process/es and scheme/s

The documentational approach leans on, and extends, the instrumental approach developed by Rabardel (2002). For performing their teaching tasks, teachers, individually or in groups, interact with a set of resources. This interaction combines two interrelated processes: the process of instrumentation, where the selected resources support and influence the teacher's activity (i.e. the resources represent an interface between the knowledge, goals, and values of the author and the user); and the process of instrumentalization, where the teachers adapt the resources for their needs (i.e. the resources require craft in their use; they are inert objects that come alive only through interpretation and use by a practitioners). This productive interaction between an individual teacher, or a group of teachers, and a set of resources, guided by a teaching goal, through successive stages of (re-) design and implementation in class, gives birth to a hybrid entity, a document: this consists of the resources adapted and re-combined; and the ways the teachers use them, which include the stable organization of associated activities and particular usages, and contain the 'knowledge' guiding the usages.

In the instrumental approach, Rabardel (2002) defines a *utilization scheme* as a structure organizing a subject's activity with an artefact for a given goal. In the context of resource use, the utilization scheme includes both procedural schemes (e.g. how to use particular resources) and mental/cognitive schemes (e.g. global use strategy; knowledge about the means that the resource offers; concepts for a way of using the resource for a given class of tasks). We claim that this definition of scheme/s (and hence of documents) provides us with a better understanding of teachers' interactions with resources and of the consequences of such interactions.

Rabardel (2002) emphasizes the fact that schemes have an individual aspect, as schemes of a given subject or topic area, but also an essential social dimension. Indeed, the emergence of schemes is essentially a collective process, involving the users and the designers of the artefact; and moreover, their transmission is a social process. Rabardel (ibid.) has coined the term of "Social Utilization Schemes", and considers that the schemes developed by a subject result both from an individual development and from the assimilation of social schemes. However, he does not consider how schemes develop within groups of users.

In DAD we refer to the following definition of schemes, given by Vergnaud (1998), where a scheme has the following four components:

- The goal of the activity, sub goals and expectations;
- Rules of action, generating the behavior according to the features of the situation;

- Operational invariants: concepts-in-actions, which are concepts considered as relevant (e.g. "differentiation of teaching"), and theorems-in-action, which are propositions considered to be true (e.g. "the low-achieving students need more help from the teacher");
- Possibilities of inferences (e.g. "in this class I need to adapt my scheme for differentiation because there are some very high-achieving students").

These four components capture at the same time the stability of the activity's organization and its potential for evolution, the balance between uniformity of activity and its variation. Indeed, the inferences can lead the subjects to adapt their rules of action to the specific features of the situation; the adaptations can lead to the emergence of new operational invariants, or even of new schemes.

To understand the importance of schemes, we go back to selected aspects of Vergnaud's Theory of Conceptual Fields (TCF; 1998). According to TCF, a conceptual field is, at the same time, a set of situations and a set of concepts, all interrelated. Hence, the meaning of a concept cannot be analyzed through a single situation and, reciprocally, a situation cannot be analyzed through a single concept, but through many of them, creating systems. This means that a teacher's professional learning/knowledge, and its development, is intrinsically intertwined with the situations in which the teacher develops that knowledge. Vergnaud (1998) contends that

> The theory of knowledge as an adaptation process is essential; but what is it that adapts itself, and to what? The most reasonable answer to date is that what adapts are the forms of organization of activity, the schemes, and they adapt to situations. Therefore, the pair scheme/situation is conceptually more interesting and more powerful than the pair response/stimulus. (p. 85)

In many studies, documentational genesis has been examined as an individual process. Different teachers may develop different schemes for the same type of task. However, documentational (and instrumental) genesis also has a social dimension. The teachers develop schemes in the context of the professional development community, and they may develop others in the classroom community with their students. The next section, therefore, addresses the social perspective of the instrumental approach. Considering groups of teachers working together, the social perspective of the documentational approach raises several questions concerning teachers' collective work. When groups of teachers engage in a collective work, they not only bring their own resources, but also their own schemes. What happens during the collective work? Do the interactions lead to evolutions in the individual schemes? Are common documents (including common schemes) developed?

We examine these questions in the next section, drawing on selected studies about teachers' collective work using DAD.

Empirical studies of teacher collective work and identification of schemes

In this section we illustrate and explain three different collective works of mathematics teachers, and identify the schemes brought to and developed from the collective.

The impact of collective work on individual schemes

The study by Gueudet and Parra (2017) reports on the documentation work of two mathematics teachers (Valeria and Gwen) working together. Both were experienced teachers. In 2015-2016 when we followed them, both had a grade 11 class (specialized in mathematics and economics). Together they decided to design their course about tolerance intervals (with the binomial law), and to propose a common assessment to the two classes at the end of the course. We followed the design and implementation of this course, with the reflective investigation method (Trouche, et al. 2019) associated with DAD.

We analyzed the data collected at the end of the follow-up and identified documents developed by the two teachers. In spite of their strong intention of collective work, all these documents were at least partly different. We claim that the main reason for this is that, as experienced teachers, they previously developed their own individual schemes, concerning the teaching of tolerance intervals. For example, for the aim “Teaching how to find a tolerance interval with a binomial law”, Valeria had developed a scheme encompassing the following theorem-in-action: “the students must learn to read the table of the binomial law and to find in it the endpoints of the interval”. Gwen had different theorems-in-action, in particular: “the technique to find the interval for the binomial law is too complex, and never assessed at the baccalaureate”; “students must learn to program their calculator”. Gwen proposed to work on an exercise with their students. This required to write (on the calculator) a program giving the binomial law tolerance interval, once the appropriate parameter had been entered. Valeria accepted and actually used this exercise. For her this was only an exercise, the program was not used further (she did not want her students to use the calculator as a “black-box”); whereas for Gwen, the programmed calculator was then used in each exercise requiring to find a tolerance interval. The common assessment included an exercise on tolerance intervals. They chose this exercise together; the text of the exercise incorporated a journal extract, and this corresponded to a shared operational invariant: “the students must learn to find information in a text”. However, the questions were modified, to propose the two different methods for finding a tolerance interval: with the table of the binomial law, for Valeria’s class; with the calculator, for Gwen’s class.

We retain several aspects from this example. Firstly, when common operational invariants exist, teachers can develop shared (at least partially shared) documents. This existence of shared operational invariants certainly fosters common work. Secondly, when operational invariants are different, even a collective work leads to different documents. We claim that this can also explain why teachers using the same resource (e.g. a textbook) work differently (e.g. with the textbook) in class. The operational invariants can evolve along the collective work: Valeria developed a new theorem-in-action about the interest to write a program on the calculator for tolerance intervals. However, during a short-term collective work, operational invariants are likely to remain stable.

Collective design work and new schemes

In Gueudet, Pepin, Sabra and Trouche (2016) we analyzed documentational geneses occurring along the work of a Community of Practice (CoP; Wenger 1998) designing a new e-textbook for grade 10, within the French association Sésamath. This CoP was followed from June 2009 to December 2013. We claim that this long-term design work, and the “extraordinary” nature of the aim (designing an e-textbook is not a usual activity for a secondary school mathematics teacher) produced phenomena concerning documentational geneses different from what we discussed above.

One of the shared goals of the CoP at the beginning of their work was: “deciding a structure of the e-textbook”. They first established together a list of 38 “atoms” (e.g. “Draw a graph compatible with a table of variations” in one atom), based on the official curriculum - these atoms corresponded to competencies of the new official curriculum. In the discussions between the authors, we observed a shared theorem-in-action: "we need to support teachers for organizing their courses according to [national curriculum] competencies". After this first stage, the members of the CoP discussed how these atoms should be organized in the e-textbook. The teachers of the CoP would have liked a networking structure, where all potential paths (considered by the teachers) were possible. However, they realised that this was not possible (because, if a notion of geometry is needed in an exercise about functions, the geometry course must be placed before this exercise). The discussions lead the members of the CoP to decide that the e-textbook was going to be structured in chapters. Then, each chapter would have a *kernel*, a list of techniques with a given order.

> This list determined the content of the essentials, the introductory activities, and the basic exercises. Outside this kernel, more complex exercises were proposed, which could correspond to other, or several kernels. (p.195; Gueudet, et al. 2016)

The CoP developed a shared document for the goal "choosing the structure of a grade 10 e-textbook". This document encompassed several resources, in particular the atoms, and shared schemes-of-use for these resources. In particular, the members of the CoP developed together theorems-in-action, such as "it is not possible to leave the possibility of any path to the users"; and "the techniques provide the coherence of a chapter".

In this example the members of the CoP developed common documents, concerning the design of an e-textbook. Probably their uses of the e-textbook in class would differ, because of their pre-existing schemes. But for the activity of designing an e-textbook, they developed new and common documents, in particular common schemes.

Collective scheme and interpretation and adaptation at each level of collective work

The study reported in Pepin, Gueudet, and Trouche (2017) concerns a Norwegian teacher's (Cora) documentation work within the context of a large European project (PRIMAS - https://primas-project.eu), where she worked in collaboration with a group of mathematics and science teachers of her region. The aim of the PRIMAS project was to develop capacity (in terms of inquiry-based teaching and learning) in mathematics and science education in schools; hence to develop instructional leaders ("multipliers") who in turn would work with small groups of colleagues in school. The PRIMAS project in Norway offered, and Cora participated in, professional development (PD) sessions over 18 months, in addition to her work with colleagues in her school between PD sessions. The PRIMAS project provided curriculum resources designed by 'expert designers' (i.e. academics working at the universities of the partner teams). These materials included (1) mathematics and science tasks; and (2) modules developed for teacher professional development sessions. They were seen as the basis for the work/sessions with colleagues in school.

For this paper the following data were analyzed: (1) Cora's general use of the PRIMAS modules (interview; Schematic Representation of Cora's Resource System); and (2) her adaptation and use of a particular PRIMAS module and associated tasks on 'division of fractions' (lesson preparation; video observation; interview).

Results showed that in terms of documentation work, Cora worked at three levels, where the first two relate to collective (re-)design and the third to individual (re-)design of curriculum resources: (1) the re-design of tasks and PD modules in the university PD sessions (so that they corresponded to the Norwegian and the individual school contexts); (2) the re-design of tasks (and lessons using these tasks) in Cora's school PD sessions; and (3) the re-design of tasks (and lessons) for Cora's own teaching. Hence, Cora re-designed and adapted the digital PRIMAS tasks, individually for her own lessons, in addition to her (re-) design work with and for her colleagues in her role as multiplier, as well as in the university PD sessions.

In terms of schemes, Cora's larger goal was to develop her inquiry-based learning skills, which included as general rule of action "to listen carefully to pupils" and build her lessons around pupil thinking. Hence, principles of good questioning were important for her – here we observe that initially she had operational invariants compatible with PRIMAS: e.g. "good questioning" as a concept-in-action. She used the PRIMAS modules, together with her colleagues, to develop "good questioning skills", and in turn enhance her understanding of student thinking. The digital PRIMAS resources provided her (and the group) with promising examples of how to adapt her questioning skills. These adaptations illustrated what we called *design flexibility* (Pepin, et al. 2017): e.g. by adapting the digital PRIMAS tools for lessons, Cora developed flexible ways of questioning in order to guide student thinking and to help them to make sense of the proposed activities. Her operational invariants related to the application of questioning principles to different topic areas in mathematics education, and Cora adapted her designs to the teaching context. In terms of inferences, differentiating tasks for different audiences, and with different resources, were part of Cora's expectations for attending to individuals in her class. Hence, we can claim that they were part of Cora's schemes that were reinforced and further developed by her work in PRIMAS.

From this study we retain that, first, teachers work at different levels with and in collectives. Second, it is noteworthy that whilst PRIMAS provided the resource/s and collective designs/modules, Cora worked with different shared schemes in different collectives: (1) in the university PD sessions, she developed shared schemes that linked to adaptations of resources to the Norwegian 'curriculum culture' context (e.g. schemes linked to questioning and formative assessment); (2) in the school PD sessions, particular resources and associated schemes were selected (by Cora and colleagues) and these were 'mixed' with individual schemes, in order to develop a certain coherence of instruction across the grade/s; and (3) in Cora's individual documentation work, she developed 'new' schemes by considering promising examples from PRIMAS resources and including them in her new individual schemes. It was clear that at each level the PRIMAS (designed) resources had an impact on teachers' associated scheme/s; at the same time in the different collectives Cora and the respective group of teachers shared and developed their schemes further. This became evident in the interviews and session observations, when the teachers talked about their mathematical-didactical thinking and practices.

Discussion and conclusions

In this section we explain and discuss what we have learnt from the three studies, and we forward three claims.

We have learnt the following from the three empirical studies about mathematics teachers' collective work by identifying schemes:

(1) From the study by Gueudet and Parra (2017): that the collective design of a lesson can lead to selected evolutions in participants' operational invariants, but not likely to the emergence of a completely shared scheme. When teachers engage collectively in an activity with a "usual" goal for

which they already developed an individual scheme, this scheme intervenes in the collective work. Eventually, it leads to different interpretations of the resources collectively designed, at least when the collective work only concerns a short period. Investigating the development of shared schemes for "usual" goals when the collective work develops over several years requires further study.

(2) From the Sésamath study: that in an "exceptional" collective work where the aim of the activity does not coincide with usual goals, teachers can develop shared schemes. Some initial schemes, corresponding to sub-goals, intervene in the collective work. The differences in mathematical/didactical opinions and beliefs can even generate conflicts; but the shared overarching goal leads to the negotiation of solutions, and eventually to the emergence of a shared scheme.

(3) From the PRIMAS study: that at each level of collective work, teachers can develop different schemes which link to the organization of the collective activity: some schemes become shared, others stay individual. Moreover, different kinds of resources have different impacts on the development of schemes, and the type of impact depends on previous schemes and experiences. What is not evident is how common schemes could develop; every group (and every teacher) seems to develop its own synergy in terms of schemes, even with expert-developed resources and common goals. It is likely that goals get interpreted differently at different levels of collectives, which in turn is likely to influence the schemes developed.

(4) From all three studies: that shared operational invariants influence the engagement in a collective work, and facilitate this collective work; that evolutions of individual schemes (or at least of components of these schemes) can result from the collective work.

What we claim: First, in order to provide an analysis of complex collaborative activities with a theoretical structure, we refer in this contribution to Vergnaud's work of the TCF, and in particular the notion of *scheme*. Through analyzing common documents produced by collective actions, teacher learning through collective action can enrich teachers' individual schemes. However, it might be that each individual, participating in a situation lived collectively, creates a 'collective mindset/scheme' shared by all members of the community. This collective mindset would be the *collective scheme* created together during the collective activity. At the same time, it appears that utilization schemes also acquire a social character, when schemes are elaborated and shared in collectives, and this may give rise to an appropriation by subjects.

Second, many studies referring to DAD have considered teachers' work within CoP (e.g. Sabra & Trouche, 2011; Gueudet, Pepin & Trouche, 2013). The link between DAD and CoPs is natural, since one of the important aspects characterizing the CoPs is a shared repertoire of resources. The definition of "resource" within the CoP theory includes the resources as defined by Adler, but also other aspects. The repertoire of resources in a CoP is a product of the CoP's practice, and at the same time it supports and contributes to shaping this practice. It also includes experiences, ways of addressing problems, which in DAD are situated within the "scheme" part of a document, and not within its "resource" part. Thus, we claim that using the concept of document, and scheme in particular, could be helpful to deepen our understanding of the repertoire of resources of a CoP, linking it with the concept of document system in DAD.

Third, when collective action is mentioned in the field of (mathematics) education (e.g. Community of Practice- Wenger 1998), it is mostly related to the formation of groups of teachers who intend to modify or design school syllabi, re-design mathematical tasks, and/or create new pedagogic practices. It appears that in most studies the term 'collective action' has been appropriated, meaning teachers making decisions in groups, for particular goals. It is also inferred that this collective action reflects the group's way of thinking, their collective scheme. It seems acceptable that individuals who live a certain situation in a group, act accordingly to their individual schemes, where a collective scheme appears to be the motor of the action, as Marcel (2005) contends that

> a collective scheme would be a structure which generates collective action in a given situation. The difference [between individual scheme and collective scheme], however, lies in the fact that this collective pattern is 'carried out' by a group of acting individuals, not by a single individual. The collective scheme is built and mobilized by the team, as a social and cognitive entity" (Marcel, 2005, p. 651).

Leaning on Marcel (2005), we argue that the organization of patterns in individual schemes constitutes the specificity of the collective scheme, which has a cognitive surplus that would, in return, also influence individual schemes. That is, there would be some influence (not simply a joint) from individual schemes on the constitution of collective scheme and, in return, the new individual schemes would have been influenced and modified by this collective scheme. What we claim in this research is precisely the following: the teachers participating carrying out the tasks proposed have built, through their individual schemes, a collective scheme that, in turn, modified the individual schemes, entailing movement in their learning.

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ICMI Study 25
TEACHERS OF MATHEMATICS WORKING AND LEARNING IN COLLABORATIVE GROUPS
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COLLABORATIVE TASK DESIGN AS A TROJAN-HORSE: USING COLLABORATION TO GAIN ACCESS TO THE TEACHER'S OBJECTIVES

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In this paper, I present and discuss findings from a research study aiming to investigate the activity of proving as constituted in the classroom. By drawing on Cultural-Historical Activity Theory and collaborative task design, this study explores the way the teacher is working with the students to foreground mathematical argumentation. In this paper, the collaboration between the teacher and the researcher is discussed, focusing on the design and implementation process as a vehicle to gain access to the teacher's objectives and motivations. Furthermore, the way the teacher intervened throughout the lessons is contrasted against this collaboration, so as to gain a deeper understanding regarding what drives the teacher's teaching decisions.

Introduction

It is now acknowledged that proof and proving should become part of students' experiences throughout their schooling (Hanna, 2000, Yackel & Hanna, 2003, Stylianides, 2007). Research has responded to the need to conceptualize proof and proving in such a way that it can be applied not only to older students but also to those in elementary school (Stylianides, 2007). The challenge remains however to understand how proof and proving is shaped by the practices in the mathematics classroom.

Regarding proving in geometry, it is argued that the potentialities of dynamic geometry environments (DGE), such as Cabri, for the validation of geometric constructions, can be exploited so as to support students in linking informal argumentation to formal proof (Healy & Hoyles, 2001). However, providing students with opportunities to follow the reasoning of an argument, evaluating and constructing proofs is not enough. Expressing mathematical ideas does not happen spontaneously in DGEs. The role of the teacher becomes central and critical in orchestrating mathematical situations. The teacher shapes the rules of discourse that get privileged in classroom activity, assists students in constructing mathematical ways of knowing that are compatible with those of wider society and promotes the negotiation, acceptance and development of these rules of discourse among students that would allow them to accept or reject an argument as proof (Yackel and Cobb, 1996). Concerning technology, the teacher is also responsible for guiding the appropriation and transformation of the tool by the student (Noss and Hoyles, 1996).

In the social environment of the classroom, where hypothesizing, explaining and justifying conjectures is encouraged, the tools and tasks used, the rules of the classroom, the way the students work together, the way the teacher negotiates meanings and other external factors all interact, interrelate and influence each other in forming classroom activity. The purpose of this study is to explore proving in the elementary mathematics classroom and the way the structuring resources of the classroom's setting shape this process. There is insufficient scope in this short paper to consider in detail these various levels and so this specific study focuses on the collaboration between the teacher and the researcher as a vehicle to gain access to the teacher's objectives and motivations. To be more precise, this paper aims to demonstrate how collaborative task design led to identifying the object of developing proving in the classroom.

Collaborative Design

In the recent times, a growing number of studies have focused on the collaboration between teachers and researchers. Collaboration may be examined through a community of practice perspective (Wenger 1998), focusing on the negotiation of meaning, the formation of common goals and the building of a teaching identity. Collaboration may also be investigated through communities of inquiry perspective (Jaworski, 2005) where teaching is seen as learning-to-develop-learning. The fundamental facet of collaborative design is the notion of collaboration. Studies have attempted to exemplify what collaboration entails and have investigated the social, organisational and technical issues surrounding collaboration in design. Kvan (2000), makes the distinction between collaboration and co-operation. He considers collaboration as *'to work together with a shared goal'*, which is different from co-operation, which could be defined as 'to work side-by-side with mutual goals. This distinction indicates that *'design collaboration requires a higher sense of working together in order to achieve a holistic creative result'*, (p.410).

In collaborative design, the design interactions and the exchange of design ideas are influenced by social roles, individual experience or level of expertise (Chamorro-Koc et al, 2009). Consequently, design involves the negotiation of multiple perspectives as the participants with different responsibilities, interests and competencies negotiate the object of the design. This characterisation of collaboration in design indicates that it is a demanding activity. In order to maintain this collaboration when a team or group of people work together, the issue of compromising inevitably emerges. However, this issue just makes explicit that some of the decisions made might only partially satisfy the team members and it should not be understood as a core problem of collaboration. Through dialogue and negotiation a common ground can be found and without anyone being forced to accept a solution, a conclusion can be made (Détienne, 2006). While conflicts between teachers and researchers in a collaborative group have been reported (Jaworski, 2005), these conflicts provide important information about the nature of the collaborative group and its internal dynamics.

Time is another characteristic of collaborative design approach. That is, due to its complex nature, design is not a simple process, but *'consists of a series of distinct events that occupy discrete and measurable periods of time'* (Kvan, 2000, p.412). Van Leeuwen et al (2005) conclude that creativity in teams, collective communication as well as process organisation are three important issues in learning collaborative design. These are aspects of the collaboration process that need to be taken into consideration for any difficult situations to be sufficiently resolved.

CHAT Based Theoretical Constructs

As this study is exploring the various forces that impact on the activity of proving, Cultural Historical Activity Theory (CHAT) is being employed as a descriptive and analytical tool alongside collaborative task design (a means of gaining access to the teacher's objectives), to capture the interaction of different levels, such as the actions of teachers, students and the wider field as evidenced in curricula and research documentation. The analysis and discussion in this paper draws upon the following CHAT perspectives: (i) the object of the activity and (ii) the notion of contradictions. Initially, the unit of analysis in CHAT is an activity, a *'coherent, stable, relatively long term endeavor directed to an articulated or identifiable goal or object'* (Rochelle, 1998, pp.84). Engeström (2001) introduced the activity system, a general model of human activity that embodies the idea that both individual (subject, tool, object) and social levels (rules, community, division of labour) interlink at the same time. The object of a collective activity is something that is constantly in transition and under construction, has both a material entity and is socially constructed and its formation and transformation depends on the motivation and actions of the subject indicating that it proves challenging to define it. Among the basic principles of CHAT is the notion of contradictions. Contradictions are imbalances, ruptures and problems that occur within and between components of the activity system, between different developmental phases of a single activity, or between different activities. These systemic tensions lead to four levels of contradictions (Engeström, 2001). This conceptualization, should be differentiated from mere problems or disorienting dilemmas from the subject-only perspective as they are more deeply rooted in a sociohistorical context (Engeström, 2001). Contradictions are important because they may lead to transformations and expansions of the system and thus become tools for supporting motivation and learning. This paper focuses on secondary contradictions which take the form of tensions between components of the activity.

Data Collection and Analysis

This study was conducted in a year 6 classroom in a public primary school in Cyprus. Apart from the researcher (the author), the participants were the teacher, a Deputy Principal at the school who endorses the integration of technology in teaching mathematics, and voluntarily agreed to take part in the research, and 22 students (11-12 years old) of mixed abilities. Even though using computers was part of the classroom's routine, the students were not familiar with Dynamic Geometry Environments (DGEs).

The data collection process was undertaken in three phases. Phase I aimed at identifying the system level and the teacher level, by employing documentary analysis and semi-structured interview. The system level, which remained the same throughout the study, in the broader sense, refers to the policy statements, curriculum, textbooks, research about proof and proving. The teacher level refers to the teacher's attitudes and perceptions concerning the role of proof in the curriculum and in the mathematics classroom, compared with what the teacher actually does in the everyday mathematics classroom. The main research focus of Phase II was to map the current situation of the classroom. The data collection process included video data from the classroom observations and field notes from the informal discussions with the teacher. My involvement in the classroom could be described as moderate participation. In Phase III, the researcher collaborated with the teacher to design DGE-based tasks as a means to gain access to the teacher's objectives. The tasks were the research vehicle, the window for generating data rather than any kind of curriculum intervention. The research instruments were classroom observation, informal discussions with the teacher and the DGE-based tasks. In Phase III, I had an active role in the classroom. My involvement was related with answering questions related with the tools the DGE provided, which the students had to use in order to explore the tasks, and asking probes. The content of the curriculum covered during the classroom observations was the area of triangles, and the circumference and the area of circle.

The overall process of analysis of the collected data was one of progressive focusing. According to Stake (1981), progressive focusing is *'accomplished in multiple stages: first observation of the site, then further inquiry, beginning to focus on relevant issues, and then seeking to explain'* (p.81). The systematization of the classroom data led to the evolution of two broad activities: (i) the activity of exploration which encompasses a nest of three distinct activities including the exploration of mathematical situations, exploration for supporting mathematical connections and exploration of the DGE and (ii) the activity of explanation which focuses on clarifying aspects of one's mathematical thinking to others, and sometimes justifying for them the validity of a statement. These activities were then interpreted through the lens of CHAT, by generating the activity systems of both exploration and explanation. The situation of the classroom regarding proving activity was further scrutinized by contrasting the outcome of the activity with the social context in which it emerged. The synthesis of the analysis led to the conclusion that the object of developing proving in the classroom is exploration and explanation that provide a point of reference for proof production.

Results

In this section, insights from the collaboration between the teacher and the researcher are presented and elaborated on, with respect to the form and content of collaboration regarding task design, and the appropriation of the tasks in the classroom. In doing so, tensions and conflicts identified in this context of collaboration are discussed.

Collaborative Task Design

During the initial discussion with the teacher, it was established that the teacher and the researcher would work together with the shared goal being the design of DGE-based tasks. It was decided that the nature of the participation in this collaborative design environment would be mutual collaboration. It was also agreed that the content of the geometry curriculum that would be covered would be related with the circumference and area of circle.

As this teacher endorses the integration of technology in teaching mathematics, she suggested employing Cabri, as she would have the opportunity to learn more about this specific DGE. By doing this though, the teacher said that she would feel more comfortable with me designing the tasks and do the lessons, as she feared that she might not be able to answer students' questions regarding this DGE. Through discussion it was agreed that the researcher would design the tasks on this DGE, by drawing on her suggestions and we would do the lessons together. The teacher's contribution was very essential. By taking into consideration the research's objectives, that is, to investigate the activity of proving as constituted in the classroom, the researcher argued for situations that give the opportunity to students to dynamically explore, make hypotheses, test these conjectures and justify their work. The teacher had exemplified the importance of exploration that leads to discoveries, as well as the importance of justification and proving in the teaching of geometry during the initial interview. Despite this, while the researcher was talking about tasks that would give the opportunity for argumentation, the teacher wanted simple activities, where the students could explore circle, make a table and add numbers in a table concerning the radius, diameter, circumference and area of circle so as to identify the relationship that exists between these notions. These activities could be found in the students' textbook. The teacher was very specific about the structure of the tasks. Even though this was in a way in contrast with the researcher's expectations and this research's objectives, the teacher's recommendations could not be ignored. The researcher recognized the fact that the aforementioned area of the geometry curriculum had to be taught and made an argument towards open tasks that support exploration. Our mutual collaboration led to resolve this challenge by considering designing tasks that would provide the opportunity to students to explore the mathematical ideas the teacher was interested in, and simultaneously encourage mathematical argumentation. In this specific occasion, we both understood each other's backgrounds and objectives and through dialogue we reached a common ground. Through exchange (via e-mail and phone calls) that followed, two Cabri tasks were designed: (CG,) the circumference graph and (CA,) the area graph. In CG, as a free radius point (A) is moved, the circle changes size and a linear graph of the circumference (C) against its radius is plotted. By default, the trace of the graph was off but this could be switched on at any time. CA was designed in a similar way, though in this case the graph would be parabolic. The rationale for designing the graphs was to explore the algebraic expression of the circumference and area of circle by relating the properties of the circle with the graph.

Utilisation of the DGE-based Tasks in the Classroom

On the first day of the introductory lesson regarding circle, the classroom had the opportunity to explore DGE-based Task1. The students had the opportunity to make inferences by relating the changes made in the circle with the measurements in the table. However, this exploration was closed down by the teacher. That is, the graph was not explored. During our informal discussion at the end of the first day, the teacher felt that the task was not that structured. The teacher stated that the students were noisy and asking technical questions. This, for the teacher, was an indicator of the students not properly engaging with the task. This, for myself, was an indicator of the students engaging with the task. The teacher felt that due to the fact that the students were asking questions related mostly with the technical aspects of this specific DGE, and the classroom was quite noisy, the students should be given a worksheet with specific instructions. This led to an additional emerging tension. This tension was first related to the teacher's interpretation of 'open' and 'closed' tasks. The teacher's understanding was related to the degree of independence in exploration. This was in contrast with the teacher's statement of supporting exploration and investigation in the classroom. At this point of our collaboration, compromising was inevitable. This tension was resolved by considering providing students with a worksheet as an opportunity to explore the students' written responses regarding the explanations and justifications given about their observations. While I was feeling that this action was against my objectives as a researcher, an opportunity arose to exploit this decision to fully understand the teacher's objectives. This worksheet would include steps that the students had to follow so as to investigate the relationship between the diameter and the circumference of the circle and questions related to what they would observe each time they followed a step, as well their conclusions regarding these observations.

It was noticed that, even though this handout was given to the students for the first task, it was not given for the DGE-based Task 2. Analysis of the way the teacher intervened throughout the lessons shows that the teacher gradually felt more comfortable in giving more freedom to students to work independently. Thus, she did not feel the necessity to provide an additional artefact to aid the students' activity. Therefore, it can be concluded that her unfamiliarity with this environment guided her in making the original decision to use a handout.

Another challenge that emerged through our collaboration was related to the way the DGE-based tasks were utilized in the classroom. Even though an agreement was reached regarding the goals underpinning the design of the tasks, and the general lesson plan, the teacher exploited specific aspects of the tasks in different parts of the lesson. That is, the teacher was the one deciding when to stop, and what aspects of the task not to use until later when she felt it was more appropriate. For example, for the two DGE-based tasks, the graphs were not really explored until the concluding lesson related with circle, as it seemed that this was just an additional thing that the students could know for 'circle'. One may argue that the teacher made this decision in order, perhaps, to satisfy the researcher. However, the teacher devoted enough time to fully explore the tasks and engage the students in a process of explaining and justifying. Even though the goal was, throughout the week to explore all aspects of the tasks that would allow the students to investigate in a different way the relationship that exists between the radius, the circumference, and the area of circle, it was not until the end that this happened holistically. The question regarding the opportunities that perhaps were missed by not fully exploiting the tasks from the beginning remains. By considering the fact that the teacher was supporting mathematical connections, one may also argue that the teacher fully exploited the DGE-based tasks to meet her own objectives. However, making connections did not act as an additional thing that the students could know. The teacher embraced the idea of investigating in a different way the relationship that exists between the radius, the circumference, and the area of circle. To be more precise, even though the students had no experience with parabolas or quadratics from school, they had the opportunity to gain a dynamic graphical appreciation of the formulas. Furthermore, the teacher seeked to advance the discussion by encouraging students to compare the two graphs and formulas and make an assertion. Some students recognised that it is radius squared that produces the curve.

The teacher's actions led to the conclusion that even though she supported employing the tasks, the time constraints that existed in covering this specific part of the mathematics curriculum, as well as the way she usually teaches this particular area of the mathematics curriculum, directed her decisions. Specific aspects of the tasks were initially used so as to achieve the educational goals of this geometric topic, and at a later stage, in concluding what was explored throughout the week, the tasks were fully explored. Nevertheless, the discussions we both had at the end of each lesson constitute indications of collaboration between the teacher and the researcher. What is more, one cannot neglect the fact that, as the teacher of that classroom, she was in the end responsible.

Discussion: Identifying Points of Contradiction

Introducing a new mediational tool in the classroom led to the emergence of secondary contradictions. As the teacher would in general employ DGEs in her teaching, this secondary contradiction resides on the design of the DGE-based tasks. This is also in accordance with the rule established in the classroom where both the teacher and the students are learning together from the computers.

The teacher's concern on how the tool would affect the object of the activity led her to deciding to hand out a worksheet to the students. Providing a worksheet to the students is arguably an action not consistent with attention to purpose and utility in task design (Ainley et al, 2005). Of course, in general it would depend on the nature of the worksheet but a danger is that a worksheet might be too prescriptive and constraining to facilitate the level of ownership a student needs in order to engage in purposeful activity around the task. The fact that some pairs felt that they had to follow in a strict way the steps in the worksheet and that if an answer was not provided in the worksheet, they could not move to next part of exploring the task strengthens the above statement. This observation may also be related with what Berge et al (2004) describe as complexity in tasks. Nevertheless, offering a worksheet was this teacher's way to resolve the planning paradox. Even though providing a worksheet to students automatically might restrict the openness of the exploration task, this approach, from the teacher's perspective, could lead the students to explore the environment in such a directed way that a discussion could follow regarding the mathematical relationship under investigation. The teacher used the worksheet as a reassurance that the students would work towards achieving that component of her objective. The students had to assume a new rule as they were not accustomed to sharing a worksheet in pairs. The tension was resolved by the students. That is, the students established a new social norm related with the collaboration between pairs of students when sharing a worksheet. The students were able to reach a conclusion regarding the mathematical formula for the circumference of circle.

The action of closing down the exploration activity indicates that the motives underlining the design of the tasks could not be reached. Inevitably, the outcome of this exploration activity was not the one intended. Regardless, this contradiction was resolved by the teacher. The classroom fully explored the tasks as a way to make forward connections with other areas of mathematics.

This discussion constitutes another indication that the teacher does not reject technology. The concerns of the teacher related with the integration of technology in the mathematics classroom have been identified. Would a consideration of the emerging tensions due to the duality of the object of the activity direct towards rethinking the difficulties teachers encounter when employing technology in their teaching practices? I would consider that it is essential that teachers have an opportunity to make sense of these tensions (if any). There is a danger that manifestations of these tensions that are not followed by reflection may reinforce and establish the teachers' concern influencing their confidence in utilizing technology. That is, while there are instances where acquiring computer skills is in fact the object at a given point of mathematical activity, the general object of the activity does not have to be matched with the technology itself. If an activity is closed down this may be due to the clashing objects the teacher has, or it may be that the concept explored or the situation investigated is, at that time, difficult for the students. Doing this may result in removing to an extent the pressure and concerns teachers feel when employing technology in the classroom.

Concluding Remarks

An important issue that emerged in the context of this study, is the complexity of the collaboration between teachers and researchers. In collaborative task design, the emergent conflicts were related to what to select to be approached in the task and how to do that. In the decision making process of task design, the conflicts were related to the mathematical context of the tasks, the degree of openness, selecting the content to be focused on the task, the language used as well as the way the task would position the teacher and the students. Identifying existing meaning and negotiating new ones, and establishing a form of consensus is an indication of the teacher's and the researcher's commitment to explore each other's perspectives, with the ultimate goal for the whole process to be effective. That is, the teacher gradually moved from a defensive attitude to one that involved adjusting interpretations. As for the researcher, she became more conscious on her effect on teacher's engagement and looked more critically at her role. Through engagement in the design of tasks and opportunity for participation, a community grew. It can be argued that dealing with issues, tensions and contradictions, also allowed for expansion within this activity of collaboration.

In the context of this process, collaborative task design functioned as a Trojan Horse, as a means of gaining access to the teacher's objectives and motivations. Employing CHAT, alongside collaborative task design, made this possible. That is, it was possible to identify the object of developing proving in the classroom and explore how the structural resources of the classroom shape this process. The object for the teacher is related with exploration that leads to conclusions related with parts of the mathematics curriculum. However, this object is being conflicted as, while a play-like exploration can facilitate learning, this can prove quite challenging for the teacher, as she wishes to maintain focus and is worried that exploring detracts from that focus. Would exposing teachers to the multidimensionality of objects of activity, enable teachers to effectively develop coping strategies and find creative ways so as to overcome these dilemmas?

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AN OPPORTUNITY FOR PRESERVICE TEACHERS TO LEARN FROM INSERVICE TEACHERS' LESSON STUDY: USING META-DIDACTIC TRANSPOSITION

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This study aims to explore and explain preservice teachers' lesson planning skills and the changes they experienced through their participation in an educational programme involving both inservice teachers and researchers in different capacities. A meta-didactic transposition is used as a theoretical model to describe the dynamic processes of preservice teachers' learning, in addition to inservice teachers' and researchers' activities to conduct the lesson study. The results show how preservice teachers adapt lesson plans and in what way this adaptation is affected by the inservice teachers' lesson study practice. The role of researchers in this context is also discussed.

Introduction

Jaworski et al. (2017) reviewed over 300 publications released between 2005 and 2015 about mathematics teachers working and learning through collaboration; they identified three fundamental themes and four research questions that this ICMI Study adopts (see also Robutti et al., 2016). According to these reviews, different forms of – and opportunities for – collaboration can contribute to teachers' professional development, in which various other parties (such as teacher educators, researchers, supervisors and policymakers) can be involved to promote teachers' learning in different institutional settings. Lesson study, which originated in Japan but has become increasingly popular in various counties, is a well-established form of teachers' joint work in the context of professional development. Since lesson study in Japan emerged as a practice-oriented activity rather than a research-based project, there are still contexts which might reveal significant outcomes of teacher collaboration within the practice of lesson study but which have not yet been investigated in depth in the field of mathematics education research.

Lesson study has been used to support professional development in numerous ways, for inservice and preservice or student teachers (e.g. Fernandes & Zilliox, 2011; Murata & Pothen, 2011; Nakamura, 2019; Rasmussen, 2016). Forms and contexts of preservice teachers learning differ from those of ordinary inservice teachers working. For example, Nakamura (2019) contrasts a general model of lesson study with one of the preservice teachers carried out during student teaching. The latter comprises three phases (lesson planning, research lesson and post-lesson discussion), while the full cycle model (Fujii, 2016) entails two additional stages (goal setting and reflection).

In the literatures on collaboration among mathematics teachers, 'many papers [do] not declare explicitly the theoretical perspectives behind a project' (Jaworski et al., 2017, p. 267), although there are some exceptions (e.g. Goos & Bennion, 2008; Miyakawa & Winsløw, 2019; Sakonidis & Potari, 2014; Trouche et al., 2019). To find common ground in this growing area, it is important for researchers to propose and delineate a theoretical framework to gain a deeper understanding of the type of teacher collaboration in question. To address this need, we came up with a theoretical perspective using *meta-didactic transposition* (Arzarello et al., 2014; Prodromou et al., 2018; Robutti, 2018), which is based on the Anthropological Theory of the Didactic (ATD) (Bosch & Gascón, 2006; Chevallard, 2019) and can be harnessed to describe teachers' dynamic process of professional growth.

In this study, we aim to explore and explain preservice teachers' lesson planning skills, and the changes they experienced, through their participation in an educational programme in which both inservice teachers and researchers are involved in different ways. To achieve this objective, we the researchers (or teacher educators) created an educational programme at a university to engage preservice teachers in lesson planning, observing a research lesson and a post-lesson discussion. In terms of a meta-didactic transposition model, we describe what and how Japanese preservice teachers learnt from lesson study practice by tracing the evolution of their lesson planning knowledge and skills in the programme.

Theoretical framework

The didactic transposition is a theoretical construct of ATD, which posits that certain mathematical knowledge exists in specific institutions such as a community of mathematicians, an educational system, a mathematics classroom or a community of study. In this study, we adopt a meta-didactic transposition as a principal theoretical framework to examine the process of preservice teachers' professional learning during the educational programme, which the researchers at the university designed and coordinated. The basic idea of meta-didactic transposition is as follows:

> This framework is useful to describe a process – analogous to the didactical transposition – that occurs when a community of researchers work with a [group] of teachers in a professional development activity. The term 'meta-didactical' refers to the fact that important issues related to the didactical transposition of knowledge are faced at a meta-level. (Robutti, 2018, p. 4)

Based on this tenet, according to Arzarello et al. (2014) and Robutti (2018), four other features are summarised in Table 1 and Figure 1.

Table 1: Main features of the meta-didactic transposition model (descriptions adapted from Arzarello et al., 2014 and Robutti, 2018)

Meta-didactic praxeologies	- Praxeologies consist of two blocks (praxis and logos). Each block has two components (Chevallard & Sensevy, 2014): *task type* (T) and *technique* (τ) for the praxis block, and *technology* (θ) and *theory* (Θ) for the logos block. *Task type* indicates the problems of a given task. *Technique* is a way of performing the task. *Technology* is a form of explaining and justifying the technique. *Theory* is employed to explain or justify the technology. - Didactic praxeologies describe teachers' didactic activities. - Meta-didactic praxeologies describe researchers' (or teacher educators') activities related to those of teachers.
Double dialectics	- The first dialectic takes place in the classroom, entailing the personal meanings that students attach to an activity in which they are engaged, and its scientific significance. - The second dialectic lies in the teacher's personal interpretation of the first dialectic, and the meaning of the first dialectic among the community of researchers.
Brokering processes	- Brokers facilitate the transition of mathematical concepts from one community to the other. - Brokering is a common habit. Researchers frequently play a brokering role between the two communities involved.
Internal and external components	- The praxeological components can be considered internal or external to a community. - The members of the community share and use internal components, but do not typically utilise external ones.

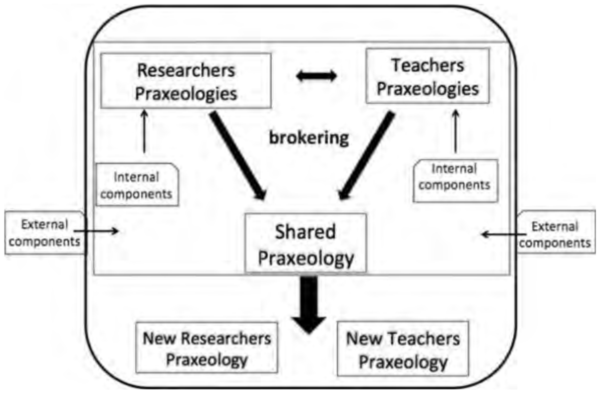


Figure 1: Meta-didactic transposition model (Arzarello et al., 2014, p. 355)

Previous studies using this model show how it works in the context of Italian mathematics teacher education. To adapt it to various settings, we applied these elements of the model in a more generic way. For example, 'researchers' praxeologies' and 'teachers' praxeologies' in Figure 1 can fall under 'personal praxeologies (A)' and 'personal praxeologies (B)', whereby each of them is involved in different communities, and 'shared praxeology' can be 'institutional praxeology'. This is consistent with the notions of 'personal and institutional relations' (Chevallard, 2019). In doing so, our analysis mainly identifies preservice and inservice teachers' didactic personal praxeologies, and their evolution into an institutional praxeology. In the discussion, we elaborate on the double dialectics and the brokering process, with some modifications. However, we cannot report on our examination of internal and external components due to limited space.

Methodology

Context of the study: An educational programme for preservice teachers

This study is rooted in an educational programme for preservice teachers at a Japanese university. Preservice teachers enrol in the university's Faculty of Education, where they receive four years of training. We focus on the educational programme 'Practical seminars for the teaching profession', which students take in their fourth (and final) year as undergraduates; it offers opportunities to identify and reflect on weaknesses in terms of professional knowledge and skills for teaching and helps students strengthen these aspects. The programme consists of two main courses (I & II); each includes a variety of topics on professional development. Preservice teachers need to take one seminar from Course I and another from Course II.

We presented a rough sketch of a seminar titled 'Designing a primary school mathematics lesson', included in Course I of the programme. We organised it through a mathematics lesson study conference held at a designated school, which is an institutionally attached school to the university (hereafter, a 'university school'). In Japan these 'schools also play a leading role and offer various settings for teacher learning at [the] national and regional level[s]' (Miyakawa & Winsløw, 2019, p. 287).

Lesson study setting

The lesson study setting in this paper has some special features. First, the lesson study took place at an annual, half-day conference held at a university school. Second, all 'open lessons' in the lesson study are centred on mathematics (five lessons for primary, three for lower secondary and one for the upper secondary level). Third, the conference is arranged by primary and secondary teachers from university schools, and university professors of mathematics education.

If participants joined the lesson study conference, they were allowed to observe one or more research lessons and then attend a post-lesson discussion. In the educational programme, before the participants joined the conference, we assigned a lesson planning task to the preservice teachers. We asked them to draft a lesson plan for Grade 3 students, referring to curriculum documents and textbooks. We only provided the goal of the lesson and a word problem used therein, which we excerpted from the lesson plan of the practicing teacher. The problem and lesson are in line with the standard Japanese curriculum. Figure 2 shows the textbook pages (translated into English) related to the lesson.

- The goal of the lesson: *Through activities such as representing two quantitative relations using diagrams, students are able to think about the idea of multiples and to understand the associative law.*
- The word problem: *You compared the heights of monkey bars, a tree and a school building. The height of the monkey bars is 2m. The tree is 3 times taller than the monkey bars, and the school building is twice as tall as the tree. How tall is the school building?*

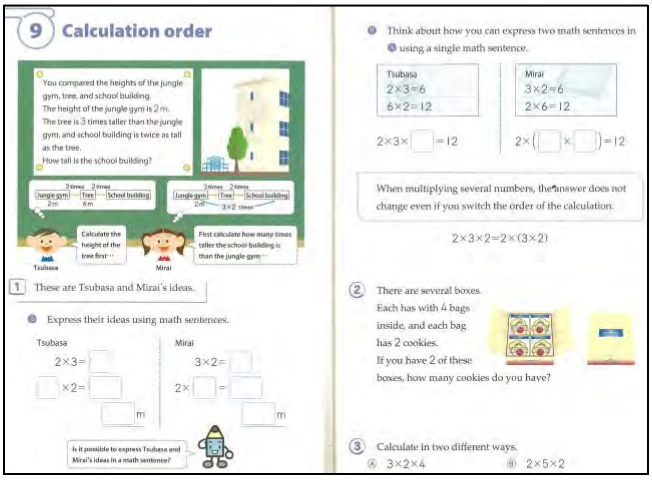


Figure 2: Textbook pages (Shimizu & Funakoshi, 2012, pp. 102-103) [1]

Notes on the lesson

What you noticed about the mathematics subject-matter	What you noticed about the pedagogical ways
…	…

Notes on the post-lesson discussion

- Summary of the comments from the teacher
- Summary of the questions and comments from the participants
- Your questions and comments

Figure 3: The format of the notes for lesson observation

Data collection

This study used three sources of data that were collected from the eight preservice teachers: (1) a rough draft of the lesson plan ('pre-lesson preparation'); (2) written notes from their observations of the lesson and the post-lesson discussion; and (3) a lesson plan for 'the next' lesson ('post-lesson preparation'). The second source was a standardised form filled out by the participants (such as that shown in Figure 3). The third source, the post-lesson plan, was a follow-up assignment wherein the participants considered, 'What would you teach, followed by the observed lesson'? We also referred to the lesson plan written by the practicing teacher, and to the textbook pages related to the lesson (Figure 2).

Table 2 compares our context with a general model of lesson study (Fujii, 2016) and a specialised one for student teachers (Nakamura, 2019). Eight preservice teachers participated in a 'research

[1] Figure 2 is an English translation of the Japanese textbook. Since it is not the newest edition, the problem is slightly different from the one used in the lesson. Nevertheless, most parts of the pages (Figure 2) are the same.

lesson' and a 'post-lesson discussion' at the lesson study conference, and also engaged in 'lesson planning' and 'reflection', which the researchers at the university organised. Since we focused on preservice teachers' lesson planning skills, the 'reflection' phase took place for looking back on the former three stages and planning the 'next' lesson that they observed. The pre-lesson preparation occurred during 'lesson planning', while the post-lesson preparation refers to 'reflection' in Table 2. Brackets indicate that the participants did not enact their own practice, but rather observed the inservice teachers' practice. Nevertheless, preservice teachers often interact with resources such as curriculum documents, textbooks and lesson plans to understand what inservice teachers do during lesson study. In this way, even if a preservice teacher does not directly work with other preservice or inservice teachers face-to-face, our setting may constitute an implicit – but crucial – 'collaborative space' for developing mathematical and didactic knowledge.

Table 2: The research context related to the lesson study setting

A full cycle model (Fujii, 2016)	Goal setting	Lesson planning	Research lesson	Post-lesson discussion	Reflection
A specialised model for student teachers (Nakamura, 2019)		✓	✓	✓	
Our study		✓	(✓)	(✓)	✓

In this study, we examined pre- and post-lesson preparation by referring to the participants' written notes. We also compared these productions and the lesson plan created by the inservice teachers, who practiced the lesson they observed. In the next section, due to limited space, we look at one male preservice teacher's productions and analyse them in terms of the meta-didactic transposition model.

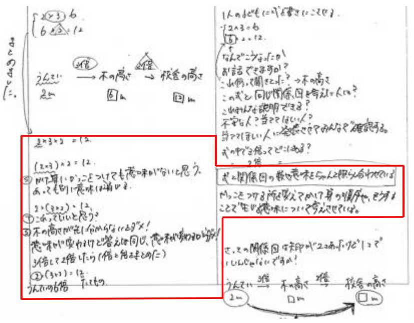
Results

Personal didactic praxeologies

Table 3 presents the preservice teachers' personal praxeology (A) and the inservice teachers' personal praxeology (B) evidenced in their lesson plans. Both praxeologies share the same task (T). To scrutinise the inservice teachers' praxeology, the techniques and logos block can be interpreted as shown in Table 3 (based on the lesson plan). Figure 4 displays what the preservice teachers noticed during the lesson, namely, students' different sense-making of the two mathematical expressions ('2×3×2=12' and '2×(3×2)=12'), with only one part translated into English (framed). Therein appear the differences regarding both the preservice and inservice teachers' expectations of the lesson.

Table 3: Personal praxeologies

(A) Personal praxeology	(B) Personal praxeology
$T_{A\&B}$: To design a lesson for students to understand the associative law of multiplication.	
τ_A: Using the textbook approach as is. θ/Θ: (not specified)	τ_{B1}: Comparing the two expressions (2×3)×2 and 2×(3×2) by using diagrams. τ_{B2}: Overcoming possible misconceptions by students such as '3 times by 2 times' is '5 times'. θ/Θ_B: This lesson provides a basis for understanding the idea of multiples and proportions that have to be taught to higher grades.



$\underset{\sim}{2}\times3\times2=12$ $(\underline{2\times3})\times2=12$ S: I don't think the brackets are useful for the multiplications. But it makes sense, even if you use it. $2\times(\underline{3\times2})=12$ T: Do you agree with this? S: No! We first need to find the height of the tree! The answer is the same, but the meaning is different. The meaning shouldn't be changed! First, multiplied by 3, and then multiplied by 2. (It represents 'times' and 'times') 2 ×(3×2) = 12 Monkey bars 6 times Building	(The teacher intends for…) the mathematical expressions to refer to the numerical relations in the diagram and their meanings. (The teacher facilitates students to think about) the calculation order and its meaning when brackets are placed in different ways.

Figure 4: Notes taken by the preservice teacher during the lesson observation

Institutional praxeologies

Table 4 indicates the institutional praxeology (C), which can be perceived as a shared praxeology, and the preservice teacher's new personal praxeology (D). The institutional praxeology is affected by (at least) three kinds of experiences: (1) the lesson observation (Figure 4); (2) the post-lesson discussion; and (3) the reflective discussion of the lesson study at the university. This information reveals that the technique (τ_C) and the logos (θ/Θ_C) are based on actual classroom practice.

Table 4: Institutional and new personal praxeologies

(C) Institutional praxeology	T_C: To design a lesson for students to understand the associative law of multiplication. τ_C: How to respond to students' different sense-making processes regarding the two expressions. θ/Θ_C: Students tend to understand expressions with 'meanings' by relying on the word problem.
(D) New lesson planning (New personal praxeology)	T_{D1}: To design a lesson for students to understand and apply the associative law of multiplication. τ_{D1}: Focusing on the calculation order to make the calculation easier. τ_{D2}: Considering multiplications with bigger numbers (e.g. multiplied by 15). θ/Θ_D: This lesson is meant to apply the associative law to promote mathematical thinking skills.

A set of components in new personal praxeology (D) is identified by analysing the preservice teacher's new lesson plan, featuring a new (but similar) problem: '*You compared the weight of a kitty (cat), a dog, and a bear. The weight of the kitty is 2kg. The dog is 3 times heavier than the kitty, and the bear is 15 times heavier than the dog. What is the weight of the bear?'* Using this problem, it seems that the teacher intended for students to calculate '2×3×15' by breaking 15 down into 5×3, as follows:

$$2\times3\times15=2\times3\times(5\times3)=(2\times5)\times3\times3=10\times3\times3=10\times9=90$$

According to the teacher's plan, the associative and commutative laws are applied to the calculation to make it easier by including a factor of '10'. The techniques (τ_{D1}, τ_{D2}) and the logos (θ/Θ_D) explain the main focus of the lesson plan that he developed.

Discussion and conclusion

What the teacher learnt

Although the teacher's pre-lesson preparation did not articulate students' processes (e.g. sense-making, reasoning or explaining) with the content, his post-lesson preparation demonstrated an improvement in his description of the content, whereby he anticipated possible learning processes. This evolution was based on what he noticed during the lesson observed at the school, presented in Figure 4. We next discuss the preservice teacher's learning from a theoretical point of view.

Enriching the meta-didactic transposition model

Double dialectic. The first dialectic in the classroom is between the meanings that students attach to the expressions and the mathematical convention to be taught. The textbook says that 'when multiplying several numbers, the answer does not change, even if you switch the order of the calculation'. This was the teacher's intended aim, although the Grade 3 students paid a lot of attention to the fact that the *meaning* does change if one switches the order of the calculation. On the other hand, the analysis mainly illustrates the second dialectic between the preservice teacher's praxeology and that of the inservice teacher. Table 3 reveals that the inservice teacher's interpretation was based on his knowledge of the content and curriculum (Ball et al., 2008), while the preservice teacher's comprehension relied primarily on the textbook. In this way, the double dialectic can occur when a discrepancy in praxeologies is understood as the difference between the communities where the individuals involved are working and learning.

Meta-didactic praxeology and brokering. In this study, the researchers ourselves played a brokering role between the preservice and inservice teachers in and out of the lesson study setting. Institutional praxeology (Table 4) was shared through brokering, based on the observed lesson and the post-lesson discussion. The researchers have two main roles in meta-didactic praxeologies. One is to organise the educational programme by incorporating the lesson study and to provide an opportunity for teachers to learn from the actual lesson. This means that the meta-didactic techniques for that preservice teacher must entail a collaborative space in which one's personal praxeology can shift to align with another's praxeology, and eventually evolve into an institutional one. The other role is concerned with the meta-didactic type of task to offer the participants a chance to engage in post-lesson preparation. This may allow them to reflect upon their existing praxeologies and cultivate them into new, personal ones.

Implications and limitations

The approach and perspective of this study imply that some elements of the meta-didactic transposition model could be more generic when incorporating different contexts in mathematics teacher education. One of the main features of ATD is the institutional approach to didactics. The adaptations we made are based on the idea of 'personal and institutional relations' in ATD. Although lesson study is normally considered an appropriate approach to inservice teachers' collaborative work and professional development, their personal praxeologies may contribute to the evolutions of preservice teachers' praxeologies, and researchers might play a mediating role between personal praxeologies and institutional ones. Our research context enriches what previous studies call the 'paradidactic infrastructure' related to the lesson study (Miyakawa & Winsløw, 2019; Rasmussen, 2016). The meta-didactic transposition model with generic elements can be used to understand the difference between capacities or positions (such as a practitioner or scholar) and the divergence between various persons from unique communities. For example, teachers from different counties might have different praxeologies and build institutional ones through their interactions.

Further research is needed to address the following limitations. Firstly, we focused on a single experience of preservice teachers, but there are similarities and differences among the learning experiences of other preservice teachers who took part in the programme. Secondly, we explored what the preservice teachers learnt, but the process and production of the inservice teacher's learning is not very clear. Thirdly, relating this meta-didactic transposition to other theories can be useful in characterising didactic and meta-didactic praxeologies more precisely, but more theoretical and empirical considerations will be necessary to do so.

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A SOCIAL PRACTICE THEORETICAL PERSPECTIVE ON TEACHER COLLABORATION IN LESSON STUDY

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There is a lack of research on how cultural, social and power aspects affect teacher collaboration when teachers adapt lesson study in countries outside East Asia. In this theoretical paper, I address the issue of how to enhance culturally sensitive and power related understandings of such processes of teacher collaboration. I am inspired by social practice theory particularly the concept of Figured Worlds which I supplement with an emerging perspective on teacher participation in social interactions. I propose four potential benefits of this theoretical approach: 1) it offers new theoretical conceptualizations of teacher collaboration when adapting lesson study, 2) it allows a focus on the individual teacher in collaborative processes, 3) it offers rich interpretive descriptions of micro-level interactions, 4) it takes into account also broader cultural, social and power aspects beyond teachers' local collaborative setting. I exemplify these four benefits by drawing on my colleagues and my long-term research on lesson study in a Danish educational context, and I discuss them in relation to some of the few culturally sensitive studies on lesson study adaptations in countries outside East Asia.

There are various forms of teacher collaboration. This paper focuses on one central form, *Lesson Study* (LS), a teacher-led practice based approach originating in China and Japan that has spread globally (e.g. Saito & Atencio, 2013). In their book introducing LS to the English-speaking audience, Stigler and Hiebert (1999) propose to view teaching as a cultural activity, which is learned through informal participation over long periods and is shared by people within a culture. In line with this, I assume teacher collaboration to be a cultural activity that rests on a set of implicit understandings about the nature of the content of the collaboration, the roles of each participant, and the expectations to the outcome of it. As a cultural activity, teacher collaboration is thus seen to be shaped by and shaping the local school context in question, but also to be affected by and affecting broader cultural, social and power aspects reaching beyond this context. However, only a few studies have researched how such local and broader contextual aspects influence teacher collaboration, when they adapt LS in countries outside East Asia (Ebaeguin & Stephens, 2014). In this theoretical paper, I will address this issue of how to enhance culturally sensitive and power related understandings of such processes of teacher collaboration. In order to do that, I draw on my colleagues and my long-term research on LS in a Danish educational context and use a social practice perspective inspired in particular by Holland, Lachicotte Jr., Skinner, and Cain's (1998) concept of *Figured Worlds* (FWs), which I supplement with J. Skott's (2013) emerging perspective on teacher participation in social interactions.

Initially, I highlight two culturally sensitive approaches in research on LS adaptations in countries outside East Asia in order to position the theoretical perspective that I take in this paper. I then present this emerging perspective on the framework of FWs and provide a narrative from our LS research analyzed using it. I conclude by proposing four benefits of the emerging FWs-perspective based on the narrative and discussing these benefits particularly in relation to the two research approaches.

Culturally sensitive approaches in research on lesson study adaptations

In the rare discussion on how to adapt LS successfully in countries outside East Asia, we have identified different research approaches (C.K. Skott & Møller, 2020), of which I now highlight the two most context sensitive.

The first research approach seeks to identify underlying cultural differences between Japan and the adapting country in order to support culturally sensitive LS adaptations (e.g. Ebaeguin & Stephens, 2014). The authors use Hofstede's five dimensions of a national culture to compare Japan and the Philippines in relation to their cultural capacity to conduct LS. They show that the cultural differences in particular seem to relate to aspects of teacher collaboration such as the Japanese traits to work steadily for small improvements over long periods and to avoid uncertainty (i.e. the extent to which people try to avoid situations of ambiguity), which match the thorough planning of LS. This research approach provides deep understandings of broader cultural and social aspects underlying teachers' LS collaboration in Japan.

The second research approach focuses on power issues at the micro-political level; that is the group dynamic and interactions that emerge when LS is introduced into existing school cultures and stakeholder relationships (e.g. Saito & Atencio, 2013). These authors use post-structuralism inspired by Foucault to reason about teachers' negotiation, submission and exercise of power in LS interactions. Assuming that teachers have different possibilities to construct identities, to speak, and to act depending on their position to power, the authors provide examples from Singapore, Indonesia and Vietnam indicating that LS challenges teachers' work and power relations. This research approach contributes new insights into the complexity of micro-level LS interactions and suggests power issues to be an inevitably but overlooked part of such interactions.

The emerging FWs-perspective

In order to investigate the mutual and transformative relationship between persons and their social worlds, Holland et al.'s framework (1998), especially their concept of FWs, seems appropriate. The framework situates people's (e.g. teachers') practices, discourses, activities, meaning making and structures of privilege in socially and culturally constructed "as-if" worlds that are also contingent on broader social forces. The framework thus renders it possible to capture how people's behavior form and are formed by their cultural and social contexts including aspects of power, and how they learn to orient and respond to prevailing interpretations of these settings. A FW is defined as "a socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al. 1998, p. 52). Holland et al.'s recurrent example is *Alcoholics Anonymous*.

In this paper, I specifically draw on the concept of a *cultural model* (Holland et al., 1998) to interpret the ways teachers realize and come to participate in new FWs (e.g. a world of LS). I use cultural model as an overriding term for the aspects and social behaviours that are taken for granted by people from the same culture. People learn these taken for granted standards implicitly through observation and participation in cultural activities within and across different lived social worlds by generalising such standards from their experiences. Hence, the cultural models underlying LS in Japan and China comprise implicit standards for how to engage in collaborative activities, such as valuing small improvements, which orient teachers' actions and self-understandings. The cultural models also support teachers in reinterpreting their past experiences when becoming actors in a lived LS world.

Two further aspects of Holland et al.'s work inform my focus on how people learn to realize and participate in new FWs. The first aspect concerns how people form the necessary *cultural knowledge*. Viewing FWs as abstractions carried out under guidance, people cannot learn to make and engage in a world institutionally, rather the worlds must "move through us as spoken discourses and embodied practices" (p.251). This means that people learn heuristically by continually rehearsing how to participate in a new world. However, this knowledge formation becomes particularly difficult when realizing a new LS world in educational contexts that do not share the features of the cultural models underlying Japanese or Chinese LS. In such contexts, the second aspect that inform my focus becomes important. This aspect concerns how people behave in terms of *improvisations*. When confronted with a specific situation for which a person has no set responses (e.g. engaging in a reflection session for the first time) he/she reacts impromptu. However, a person is not acting freely. On the one hand, his/her improvisations are based upon the sediments of his/her experiences (e.g. aspects of a cultural model), and on the other hand, he/she must use the cultural resources available in response to the subject positions (see below) afforded him/her in the moment. A person may be able to exceed certain cultural forms by improvising. In this sense, realizing a new FW or reengaging in a past one may form a person's identity. Considering FWs as contexts for identity formation, two interdependent, but not necessarily coincident processes are at play. The first process, *figuring* concerns the general narrative of a FW, for instance the storyline of a Japanese LS, thus imagining its characters and actors, and how it would be to participate in the world. The second process, *positioning* involves the "day-to-day and on-the-ground relations of power, deference and entitlement, social affiliation and distance"(p.127) between people, who occupy or claim different *subject positions* afforded by a FW. Positioning is, thus, about a person's apprehension of her social position in the lived world.

By supplementing the framework of FWs with J. Skott's emerging perspective (2013), we take into account that teachers relate dynamically to a multitude of shifting prominent FWs when participating in social interactions (e.g. collaborative processes). In line with this perspective, we conceptualize participation as a teacher's emerging patterns of engagement in past, present and new FWs, which is influenced by his/her interpretation of the immediate social situation and by his/her simultaneously meaning making of how to contribute meaningfully (J. Skott, 2013). One example is a teacher that during teacher education was introduced to a FW of *Reform mathematics teaching*. Later as a teacher, he/she might reengage in practices related to this FW, if his/her interpretation and meaning making of a social interaction with his/her students prompt him/her to do so.

Skott's emerging perspective also inform our analytical approach as "it is not apparent at the outset what practices and FWs are significant for the teacher in question" (Skott, 2013, p. 552). It is further assumed that the teacher's mode of participation is continuously transformed by broader social and cultural worlds at the school in question and beyond. This means that the analytical objective is not to describe overarching FWs that relate to being a teacher in Denmark. Quite the contrary, the objective is to deduce empirically the FWs a teacher tends to engage in during social interactions and to investigate the fluctuating relationships in the prominence of these FWs. For this purpose, we conduct a double analysis in our research. First, we construct a preliminary set of possible prominent FWs by identifying practices, discourses and values in our data and assembling these into various "as-if" realms. Second, we thoroughly analyze data both to confirm, possibly delete, or add new FWs to the set, and to investigate shifts in prominence of their roles for the teacher's participation. We conceive these shifts as teacher learning.

A narrative of lesson study adaptations in a Danish educational context

I will provide a narrative from our long-term research on teacher learning and collaboration (see C.K. Skott & Møller, 2016, 2017, 2018, 2020). We conducted a four-year LS project at an urban school in the Copenhagen area that had no prior LS experiences. We studied in particular the learning of two teachers, John (mathematics) and Petrea (Danish-as-mother-tongue), as they each collaborated with two colleagues and two facilitators (i.e. teacher educators) in two LS cycles (i.e. one cycle consisted of three two-hours planning sessions and three rounds of reteaching the research lesson (by each teacher) followed by an one-hour reflection session). Focusing on John, I first characterise him as a teacher prior to the project, and then I highlight three major shifts in his ways of contributing to the LS collaboration. To keep the flow of the story, I describe the most prominent FWs continually as they become important for the narrative (contrary to our analytical approach) and on a general level.

John – a Mathematics teacher

John has worked for 15 years at the school in question. He is finishing a coaching program on digital technologies, and already he is working as such at the school. John has a high professional self-confidence. He distinguishes himself by valuing process aspects of mathematics teaching and learning such as engaging students in processes of problem solving. Therefore, we identified a FW of *School Mathematics*, which relates to school-based interpretations of reform-oriented approaches to mathematics teaching and learning. However, the majority of John's colleagues primarily turn to a more general FW of *Schooling* that focuses on the well-being of the individual student and on pedagogical aspects. Due to John's distinctive orientation, he is positioned and positioning himself as a competent Mathematics teacher among his colleagues. This period is also one of curriculum changes. New policy documents encourage teachers to plan and teach with a focus on student learning outcomes, which we conceptualize as a FW of *Policies*. However, at that time John considered that his possibly alignment with *Policies* would not add to the quality of his classroom teaching.

Major shifts in John's ways of participating in collaborative processes

I highlight three significant shifts in John's ways of participating in the collaborative processes from the first to the second LS cycle (see Tabel 1). The first shift is that the new in-the-making world of *Lesson Study* played a more prominent role for John's participation in the second cycle. He engaged more often its practices such as designing meticulously tasks for students and aligning these tasks with lesson goals. Combined with his central position among his colleagues, he took on a more leading and authoritative role in the collaboration (C.K. Skott & Møller, 2017).

The second shift is that a FW of *Teaming* changed to play an even more prominent role for John's engagement. Its actors are John and his colleagues, and one valued outcome is to keep a friendly atmosphere among them. From the first to the second cycle, John is gradually positioned and positioning himself more strongly in *Teaming* primarily due to his claim of the two subject positions: LS experienced (*Lesson Study*) and process-oriented (*School mathematics*). I outline the impact of his strong position on a critical episode, where he taught the third research lesson (second cycle). In the episode, John unexpectedly compromised the problem solving potential of the jointly developed lesson plan by deliberately telling the students what to do. Afterwards, in the reflection session he compared this lesson with the two previous ones taught by his colleagues, claiming the new plan to be perfect. Despite this, his colleagues praised him without any critical comments (C.K. Skott &

Møller, 2016). I shall return to the episode later, and here make do with stressing that John got away with ignoring the process aspect by managing to position himself even more strongly in *Teaming*.

The third shift is that *Policies* and the facilitators gradually came to play important roles for John's participation. In the second cycle, we identify a recurring pattern in his participation as he experienced conflicts between the practices and FWs that he could turn to. I exemplify this pattern by John's use of lesson goals when planning. Normally, John chose goals of pragmatically reasons and did not use them to inform instructional decisions (*Schooling*). However, his reengagement in this practice was disturbed, as he felt obliged to turn to *Policies* that encouraged a recipe-based approach to goal-oriented planning. We interpret that his new engagement was then challenged by both *Lesson study* and the facilitators prompting him to participate in acts of choosing and using goals with the explicit aim of supporting student learning (C.K. Skott & Møller, 2018). In these conflictual experiences, the facilitators gradually came to play the most prominent role for John's participation.

	School mathematics	*Schooling*	*Lesson study*	*Teaming*	*Policies*
First LS cycle	Major	Major	Minor	Minor	No
Second LS cycle	Major	Minor	Major	Major	Minor

Table 1: The role played by prominent FWs for John's participation in LS cycles

In the longer term, we find it likely that the first and third shifts become a continuing part of John's ways of collaborating with his colleagues. The plausibility of this is primarily due to John's strong position in *Teaming*. In contrast, Petrea's experiences seems to be counterproductive for her otherwise promising learning as she lost control when teaching in the second LS cycle (C.K. Skott & Møller, 2017). Losing face among her colleagues, she is likely to reengage in previously dominant FWs (*Schooling*) in order not to risk further her strong position as an experienced teacher in *Teaming*.

Potential benefits of the emerging FWs-perspective

Based on the narrative, I propose four ways in which the emerging FWs-perspective might contribute culturally sensitive and power related understandings of teachers' LS collaboration. I discuss each proposal in relation to the LS literature, in particular to the earlier mentioned two research approaches on respectively underlying cultural aspects and power issues in LS interactions at the micro level.

New theoretical conceptualizations of teacher collaboration in LS adaptations

The emerging FWs-perspective offers new conceptualizations of collaboration processes when teachers adapt LS in countries outside East Asia. It does so by theoretically conceiving LS adaptations as teachers' collective realization of a new FW of LS (inspired by lived LS worlds in Japan and China) into their ordinary collaborative context. This context is dominated by other past and present FWs. Often the significant acts and valued outcomes of these worlds will not be aligned with those of the new LS world as well as the subject positions normally occupied by the teachers will not be similar to those afforded by the new world. Thus, the emerging FWs-perspective transforms LS adaptation questions into questions about which FWs dominate teachers' ordinary collaborative contexts, and how teachers' realization and coming to participate in a LS world will influence their ways of contributing and positioning themselves and each others in the collaborative processes due to shifts in the prominence among their FWs (C.K. Skott & Møller, 2020). In the narrative, the teachers' collaborative context was dominated by *Schooling* and *Teaming*, where for instance strong subject positions were created in terms of seniority as experienced teachers claimed the strongest positions. In C. K. Skott and Møller (2020), we show how the teachers' realization of *Lesson study* profoundly challenged these work and power relations as new collaboration forms were required in this new world and strong positions were created differently which led to displacements in the teachers' traditional hierarchy of power and almost turned it upside down.

I now compare the emerging FWs-perspective with the theories used in the two research approaches mentioned earlier. Based on Hofstede's dimensions of a national culture, Ebaeguin and Stephens (2014) conclude that the general profile of the Japanese culture and the collaborative features of LS seem to coincide, unlike the Philippine case. Although a compelling conclusion, the authors' use of an overriding cultural approach raises questions about the relationships between a national macro-level culture and the practices teachers establish in their local micro-level collaborative setting. That is, does such a macro-level perspective provide sufficiently sensitive contextual insights into how micro-level processes are formed? In contrast, Saito and Atencio (2013) use post-structuralism to focus on teachers' micro-political interations and the ways power are attained by certain individuals and deployed to affect the manner in which key issues are understood and governed in LS collaboration. Though providing compelling examples of such complex interactions, the authors call for theoretical concepts that might provide for more concise analysis as well as broader explanation. The emerging FWs-perspective takes both concerns into account. It does this by empirically exploring which and how FWs in and beyond teachers' local setting prove to play significant in their LS interactions and by allowing extended cultural and social analyzes and explanations of the social dynamics of the teacher groups and their mutual power relations.

Focus on the individual teacher in collaborative processes

The emerging FWs-perspective allows us to study the learning of the individual teacher when he/she participates in collaborative processes. When using a purely participatory approach, an individualistic focus requires that we be able to track the contributions of specific teachers to the LS interactions and to identify shifts in their ways of participating. By using such a double-analysis we contructed a learning trajectory for each of John and Petrea (as indicated in the narrative), which comprises their constellations of prominent FWs at critical points during their LS engagement. The trajectories show how John and Petrea differ significantly in terms of how and what they learned. For instance, they participated differently in the initial realization of a LS world. While John participated with the explorative goal of researching different instructional approaches, Petrea participated performance-oriented aiming to show the facilitators how to teach well. Although these differences may be explained in terms of the teachers' seniority (i.e. being midway or at the end of their careers), they profoundly influenced and conditioned which acts and discourses each of John and Petrea (re)engaged in and what was possibly for each of them to learn (C.K. Skott & Møller, 2017).

In general, the LS literature calls for deeper theoretical underpinnings of teacher learning (e.g. Huang & Han, 2015). The intial tendency was to describe what teachers learned without applying a learning theory and to focus on groups of teachers without distinguishing between individuals. Among the theories now used, acquisitionist approaches are most common, while purely participatory approaches are rare. However, it is worth considering why research studies tend to stick to such individual, cognitive perspectives, when key features of LS are collaboration and participation. There thus seems to be a need of theoretical approaches such as the emerging FWs-perspective that allows us to study the learning of the individual teacher while he/she participates in collaborative processes.

Rich interpretive descriptions of micro-level interactions

The emerging FWs-perspective allows rich insights into teachers' micro-level interactions, their social dynamics and power relations. We obtained such insights by empirically identifying significant FWs in the teachers' LS interactions and exploring the shifting interplays among them. We did so by abstracting actors, subject positions, acts and values from micro-level data and exploring teachers' dynamic orientations and mutual positioning. Hereby we could catch a high degree of complexity in the teachers' participation in the form of their multiple engagements in simultaneous and often conflicting FWs. One example is the critical episode where John ignored the key problem solving potential of the plan. I interpret the episode as a conflict between which of John's prominent practices and FWs he should turn to. Throughout the LS collaboration, John, on the one hand, strived in line with *Lesson study* to explore how to teach problem solving (*School Mathematics*); even to the extent that he as the only one insisted on keeping this process aspect when it was challenged by observations from the two previous lessons. On the other hand, he was concerned that all students could do the tasks (*Schooling*) and not to lose face in front of his colleagues (*Teaming*). The richness of this interpretation of John's constellation of prominent FWs renders it possible to understand and explain in-depth his ways of participating. Often these ways were strung between conflicting social, cultural and power aspects that related both to the immediate situation and beyond.

Such rich interpretations of micro-level LS interactions can help us as researchers to consider to what extent the conditions we identify for successful LS adaptation across countries also look similar when we look behind the processes of teacher collabortion. It is well-known that one counterproductive factor for successful LS collaboration is teachers' resistance to open their classrooms. Saito and Atencio (2013) claim that Vietnamese teachers are used to opening their classrooms, but that their main concern is that a very critical observer positioned more powerful than themselves should enter. At some schools this has led to mistrust and lack of respect. In contrast, Danish teachers' resistance is rooted in their fear to expose themselves as teachers by showing their relationship with the pupils and their management of the classroom. John shows this kind of fear in the critical episode. Thus, although this factor of resistance is recognizable as counterproductive in both the Vietnamese and the Danish context, the reasons for it and the future actions required are different as these reasons and actions relate to the cultural models that underlie the adapting country and the FWs that turn out to be significant in the teachers' LS collaboration in the local educational context.

Broader cultural, social and power aspects beyond teachers' local setting

The emerging FWs-perspective allows us to study the influence of cultural, social and power aspects stemming from broader cultural models or FWs on teachers' collaborative processes. I highlight three such influences. Firstly, the lived world of *Teaming* relates to an overriding FW of classroom teaching in Denmark that is characterized by family-like traits such as keeping a cozy atmosphere and avoiding disagreements and conflicts (C.K. Skott & Møller, 2020). In the narrative, these wider traits are reflected in the affirmative feedback from John's colleagues in the critical episode and generally in the teachers' interactions where criticism, if formulated, is only vague. Secondly, the teachers act in response to issues beyond their local setting as when John engages in the acts of *Policies*. Thirdly, our constructed FWs are imbued with acts and values socially evolved over time within particular cultural settings and models. Thus, all the FWs relate to aspects in the local setting and beyond.

In contrast to the Danish teachers' family-like work relations, Saito and Atencio (2013) stress teacher contests and surveillance as central aspects of the hierarchical and authoritarian educational system in Vietnam. Consequently, awarded teachers are particularly in a position to use LS to enact their power upon younger colleagues; for example, by requiring them or other less powerful teachers to internalise and further reproduce prevailing discourses and ideas. In both cases we see how broader cultural, social and power aspects affect the teachers' micro-level LS collaboration either by avoiding to respond critically to colleague's teaching or by issuing less powerful teachers with harsh criticism. Hence, none of these cases provides ideal settings for teachers to collaborate productively.

Conclusion

I propose and discuss four ways whereby the emerging FWs-perspective might contribute culturally sensitive and power related understandings of teacher collaboration processes. While these proposals and the narrative stress LS as one essential way to support teacher development in countries outside East Asia, they also suggest that teacher collaboration in these countries is a complicated endeavour. My key point is that we need culturally sensitive approaches such as the one taken in this paper to identify aspects of the cultural model of the adapting country and the social worlds that turn out to be significant when teachers engage in LS. In particular, we need to identify and transform the implicit and contextual aspects of these models and worlds that seem to hinder productive LS collaboration.

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USING A PROFESSIONAL GROWTH MODEL TO ANALYZE INFORMAL COLLABORATION AND TEACHER LEARNING

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Reform movements in mathematics education keep changing expectations for teachers and for the mathematics classroom. The long arc of a professional teaching career means that teachers must adapt to a number of waves of change and to do so while also continuing their full time work as mathematics teachers. This study examined how informal collaboration within a high school mathematics department led to professional growth. The Interconnected Model of Professional Growth, developed by Clarke and Hollingsworth (2002) was used to organize and analyze the data. Although the model was developed around the professional growth of the individual, it serves as an excellent tool for analyzing how collaboration affects the domains of practice, personal knowledge, and consequence (effect). This paper argues that collaboration plays a pivotal role in the professional growth of teachers even when that collaboration is unstructured.

Reform movements in mathematics education keep changing expectations for teachers and for the mathematics classroom. The long arc of a professional teaching career means that teachers must adapt to a number of waves of change and that they make these adaptations largely on their own. Although the teachers have finished their formal schooling, collaboration with other teachers helps mediate the process of change. This study was aimed at Improving our understanding of the processes through which teachers incorporate new knowledge and adapt and improve their practice. This study examined members of a high school mathematics department over the course of a semester and used a model of professional growth to analyze the manner in which the teachers worked separately and together on their professional practice. Although the model is focused on individual change, it serves as an excellent tool for analyzing how collaboration affects the domains of practice, personal knowledge, and consequence (effect). This paper argues that collaboration plays a pivotal role in the professional growth of teachers even when that collaboration is unstructured.

The study

The purpose of this study was to examine the process through which teachers direct their own learning and professional growth while working within a supportive, collaborative environment. The study was guided by two goals: 1) analyzing the arc of teacher learning and development within a small collaborative environment and 2) determining the degree to which an existing model of teacher learning was useful in organizing and making sense of the complex interactions that were taking place over time. Good models for teacher learning do more than simply represent that learning; they also serve as roadmaps to encourage and guide the process (see, for instance, Ebert & Crippen, 2010; Chapman & Heater, 2010; Gningue, Schorder, & Peach, 2014; and Goldsmith, Doerr, and Lewis, 2014). Shared frameworks for teacher learning are an important part of advancing the study of collaboration and represent one of the four strands of this conference.

This research study examined high school mathematics teachers in an educational setting. This focus on the constructed meanings of the experience make qualitative research the appropriate methodology. The researcher is employed at the site as a mathematics teacher and department chair. As department chair, the researcher represents department interests and serves as an instructional leader but has no role in evaluating teachers. The department chairs are not members of the administration and the researcher has no line authority over other members of the department. The ready access to the environment of the participants and their willingness to share their insights enhance the role of the researcher as the primary instrument of data collection. This immersion in the experience is an inductive process that allows understanding to emerge from the steady collection and analysis of observation, interview, personal interactions, journals, and documents.

Study Participants

The study participants are members of the high school mathematics department for a school located in the suburbs of New York City. There are seven full-time mathematics teachers, including the researcher, in the high school mathematics department at the study site. Everyone in the department participated in the study although data from one member was excluded because the model used for analysis was developed around experienced teachers and the excluded member is in her third year of teaching.

The participants of this study fit the profile of the teacher for whom Clarke and Hollingsworth (2002) developed the Interconnected Professional Growth Model (IMPG) used in this study; they are "active learners shaping their professional growth through reflective participation" (p. 948). Each participant expressed an interest in participating in the research study as a way of exploring their own pedagogy. The study participants are purposefully chosen to fit the profile of an active, knowledgeable, and self-aware proponent of continued professional growth.

Methodology

Data was collected through interview, observation, and document collection. Formal, semi-structured interviews were conducted at the beginning and end of one semester, in September and December. Observation and discussion throughout the semester between the researcher and the study participants provided an additional avenue of data collection. Classroom observations were conducted so that teachers could illustrate to the researcher how changes in their professional learning were playing out in the classroom. A third data stream was comprised of document collection and analysis. These documents included lesson plans, student work, and curricular documents that the teachers worked on together. This study relies on multiple sources of data in an effort to ensure that the conclusions are reasonable.

The Model

This study used the framework of the IMPG (Clarke & Hollingsworth, 2002). This model provides a critical lens for the analysis of change in learning by separating the components of change into four domains representing outside knowledge, inside knowledge, changes in practice, and changes in outcome (figure 1). Although roughly circular, the model is not cyclical, since the domains are neither causal nor sequential (as they would be if the domains led from one to the next). Instead, arrows represent how change in one domain can influence another, either through reflection or through enactment. These arrows are vectors representing both the direction and type of influence and make it clear that learning is an iterative process that takes place over time.

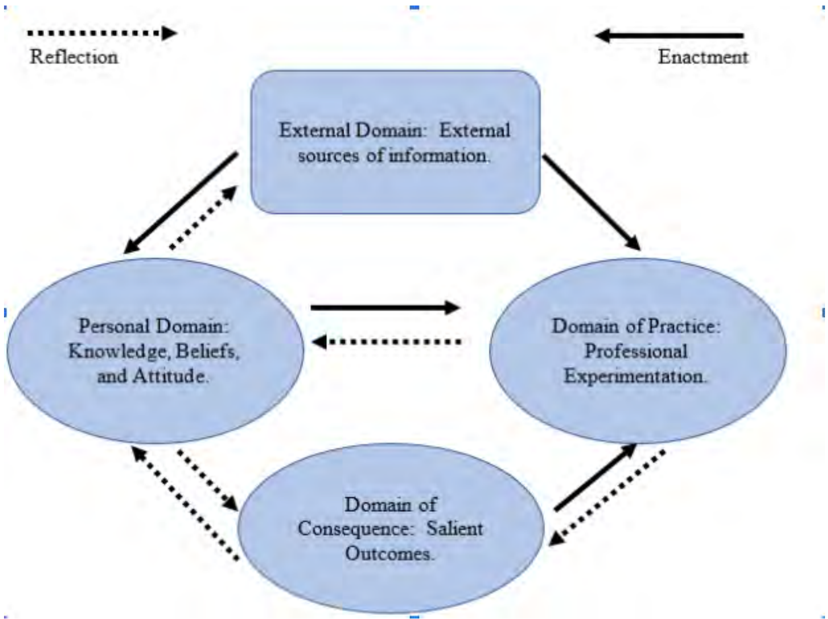

Figure 1: Interconnected Model of Professional Growth. Clarke and Hollingsworth, 2002

The model is particularly useful to the study of teacher growth because of the centrality of the teacher as the key mediating body between the external domain and the domain of consequence. The model casts the teacher as the central agent of change rather than suggesting that change is the culmination of the effects of outside forces. There are only two mediating forces in this model: reflection and enaction. Put simply, enaction is more than acting; it is the process of incorporating a new idea within the structure of existing ideas or putting into place a new practice within an existing routine. The second process, reflection, is conceived of in much the same way as Dewey described it a century ago, as a process of active and persistent consideration (Dewey, 1910).

The inter-relationships the model attempts to capture occur within a larger context, which the authors refer to as the change environment. Although the model is focused on the individual, the study participants operate within a collaborative environment. While the model speaks of the 'change environment' as the context for the individual's professional growth, it became clear over the course of the study that collaboration was the key 'change environment' for the study participants. This represents a much more active interaction with the change environment than the model was intended to capture. The model emphasizes that changes in the external domain create a cascade of interrelated effects that reverberate through the other domains. In practice, the study participants continually returned to the external domain, whether it was through discussion with their colleagues or through the concrete exchange of material. This dynamic -- and continual -- interaction with the external domain became a key characteristic of the processing that the teachers did as they worked.

Findings

Several important themes emerged through the analysis of data. These themes are as follows:

1. The influence of outside knowledge;
2. Knowledge sharing peer-to-peer;
3. Reflection and enaction are social processes;
4. The IMPG and the change environment;
5. Material exchange promotes collaboration and learning.

The Influence of Outside Knowledge

The IMPG specifies two pathways through which outside knowledge affects teachers. In the first, changes in knowledge from the outside (the external domain) are internalized and effect a change in the teacher's internal system of knowledge and belief. From this, changes in practice or outcomes may follow. In the second pathway, changes in the external domain can effect a change in practice first, as teachers test out new ideas or practice in their pedagogy. These changes in practice can then lead to changes in internal knowledge and belief (the personal domain) or in outcomes (the domain of consequence).

This study found that the influence of outside knowledge was powerful. Several teachers referenced the continued influence of workshops they attended several years ago. The workshops were conducted as part of the annual meeting of the National Council of Teachers of Mathematics (NCTM) and the teachers went to a number of workshops over the course of two days. Teachers commented on how much they learned while interacting with other teachers during the workshops. One teacher related how effective it was to see the presenters of a workshop running the activity as if she and the other attendees were students. This made it easier for her to envision how she might use the activity as her own. Another teacher talked about adapting an activity that focused on student-centered exploration. She made changes to the content level and focus but kept the flow of the activity and the student-focus. The participatory characteristics of the workshop activities allowed teachers to see the workshop as a model, or template, for their own practice, while the narrative helped them connect what they were learning to their existing practice.

Knowledge Sharing Peer-To-Peer

A recurring theme from the interview data was the degree to which new ideas and new knowledge were garnered from peers. Addressing what collaboration looks like, one participant described how she might talk about creating a group activity with one person and share a worksheet with another. She is using quiz questions from one colleague and sharing her presentation notes with another. Another teacher pointed out that the activity she invited the researcher to observe came from another teacher, which she adapted to fit her style and needs.

Informal collaboration also provides opportunity to experiment. One of the teachers discussed his apprehension at changing the way he taught calculus. He and another teacher taught separate sections of the class and the other teacher suggested several laboratory experiments that could be incorporated to help students explore key topics. The idea of incorporating experiment felt alien, but he agreed to try them and the two teachers planned out the activities and an approach to introducing them. This teacher is now teaching both sections of the class; there is no more lesson planning with the other teacher. Nonetheless, he was eager to explain that he is still using those experiments, and more, because it became clear that the students are engaged during the labs and that they remember the material better by participating in the activity. This anecdote illustrates the utility of the IMPG model in exploring professional growth within a collaborative environment. The teacher is describing an idea that came from a colleague; it is a change in the external domain. The pathways of enactment and reflection carry the influence of change from this domain to the personal domain and then the domain of practice. The teacher has seen a change in student learning, the domain of consequence, which has solidified the changes to the domains of both knowledge (personal) and practice.

Reflection and Enaction are Social Processes

A recurrent theme in the data was the degree to which the study participants shared their experiences with each other. Although the model is designed around the learning process of a single teacher, the teachers continually acted as part of a larger social unit. The model uses the dual processes of reflection and enactment as the forces that bring the influence of change in one domain to another within the individual. Nonetheless, social versions of reflection and enactment were also readily observed as teachers repeatedly discussed their work with each other, sharing how a lesson went, sharing ideas about approaches, and even sharing their papers, notes, lesson plans, and assessments. They tested out their ideas in conversation, reflected aloud on lesson and student outcomes, and generally shared their thinking aloud, throughout the day, and across all of the haphazard combinations of who was available.

The degree of interaction among the teachers suggests that reflection and enaction should be seen as both personal and as public processes. The study participants used each others' experiences to revise worksheets, change their lesson structure, and amend explorations and then shared their own results with each other. The communal nature of reflection was not just apparent in observing the teachers but also in how easily teachers could trace the pathway from something they were doing to how much it was the result of other teachers helping them think through ideas and approaches.

The IMPG model does not explicitly address this social component of the learning process but it is clearly present and influential. The model implies a cascade of learning events as changes in each of the four domains trigger changes in the others. The behavior of the participants in this study suggests that this cascade of learning is at least as much a collaborative event as it is a private one.

The IMPG and the Change Environment

The focus of the IMPG model, as designed by Clarke and Hollingsworth (2002) is on the process of professional change and growth in the context of the individual teacher. Nonetheless, the model recognizes that there is an outside environment within which the learning process is taking place. This contextual background is referred to as the 'change' environment Although this change environment is not directly captured through the model, the authors recognize the importance this environment can have on the how teacher learning evolves. Each of the teachers made reference to the importance of the environment within which they worked, specifically mentioning the degree to which this change environment supported their professional growth.

The teachers attribute much of their success to the collaboration they enjoy with each other. They talked about sharing resources among themselves, discussing pedagogy and content together, and are quick to ask each other for help. During the follow up interview, one of the teachers captured this emotionally, saying that she loved working with her colleagues and she really benefited from talking through her ideas with them. Several teachers mentioned the importance of having common space as a department. One of the teachers suggested that they were able to work on teaching so much because everyone shares a common office space. Other teachers made similar comments, noting that teachers are always talking about how a specific lesson went, or what they're designing for a class, and that the conversations are all about teaching mathematics because all the math teachers are sitting in the same office.

Material Exchange Promotes Collaboration and Learning

The study participants made numerous references to the concrete exchange of material. This included physical objects such as a worksheet or test, semi-material objects, such as lesson plans or the best sequencing of content, and totally abstract material, such as suggestions on how to frame a concept or create an exploration that would drive student learning. This exchange was continuous at all levels and appears based on the assumption that there are common elements of good teaching that transcend a particular classroom audience or content. Teachers used this continuous exchange to improve their work and to try out new ideas. Other researchers (see, for instance, Coutinho & Lisbôa, 2013), have also found that collaborative learning is enhanced by the creation of shared artifacts. The model details the cascade of effects that occur after a change in outside knowledge, but the constant flow of information among the study group supercharged this cascade by continually introducing new knowledge.

Conclusion

In addition to the specific findings outlined above, this study concludes with the idea that the IMPG model is a useful tool in framing teacher change within a collaborative environment. Goldsmith, Doerr, and Lewis (2013) posit the need to develop shared frameworks to use in studying teacher learning as a way to advance the study of professional growth. They suggest that the IMPG model might be an appropriate choice because it appears to be sufficiently robust to allow different research foci and capable of becoming more and more detailed as additional research findings expand and articulate the model.

The data from this study match the model well. Teachers readily identified sources of new knowledge, such as workshops they had attended, research they conducted in developing lessons, as well as the ideas, worksheets, and practices that they get from each other. Similarly, teachers differentiated between things that they were doing differently and differences in how they thought about teaching. The evaluation of consequences (outcomes) played a major role in the iterative process of change; teachers were candid about their reflections on what had made a successful lesson or why they were making changes to a specific worksheet or exploration. These four domains -- external, personal, practice, and consequence -- were useful categories in examining the processes and events that were gathered as data during the study.

Interestingly, the study participants do not connect their own work to the specifics of the model. The teachers were clearly engaging in the processes of reflection and enactment. They thought deeply about what they wanted to teach, how they wanted students to experience the class, and about what was working or not working. Nonetheless, teachers did not characterize their work as reflection or enactment. Instead, they used phrases suggesting that these processes were subconscious rather than explicit. Teachers took for granted the idea that the teaching profession involved continuous adaptation and that their own learning and professional development would take place over time. In essence, the teachers are acting the part of the lifelong learner without concerning themselves with the structure of the work. This suggests that the utility of the model lies with researchers rather than to the members of the collaborative teams.

Participants were candid in their discussions and open with their classrooms. They expressed delight in sharing their experiences and were interested in understanding how their own experience compared to the interconnected model. The power of collaboration was inherent in their stories and in the materials and practices they use professionally use every day. The lens of the IMPG allows the exploration and study of professional learning as a dynamic process. Where the focus of collaboration is on the group, the model helps highlight the effect of the group on the individual. The model is a useful research tool in furthering our understanding of how growth happens and where it is most subject to influence and support.

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FRACTALS: MODELS FOR NETWORKED TEACHER COLLABORATION

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This paper draws on several research projects focused on teacher collaboration to highlight the use of the fractal as a model for networked teacher collaborative communities and research with such communities. The paper primarily addresses Theme A which discusses theoretical perspectives but the paper also crosses into other themes. Through the discussion of several research projects, a fractal will be shown as a model which highlights the iterative and self-similar nature of purposeful collaborative teacher communities. The fractal also emerges as a conceptual model for engaging in research with teacher collaborative communities as it allows for iterative processes that respects the reciprocal learning that occurs between researchers and teachers.

Mandelbrot (1977) suggests that fractals, geometrical objects generated through iterative techniques with self-similar characteristics, can be used as conceptual models in diverse fields. This paper uses the fractal model to explore the iterative dynamic of collaborative learning communities of teachers that in turn, support communities of learners in mathematics classes. In particular this paper considers several studies (Graves, Suurtamm, & Benton, 2005; Suurtamm, 1993, 2017; Suurtamm & Graves, 2010; Suurtamm, Koch, & Arden, 2010; Suurtamm & Koch, 2014) that examine the role and working of teachers' collaborative work to understand and implement pedagogical and assessment shifts in their mathematics teaching practices in their classrooms. The goal of this paper is to examine the variety of ways that the model of a fractal might inform and emerge from collaborative work with teachers.

Theoretical Framing

My work aligns with others who see the teacher as the key element in educational change and improvement (e.g. Fullan, 2007; Hargreaves, 2009). What teachers do and think, the mathematical tasks they use, the tools they make available and the way they respond to students' thinking are critical to the development of mathematical understanding (Ball & Even, 2004; Boaler, 2002; Boston & Smith, 2009). Yet, teachers often work in isolation with little support and thus may not be in a strong position to explore new pedagogical practices on their own. Many suggest that as teachers adopt new practices in their classrooms, their assumptions and beliefs about teaching mathematics are challenged. Communities of practice (Lave & Wenger, 1991; Wenger, 1998) can support new learning and combat the isolation that teachers often experience as they test out new ideas in their classrooms.

Collaborative inquiry has shown great promise in supporting teachers (Bruce, Flynn, & Stagg-Peterson, 2011). Teacher dialogue and collaboration are vital aspects of encouraging change in classroom practice (Hargreaves, 2009; Lachance & Confrey, 2003; Putnam & Borko, 2000). Teachers derive support, motivation and direction from one another. My stance of inquiry (Cochran-Smith & Lytle, 2009) creates situations for teachers to collectively establish what they want to work on and how they want to work. As a researcher, I may offer support and facilitation but teachers determine the ultimate focus of the inquiry.

My work is informed by theories of constructivist learning (Cobb, Yackel & Wood, 1993) and sociocultural theory (Vygotsky, 1978, 1986) as I consider how teachers and students construct new understandings of mathematics teaching and learning. Further, I lean on complexity theory to examine and describe the dynamic and complex systems of students' and teachers' learning communities and the connection between these communities. The work of Davis and Simmt (2003) helps to consider the connected and dynamic components of these complex systems. In turn, the examination of the dynamic of collaborative learning communities provides significant understanding of structures that promote teacher and student learning. In speaking about learning systems, Davis and Simmt (2003) use the metaphor of nesting and refer to nested learning systems. It is this notion that has lead me to the vision of a fractal as a model for working with collaborative teacher learning communities.

Fractals as models for learning communities

I use the image of a fractal to discuss the iterative and self-similar nature of the collective work of researchers and teachers, as well as the collectives created in classrooms. A fractal is an object that is created through a feedback process that occurs in living and nonliving natural systems (Mandelbrot, 1977). An important property of a fractal is that it is self-similar; that is, on close inspection of a fractal object, each part of the object resembles the object as a whole, often through several orders of magnitude. Fractals seem to be formed where complexity, such as the random motion of atoms, is constrained by simple forces. This formation is thus the interplay of chaos and order and displays the geometric implications of iteration. Figure 1 shows a computer-generated fractal image that was created through an iterative process that, in simplified terms, starts with a complex number, performs an operation on that number and then extracts part of that result to begin the process again. The results of each iteration determine a point on the plane which taken together, create the fractal image (Suurtamm, 2011). Throughout my career I have used the metaphor of a fractal as I facilitate and research self-similar, nested, communities of practice and the iterative dynamic of teaching, learning, and research within such communities. I use several examples to illustrate.

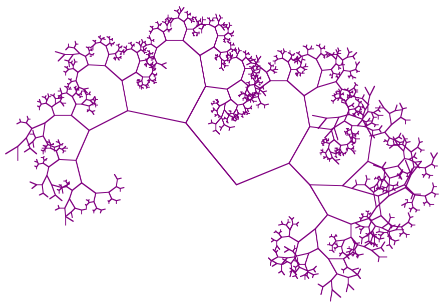

Figure 1: Fractal Image

Fractals as models for collaborative learning communities

The first study that I discuss is from the beginning of my career and was an interpretive study of seven teachers in a secondary school mathematics department (Suurtamm, 1993). These teachers worked collaboratively as they grappled with a move from traditional, teacher-directed classroom strategies to a more inquiry-oriented model of teaching that included increased problem solving, connections to the students' outside experiences, cooperative learning, and alternate assessment and evaluation strategies. The teachers had decided that they would like to make changes within the Grade 9 mathematics program and would work collaboratively to develop and implement these changes. The changes were not mandated but rather were changes that the teachers determined they would like to make. They were concerned with the lack of confidence that students displayed in mathematics. The teachers were concerned that this, coupled with students' perceived weak understanding of mathematics concepts made it difficult for the students to take risks to engage in a problem-solving approach in mathematics. The teachers saw an opportunity to make changes through a new teaching approach.

The study followed these teachers over a period of one year. Data was collected through three interviews with each of the teachers as well as participant observation at six of their planning meetings. The findings of the study consisted of two parts. The first part dealt with the reporting of what the teachers were doing within their classrooms. Descriptions were made of the types of manipulatives that were introduced in the classroom, the assessment methods used, the co-operative learning activities that were developed, and the tasks and units that were designed on particular themes. The second part of the findings dealt with how the teachers were experiencing the implementation process. Teachers had a variety of concerns but common themes between the concerns were recognized. The data suggested that teachers often felt uncomfortable with many of the new methodologies. Some of their discomfort came from the unfamiliarity of the teaching methods and some of it came from their uncertainty as to whether it was the best approach for the students. Teachers reported that course content did not seem to be covered as quickly as it had been with a previous traditional, teacher-directed style and each teacher had to struggle with justifying what was gained when less content was covered. For the most part, teachers felt that, as an outcome of this work. the students had a better grasp of the mathematics that they did learn as well as a more positive attitude towards mathematics.

Another important finding from this study was that the support of working collaboratively with their colleagues helped the teachers to share their concerns and to be more willing to take risks to try new ideas within their classroom. Further, they developed more confidence as they progressed with the project. In fact, as these teachers worked collaboratively to develop classroom approaches that would engage their Grade 9 mathematics students, the teachers exhibited many of the risk-taking qualities that they hoped to encourage in their classrooms. They leaned on their own collaborative group of teachers for support as they took these risks, in similar ways to how they saw students' collaborative work support new understandings. The data suggested that teachers' development of new pedagogical and mathematical understandings emerged in similar ways to how they saw their students' mathematics understandings develop. I saw self-similarity between the collaborative community of teachers and the collaborative communities they were developing in their classrooms. This self-similarity was coupled with a reciprocal process of feedback: classroom experiences feeding collegial dialogue and collegial dialogue feeding new classroom experiences. The fractal image, with its view to self-similarity and iterative dynamic, emerged as a model of the reciprocal feedback, the iterative processes and the self-similar characteristics of the complex web of teaching and learning that occurred in this project.

Fractals as models for engaging in research with teachers

In another project, the Curriculum Implementation of Intermediate Math (CIIM), the fractal image emerged as a way to think about how I and my colleague, Barbara Graves, conducted the research. This large-scale, multi-year project examined the way an inquiry-oriented curriculum is understood and taught and included multiple phases and types of data collection (Suurtamm & Graves, 2010; Suurtamm, Koch, & Arden, 2010). Our study examined teachers' experiences, beliefs, and instructional practice through teacher questionnaires (n =1096), individual and focus group interviews with teachers and leaders in mathematics education, and nine 1-week case studies of environments where there was evidence of inquiry-oriented classroom practices. Participants came from 42 of the 60 English-language school districts in the province. The findings showed that, while, in some classrooms, the enacted and the written curriculum may be somewhat dissimilar, many classrooms enacted inquiry-oriented practices, particularly in the case studies. The work also provided details of the dilemmas that teachers face as they shift practice, and the vital role that administrative support, teacher collaboration, and program coherence play in supporting teachers as they develop new pedagogical practices.

The research design was informed by complexity theory and used the model of the fractal to reinforce the use of iterative rounds of data collection, with each round building on the previous one. For instance, interviews with provincial and school district leaders in mathematics education not only provided us with their perspectives, but also gave us insights to better develop questionnaire items for teachers. The questionnaire asked about teachers' beliefs about mathematics teaching and learning, comfort with a range of classroom practices and mathematics content, their use of a variety of classroom practices, and their professional development experiences and needs. The work that we did with the mathematics leaders prior to the questionnaire helped to present options on the questionnaire that might most resemble teachers' possible responses.

The research design also allowed each iteration of data collection to feed into the next. For example, once initial focus group interview and questionnaire data were analyzed, we held a two-day retreat to share these results with mathematics coordinators, consultants and policy makers, and as they discussed the implications of the results, they also became research participants as we gathered data about what they saw in the data as expected, surprising, and next steps in supporting teachers' work. The fractal image worked as a model but this time, it not only represented the interconnectedness of communities of practice, it also represented the iterative dynamic represented in our research design.

Networked learning communities

In a more recent project, the fractal model emerged in the design of a professional learning project, in the makeup of the 10 professional learning communities (PLCs) involved, and in the way that the research was conducted. In 2015, the Ontario Ministry of Education (OME) and the Ontario Association for Mathematics Education (OAME) established a project with a view to enhance the teaching of Grade 9 Applied Mathematics in Ontario. I was invited to design and carry out research to support and document teacher learning through this project. We created a Steering Committee that involved OME, OAME, myself, and other stakeholders so that practitioners, researchers, and policy makers were represented. The Steering Committee began by inviting schools to apply by indicating the schools' interest in developing a professional learning community (PLC), suggesting PLC goals, and being willing to take part in research. Each PLC was to be comprised of: a school administrator, a school or district mathematics lead, Grade 9 mathematics teachers, a Special Education Resource Teacher, and others who would support Grade 9 mathematics. Funding was provided to support 10 school PLCs to meet one day per month for two years and to have the support of my research team consisting of myself, research collaborators from other universities as well as 8 research assistants. As a university research team, our purpose was 1) to help design the professional learning and support PLCs through their process, and 2) to document the process and address the question: How can teachers enhance their understandings of the interactions between Grade 9 Applied Mathematics curriculum, pedagogy, and students' needs? The focus of the initiative was on teachers deepening their understanding of the mathematics curriculum with an emphasis on supporting learners' needs.

Loucks-Horsley and Matsumoto (1999) identify several key components of effective professional learning: curriculum-based; focused on student thinking; collaborative and within supportive school and district environments; and aligned with other elements in the system. These were all considered as we designed our meetings with the 10 PLCs, suggested professional learning models to them, and carried out our work. We began with a project launch where all PLCs met together. We discussed possible PLC activities, presented research on professional learning models, engaged in activities connected to the mathematics curriculum, and provided time for PLCs to discuss their plans amongst themselves and with other PLCs.

Following this, during the two years of the project, each PLC met one full day per month in its own school, with the date and agenda determined by the PLC based on their focus. During each meeting, a researcher or research assistant (RA) documented the PLC process and provided support (e.g. resources, articles) when required. As well as individual monthly PLC meetings, there were times over the two years when all the school PLCs were brought together. These meetings often emerged when particular needs were identified. For instance, at the end of Year 1, all the PLCs, research team, and steering committee got together for a 2-day retreat to share their work, take part in workshops on areas of need, and plan for Year 2. This meeting was prompted by the research team noticing the number of similarities between the teams, both in terms of what they were working on and areas where they had expressed needs. Table 1 provides an overview of the schedule of the two years.

Table 1: Schedule of Project Events

Event	Date	Participants
Year 1		
Project Launch	October 2014	All PLCs, steering committee (SC), research team (RT)
Regular monthly PLC meetings	Nov 2014 – May 2015	PLC members, RT member
Adobe Connect sessions with PLC representatives	Nov 2014 – May 2015 (every 6 weeks)	SC, RT, representative of each PLC
Steering committee meetings	Sept 2014 – July 2015	SC members
Research team meetings	Sept 2014 – Aug 2015	RT members (researchers & RAs)
Presentations at OAME conference	May 2015	Various members of PLCs and RT
Wrap-up of Year 1	May 2015	School PLCs, RT, SC
Year 2		
Summer Institute	August 2015	School PLCs, RT, SC
Regular monthly PLC meetings	Sept 2015 – May 2016	PLC members, RT member

Adobe Connect - PLC represent	Oct 2015 – April 2016	PLC members, RT member
Steering committee meetings	Sept 2015 – July 2016	SC members
Research team meetings	Sept 2015 – Aug 2016	RT members (researchers & RAs)
May Institute	May 30 & 31 2016	School PLCs, RT, SC
August Provincial Conferences	August 22, 23, 24, 25, 2016	600 Ontario educators, PLC members, RT, SC

Each school PLC was considered to be a case in this multiple case study project (Stake, 1995, 2006) and was supported and documented by a designated research assistant (RA) who attended all meetings of the group. Data was collected through observation notes and recordings of each monthly school PLC meeting, short focus group interviews, artefacts from PLC meetings, and individual interviews.

The research team met each month and the RAs shared their summaries of the school PLC meetings. These meetings helped to inform the direction of the project, to explore ways to further support the PLCs, and allowed the research team to see the full picture of the project as it unfolded. The research team meetings also helped with the cross-case analysis, while supporting and mentoring the RAs. Over the two years the teachers worked on various pedagogical practices and ways of approaching mathematical topics. Some of their foci included: the use of rich tasks, alternative assessment, creating a positive learning environment, going deeper with the curriculum, or re-organizing the curriculum sequence. There were several aspects of the project that the data suggested supported teacher learning. Here we focus on two of those research results.

Self-direction of PLCs

The data provide evidence that each PLC set its own direction and determined how to use their meeting time. Although the research team suggested different types of professional learning models, each PLC determined what they would work on and the ways that they would work. Analysis of the data shows that the PLCs engaged in a variety of activities within their meetings that included: classroom observations, co-teaching, co-planning, lesson studies, book studies, co-designing new tasks, moderated marking, examining data, anticipating students' responses, examining students' thinking through responses to tasks, participating in interviews with students, examining video recording of lessons and/or students' thinking, as well as discussions with teachers from other grades or schools. The value of this self-direction in terms of what PLCs worked on and how they worked emerged from the data, as this quote from a mathematics department head demonstrates:

> The biggest thing is the fact that it is teacher-driven, the fact that we have time to work together, like we meet monthly. We have time to sit down and plan, and then reflect on what we've done and say "well this went well, but this didn't, here's where we should go next". Just that, is huge! The fact that it's changed all of our teaching comes out of that, cause when you do have time to properly reflect and learn from other people ... we've learned from other people in the province as well (Case 8).

Evidence of the structure supporting learning

The data analysis signals several components in the structure of the PLCs and the structure of the entire initiative that supported learning. Results show that the diversity of roles in the school PLC contributed to members' learning as participants brought different knowledge and experience to the discussions. Participants reported that learning was "messy" and there were tensions at times. For instance, although participants might have been working on the same thing, such as a task or a new assessment, PLC members noted that they were working on them in different ways

Data, particularly from individual interviews at the end of the project support that the responsiveness of the overall project allowed for the emergence of new structures and ideas. Table 1 shows the multiple times that the PLCs met on their own, together with other PLCs as well as ways the Steering Committee and Research Team created opportunities for support. The frrquent meetings of the research team and the steering committee provided the opportunity to share the work of the PLCs and respond to their needs. For instance, the Summer Institute of 2015 was not part of the initial project plan but emerged as a structure for PLCs to share their work, receive targeted professional development, and have time to plan for Year 2. Networking with other PLCs helped to broaden the ideas that PLC members were exposed to. As one participant suggested: "the members who were initially more conservative in their thinking have become more open and positive about trying new ideas" (Teacher, Case 10). Another participant saw this networking as critical to the learning of the PLCs:

> I think that's it's nice that we were able to cross-pollinate with other schools at the anchor sessions…because this is where that [the way their course was structured] came from. This came from [Grade 9 teacher] seeing the spiralling…at the summer institute and so this is where the idea was born and so I think that the cross-pollination this project has provided really got what we needed into [our school district] (Instructional Coach, Case Study 5, 2016)

In this project, we see many different groups, the 10 PLCs, the steering committee, and the research team, connecting and growing together, resembling the image shown in Figure 1. Drawing on complexity theory we see the emergence of new ideas being facilitated by the connections within the PLC and the entire network of communities and individuals witihn the project, modeled by the self-similarity of the connected communities, and the iterative dynamic of ideas moving in and out of different communities and phases of the research.

Concluding comments

The self-similar and iterative properties of fractals appear in research and collaborative work with teachers in a variety of ways in this paper. First, in the collaborative work of teachers, we see self-similarity between teacher learning in communities and student learning in classrooms. Teachers work together trying out new mathematical and pedagogical ideas, taking risks in a safe environment, and sharing ideas and learning through dialogue and discussion in a similar way that they hope their students test out new mathematical ideas and engage in dialogue in their classrooms. Second, in terms of research, engaging in collaborative research with teachers provides a space to allow research designs to emerge or shift based on the interactions with participants, demonstrating the iterative feedback similar to the way such looped iterations are used to create fractals. Third, in large-scale projects with multiple teacher collaborative learning communities, the full community of networked teams (e.g. teacher teams, research teams, steering committees) can interact with one another in ways that support everyone's professional learning and allow for responsiveness to needs and new learning. In all cases, new ideas and actions emerge through this dynamic and iterative process of reflection and action. The fractal image models the reciprocal feedback, iterative processes and self-similar characteristics of the complex web of communities of learners, teachers, policy makers, and researchers working together.

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LESSON STUDY: A CASE OF EXPANSIVE LEARNING

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Lesson study is increasingly used throughout the world as a model of collaborative professional learning. In this article we describe the theoretical foundations of an ongoing study which seeks to explore how we might inform collaborative learning with a focus on the development of curriculum coherence. The study involves collaboration between teachers and researchers in both the United Kingdom and Japan. We draw on the theory of expansive learning as a development of Cultural Historical Activity Theory and give insight into the potential this has to inform our work with a range of didactical devices or artefacts that provide connections across topics of conceptual understanding of mathematics and development over time. Expansive learning as a theoretical lens helps us understand our exploratory work in this area and provides insights into how this may be taken forward over the course of the study.

Introduction and Context of the Study

Lesson study is a long-established model for teacher professional development that originated in Japan and involves a process of identifying a professional research question, collaborative lesson planning, observing the lesson, analyzing what takes place, and reflecting on outcomes in relation to this question in a "research lesson" (Lewis, Perry, & Murata, 2006; Lewis & Hurd, 2011). Thus lesson study provides for collaborative professional development that "ties both theoretical and practical learning together in a most authentic way – *through teaching*" (Pothen & Murata, 2006, p. 824).

This article originates from the work of an ongoing study that brings together teachers and researchers in the United Kingdom and Japan seeking to understand how in collaboration (of teachers in each country separately and of researchers between countries) and working within a lesson-study paradigm they might better understand how to develop curriculum coherence. Put simply, we wish to establish how we might gain insight into how to organise teaching to provide students with a more connected experience of learning mathematics. The current focus has been informed by observations and discussions of the carefully designed textbooks used by teachers in lesson study activity in Japan.

In this article we provide insight into the theoretical underpinnings of the study. In particular, we draw on the theory of expansive learning (Engeström, 1987, 2001) to provide insight into how we have considered the work to date and how this is informing the design of future approaches that support our aim to develop professional learning for curriculum coherence. The theory of expansive learning builds on the more general cultural-historical activity theory (CHAT), which is firmly rooted in the Russian school of social psychology originally developed by Vygotsky, Leont'ev, Il'enkov, and Davydov. In the next and following sections, we explicate the ideas appropriate to the study that relate to CHAT and the theory of expansive learning. We provide a brief illustrative case showing how these theoretical ideas might be harnessed to better understand how we might use lesson study to develop coherence in curriculum implementation in mathematics.

Cultural Historical Activity Theory, Boundaries and the Potential of the Role of Didactical Devices

Cultural Historical Activity Theory

Cultural Historical Activity Theory (CHAT) has been increasingly used to make sense of the complex activities that are involved when groups of individuals come together in a group to work with common purpose to achieve some particular outcome(s). The theory provides a useful lens through which to view how both the actions of individuals and the joint activity of the group are mediated by a range of factors in pursuit of individual and jointly-shared goals. Particularly helpful in the case of the study reported here is *third-generation* CHAT, which provides insight into interactions between two or more systems. Consequently, ideas of boundaries, boundary crossing, and boundary objects prove useful (Wake, Foster & Swan, 2013).

Fundamental to CHAT is the Vygotskian notion of goal-directed actions of the individual (Vygotsky, 1978). This considers how the actions of individuals are mediated by *instruments*. It is important to our later analysis that we distinguish carefully at this point between the terms *artefacts* and *instruments*. The word *tool* is often used instead of *instrument*, and this we find unproblematic, but it is important to draw a distinction between the use of *artefact* and *instrument/tool. Artefacts* are devices that are generally available with multiple potential uses, whereas instruments/tools are being used for a specific purpose (Daniels, 2001). For example, the pedagogic strategy of *think-pair-share* is used by teachers in classrooms around the world to facilitate constructivist, dialogic approaches to teaching. However, the teacher's decision to use such a device shifts it from being an available artefact to being an instrument and we consider this as an artefact being made instrumental in its use.

A wide range of artefacts is available to teachers in classrooms, including, for example, diagrammatic representations, texts, presentation slides, manipulatives, and non-material objects such as discourse/language in the sense of Werstch (1991). Our study identifies three important classes of artefacts available to the mathematics teacher: general pedagogies, mathematics-specific pedagogies (i.e., didactics) and what we have termed *didactical devices*. As explained above, we see something like *think-pair-share* as an example of a general pedagogy. An example of a mathematics-specific pedagogy would be the strategy of seeking to draw learners' attention to mathematical structure by asking students in their lessons to give an example that demonstrates understanding of a particular concept and then asking for "another", " and another", "and another" (Watson & Mason, 2006). This particular pedagogic strategy has generality of use in mathematics teaching, but may be of perhaps lesser value in the teaching and learning of other subjects. In this study we are concerned with the construct of didactical devices which are more closely connected with specific aspects of mathematical knowledge and its structuring. We will provide examples of such devices later in this article. It is our emerging belief that this latter category of artefact has the potential to provide an important set of devices that can facilitate teacher learning in relation to teaching for curriculum coherence. Importantly, here we consider the learning of both individual teachers and researchers as well as that of the collaborative collective lesson study research group.

The notion of the goal-directed action of individuals (upper triangle in Figure 1) was expanded by Leont'ev (1981) in second-generation activity theory to include the community in which individual actions are aggregated in pursuit of joint activity. The lower triangles highlight the additional mediating nodes considered by Leont'ev, indicating how ideas of *community*, *division of labour* and *rules* mediate the actions of the subject as an individual in relation to the object of activity of the collective.

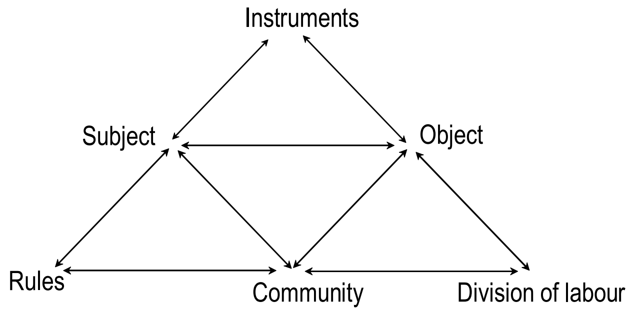

Figure 1: Engström's Cultural-Historical Activity Theory model that introduces mediating factors of the activity system proposed by Leont'ev that brings together individuals in pursuit of a joint enterprise.

There are many rules, both explicit and implicit, that regulate the behaviour of individuals in collaborative enterprise. Important to our study is the joint enterprise of lesson study and the community of teachers and researchers that are brought together in the lesson study group. In this case, the group develops ways of working that over time lead to expectations and rules in relation to all stages of the lesson study cycle: identifying issues, developing a research question, planning the research lesson, teaching and observing the research lesson, post-lesson reflections and discussion and planning future activity. Leont'ev (1981), in his consideration of the activity of the collective, also drew attention to the division of labour between participants, and how a sense of community begins to determine who has agency and control over deciding the direction of the activity. Although in lesson study as a collaborative professional activity we seek to provide a safe space in which the views of all are respected, it is important that some responsibility for group leadership is invested in certain individuals and that they foster a strong sense of community.

Boundaries

As individuals, we are all involved in multiple Activity Systems. Classroom teachers of mathematics, for example, in secondary education may be organised to work collectively in a distinct mathematics subject department. An individual teacher in this activity system is involved in different actions from those that they carry out in other activity systems, such as classrooms. For example, as a member of a mathematics department in a school a teacher may have a role in developing a scheme of work that organises the curriculum for the group of teachers with whom they work, whereas in the classroom they might directly interact with students.

Third-generation Activity Theory (Engeström, 2001) models relationships between the multiple activity systems which individuals inhabit as they work towards the different goals that result from the different overall objects of activity of the different collectives (for further discussion see, for example, Wake, Swan & Foster, 2016). In CHAT, this simultaneous membership of, and participation in, multiple activity systems leads to *boundary crossing* "as a socio-cultural difference leading to a discontinuity in action or interaction" (Akkerman & Bakker, 2011, p. 133).

Boundary crossing is often facilitated by *boundary objects* (Star & Griesemer, 1989), which are artefacts that have different meaning and use in two or more different Activity Systems, while retaining a common essence across systems. Didactical devices are examples of such boundary objects: in the classroom they support the teacher's work of didactical transposition (Brousseau, 1997) as they facilitate building on students' current understanding to develop new conceptual understanding, whilst in the lesson study group they provide teachers with devices that assist their consideration of epistemological issues.

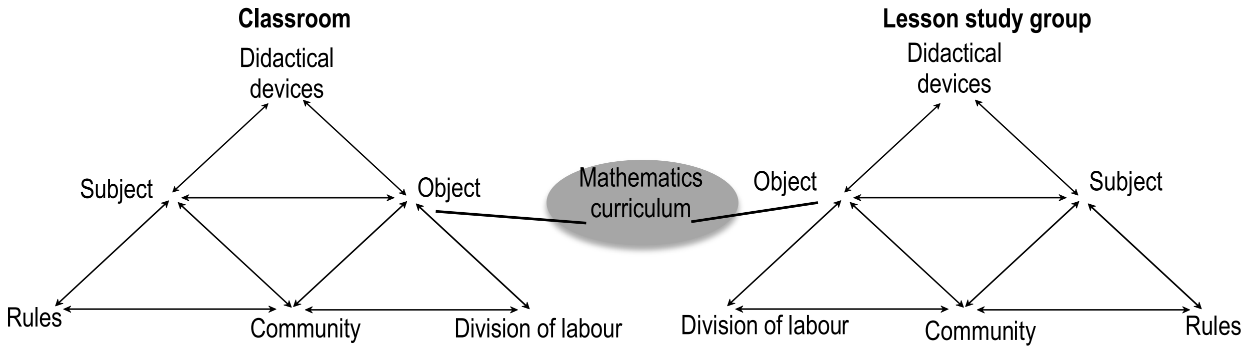

Figure 2: Interacting activity systems of classroom and lesson study group

The potential of the role of didactical devices

The notion of the transposition of 'everyday' knowledge of mathematics into mathematical knowledge for teaching has been acknowledged as an important, and often problematic, issue and explored, primarily in Western education research, for a number of decades (for example, Chevallard (1985)). The most concise and logical way to present a curriculum specification, often referred to as the *intended* curriculum (Schmidt et al., 1997), as opposed to the curriculum as enacted in the classroom, is often far removed from how it might best be sequenced and taught. In particular, the didactic devices that underpin specialist content knowledge for teaching (Ball et al., 2008) are rarely explicit in curriculum specifications. In the UK, current curriculum specifications are primarily lists of content. These are commonly organized, as in many nations, into generally accepted sub-domains of mathematics: number and measure, algebra, geometry, probability and statistics, with many countries drawing on the Organisation for Economic Co-operation and Development's framework that specifies mathematics as a domain of study (2016). The work of teachers then involves a process of didactic transposition (Brousseau, 1997), whereby knowledge as presented in curriculum documents is adapted in ways that make it suitable and sequenced as learning objects.

It is clear, although not explicitly emphasised in the Japanese texts, including student text books and teaching guides, that boundary objects that support didactic transposition are introduced (what here we are calling didactical devices). For example, the number line as a representation is used in increasingly sophisticated manifestations again and again across the curriculum to provide a coherence in approach to support students develop common ways of seeing concepts with which they are familiar and to help support insight into new concepts that are introduced as objects of learning. For example, the development of the number line as the *double*-number line can be used by both teachers and learners to gain insight into their different conceptualisations of proportionality and its effective application in a range of contextual problems. In other work, the UK group has explored how the double number line used in this way as a didactical device can be developed as a boundary object to support teacher learning in a modified version of lesson study. We have found that, in Drijvers & Trouche's terms (2008), such a didactical device becomes an *instrument* that both shapes the thinking of the user, in this case the teacher (the instrumentation process), and is in itself shaped by the user (the instrumentalisation process). From this theoretical perspective, we are considering how didactical devices might inform the process of curriculum design for coherence in ways that allow for teachers to engage in co-construction of knowledge and documentary work as they engage in the process of didactical transposition for effective learning.

Expansive Learning: an illustrative case

In an attempt to capture the multidimensional and complex nature of learning, Engestrom (1987) draws on the theory of expansive learning which "puts the primacy on communities as learners, on transformation and creation of culture, on horizontal movement and hybridization and on the formation of theoretical concepts. Fundamental to this is consideration that through collaboratiion participants construct a new object and concept for their collective activity and implement this in their practice. Engestrom and Sanino (2010), point to a variety of interventions/studies that have drawn on the theory of expansive learning, such as those by FitzSimons (2003) that considered adults learning mathematics in workplaces and learning as boundary crossing in a school-unversity partnership (Tsui and Law, 2007). Important to the theory is that the subject of learning is transformed from the learning of the individual to the learning of the collective in ways that leads to redefinition of the object of activity of the collective. This is perhaps best explained by reference to a specific example arising from the work of the study to date.

Following an intial analysis of the series of Japanese text books, developed over many years by the Japanese research team, and discussions between both research teams, three experimental lesson studies were held in the UK in 2019 and attended by both research teams working collaboratively with the UK teachers. The focus of the lessons was the potential of the construct of vectors as a didactical device to support the development of mathematical knowledge across a range of lessons in different topics in mathematics. Here, we briefly illustrate the use of the notion of "vector" as a didactical device in relation to a lesson on the addition of directed numbers (positive and negative numbers) for students aged 11-12. Important here is how the introduction of the construct of the didactical device, a device that is fundamentally concerned with providing an underlying focus on mathematical knowledge and mathematical knowledge for teaching, expanded the object of the collaborative activity of the planning team, in this case two teachers from one school and the first author as researcher. In lesson study, the object of activity of the planning team is usually a plan for a single lesson that seeks to answer a particular research question by prompting student and teacher activity when the lesson is taught and observed by the wider lesson study group.

We have also had conversations around the idea that the use of a vector representation here is helpful for future learning such as finding the position vector of an object. Specifically, if the position vector of an object at time t is given by the vector **r**, the initial position of that object at time t = 0 is $\mathbf{r}_0$ and the object undergoes a displacement of **s** then $\mathbf{r} = \mathbf{r}_0 + \mathbf{s}$.

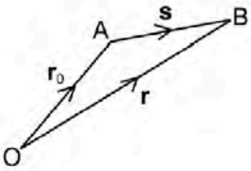

Figure 3: Discussion of vectors in general terms from the teachers' research lesson plan for a lesson on adding and subtracting positive and negative numbers

The activity of the lesson planning group, whilst also focusing on the research lesson in the study reported here, was also informed by the new construct of the didactical device in general, and more specifically the construct of "vectors" in this particular case. This resulted in a new expanded object of activity that focused more than previously on curriculum coherence in terms of how understanding of the underlying mathematical structure of a vector might be used to inform this area of the curriculum (adding and subtracting directed numbers). Whilst the intention is not to study vectors as mathematical objects in this phase the introduction of their underlying properties is found to have potential value as preparation for the future work of students such as when decomposing vectors into horizontal and vertical components in the curriculum, some two years down the line (and the subject of another lesson study in the sequence of three). Figures 3 and 4 show how the teachers' thinking in relation to "vector" as a didactical device was presented in the lesson plan for adding positive and negative numbers. Figure 3 illustrates how the teachers rationalised they would work with vectors later in the curriculum, and Figure 4 provides insight into how their more general thinking about the use of "vector" thinking was transformed to their plan for adding and subtracting positive and negative numbers.

We aim to use vector type representations to first illustrate 'journeys' and then move to representing and solving addition of directed number calculations.

In doing so we aim to draw out the following key points as we progress through the lesson:

- One leg of a journey must start from where the previous one ended (*building towards ALWAYS adding vectors*)
- The destination of a journey can be different for the 'same' journey (*the idea of free vectors*)
- The different destinations for the 'same' journey are dependent on the starting point (*link with the initial position vector and the need for an origin*)
- It is often helpful to have a fixed starting point
- In maths we have a ready-made starting point in zero
- Addition calculations involving directed numbers can be represented using one dimensional vector type diagrams
- Vector type diagrams can help us solve and explain how to add directed numbers

Figure 4: Key points of the teachers' research lesson plan

Figure 5 show some of the work that students undertook exploring "vector style" journeys in preparation for initial work in adding and subtracting positive and negative numbers.

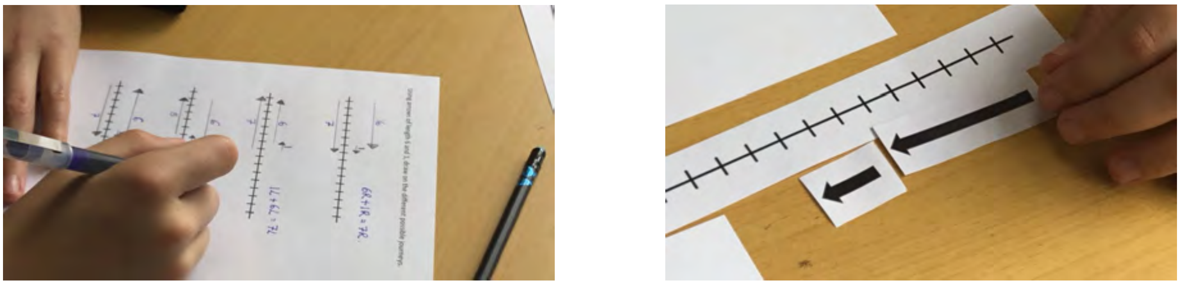

Figure 5: Student work using vector thinking in the early stages of developing conceptual understanding of adding and subtracting positive and negative numbers.

Discussion

Here we have exemplified key aspects of theoretical ideas relating to how the work of teachers collaborating through lesson study might be considered from a CHAT perspective. In particular, we have used the theory of expansive learning, whereby the multifaceted dimensions of learning of both individuals and the collective might be considered as expanding the object of their activity, in order to better understand how we their teaching may better ensure student experience of conceptual connectedness. Our claim is that the introduction of the construct of didactical device as a boundary object with meaning in both classroom and lesson study group has the potential to facilitate such expansive learning in terms of developing teacher knowledge of curriculum coherence. Further, it is our contention that socially distributed teacher knowledge of this type has the potential to improve student learning outcomes as over time they experience mathematics as a conceptually connected knowledge domain. For example, in the brief illustration given here we see students informally working with underlying ideas of vectors that will eventually support their future work where the focus will be on using vectors both explicitly and more formally.

Our introduction of the didactical device into the activity system of the lesson study group we consider as essential in refocusing the work of the group so that the object of study is developed beyond the consideration of a single lesson focused on teaching a particular mathematical concept at a particular time. The didactical device itself seeks to provide a potential tool for teaching (and learning) both across traditional topic boundaries and to support development. over time. Here, we have drawn, briefly, on two such devices: the number line and vectors. Although the latter of these at a later stage becomes an object of study in its own right, here we focus on its potential to provide an underlying structure and way of thinking that can underpin other areas of the curriculum, such as exemplified here: the adding and subtracting of positive and negative numbers. Our work in this area is a work in progress, although we have identified a number of potential other devices, such as the use of units, the circle as a locus of a set of points, the idea of the unit (one-ness).

In activity theory terms, the introduction of the didactical device provides a secondary contradiction, as an old artefact/instrument meets a new object: in the example here, the old artefact of vector, now considered as a didactical device, underpins the new object of the group as it works to better understand curriculum coherence. Contradictions of this type are necessary but not sufficient drivers for expansive learning (Engestrom, 2016). Our theoretical analysis of our work to date suggests that there is much potential for continued collaborative, and expansive, learning facilitated by focussing on further didactical devices. Our analysis highlights the importance of the introduction of the new construct as being central to supporting learning of both individuals and collective as a whole.

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A MOVE TOWARDS TEACHER COLLABORATION AMONG IRISH MATHEMATICS TEACHERS – IS IT FEASIBLE FOR ALL?

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Teacher professional development has a checkered history in the Irish education system. The first ever policy for teacher professional development in the state was published in 2016 and while collaboration is deemed important in this policy, it is not deemed a necessity for teacher learning. Despite this, state providers for professional development opportunities for teachers of mathematics in Ireland have embraced teacher collaboration in the form of Lesson Study as a mechanism for teacher collaborative learning. This paper reports on the implementation of Lesson Study in Ireland and discusses the limitations of this approach in this context. The paper concludes by outlining an alternate approach to teacher collaboration for Irish mathematics teachers that is currently under construction.

Introduction

This paper aligns with Theme A: theoretical perspectives on studying mathematics teacher collaboration. The paper begins by outlining a particular context, the Irish secondary education system, within which attempts at mathematics teacher collaboration are beginning to emerge. The paper, first, discusses the current form of teacher collaboration being utilized by providers of professional development for Irish mathematics teachers. The paper then moves to question the limitations of this format in the given context before addressing the theme question "what are promising research designs and data collection methods to study teacher collaboration" through the outlining of an alternate model of teacher collaboration that is currently being developed by the author. First, given the turbulent history of continuous professional development and teacher collaboration in general in Ireland, a brief background to both is provided.

Teacher Professional Development in Ireland

In the past century Ireland has undergone a radical reformation following the gaining of independence from British rule, a reformation that has had major impacts on our education system at large. Education has always been at the core of our national identity, predominantly due to the struggles regarding the control over the system when under British rule (Harford, 2008), and Ireland is proud of both the high-quality system in place and the high caliber of teachers within (Coolahan, 2003). Since gaining independence in 1922, a number of reports, acts and policy papers have been produced in an effort to further improve, develop and reform the system, a process that has had continuous professional development (CPD) for teachers at the core theoretically, however these good intentions have not always produced results in this regard. Despite the advancements made in legislation, the reality of teacher professional development in Ireland remained unchanged for many years. CPD was described as "fragmented" and "ad hoc" by teachers, with CPD itself described as "narrowly defined, lacking in theoretical basis, and rolled out in stops and starts rather than in any coherent or sustainable way" (Harford, 2010, p.355).

In March 2006 the Teaching Council of Ireland was established. The primary objectives of the council are the regulation, maintenance, improvement and promotion of the teaching profession. In particular, the council are tasked with promoting “the continuing education and training and professional development of teachers” (Teaching Council Act Part II: 6, a, ,b ,c), conducting research into same (Teaching Council Act Part IV: 39.2) and to “review and accredit programmes relating to the continuing of education and training of teachers” (Teaching Council Act Part IV: 39.2, c). Despite these objectives, the focus of CPD in Ireland to date has been largely reactive rather than proactive, with the main focus on preparing teachers for curriculum change rather than promoting new pedagogical approaches or reflective practice (Harford, 2008). In 2009, in an attempt to create a system-wide approach to CPD, the Professional Development Service for Teachers (PDST) was formed by amalgamating many existing services and programmes for CPD into one body. However, to create meaningful and sustainable change a more strategic policy approach was called for. Conway, Murphy, Rath & Hall (2009) expressed an immediate need for “comprehensive policies for the continuum of teacher education in order to meet the challenges of globalization, sustainable development and the knowledge society, and of the changing social and demographic context in which teachers work” (p. 171). Ten years after their formation, and 15 years after the initial Teaching Council Act of 2001, the Teaching Council responded to this need by producing “Cosán”, a framework for teachers’ learning, in 2016.

The Cosán framework (The Teaching Council, 2016) is self-described as a “flexible framework which provides a long-awaited opportunity to affirm the value of teachers’ learning” (pg. 2). Professional development opportunities are defined as “the full range of learning activities that teachers undertake for their own benefit and that of their students” (pg. 2). As per the framework, these activities can be formal or informal, personal or professional, collaborative or individual, school-based or external. Teachers are viewed as “professionals who are intrinsically motivated to take ownership of their professional development and steer the course of their own learning journeys” (p. 7) and thus the Council will “allow them to exercise autonomy in identifying, and engaging in, the types of professional learning opportunities that benefit them and their students most”. Given that Irish teachers spend over 95% of their working time in school teaching lessons, in comparison to the OECD average of 63% (OECD, 2017), the framework acknowledges that it may be challenging for teachers to find time to partake in professional development opportunities during the school day. Hence, the Council call on the government and school leadership/management to support teachers in this regard.

The Cosán framework was developed and guided through a series of consultation processes with teachers and invested stakeholders (e.g. Education Centres, etc.). During this consultation process, collaborative teacher learning was one of the most important aspects of successful CPD from the viewpoint of the teachers. “Teachers valued the sharing of new ideas, methodologies and resources, and the support they received from colleagues” (p. 12). While the Council recognized that many theorists have argued that “all learning is social and that teachers’ learning should be socially constructed in an environment that supports teacher interdependency” (p. 12), the Council encouraged teachers to also consider individual learning, as “the available evidence does not support the idea that collaboration with other teachers will always be the best way for every teacher to improve his or her practice” (p. 12). This warning aligns with a history/culture of privacy in Irish schools. A 1991 OECD report referred to Irish teachers’ levels of autonomy in this regard as “legendary” (OECD, 1991). Twenty years later, not much had changed as TALIS (the OECD Teaching and Learning International

Survey) reported that proportionately fewer teachers in Ireland participated in mentoring and peer observation (18% vs. 35% TALIS country average) (Shiel, Perkins & Gilleece, 2009). The Irish education system has been described as having a "predominant cultural norm of non-interference with professionals" (O'Sullivan & West-Burnham, 2011) with "pedagogical solitude" (Conway, Murphy, Hall & Rath, 2011) continuing to prevail.

Despite this history, the importance of collaboration from the viewpoint of teachers is encouraging. While the practice of collaboration may be new to many in the system, a shift towards teacher collaboration as a form of professional development is evident, particularly among Irish teachers of mathematics. The next section outlines efforts currently being made by the Maths Development Team (a sub team within the PDST) to facilitate and encourage collaboration among secondary school mathematics teachers in the aftermath of a curriculum reform.

An Attempt to Collaborate: Lesson Study in Ireland

Over the past ten years, the Irish secondary school mathematics curriculum has undergone a major reform. The curriculum has moved from one that was often criticized for being procedure focused, over reliant on textbooks (Lyons, Lynch, Close, Sheeran & Boland, 2003) and very abstract (Oldham, 2001) to one that encourages higher-order mathematical thinking, mathematical problem solving and real world contexts (National Council for Curriculum and Assessment [NCCA], 2011). The Department of Education and Skills (DES) mandated ten day-long structured professional development workshops for all teachers of mathematics to help with this transition. These workshops were available to teachers from 2010-2014 but a NCCA report in June 2014 recommended that further professional development related to this reform was necessary as these workshops focused on the change in topics of the mathematics in the curriculum and teachers needed greater support in changing their approaches to teaching and learning (NCCA, 2014).

Lesson Study (Takahashi & Yoshida, 2004) was introduced as an option for professional development for mathematics teachers nationally in September 2014 by the PDST. Participation in Lesson Study, as with most other forms of CPD for Irish teachers, is on a voluntary basis and each individual school interested in engaging with the process must apply to the PDST to partake. The aim of the PDST, as outlined in their current Lesson Study handbook (Maths Development Team, 2017), was to use Lesson Study as a mechanism to improve the teaching of mathematics through problem solving and students' confidence and experience as problem solvers. Teachers were encouraged to create teams of 5-6 teachers of mathematics within their school and work collaborately through the seven components of Lesson Study – define research themes and learning goals; meitheal machnaimh (Irish translation of the Japanese *Kyouzai kenkyuu*); write a research lesson proposal; conduct a live research lesson; engage in a post-lesson discussion; seek contributions by knowledgeable others; share learned outcomes. Teachers are instructed to meet five times outside of school time for two to two and a half hours. Teachers are supported by the DES through the provision of substitution to allow the research group to observe the teaching of the designed lesson to students once. No support or payment is provided for the five out of school time meetings. Since its initial implementation in 2014, Lesson Study continues to be offered as the main form of continuous professional development for Irish teachers of mathematics by the PDST and has also become the main focus of the annual Maths Counts teacher conference organized by the PDST.

Reviewing of the Effectiveness of Lesson Study in Ireland

A report on the effectiveness of Lesson Study as a form of in-school professional development for Irish mathematics teachers was commissioned in 2015 (Ní Shúilleabháin, 2015). The report was based on a case study research project involving twelve mathematics teachers in two secondary schools over the course of one academic year. Two groups of 6 teachers participated in separate cycles of Lesson Study four times in their individual schools. The teachers met, on average, every two weeks for an hour to discuss and plan for Lesson Study collaboratively. Qualitative data in the form of audio recordings of teacher meetings, teacher interviews, teacher notes, lesson plans, field notes, samples of student work and a researcher log were collected and analyzed. This research reported improvements in teachers' pedagogical content knowledge and an increase in the incorporation of reform teaching approaches, however this increase is teacher reported with no empirical data to justify the claim. In addition, participating teachers reported improvements in relation to their confidence in teaching the new syllabus and improvements in relation to the sense of community within their respective departments. While these are positive results, the limitations of the study as outlined by the report's author, need to be acknowledged. The two schools reported on in this research are not representative of all mathematics departments in the country. Both schools would be regarded as "large schools" with 550 and 800 students respectively. In Ireland, 52% of secondary schools have less than 500 students and 25% have less than 300 students. In many of these schools mathematics departments would have less than the desired five teachers needed for Lesson Study, with some schools having as few as one or two mathematics teachers. In addition, it is envisaged that finding 5 teachers in these smaller schools that would be willing to engage in numerous hours of out of hours work may be difficult. It is important to note that the two participating schools in this research received financial assistance from the NCCA to cover any necessary substitution costs so that the teachers could meet to plan Lesson Study during the school day. As outlined earlier, this is not the norm for teachers who wish to engage in Lesson Study. Even with this added incentive, teacher take up in the schools was 56% and 70% respectively. While Lesson Study may be a credible option for collaborative professional development for some Irish mathematics teachers, it is not necessarily an option for all teachers. The next section outlines an alternative approach to collaborative professional development that could be suitable for all teachers of mathematics, no matter the size of their school or the interest/motivation levels of their department colleagues.

An Alternate Approach to Collaboration for Irish Teachers

DuFour (2004) defines a Professional Learning Community as a group of teachers who recognize the need to work collaboratively with a common purpose of improving student learning and achievement. Newmann et al. (1996) outlines the five essential characteristics of professional learning communities as: shared values and norms among the group of teachers in regard to student learning; a constant and clear focus from the group on student learning; engagement in reflective dialogue on student experience and learning; the deprivitisation of practice; and a focus on teacher collaboration. More recent literature from the UK, the US, Canada and Australia align with and build on these characteristics as summarized in Table 1 by Scott, Clarkson & McDonagh (2011).

Table 1: Overview of elements of effective PLCs from the literature

PLCs promote and operate effectively when members...	**Bolam et al. (2005)**	**Johnson (2009)**	**Darling-Hammond & Richardson (2009)**	**Coburn & Russell (2008)**
have shared values and vision	•	•	•	•
have formal and widespread leadership	N/A	•	N/A	×
have collective responsibility for pupils' learning	•	•	•	N/A
attend to school teacher-learning challenges	•	•	•	N/A
focus on student learning	•	•	•	•
take an inquiry stance	•	•	•	•
goal set and design action plans	N/A	•	N/A	N/A
make teaching more public	•	•	•	•
share experiences and expertise	•	•	•	•
are willing to experiment with alternative strategies	•	•	•	•
engage in reflective dialogue	•	•	•	•
engage in high depth interactions (about how students learn content, pedagogical principals, curriculum content, etc.)	N/A	N/A	N/A	•
have mutual respect and support for teachers	•	•	•	N/A
have inclusive membership	•	•	•	×

• = explicitly stated; • = implicit; × = questioned practice; N/A = not referred to in this document

When the above mentioned characteristics are present, well-developed professional learning communities have been witnessed to impact positively on both teaching practice and student learning (Vescio, Ross & Adams, 2008). While research on professional learning communities is most commonly reported on an intra-school level, positive results relating to inter-school professional learning communities are beginning to surface (e.g. Scott et al., 2011). The research project outlined in the next section relates to an attempt to utilize inter-school professional learning communities, following the characteristics outlined previously, in an attempt to provide Irish secondary school mathematics teachers, from all schools around the country, with an opportunity to collaborate with other teachers to improve classroom practices and student learning.

Reimaging Professional Development for Irish Mathematics Teachers

The aim of this research project is to design a collaborative professional development programme that allows teachers from numerous schools to form a professional learning community. The objectives are to examine the classroom realities of a group of 4 teachers from 3 schools in a geographically similar area, analyze how they are currently incorporating mathematical problem solving into their classrooms, and thus design a longitudinal, evidence-based, support-centered professional development programme specific to the needs of participating teachers and monitor its impact on both participating teachers and their students. The programme activities will be structured on Loucks-Horsley and Matsumoto's (2010) model, at the core of which lies teacher collaboration. Teachers will gather on five occasions, out of school time, over the course of one academic year (2019/2020). The group will weave collaboratively through the stages of immersion, curriculum implementation, curriculum adaptation, curriculum design, and examination of practice all while forming a professional learning community, engaging with the literature and supporting one another through open communication. Teachers will be supported regularly throughout the programme by the other participating teachers, and the researcher in the role of facilitator of the programme, through continuous communication and school visits. The content of the programme will be designed and redesigned based on the feedback and data gathered from both teachers and students but will be guided by the work of Schoenfeld and the Teaching for Robust Understanding (TRU) Project (2016).

Educational design research (McKenney & Reeves, 2012) will be utilized in this study as it allows for both the development and redevelopment of solutions to educational problems while also providing a context for empirical investigation. As the intervention progresses, teachers will partake in increasing levels of collaborative design of curriculum materials. The goal of this professional learning community will therefore be the improvement of teaching through the design of tasks and lessons that the participating teachers can use immediately in their individual classrooms. The design process will be scaffolded by the facilitator to ensure that the resulting materials are coherent with evidence-based good practice. Much research has been conducted on the charateristics of impactful CPD which has been summarized comprehensively by Loucks-Horsley and Matsumoto (2010) and Hunzicker (2011). These strategies and checklist will be used consistently by the facilitator to ensure that the structure and activities of the programme also remain to be consist with evidence-based best practice. This intervention programme will contrast vastly to the traditional, generic, once-off workshops that are common place in the Irish CPD landscape, a landscape that has been shown to be ineffective (Hawley & Valli, 1999).

Each gathering of the community will be planned for by the facilitator/researcher based on the concerns/needs expressed by the participating teachers and/or witnessed by the facilitator. All activities will be designed so that teachers are working together in pairs or small groups. For example, during the stage of immersion, teachers will solve mathematical problems in pairs, experiencing what their students may experience when asked to do the same in their classrooms. During the stage of curriculum implementation, while teachers will be individually implementing sample lessons, teachers will be encouraged to write about their experiences with each lesson in a shared forum and share any "hints/tips" that they may have gained from that experience with the other teachers in the group. This method of communication aims to further create a sense of collaboration even when teachers have returned to their individual schools. Discussion on this implementation experience will also be facilitated at the beginning of the next meeting to continue this building of community and to deprivitise practice. As the teachers begin to design their own problem solving lessons/materials in meetings, participants will be grouped based on what topic they wish to plan for. All work completed by the particpants will be shared in the communal forum and teachers will be encouraged to continually add ideas/content that they create/find to this forum in order to build a bank of material together.

Impact on teachers' classroom pedagogy and opportunities for student learning will be monitored empirically through the use of video recorded lessons and analysed using the TRU framework. Five lessons depicting each teacher introduce and teach a topic to one class of their mathematics students will be recorded before, directly after and six months after their engagement in this collaborative professional development programme. Students beliefs about mathematical problem solving will also be monitored through the use of the *Indianna Mathematics Belief Scales* and the *Usefulness of Mathematics Scale* (Kloosterman, 1992). This quantitative data will be accompanied by qualitative data in the form of teacher entry and exit semi-structured interviews, teacher reflective journals, and teacher lesson plans. This combination of data sources should provide triangulated evidence on the impact of this form of teacher collaboration, inter-school professional learning communities, on both teachers' classroom practices and student learning in the particular context that is the Irish education system. While teachers in this context may have been exposed to short periods of collaboration during CPD sessions in the past, this level of collaboration on a longitudinal scale is highly likely to be new to them. Teacher exit interviews and the teacher reflective journal will provide data on this experience from the participant perspective, while the empirical data will show what impact this experience has had on their day to day classrooms.

The inclusion of the collection of empirical data in this study from both the perspective of teachers and students responds to the claim that research on teacher collaboration needs to begin collecting and reporting on data that shows changes in teaching practices and student learning that goes beyond participant reporting (Vescio et al., 2008). This study has the potential to shed new light on the impact of teacher collaboration in this regard and find answers to the following questions: what impact does inter-school collaborative teacher professional development have on teacher classroom practices? What impact does inter-school collaborative teacher professional development have on student learning? Is the establishment of inter-school professional learning communities a credible alternative to Lesson Study for Irish teachers of mathematics?

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Theme B

Theme Chairs: Cristina Esteley & Rongjin Huang

Contexts, Forms and Outcomes of Mathematics Teacher Collaboration

LEARNING TOGETHER WITH MATHEMATICS PROSPECTIVE TEACHER

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The prospective teacher Course of the Faculty of Education of the University of Campinas (Brazil) is interdisciplinary, bringing together trainees from different degrees in the same conetxt to understand and confront the reality of the classroom in a school setting. This is a qualitative research, that aims to understand the teaching professional practices of prospective mathematics teacher from the participation in interdisciplinary experiences. The research has a theoretical framework from the Situated Learning, and the participation in Communities of Practice. To analyze this information, two enterprises of the Communities of practices were found, which were characterized as actions (the construction of interdisciplinary intervention project in the prospective teacher Course, and construction of final report of prospective teacher Course). The participation in the different interdisciplinary contexts of teaching and learning in the school helped the trainee to problematize their professional practices as a teachers, with a re-meaning of the contexts of school practices, knowledge and specific processes of school mathematics.

Introduction

The prospective teacher course is a fundamental place in the formation of the future math teacher as evidenced by several researches (Ludwig, 2007; Levy, 2013; Pimenta Pimenta e Lima, 2012; Fiorentini, 2000; 2012; Melo, 2013). Most of these studies have taken as their research context the disciplinary stage in mathematics in which they emphasize the importance of mastering the knowledge of the disciplinary teaching content and the didactic-pedagogical knowledge of the content, according to Shulman's original categorization (1986). Since 2008, Education Institute at Unicamp has also adopted a prospective teacher course as a multidisciplinary or interdisciplinary, bringing together trainees of different degrees in the same setting to experience, intervene and understand the reality of the primary school classroom. This research presents some results obtained from the doctoral dissertation describing and discussing the situated and differential teaching learning (Acevedo-Rincon, 2018).

Theoretical framework

Several researches with the Social Learning Theory proposed (Lave & Wenger, 1991; Wenger, 1998) has been developed in Mathematics Education. Although this perspective was created to describe learning in non-formal educational environments, it has also been used as a reference for analyzing learning in Communities of Practice (CoP), such as teaching communities or school describing teacher or student learning in CoP (Adler, 1996; 1998; 2001; Boaler, 1997; 1999; Lerman, 1998; Santos & Matos, 1998; Graven, 2002; 2004). Specially in Brazil this perspective demonstrate their applicability and sustainability in the area of mathematics education research (Fiorentini; 2009; 2013; Braz & Kato, 2014; Oliveira, 2014; Cyrino, 2013; Garcia, 2014). For example, accompanying a classroom in the elementary school implies the recognition of a world of actions, relationships and interactions between the agents of this practice and that happens in a limited time. In turn, the teaching professional practice is one direction of social practice, as well as the practices of being a student in the classroom of the course, or in the virtual environment (TelEduc) when reflecting on the teaching and learning practices in elementary school. Thus, practices “do not develop in isolation, but are part of relationships system in which they have meaning” (Lave & Wenger, 1991, p. 169).

The CoP theory recognizes that learning is developed within systems of human activity. The relationships and interactions between people and their practices produce learning and, although they participate in the same situation, their experiences and interests are not homogeneous, as they may have different social meanings. Thus, the practice of learning occurs as “processes of change in human activity” (Lave and Wenger, 1991, p. 25). The processes of human activity always guarantee the heterogeneity of experiences, because the participants' social places, being equal, their interpretation and negotiation of meanings of experiences are personal, and correspond to the meaning given to each one of them. Therefore, we can identify that knowledge is produced as people, as social beings, go through different experiences, which gain meaning through participation, and learning is due to the process of negotiation of meanings that develops in practice. In developing the practice within a system of relationships, meaning depends on a social, historical, and cultural context that, for the particular case of research, is determined by the trainee's four participation contexts.

According to Wenger, McDermott and Snyder (2002, p.4) “CoP are groups of people who share“ a concern, problem set or passion for a topic and deepen their knowledge and experience in this area by interacting continuous way ”. According to Wenger, CoP can be conceived of as “shared learning stories” (1998, p. 93). However, not all groups of people work as does the CoP. In addition, Wenger (1998, p. 95) argues that practices are “stories of mutual engagement, joint enterprise negotiation, and development of a shared repertoire and have implicated processes”. As a result, learning in practice involves three dimensions in the community: i) developing forms of mutual engagement ii) understanding joint enterprise, and iii) developing shared repertoires.

Method and context

The prospective teacher course is developed in person at the University of Campinas-Unicamp classroom and two credits developed at the internship field school. Thus, the 60-hour semester internship has 30 theoretical hours (at University), and the other 30 hours in School practices (public schools in São Paulo state). The Prospective Teacher participants came from Mathematics, History, Literature, Social Sciences, Sports, Arts and Biology Institutes. They had already, another experiences as “teacher”. Prospective teacher identifies similar issues as they developed their practices at school and university. Thus, the experiences observed in the classroom of the field school allowed individual and group analyzes, contributed to deepen their interpretations. Even following the school practices in different schools, the trainees problematized situations that were common to other classmates.

In this course, the prospective teacher was accompanied with the teacher (at school) seeking to approach the analysis and investigation of the school problems and thus recognize and/or understand the educational actions developed in this field. From the observations and reflections on the teaching practices of learning in the internship field school, the course's trainer proposed the trainees to design and develop an intervention plan for/in the school, which was developed in a interdisciplinary joint enterprise. This allowed prospective teacher to identify common problems beyond the disciplinary perspective in most groups.

Participation and reification contexts

The contexts in which the prospective teacher actions, relationships and interactions allowed them to negotiate meanings throughout the participation and reification processes. Four contexts were identified: Context 1 (C1), was the prospective teacher course; context 2 (C2) corresponded to the interdisciplinary group, involving interns from at least two different degrees; context 3 (C3) was determined by the school contex; and context 4 (C4), were the Open Learning virtual environment (TelEduc). As a result, different CoP were formed: CoP-C, Community in the prospective teacher course, including also the trainer and the trainer from the Teaching Internship Program-TIP configuring C1, CoP-I (Interdisciplinary Community of prospective teacher) belonging from C2, and CoP-E, Community of teachers, students and prospective teacher from the C3.

As six subcommunities were formed, it was necessary to name each of them and thus differentiate them, for example: Alnala (from Mathematics Institute) and Mari (from Biology Institute) belong to CdP-I4 (Interdisciplinary Community of Practice # 4).

Actions, enterprises and data collection

The data collection instruments used in this research aim to recognize, transversally within those contexts, which learning experiences the four selected trainees of the Mathematics Degree had when participating in the practices of the three Communities of Practice (CoP-C, CoP-I, CdP-E) in which they were inserted. Subsequently, together with a longitudinal analysis of such experiences, we sought to understand which specific moments allowed them to be professionally constituted as prospective mathematics teachers.

We emphasize that the tools for the production of understanding about teaching learning during the interns' participation belong to one or several context of their participation in the practices, within the three CoP. These were: field diary, trainee narrative diary records, trainee intervention plans, trainee questionnaire, final report, CoP interviews, and individual interview to feed trainees' individual learning trajectories. The analysis of the data produced during the field research aims to build understanding about the learning and professional constitution of a group of interns by participating in the experiences of Supervised Internship I, demarcated by the practices of the CoPs. For this research, to analyze this information, we can identify two CoP joint enterprise were found, which were characterized as actions. The enterprise were: (i) the construction of interdisciplinary intervention project in the prospective teacher Course, and (ii) construction of final of prospective teacher Course report. About the firs enterprise, were analized three actions: 1st) identification of the school context, (2nd) observation and registration of teaching mathematics in school; and, (3rd) identification of interdisciplinary experiences in the school; and for the second, were identified the actions: (i) being a teacher in the school and (ii) reflections about the teaching profession. In this sense, the key points of meaning negotiation are presented here, with the purpose of understanding the narrative line proposed for the analyzes (Table 1). For this, we propose a dialogue between the individual learning trajectories and the dialogue made with their CoP-I. Throughout the narrative analysis, we take into account the processes of identification and non-identification of the interns with the practices experienced in the different contexts during the prospective teacher course.

Table 1- Negotiation of meanings in two joint enterprises identified

Joint enterprise	Actions	Meaning negotiation focus
Construction of interdisciplinary intervention project in the prospective teacher Course	Identification of the school context	Context of school practices (public education, course or ESIP).
	Observation and registration of teaching mathematics in school	Specific knowledge of mathematics.
		Processes of teaching to learn mathematics.
		Intellectual activity at school.
	Identification of interdisciplinary experiences in the school	Common situations in school practices.
Construction of final of prospective teacher Course report	Being a teacher in the school and (ii)	Be a teacher in C3 (public education, cram or ESIP).
		Interdisciplinary intervention actions at school / classroom
	reflections about the teaching profession.	Professional value of the teacher

We adopt Riessman's definition of narrative in which "storytellers interpret the past rather than reproduce it as it was. [The reports offer] a way to rethink or re-imagine lives (as narratives do for the nations, organizations, ethnic/racial groups, and others that form collective identities) (Riessman, 2005, p.6). It is important to add that what is intended through narratives is to interpret and analyze a past that occurred through the trainees' past experiences in their CoPs, and which reflect on those representations of the world of "being an teacher" in the different contexts. Of course, the present work is not intended simply to reproduce the past through the textualization of the materials produced, but to interpret and analyze those stories based on the abstracted teaching categories of the (future) mathematics teacher. In particular, this narrative will be from Anala, a prospective mathematics teacher who did her practices with adults education.

Anala's learnings in the first joint enterprise

Although Anala's participation in ESIP has been around for four years, it does not mean that it has all the answers ready for the situations that happen. Her narratives showed her more engagement with that project, as well as imagining herself as a teacher (in the future) of the same kind of population. Almost five years in the undergraduate degree, and of these four years in the project, show her personal and professional growth as a Youth and Adult teacher in the ESIP project. From the previous prospective teacher course, Anala has developed work on researching her own practice. From this she gained experience in reflecting on the practices of teaching and learning, “especially in the practice of reflecting, the practice of sharing my experience with others, even to come up with new ideas, the kind of ideas that the trainer gave (to her)” (Anala’s interview in CoP -I4). For this reason, it was easier for her to write in the diaries than for the other CoP-C trainees.

Anala highlighted some experiences of what it means to be a teacher at ESIP, such as: classroom follow-up, student learning pace, small numbers of students per class, changing plans and dynamics in the classroom, as well as as the study dynamics of the students.For example, the age difference between the participants in her project interferes with the teaching methods of mathematics. With each explanation students have the possibility to talk to the teacher because they feel comfortable saying what they are enjoying in class and what they are not liking. This is, when she wanted to introduce new content, she asked, "If they wanted me to introduce something new or continue with sets, they agreed to see something new, I promised that next semester I will resume content at the beginning of the semester." One of the examples she narrates has to do with how they learn best as in the case of sets, because they “even commented that they found it easier to visualize 'sets' with numerical examples rather than the ones I used when introducing them using the project classes as subsets ”.

About teaching mathematics in school, Anala reported an episode of sequences in Class 13 after the theme of sets, in which: given a numerical sequence in which all numbers began with the letter d (two, ten, twelve, sixteen, seventeen, eighteen and nineteen), when asking students about the next number in the sequence “they couldn't solve it either, nor gave hints (saying) that all terms started with the same letter D, so they couldn't solve it either” (Anala’s voice in C1). Students said 20 was the next number in the sequence. Anala's estrangement and questioning were immediate, in which she said: “twenty starts with the letter D? Not! So damn, it's not 20. Then they kept thinking and thinking, they couldn't and they gave up ”(Anala’s voice in1). It seems that the students gave up looking for the answer to the problem and so did Anala, as their reaction was to give the answer promptly, since it was their last class in the semester. Faced with this, the trainer asks: “But there was no other way to try?”, To which Anala responds: “I think so, but as it was the last class, there was no way of thinking” (Trainer and Anala’s voices in C1). In this episode, the trainer proposes the question so as not to despair to the point of giving the ready answer.

Possibly, Anala did not find another question that would guide the students to come up with the answer, such as drawing on the board, writing the sequence numbers in letters, and highlighting the letter d on them, and exploring one by one until she could. let them say the number nineteen and then explore by the tens, hundreds, until they reach the number “two hundred” that was Anala's expected response. Then the trainer thought of other suggestions so that they could come to the right conclusion, such as: enjoy the conversations with them in jest, but also instigate the negotiation of meaning of the sequence. In this dialogue, trainer observations refer to the language games of the school, and relate them to their cultural practices. It is interesting to point out the kind of mathematics Anala conceives in the classroom, skipping some explorations with the students, while expecting from them quick thinking for a simple problem in her vision.

From the multiple experiences Anala has experienced during the four years, she finally defines what it means to be a teacher at Educational Social Integration Project (ESIP) as continuous learning as stated in the CoP interview: “(...) is learning all the time. Not just learning from your own practice, the area (discipline) you work in, but (also) things like that, from life. ” Anala assesses questions from her own practice when teaching, but also asks, throughout the narrations, the student's learning and sees him influenced by multiple factors, such as the type of population, their interests to attend the project, as well as the difficulties caused by the wrong entries in the notebooks. Other practices, such as not exercising at home, are part of the school reality of the youth and adult population. In the interview, Anala is emphatic in saying “this is my place, I want to be a teacher. It's a place where I still see myself. It is acting as a teacher in the course I am. I can't see myself anywhere else ”(Anala's voice in interview in C3). These final reflections present Anala's desire to continue in the teaching profession, as part of her imagination and her engagement with practices, in spite of the difficulties one may have in teaching a simpler school mathematics than the Institute's mathematics. But also, this may be because they have not known other options either, for example, school math in formal education, or simply other experiences with other math (s).

In general, the experiences of the four trainees show difficulties inherent in teaching to learn a school mathematics, which is different from the mathematics learned at the Institute of Mathematics. However, in these practices they also find a way to reflect on the practices they observed during the initial period of the internship, and to implicitly compare what they experienced as teachers, as presented by three trainees in C1 and C2. In Anala's case, the reflection on her own practice makes her confront the reality that implies taking on a classroom, with students with other interests, such as socializing, when attending the project. Some difficulties identified were in teaching and others in learning, but always had as reference the previous experiences of other stages, which helped to look at school practices more critically and reflexively.

Conclussions

The context and the problems and challenges of the current school were the object of study and discussion between trainees about the scholar experiences. In general, the course was intended to motivate the problematization of the practices of its own disciplines, but always bearing in mind the interaction with interns, teachers, trainers and advisors, who did not necessarily belong to the same discipline. The prospective teacher course proposed differs from other proposals developed by Education Institute Unicamp, especially in relation to the configuration of differents contexts, in order to have contact with professional work, mediated by a process of reflection, problematization and practice investigation. , without dissociating theory and practice, having in context the problems and challenges of the current school. The investigated stage, therefore, is a place of convergence of formative experiences that permeate four formative context. This configuration allowed the four mathematics trainees, investigated by this study, to live and discuss their formative experiences related to teaching and learning practices in elementary school.

An advantage of this non-disciplinary prospective teacher course was contribute to the participants crossing the boundaries of their basic training subjects, an aspect that does not happen in their institute courses. The internship practices in these institutes were generally limited to observation and some pedagogical intervention in school practice in a single subject for the whole class and without promoting a reflection, problematization and researchs context for the prospective teacher pedagogical practices.

The interdisciplinarity in prospective teacher course in the Education Institute-Unicamp, developed from the perspective of the PraPEM research group, contributed to the learning of the practice being built collectively and collaboratively in communities. This had strong implications for the training of the prospective teacher, in particular for future mathematics teachers, since, through the prospective teacher practices, some gaps or needs were highlighted in their initial education degree: imbalance between school subjects and disciplinary training. But not pedagogical and didactic courses of the different undergraduate courses. That is, the need to reduce the distances between the disciplines, didactics and pedagogy, which suggests the curricular restructuring of the undergraduate courses aiming at teacher training. The high disciplinary teaching load in undergraduate degrees reveals the power that pure science over pedagogies and didactics. This does not seem to be consistent if we consider that Licensee's main field of activity is the primary school classroom. The fact that students are from different degrees promoted, within the course context (C1), a diversity of discourses and positions of future teachers in relation to class situations reported by colleagues, or against the concepts and propositions arising from the literature privileged by the trainers. In general, each of the readings made from the perspectives of different disciplines (undergraduate) contributed to the problematization and negotiation of meanings about the meaning of school, public school, polysemias and polyphonies (Bakhtin, 2003), the relationship with knowledge (Charlot, 2013), the pedagogical intervention plan, being a teacher and the teaching profession.

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Teachers' Collaboration in Dealing with Mathematics Curriculum Development A Report on a Project Carried out by Isfahan Mathematics House[1]

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In a centralized educational system like Iran, mathematics educational contents are developed by curriculum developers to be implemented by curriculum implementers, that is, teachers. In such a process, separation of curriculum developers and curriculum implementers leads to a gap between the intended curriculum and implemented one. This study aimed to assess a project entitled "Enhancing Knowledge and Skills of Mathematics Teachers in Mathematics Curriculum Development." It is used as a design-based research method, in which mathematics teachers and curriculum developers collaborated in groups to develop a mathematics course pack. The findings revealed that the teachers' knowledge of the curriculum content was enhanced, and their beliefs about the role of teachers in curriculum development have been changed. It was found that the teachers believed in the effectiveness of team working in enhancing knowledge and exchanging their experiences. This study also introduced a model for teachers' collaboration in mathematics curriculum development.

In Iran, the Ministry of Education is in charge of developing textbooks to be distributed among teachers and students throughout the country. The Iranian centralized educational system expects teachers to be faithful to curriculum implementation, and curriculum developers expect teachers to implement the curriculum as they intended. Although this may seem to be a teacher proof curriculum, the literature has revealed that mathematics teachers are not as faithful to this type of curriculum as expected. For example, Asgari et al. (2019) revealed that only 20 percent of the teachers followed the official textbook methods in an algebraic subject for the seventh grade students. Moreover, Ataei (2015) reported that if the teachers are not convinced to implement the course based on the goals of curriculum developers, then textbooks would not have a significant effect on their teaching process. Therefore, it seems necessary that teachers' attitudes change consistently based on changes in the curriculum.

To help the Iranian educational system in enhancing the professional knowledge of teachers and helping teachers to be consistent with changes in the curriculum, Isfahan Mathematics House (IMH) has designed various teamwork activities for teachers (Kenderov et. al, 2009). The participants in these team work activities include experienced teachers, IMH members, novice teachers, and future teachers (student-teachers). Moreover, IMH workshop instructors collaborate in developing and implementing these team work activities. One of the most important activities of IMH is holding workshops for teams of teachers to enhance their knowledge of content and pedagogy and exchange their experiences. Since mathematics textbooks have been redesigned recently, using new approaches, and incorporating major changes, some IMH members now collaborate in team working groups for reviewing the draft of these newly-developed mathematics textbooks and creation of

[1] This study was conducted under the supervision of Ahmad Reza Nasr, Professor of the University of Isfahan and Ali Rajali, Professor of the Isfahan University of Technology and Isfahan Mathematics House.

materials for the planned workshops for teams of mathematics teachers. Based on the observations of the Scientific Committee on the activities of IMH, it was found that the teachers' knowledge of content and pedagogy is not sufficient for achieving the goals of the workshops (for reviewing the draft of the newly-developed mathematics textbooks), and that they also need knowledge for the process of curriculum development.

On the other hand, the literature on teachers' use of curriculum content has revealed a significant variation in teachers' use of such contents (Stein, Remillard, & Smith, 2007; Tarr et al., 2008). For example, Remillard et al. (2014) reported that teachers tend to implement the approach recommended by curriculum developers. In contrast, Lloyd (2009), in explaining the mutual relationship between teachers and curriculum content, revealed that teachers select and change curriculum recommendations based on their knowledge and beliefs. Therefore, it seems that if the teachers are involved in curriculum development, they would be more faithful to implementing them. Moreover, teachers would be more successful in the implementing educational approach of the curriculum. In other words, teachers' collaboration in curriculum development not only improves the quality of curriculum outcomes, but also constitutes a powerful means of teacher professional development (Deketelaere & Kelchtermans, 1996). Given the lack of participation in curriculum development by mathematics teachers, their unfaithfulness to the implementation of the curriculum, and the need for enhancing teachers' knowledge about the curriculum, IMH designed the project "Enhancing Knowledge and Skills of Mathematics Teachers in Mathematics Curriculum Development". Since one of the functions of teacher collaboration can be resource development (Gueudet et al., 2016), a group of experts consisting of a curriculum developer, a mathematics instructor, and an expert teacher supervised teams of teachers in the development of a mathematics course pack, consisting of published and digital contents, student's content, teacher's guide, and education media for each textbook.

This study, as a part of the above mentioned project, investigates the effect of collaboration of mathematics teachers and curriculum developers on enhancing teachers' knowledge and changing their attitudes toward the curriculum. It also introduces a model for collaborative participation of mathematics teachers and curriculum developers in developing mathematics curriculum contents.

Method

This study has used a design-based research method. We have followed the Wang & Hannafin (2005), design-based research method which aimed to "improve educational practices through systematic, flexible, and iterative review, analysis, design, development, and implementation, based upon collaboration among researchers and practitioners in real-world settings, and leading to design principles or theories."

Before conducting this study, a pilot survey study was conducted on curriculum content, organization of the seventh grade mathematics textbook, and teachers' knowledge of curriculum contents and teaching methods in a sample of 108 teachers (Asgari et al., 2016). The findings revealed that the teachers' scores related to the algebraic contents of the 7th grade mathematics textbook, among the fifteen subjects, were significantly lower than the mean score. While the teachers assigned a higher score for their knowledge of algebra than this mean score. This was the main reason for the selection of algebra for the project in this study. Also, one of the questions in the questionnaire was related to whether the teachers were willing to collaborate in mathematics curriculum development. 21 teachers who answered yes to this question and also discussed their views about the curriculum subjects were selected to participate in this study.

The group of experts consisting of a curriculum developer, a mathematics instructor, and an expert teacher had made plans for the theoretical foundations of the project and had designed the contents of the workshops, three months before the implementation of the project. However, during the process of implementation of the workshops, some parts of the plans have been changed according to be adjusted to the participants and their needs.

The group of experts designed 12 sessions of 2-hours workshops for the project. The workshops were based on the group discussion technique. According to the group discussion technique, the participants are expected to rely more on group discussion than on individual work in making decisions because in the actual curriculum development process, the participants are not involved in the decision-making about curriculum content unless they collaborate with each other (Glatthorn, et al., 2016). All the group discussions were video recorded, and the within-group interactions were watched and written down by the group of experts.

The teachers were grouped heterogeneously, and the inclusion criteria included work experience, gender, and place of service. In the first workshop activity, the teachers were asked to write down the content and the teaching method they considered appropriate for the instruction of algebra in the 7th grade in a separate worksheet. Then they were assigned to groups. After that, they were trained on the technical process of curriculum development, consisting of the following steps: 1) determining the course parameters (fundamental aspects of the course, general goals, course scheduling), 2) learner needs assessment, 3) setting the ultimate desirable goals based on needs assessments, 4) determining the sequence of goals, 5) determining learning activities, 6) selection of curriculum materials, 7) determining assessment methods, and 8) designing a curriculum guide (Wolf & Shiv, 1984, as cited in Glatthorn, et al., 2016). After the training was completed in each step, the teachers were asked to develop in groups the algebraic content of the 7th grade mathematics textbook related to that step. Initial discussions between the groups revealed siginificant differences in opinons about policy making, the role of the teacher in the curriculum, and the methods of curriculum development. Individual differences in opinions about developing the algebraic content were also evident in for example individual worksheets. Different teachers used different approaches to the instruction of algebra, some of them used the obsolete method of fruit salad algebra, which proved to be controversial. Therefore, the teachers presented the theories related to the curriculum, the role of stakeholders in the curriculum, and appropriate approaches to the instruction of algebra before determining learning activities. Such knowledge decreased the within-group differences in opinions to a great extent. Then, the teachers developed contents in their groups. In each step, if the teachers had still unresolved differences in opinions or needed guidance, they would share their ideas and views with the group of experts, in that one expert in the given subject participated in the group and guided the discussion indirectly.

At the end, all the developed packs were reviewed and integrated into a new course pack by the group of experts, which was then presented to the groups of teachers for assessment and validation. The assessment was carried out in two steps: 1) individual responses to a validated questionnaire, 2) group responses to the same questionnaire in addition to the teachers' written responses. Then, the assessment forms were statistically analyzed, and the teachers' opinions were extracted.

In the next workshop, the teachers' extracted opinions were presented to all groups of teachers, and a consensus was reached on the final course pack, based on the discussions among the group of experts and the groups of teachers. The final course pack was delivered to a number of interested volunteering teachers to be pilot tested in schools. During the teaching of the developed content by four teachers, one member of the group of experts participated in the classroom to watch and evaluate the efficiency of the developed course pack.

At the end of the workshops, the teachers individually responded to self-assessment questions. The findings were coded by two of the researchers and an external coder. The kappa coefficient for text types was 0.90, p , 0.000, 93.1%. These findings indicated a desirable reliability and a reliable coding (Landis & Koch, 1977). The results of the content analysis are presented in part in the section on the findings.

Findings

As it was mentioned in Introduction, this study made an attempt to investigate two important factors: enhancing the teachers' knowledge of the curriculum, and examining the effectiveness of collaborative participation of the teachers in curriculum development. The findings of the self-assessment of the participating teachers in the workshops revealed an improvement in their knowledge of the theories of mathematics curriculum development, criteria for identification and organization of the contents, textbook development methods, types of assessment, school algebra, and organization of the algebraic contents of mathematics curriculum.

Also, some teachers reported that they gained some practical experience in writing and organizing the content and in scientific criticism of textbooks. According to the teachers, their attitudes were affected by the participation in this project. For example, they pointed out that their attitudes were changed in respect to the nature of the curriculum, development methods of the content, criticism and assessment of textbooks, the difficult and time-consuming nature of the development and assessment of textbooks, and most importantly the role of teachers in the development of textbooks. The frequency percentages of the above coded categories are presented in Table 1.

The coding of the teachers' self-assessment forms related to the effect of team work activities on knowledge, experience, and attitudes of the teachers participating in the development of the course pack revealed that group discussion and sharing their opinions had an effect on enhancing the quality of learning and deepening their knowledge of curriculum development. The results of this study were in line with those of Huang and Shimizu (2016) and Xu and Pedder (2015), who showed the effect of group work on teachers' beliefs. In this vein, the results revealed that teachers' collaboration in the development of the course pack had a positive effect on their attitudes and beliefs. One of the most important of such changes was that the teachers realized that it was possible for the teachers to participate in the curriculum development process, which was one of the important goals of this project. The results of the coding of the teachers' self-assessment in relation to the effect of team workings are presented in Table 2.

Table 1: Results of the coding of the teachers' self-assessment related to enhancing their knowledge of and changing their attitudes toward mathematics curriculum

Main Category	Frequency Percentage (All codes=93)	Effective themes proposed by the teachers	Frequency Percentage
Familiarity with the theories of curriculum development	59%	I am familiar with the theories of curriculum development.	10%
		I became familiar with the criteria for identifying and organizing the content.	8%
		I became familiar with development methods of textbooks.	18%
		I learned all assessment types of the curriculum.	5%
		I realized that I should pay attention to all the parameters in developing the content.	12%
		I became familiar with the concepts of and approaches to school algebra and of the misconceptions.	30%
		I became familiar with organization methods of the algebraic content of the curriculum.	17%
		Total=55	100%
Practical applications of the theories of the curriculum	6%	I gained some experience in writing the content of mathematics textbooks.	50%
		The practical experience in the organization of the content of mathematics textbooks was appealing to me.	50%
		Total=6	100%
Familiarity with the criticism and assessment of the content of the curriculum	8%	I realized that in the criticism of the curriculum I should pay attention to the goals of the curriculum.	85%
		I learned the scientific criticism of mathematics textbooks.	15%
		Total=7	100%
The teachers' attitudes toward the curriculum	27%	Now I know that knowledge of the content and of the curriculum are prerequisites to the development of the content.	24%
		My attitude toward the nature of the curriculum has changed.	8%
		So far I was unaware that the development of the content was so difficult.	38%
		I found that the development of the content was so time-consuming.	8%
		I found that teachers can also participate in the development of textbooks.	6%
		I found that the criticism of the content is not an easy task and requires the knowledge of the curriculum.	16%
		Total=25	100%

Table 2: Results of the coding of the teachers' self-assessment in relation to the effect of group work

Main Category	Frequency Percentage (All codes=42)	Themes Effective in group work proposed by the teachers	Frequency Percentage
Knowledge	36%	Group discussion had an effect on my learning about mathematics curriculum development and algebra.	33%
		Sharing of opinions in group work activities led to the deepening of my knowledge of the development of mathematics textbook content.	67%
		Total=15	100%
Experience	28%	I gained an invaluable experience in developing the content in groups.	25%
		I gained new experiences through discussion.	33%
		Group work helped me experience how to convert knowledge (theory) into practice.	9%
		Group interaction helped us to carry out a more comprehensive criticism and assessment of the developed course pack.	33%
		Total=12	100%
Attitudes	36%	I think that the use of mass consciousness in the application of knowledge leads to a dynamic education.	20%
		I found that the development of the content in groups was appealing.	47%
		I think that the collaborative teaching method of the group of experts was a model in itself.	20%
		I began to believe that it was possible for the teachers to collaborate in curriculum development.	13%
		Total=15	100%

After several sessions, when the participants became more familiar with the theories of curriculum development and the algebraic teaching methods, through discussion within teams, many of them realized that not only they lacked the knowledge of curriculum development but their criticisms of textbooks were merely based on their teaching experience and capabilities of their students. For example, those teachers with low-capability students believed that the level of contents of textbooks was high, and the scheduling was inappropriate. After the workshop training sessions, many of the participants began to believe that criticisms should be based on the knowledge of mathematics curriculum and new approaches to the instruction of mathematics. Moreover, the teachers had a strong tendency to improve the developed course pack during the assessment. They showed that they developed a strong attachment to the course pack and asked to use the course pack instead of the 7th grade mathematics textbook's algebra content in the classroom.

Additionally, the comparison of their opinions at the beginning and at the end of the workshops revealed that their self-confidence in the collaboration in curriculum development was increased. This finding is consistent with that of Schleicher (2015), who reported the effect of group work among teachers on the improvement of their self-esteem and self-efficacy.

Discussion and Conclusion

Using the design-based research method, this study selected 21 secondary school mathematics teachers and assigned them to five groups to collaborate with a mathematics curriculum development group of experts. In this way, the teachers were not outside observers, but were directly involved in the process. The group of experts trained the teachers in the technical process of developing the curriculum content and the approaches to the instruction of algebra. Then, the teachers set out to develop an algebra course pack for the 7th grade. The training was based on group discussion, which enables a practitioner's perspective to be used regularly in the process of curriculum development. Group discussion often involves mutual interaction between people in small groups. It not only enables the teachers and curriculum developers to collaborate with each other and share their opinions and actions, but it also improves the informed data on curriculum, teaching, and learning (Glatthorn, et al., 2016). After the groups of teachers provided their developed course packs, the group of experts reviewed and incorporated them into one single pack and sent it back to the teachers for criticism and assessment. After the teachers applied their opinions to the course pack, the final course pack was prepared and sent to the target population –Iran's Office of Planning and Compilation of School Textbooks (textbook publisher)– and subsequently the end-users, that is teachers and students.

The technical process of curriculum development is illustrated as a non-linear model in Figure 1. This process begins and ends with the Office of Planning and Compilation of School Textbooks. Since this model is generalizable to other publishers and teachers, IMH pilot tested it for a group of high school mathematics teachers. One of the important outcomes of this pilot study was to form a research group of teachers who set out to develop a teacher's guide for enhancing the teachers' professional knowledge of algebra in the 10th grade mathematics textbook.

As the results of this study revealed, the first effect of the proposed research model was to enhance the ability of the teachers in the criticism, assessment, and selection of textbooks. In fact, the results of the coding of the teachers' self-assessment related to enhancing the knowledge of and changing their attitudes toward curriculum development revealed that 56% of the codes was allocated to the familiarity with the theories of curriculum development, 6% to the practical applications of these theories, 8% to the familiarity with the criticism and assessment of the curriculum, and 27% to changing the teachers' attitudes toward curriculum development.

Moreover, the results of the coding of the teachers' self-assessment related to the effect of group work revealed that 36% of the codes was allocated to enhance the teachers' knowledge of curriculum development, 28% to gaining experience by the teachers, and 36% to changing the teachers' attitudes. The findings also revealed that the teachers developed strong attachment and commitment to the course pack they were involved in its development. In other words, group discussion enabled the teachers to acquire the ownership of the course pack developed through group work activity and remained faithful to it (Glatthorn, et al., 2016). This faithfulness can reduce the gap between the development and implementation of the curriculum. If teachers are not faithful to the developed curriculum, there will be a gap between the intended formal curriculum and the experienced curriculum. Such a gap, as Goodlad (Saylor, et al., 1954) has argued, is as a result of the separation of ends and means, that is, the separation of goal setters and planners and implementers. Such a separation can lead to the teachers' suspicion of and mistrust in the efficacy of the curriculum, which is also known as 'pedagogical legitimacy crisis' (Mirloohi, 1990).

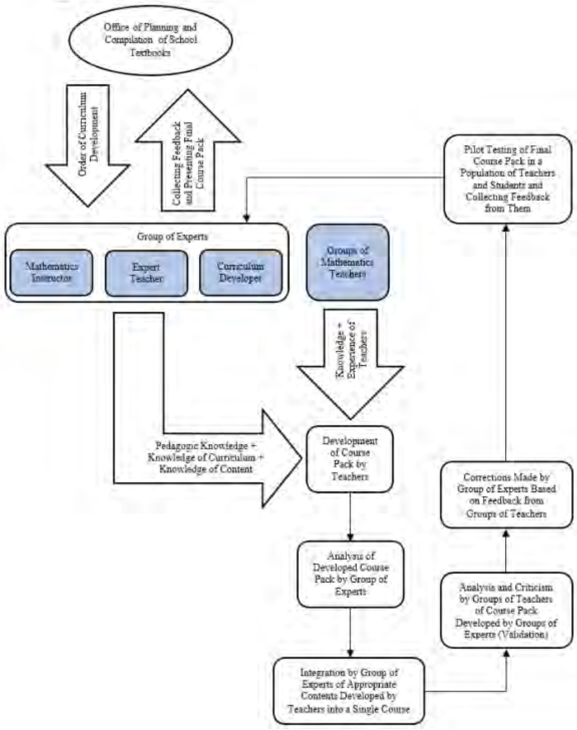

Figure 1. Model of teacher collaboration in curriculum development

Therefore, based on the results of the analysis of this model, it is suggested that mathematics curriculum developers first of all seek the opinions of mathematics teachers in the process of curriculum development. Then, teachers' opinions should be integrated into those of mathematics curriculum developers. In this way, since learning is a bilateral process, both curriculum developers and curriculum implementers (teachers) would learn from each other. This further reduces the gap between curriculum development and curriculum implementation.

The results of the model of the teachers' collaboration in curriculum development proposed in this study cannot be easily generalized to the collaboration of all the teachers in curriculum development throughout the country. If the majority of teachers are trained in the knowledge of curriculum development as part of their basic teaching knowledge (Shulman, 1986), then it would be easier to select competent teachers to collaborate on curriculum development.

This study stressed the importance of teacher collaboration in mathematics curriculum development; however, such a collaboration necessitates the training of teachers on professional mathematics knowledge (Borko, Koellner, & Jacobs, 2015). Such teachers should not only be aware of the theories of mathematics curriculum and of the instruction of mathematics, but also they should be able to manage educational workshops for teachers. On the other hand, such a collaboration requires the institutionalization of group work and critical thinking in education.

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ICMI Study 25
TEACHERS OF MATHEMATICS WORKING AND LEARNING IN COLLABORATIVE GROUPS
Theme B
Lisbon, Portugal, 3-7 February 2020

STUDY OF COLLABORATIVE WORK DEVELOPED AS PART OF DOCTORAL RESEARCH ARTICULATED WITH A TEACHER TRAINING

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This paper addresses the second theme of the discussion paper, which focuses on contexts, forms and outcomes of mathematics teacher collaboration. More specifically, we present and discuss limitations and possibilities of the collaborative work model set up to advance a doctoral research project. This research deals with the role and use of simulation by teachers after its integration in the teaching of probability and statistics in French curricula in 2010. To study this question and in an original way, the thesis draws on a training course organized on probability teaching.

Introduction

This paper addresses the second theme of the discussion paper, which focuses on contexts, forms and outcomes of mathematics teacher collaboration. More specifically, we present and discuss the limitations and possibilities of the collaborative work model that one of the authors has set up to advance her doctoral research project (Masselin, 2019). This doctoral research deals with the role and use of simulation by teachers after its integration in the teaching of probability and statistics in French curricula in 2010. To study this question and in an original way, the thesis draws on a training course organized by the IREM group in Rouen on probability teaching. The IREMs (Institut de Recherche sur l'Enseignement des Mathématiques) (Trouche 2016, Lagrange 2016) were founded in 1969 to provide in-service training for mathematics teachers faced with the implementation of the modern mathematics reform. Their particularity is to facilitate networking and exchanges between researchers, teachers and teacher trainers. The working groups thus formed meet regularly to address educational issues that are often related to the integration of new mathematical content into the curriculum. In addition to producing educational resources, these groups also provide short training courses (usually three days) for middle and high school teachers. It is in this context that the doctoral research was carried out.

In the paper, the doctoral research topic of Masselin's thesis is first presented, followed by a more detailed development of the collaborative working model implemented to both train teachers with Hartmann and to contribute to the doctoral research under the supervision of Dr. Kuzniak. Finally, this specific model of collaborative work is discussed. Doctoral research is not directly about collaborative work, but it has been enriched by the collaborative work carried out by different actors whose role will be specified later.

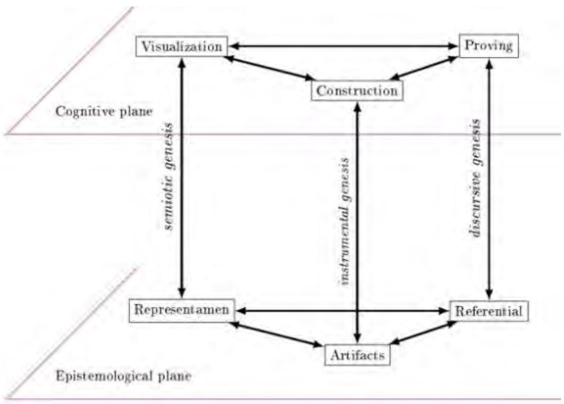
A study on the use of simulation in the teaching of probability

Study topic

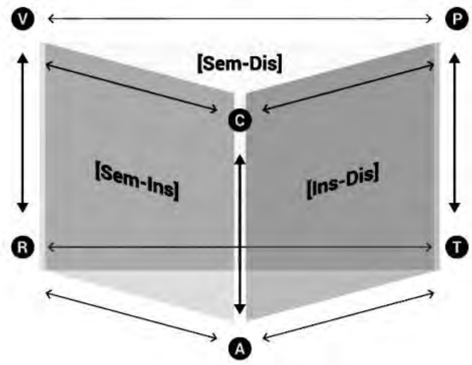
The doctoral thesis is dedicated to the study of type of use of simulation by teachers in their teaching of probability in Grades 8 and 9. Three main research questions are investigated, that focus on the links between random experiments and models, the place of artifacts, and the nature of simulation-based proof. To carry out the study, a specific task - the hare and turtle - was developed and introduced by the trainers during an in-service teacher training and the way in which the teachers (trainers and trainees) deal with and implement this task, was explored.

Theoretical background

The analysis of the mathematical work expected by the trainers and the analysis of the implementation of the sessions in class is based on Mathematical Working Space (MWS) theory (Kuzniak 2011, Kuzniak, Tanguay and Ilia, 2016). In this theory, dynamics of knowledge construction is viewed through the relationships between epistemological and cognitive planes that capture the epistemological and cognitive aspects related to mathematical work. In addition, mathematical work is interpreted through the intertwined links of three particular geneses: semiotics, instrumental and discursive. In the thesis, these three geneses are taken into account to question use of models, experience and proof.



1.a MWS' model (Kuzniak, 2011)



1.b Three vertical planes (Kuzniak & Richard, 2014)

Figure 1: Mathematical Working Spaces

Mathematical work studied in the thesis is that appears in the class and is associated with *a suitable* MWS in the MWS theory. It depends on teachers' implementation of the task, but also on students' involvement in solving the task. Using the MWS diagram, it is possible to describe circulation of mathematical work by specifying the vertical planes (Fig 1, diagram 1.b) activated by students and those favored by teachers. In addition, dynamics division of roles between students and teachers is observed and characterized. One of the key points of the study on simulation is the influence of teacher's choices of a given artifact on circulation of mathematical work.

Describe with the MWSs how the task and its implementation are evolving: avatars and cognitive routes

An *avatar* is defined as a statement of the task with its questions at a given time. This term was introduced to account for the transformations the task underwent. The study of successive avatars with its associated suitable MWS helps to identify decisions that teachers take individually or collectively. In complement to an epistemological analysis, the evolution of suitable MWS associated with different avatars is described through what is called *cognitive routes* (Masselin, 2019): cognitive routes are compound of the various routes leading to achievement of the task. Moreover, roles of teachers in cognitive routes development are described through the circulation of work in the MWS.

Referring to an expected suitable MWS defined a priori (Masselin, 2019), transformations carried out during and after the training are identified by describing the different cognitive routes that occurred when the hare and turtle task was implemented. These various theoretical tools allowed us to identify changes made by various collaborative groups.

The task "the hare and turtle"

The task on "the hare and turtle" game was used in the doctoral study to introduce links between probability and simulation. The collaborative work of various groups of teachers and researchers was dedicated to preparation and implementation of the task in classroom. Various adaptations in different contexts of the task are possible and below is presented the first avatar proposed in teachers' training.

> *A race takes place between a hare and a turtle: there is a 6-square course online. We roll a six-sided balanced die. If the 6 comes out, the hare wins, otherwise the turtle advances one square. The turtle wins when it reaches the 6th square. Who has the best chance of winning?*

Two different probability laws are possible to model the task and generate two different models:

- the truncated geometric law, where the sequence of throws are not independent of each other (in particular to obtaining a six or not),
- the binomial law, with the need of six dice rolls, even if a six is already down.

The geometric law describes the chronology of the task in a congruent way, whereas the binomial law does not. In addition, different artifacts can be used to perform or simulate the random experiment (dice, spreadsheet or programming language such Scratch). Models and artifacts are not independent. For example, if a spreadsheet is used, then simulation of binomial law is much easier than that of geometric law, whereas its associated model is less congruent with the statement. This is a possible source of difficulty for students.

A collaborative model for research and training

On research and training methodology

Methodology used for the doctoral research is based on a collaborative working model. The model should enable each of the actors to achieve their specific objectives related either to training, research or a combination of both. The purpose of research methodology is to obtain "in-depth" information on teachers' conceptions of the role of random experiment simulation in their teaching. The point is to go beyond claims and justifications usually given, which are often superficial and formal in relation to lack of time, material problems....

Moreover, in the specific case of the new way of teaching probability teaching based on use of software and simulation, trainers themselves do not have a well established position and also have difficulties with this new teaching. The collaborative work set up should enable joint development of sessions that allow everyone's ideas on the subject to be expressed, which gives hope for a richer collection of data for research.

The collaboration model is structured in three loops involving different collaborative groups (Figure 2). Loop 1 is located before the formation, loop 2 is that the formation and loop 3 is located after of the formation. Each of these loops gives rise to different avatars associated with suitable MWSs. These couples (avatar, suitable MWS) were the subject of an in-depth study in Masselin's thesis.

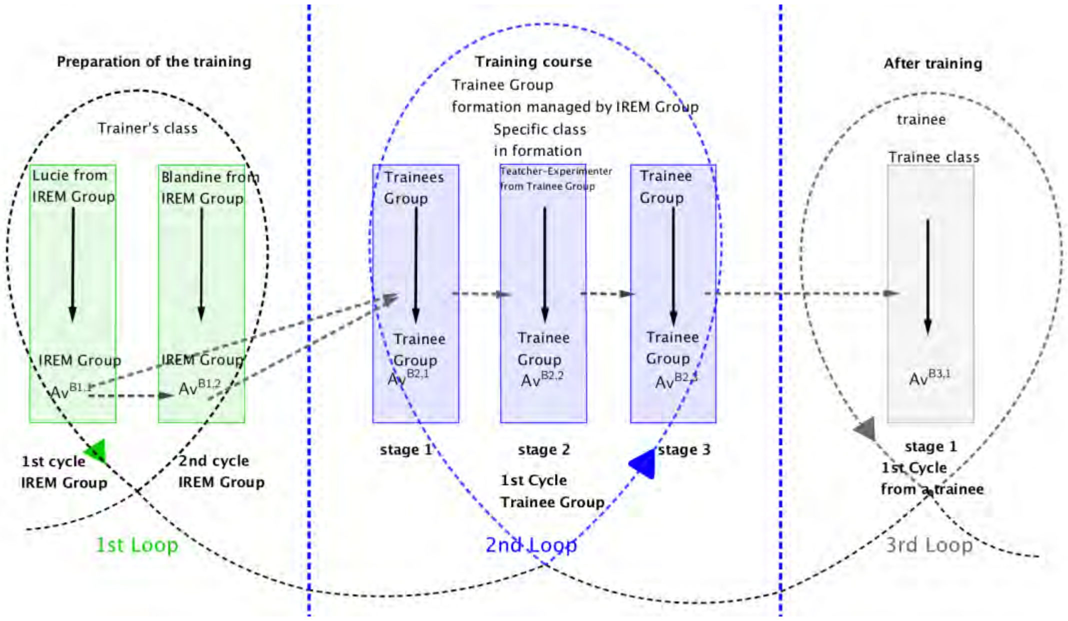


Figure 2: Collaborative model associated to doctoral research, Masselin (2019)

Three forms of collaborative groups

Three collaborative groups are involved in the study with different caracteristics and objectives.

The first group (IREM Group) is a group composed of four high school mathematics teachers (including Hartmann and Masselin), who meet voluntarily and regularly on their personal free time (every 15 days). This group is of a long-term nature and has been in place since 2008. One of its goals is to design and implement tasks in line with curriculum reforms and to provide in-service training on the new syllabuses. Members' involvement in the group is high.

A second working group (Trainees Group) is composed of a set of trainees from Rouen middle and high schools. Teachers in this group participate voluntarily in in-service training and do not know each other before. During the three days of formation, they must collectively prepare a class session, a *lesson*. The task “the hare and turtle” is used as support of the session and its first avatar, eventually subject to change, is proposed by IREM Group. Then, one of the members of Trainee Group has to implement the lesson in a classroom while other trainees and the trainers observe the session. This class session is then analyzed and modifications of group's initial choices can be made. The aim of Trainee Group’s members is to train themselves in probability and to develop their own teaching practice.

The third group (LDAR Group) is organized around Masselin's doctoral research. It is composed of Masselin and his two research directors as well as two other researchers from the Laboratoire de didactique André Revuz (LDAR). The research of this group deals with challenges of probability teaching by simulating random experiments and takes into account the teacher's work perspective.

Each of these groups is involved in different ways into the loops of the model presented above. The IREM group of trainers is concerned by the first two loops relating to the actual formation. Its purpose is to obtain material to use in carrying out training sessions with trainees after a first implementation by trainers in their class (student productions, spreadsheet files, video extracts from sessions conducted by group members). The LDAR Group of researchers is a priori interested in all loops insofar as they can provide it with information on the question of the use of simulation by teachers. For example, during the first loop, which led the members of the IREM Group to imagine a new implementation of the same avatar with Scratch software (instead of a spreadsheet), it was possible to highlight a strong link between artifact choice and model coupled with teacher's role. The Trainee Group is essentially concerned by the second loop. Masselin is the only person who belongs to two groups (IREM and LDAR) and is involved in both training and research and is directly concerned by all loops of the model.

Data from the first loop on the preparation of the training

The IREM Group

During the first loop before the training, the IREM Group focused its reflection on the use of digital artifacts to develop probability simulation. Without first discussing it with the group, one of its members, Lucie, set up a first avatar (Figure 3) in her grade 9 class. The session was filmed by Masselin, who was invited to observe the session as a member of the LDAR Group of researchers.

A hare and turtle race is carried out with a 6-sided die on a 6-square course.

					Arrival

This race takes place as follows.
- At each round of the race, one die is rolled.
If the die falls on 6 the hare reaches the finish line directly.
Otherwise, the turtle moves forward one square.
- The first to reach the "Arrival" box wins.
- We make as many rounds as necessary to have a winner.
Who of the hare or turtle has the best chance of winning this race?

Figure 3: Lucie's avatar (Masselin, 2019, Figure 3.2 p.98)

Lucie plans to ask students to do a simulation with a spreadsheet by using an empty file. She decides to impose on students the model associated with the binomial law that allows an easier implementation in the spreadsheet. However, this model is not congruent with the situation, unlike the geometric law model that follows the stages of the game. During the implementation in class, some students chose this second model. Starting from a blank file on a spreadsheet, the students try to simulate samples according to the truncated geometric law but they have touble to program the second roll of the die, that is conditioned by the result of the first roll. Lucie intervenes by proposing to simulate a six dice roll each time and imposes the model with the binomial law. The avatar and its associated suitable MWS reveal blockages due to the choice of the binomial law model imposed by Lucie.

During the preparation of the training, Masselin shown a video excerpt on the blockage that appeared in Lucie's classroom. This extract raises the issue of the non-uniqueness of models and links with simulation. The video excerpt illustrates blockage of students who are unable to change their initial choice of model. The confinement and blockage of circulation of student's work in the MWS associated with this avatar, has been retained by the IREM Group to be shown in training in order to raise models and simulation issue and to question the Trainees Group about teacher's role. The use of video excerpt in IREM Group revealed a confusion between modeling and simulation that was detected by the Group of researchers who suggested to Masselin to clarify these concepts before training with help of Blum & Leiss (2007) modeling cycle

Hartmann: Personally, I would not know what to place in "modeling" and "simulation". What are we expecting from them (the teachers)? (...) I'm not clear on the simulation, in fact I was confusing the real experience and the simulation before the last time. Is it still a simulation with ICT?

Stéphane: It's the same for me.

With a view to training trainees and in face of the blockage identified in the appropriate MWS proposed by Lucie, the choice of the model is questioned and choice of other digital artifact than the spreadsheet is evoked:

Hartmann: She doesn't want to roll again when she's already had a six. In fact, it is another game that has the same probabilities as the game of the hare and the turtle.

Lucie: Interesting, do we change the statement and roll the dice?

Hartmann: No, it's important to talk about modelling, it makes me think when you solve an equation, you have a decimal as a solution and you want to put an integer. Scratch is more compliant with the rules regarding this statement problem? We roll a die until there is a six.

The question of who is in charge of performing the program with Scratch is then asked:

Lucie: The teachers who tell us "we would like to use Scratch, but we've never tried it", tell them that it takes a long time to do it and, we can give them a file already done, we won't let them start programming like Stéphane and spend too much time there, we don't have the time.

A Scratch simulation file introduced by Masselin is then debated. Two points of view are expressed on Scratch software: one member wants to keep the *elves* to visualize races while another hides them in favor of displaying lists. Collaborative work reveals various positions on Scratch use and shows distinct perceptions of elements influencing the simulation related to the graphic displays of the software.

Stéphane: Ah, I understand why I had trouble last time, you didn't put yourself in the shoes of a student who wants to play a game with two elves.

Hartmann: No, Blandine (Masselin) has removed what I call the frills, which is what I asked my student to do, for sure. I'll "hide" the cat, you can make it disappear. I think that in fourth and third grade you have to get away from the fun part of doing maths.

Stéphane: It's easier like you did Blandine. My students are used to playing with elves so...

These first discussions turned the course of the preparation and oriented the IREM group towards a new implementation different from Lucie's and aimed at questioning the impact of a change of digital artifact (spreadsheet replaced by Scratch) to identify what this software could potentially involve during the simulation.

Implemented in class by Masselin, this new suitable MWS based on the same avatar but using Scratch instead of a spreadsheet has allowed IREM Group to clarify the role of digital artifacts in simulation by showing differences between spreadsheet and Scratch. The use of this software led students to focus on describing the game with an algorithm to solve the problem and as a result, they stopped the game after a single run without feeling the need to do several. They moved away from the probabilistic issue at stake and the LDAR group identified a classic domain shift, with algorithmic work supplanting the work on probabilities that was supposed to be prioritized.

At the end of this first loop, after having advanced its didactic knowledge on the use of different artifacts for simulation, the IREM Group developed tools for the Trainee Group (a priori analysis grid, video extracts exposing students' difficulties or relevant students' productions). This loop also has the purpose of fixing the avatar to be proposed in formation.

It is worth noting here that, thanks to Masselin, interactions between LDAR Group and IREM Group allowed a collective study of Lucie's session by IREM Group. These common work allowed IREM Group to enrich its analysis of potentialities of task implementation with identification of interesting variables for managing training sessions.

Data from the second loop during the class study

In preparing the lesson for the class study, the Trainees Group decided very early on to use simulation, with Scratch or a spreadsheet, keeping Lucie's avatar. Before the lesson, trainees spontaneously chose the binomial law (a race with always six dice throws). Difficulties related to simulation and difference in models were explained by IREM Group based on the video recorded during Lucie's implementation.

During the lesson prepared as part of the training sessions, three of the eight groups of trainees switched digital artifacts from spreadsheet to Scratch and failed to achieve a valid simulation. The question of different models used for the simulation was not perceived as an issue by trainees. They mainly focused on instrumentation problems related to instrumental dimension and genesis, thus leading them to instrumental confinement and blockage.

IREM Group members identified changes made by the Trainee Group to the proposals they submitted to trainees during the training. Thus, Scratch and the spreadsheet were both used but in two separate sessions. The trainees have also included a first race in the files prepared for students. In this case, researchers of LDAR Group noticed that IREM trainers did not ask trainees about relationships between probabilistic models and type of race chosen. IREM Group members focused on task transformations while researchers sought to characterize teacher's work on simulation during the second loop.

Conclusion

We have already given some specific results upon which the collaborative work has made it possible to move forward and we conclude this paper by specifying some general contributions of this work to the various groups involved, with a distinction between results for training and research.

Trainee Group members were not very involved in this collaborative work, with a much lower commitment than other groups members. However, they provided an external and complementary perspective on the task that served to clarify teacher's approach on simulation.

A important feedback on the training has resulted from IREM Group collaborative work and impacts their practice as trainers and teachers, as Frédéric Hartmann attests:

Hartmann: Through this class study, I have introduced the frequentist approach to probability further up in my progress. That confirmed my choice not to use the spreadsheet, I find it inappropriate, technical and not natural. Simulation seems easier with Scratch if you

> spend a little time to discover it (loop, variable, conditions). I discovered resources that I didn't know about and adapted and used in class.

Masselin's dual status as a trainer and researcher has allowed a link between Groups IREM and LDAR and has led to a reconfiguration of some didactic knowledge through the three loops, both for trainers and researchers. If Masselin's research compelled the trainers' work, it allowed IREM group to specify concepts (such as simulation and modelling) thanks to research contributions promoted by LDAR group to IREM group. For LDAR group members, this work helped to put into perspective didactic knowledge about mathematics and teachers' professional knowledge. The researchers identified a variety of cognitive routes for the proposed training task and noticed a complexity in the implementation of avatars by teachers. In particular the researchers have developed theoretical and methodological tools that they have shared with the trainers thanks to Masselin.

But to come back to the initial rationale for this specific implementation through a collaborative work device associated with Masselin's doctoral research, it is possible to assert that the model developed provided results for research on the simulation of random experiments. While it has limitations related to the constraints of doctoral research and training over a limited period of time, this collaborative work has helped to explore task simulation use and conceptualize training engineering based on class study (Masselin, 2019).

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STUDENTS' MATHEMATICAL PRODUCTIONS IN A COLLABORATIVE PROFESSIONAL DEVELOPMENT PROGRAM: A POWERFUL BUT STRESSFUL STRATEGY FOR TEACHERS

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This study occurred in the context of a national programme that took advantage of collaborative strategies to promote primary teachers' professional development concerning mathematical teaching. The programme was grounded in teachers' classrooms and placed great emphasis on teacher collaboration in schools, adopting the analysis of students' mathematical productions as focus to promote reflection. Our aim is to understand which attitudes teachers identify to have developed from the discussion of their students' mathematical productions with peers and the teacher educator. From the content analysis of the portfolios of 45 teachers, we concluded that teachers developed diverse attitudes from the analysis of their students' solutions to problems, most of them contributing to improve their mathematics teaching practices oriented by high learning standards. Attitudes like embarrassment and concern also emerged from teachers that did not feel comfortable with creative and effective solutions of their students. This suggests that this professional development strategy must be used in articulation with others that can foster teachers' confidence in mathematics.

Introduction

Guskey (2000) defined professional development as "those processes and activities designed to enhance the professional knowledge, skills and attitudes of educators so that they might in turn, improve the learning of students" (p. 16). In fact, not only professional knowledge but also teachers' attitudes, appear to be of particular importance within the context of teacher development.

Many professional development programs draw on teacher collaboration as a strategy for teachers learning (Robutti et al., 2016). Collaboration in job-embedded contexts is pointed as one of the features of effective professional development programs (Darling-Hammond, Hyler, & Gardner, 2017). Common modalities of collaboration are coaching by teacher educator to individual teachers in their classrooms or reflection about teaching experiences with challenging tasks that are being shared by peers on a group, with teacher educator support. In each one of these modalities, focusing on the analysis and discussion of students' mathematical productions on classroom is a frequently used strategy (Robutti et al, 2016). As result of having the teachers observing and discussing how students solve problems and how they reason to give sense to mathematical ideas, teachers usually raise their expectations about what students are able to do in mathematics (Kramarski & Revach, 2011). This is an important issue because the actual demands for students mathematical learning requires not only the mastery of mathematical contents, but also critical thinking, complex problem solving, mathematical communication (OECD, 2018). So, from the point of view of teacher educators, analysing students' mathematical productions with teachers seems to be a powerful strategy for teacher development oriented by high standards for mathematical learning. But how do teachers position themselves in this teacher education scenario? How do teachers feel when their students are able to produce mathematical reasoning that they never thought of and, many times, hardly understand? How do teachers react when they observe their students solving tasks differently from what they taught? Does this originate positive or negative attitudes towards the teaching practices being experienced? These are sensitive questions that need attention from research.

This study occurred in the context of a national professional development programme that took advantage of combined collaborative strategies to promote primary teachers' professional development concerning mathematical teaching. The programme was grounded in teachers' classrooms and placed great emphasis on teacher collaboration in schools, adopting the analysis of students' mathematical productions as a focus to promote individual and collective reflection (Serrazina et al, 2011). Having in mind that teachers' professional development is not only about teachers' knowledge to teach mathematics, but is also about teachers' professional attitudes (Guskey, 2000), our aim is to understand what attitudes teachers identified to have develop from the discussion of their students' mathematical productions with their peers and teacher educator. This allows to reflect on the relevance of this strategy for their professional development.

Theoretical framework

Collaboration has great potential for the promotion of teachers' professional development (Robutti et al., 2016). It may be operationalized in multiple ways, with different foci and involving different actors with distinct roles. In fact, the diversity of the role amongst group members may be seen as a factor that supports collaboration (Becker & Pence, 2003; Robutti et al., 2016).). When there are colleagues with whom one can discuss both successful and frustrating experiences, a forum is created, enabling all participants to learn from each other (Mewborn, 2003), especially when they feel "safe to talk" and to expose their difficulties (Robutti et al., 2016, p. 677). When teachers participate in education programmes with colleagues from the same school, creating a professional learning community, the implications for teacher development may be increased (Maass et al., 2015). Having peer support, teachers can create a collective curricular complicity and a dynamic of continued curriculum development in the context of a school community (Ruthven & Goodchild, 2008). Collaboration with other actors participating in education programmes is equally important, such as teacher educators that act as professional development facilitators (Maass et al., 2015). The development of teachers can be more effective if the teacher educator is able to promote a positive environment where teachers work in complicity, sharing goals and interests such as the improvement of practice (Robutti et al., 2016).

In what concerns the foci of collaboration, Robutti and colleagues (2016) point to two different ones. The first is related to aspects of innovation in mathematical, curricular or didactical knowledge, such as the learning of specific mathematical content, different pedagogical approaches and the integration of new tools. The second is not based on curricular novelties but is rather "associated with the different practices designed to foster teachers' professional learning" (p. 662). Assuming teacher practice as a central anchor for teacher development is of utmost importance. The practical and situated nature of professional knowledge makes classroom practice crucial, thus enabling teachers to develop in relation to their contexts and teaching experiences. As referred to by Sowder (2007), "teachers need to become serious learners of the practice rather than learners of strategies and activities" (p.161). Therefore, teaching practice is simultaneously an object of development and a strategy for its development.

By implementing specifically prepared lessons with his/her own students, the teacher has the opportunity to learn from that experience and to identify features that may contribute to their students' (un)success in learning. This is essential for teacher self-regulation (Kramarski & Revach, 2011). By bringing the work that teachers actually do in their daily teaching to teacher education enables them to unpack the mathematics class activity, to examine their teaching strategies and student learning and to discuss ideas for improvement (Borko, 2004).

Of particular interest for teacher development is the analysis of students' mathematical productions. When teachers examine the work of their own students, they "gain insight into their students' thinking and understanding (…), they discuss the kind of strategies used and hypothesize about the students' knowledge, the kind of instruction received, and the instructional changes needed to improve understanding and achievement" (Sowder, 2007, p. 173).

Trying to make sense of students' responses and processes of solving mathematical tasks allows teachers to expose their doubts and uncertainties; with regard to not only students' reasoning but also to the mathematical content involved and their teaching practice (Kazemi & Franke, 2004). As documented by Robutti and colleagues (2016), "teachers learned by focusing on students: student conceptions, student errors and student strategies" (p. 679).

Practice-focused collaboration may include teaching practice supervised either by peers or by a teacher educator, given that the objective of supervision is not only to improve knowledge, but also to develop reflective abilities and to reconsider attitudes that may contribute to improving practice. Indeed, this practice is influenced both by teachers' knowledge and their own personal views on the teaching and learning of mathematics (Becker & Pence, 2003). Reflection with the supervisor is essential to help teachers "analyse planning, concretization and evaluation of teaching, in the sense of developing their ability of reflection upon practice" (Martins & Santos, 2012, p. 197).

The potential of collaboration to promote reflection is well acknowledged. In the context of collaboration, "teachers learned through reflection: reflecting on their own teaching, on student learning and on others' teaching" (Robutti et al., 2016, p. 679). This reflection may be "in action", when something unexpected happens during the teaching, demanding an immediate response from the teacher during the lesson, "on action", when it occurs immediately after the lesson, or "about action", when it occurs at a specific time allocated for more profound reflection, when the artefacts of the practice such as students' productions are taken into account, and an attempt to draw conclusions is made in order to inform future practice (Schön, 1987).

Context of the study

The Programme of Professional Development in Mathematics (PFCM) is a large-scale, nationwide initiative developed in Portugal over six school years from 2005/06 to 2010/11. It involved more than 14 000 primary school teachers (grades 1-4) and 18 public Higher Education Institutions (HEI) responsible for primary teacher education in Portugal.

Teachers' adherence to this voluntary programme was highly significant (Table 1). At the end of its sixth year, an effective total of 14 414 primary teachers were educated by PFCM, corresponding to approximately 53.5% of the 26 947 primary teachers in Portugal (2010/2011). Furthermore, 3 117 teachers chose to participate in the Programme across two years (approximately 21.6% of the total participants).

Table 1: Number of primary teachers involved in PFCM

School year	2005/06	2006/07	2007/08	2008/09	2009/10	2010/11	Total*	Effective total
Teachers/year	5 229	3 646	2 771	3 026	1 703	1 156	17 531	14 114

*Including 3 117 teachers who participated in the Programme for two years.

A Steering Committee, nominated by the Minister of Education, of six higher education teachers, including the authors of this article, was assigned to design the PFCM programme. This Committee defined the development of teachers' positive attitudes towards mathematics and their acquisition of solid knowledge in order to teach mathematics as the main aims of the programme, within the scope of the innovative curricular guidelines that were being set. A new mathematics curriculum for basic education (grades 1-9) was approved in 2007. Its preliminary version was made public in 2006 and its guidelines have since become a reference for the work carried out within the PFCM.

The contents of the new mathematics curriculum included mathematical knowledge, with some non-familiar topics for the majority of primary teachers (for example, algebraic thinking, data analysis and probability), and also mathematical processes such as problem solving, mathematical reasoning and communication (NCTM, 2000). Its implementation represented a shift in the regular Portuguese teaching practices of mathematics by valuing students' mathematical understanding and promoting a dialogical perspective for mathematics learning within a climate of inquiry-based learning (Maass & Artigue, 2013). Hence, both the knowledge of mathematics to teach in primary school and the respective teaching practice were emphasized by the PFCM.

As for its dynamics, the PFCM invested in strategies that benefited from the development of collaboration among different actors, namely supervised teaching practice by a teacher educator and reflection on practice with peers.

The programme was organized at three levels: a macro level, relative to the interactions between the Steering Committee and the HEIs; a meso level, relative to the interactions between HEIs and their teacher educator teams; and at a micro level, relative to each teacher educator and the teachers respectively supported by him/her in the schools. At this level, the interactions occurred in different interrelated sessions that took place in schools throughout the school year, as follows:

1) Collaborative working sessions with each group of 8-10 teachers. These sessions occurred twice per month, with a duration of 3 hours, and had two aims: a) to prepare for teaching and b) to reflect on teaching. This option acknowledges that teachers learn when they prepare new topics for teaching, but also when they reflect on what happens in the classroom (Warren, 2008). As for preparing for teaching, the teachers worked on the new mathematics curriculum, respective mathematical contents and didactical knowledge needed for the planning of teachers' lessons on the chosen contents. They also participated in the collective planning of challenging teaching experiments, requiring new mathematical knowledge, new kinds of tasks, different teaching strategies and new resources in the classroom — which is fundamental for teacher development (Franke, Kazemi, & Battey, 2007). As for reflection on teaching, teachers discussed the questions arising from teachers' supervised practice (explained in the next section). Teacher educators played a highly important role in these discussions, promoting the sharing of the significant classroom episodes experienced by different teachers, acknowledging the role of the peers from the same school or school organization (Maass et al., 2015).

Their reflections were focused on students' productions, achievements and misconceptions — an effective strategy for teachers' development (Kazemi & Franke, 2004; Sowder, 2007).

2) Supervision sessions of the classroom practice of individual teachers. Each teacher experienced to put in practice a number of selected tasks from the previous group working sessions. During the school year, each teacher had four/five sessions of supervised practice by the teacher educator, seen as an opportunity to receive specialised support to put the lessons planned in the working sessions in practice in their own class. The teacher educator had varying degrees of participation, ranging from observing and taking notes for subsequent reflection to delivering a part of the lesson (for example, orchestrating a discussion of student strategies to solve a given problem). His/her intervention was arranged with the teacher beforehand. Each session took around 2,5 hours, including 90 minutes in the classroom and one hour for reflection on action regarding the development of the lesson, identification of students' learning and of the factors affecting their achievement. This strategy enabled all the teachers to relate effective training to their practice, a well-valued principle of the PFCM (Franke et al., 2007)

Methods

For the purpose of this study, we choose to focus on one of the HEI where the programme was developed. This option justifies because we need to do content analysis (Bardin, 1977) to be able to understand what attitudes teachers identify to developed from the observation of their students' mathematical productions. The first author of this study was the coordinator of the selected HEI in 2009/10 so we had access to the data we wanted to analyse: teachers' portfolios of the 45 participants on PFCM. At the end of the year, each teacher was asked a reflective portfolio intended to foster teachers' meta-reflection and learning (Bednarz, Fiorentini, & Huang, 2011). The portfolio included a personal reflection on the challenges of teaching within the scope of the new curricular guidelines and also on how the PFCM has contributed to meet the teacher's needs.

We followed an exploratory approach, capturing the attitudes teachers revealed by the teachers in their written reflections about the professional development strategies followed in PFCM. The first author of this study analysed all the portfolios but before, a systematic reading of 20% of the portfolios was made by both the authors. Both coded 9 portfolios in terms of the emerging ideas concerning the attitudes expressed by the teachers about analysing students' mathematical productions. The coding was compared and an agreement of 92% in this coding process where obtained.

The analysis was then completed, being set five categories: attitude of curiosity, attitude of openness, attitude of embarrassment and concern, attitude of awareness of the influence of each teacher's practice on what students learn and attitude of awareness of the importance of taking what students learn as a measure of the success of the practices.

A set of representative excerpts were selected from the teachers' testimonies, as illustrative evidence of the trend of the results, which has been included and referenced in the results section. These excerpts were essential to capture teachers' meanings "through teachers' own words" (Robutti et al. 2016, p. 680).

Results

This section summarizes the diversity of attitudes that we captured from the teachers concerning the discussion with their peers and teacher educator of their students' mathematical productions in response to the tasks proposed in the context of the supervised teaching practice in PFCM.

One attitude revealed by teachers was of getting surprised. Some teachers reported a strong sense of surprise with the processes of solving problems adopted by some of their students and this had as consequence the rising of expectations about the diversity of reasoning and creativity of the students when solving open tasks. Nevertheless, we may wonder about the potential of this finding. In fact, some hypothesize that the observed diversity of solution processes was due to particular students' characteristics:

> I was very surprised by some solution processes my students made to some problems. I am curious to see if in another year, with other students, I will also have so many surprises.

The sharing of the solutions of other teachers' students in the context of collaborative sessions contributed to make clear that in all classes, no matter the characteristics of some particular students, students show different and correct reasoning, some quite sophisticated or very effective.

Other attitude observed on teachers' reflections was of openness. This corresponds to a predisposition of the teachers, when observing and confronting several students' responses in the collaborative sessions, to accept and to respect students' solutions of problems that are different from the strategies that the teacher him/herself explained previously, recognizing legitimacy to students' ways of thinking:

> Most of the time we are prepared to solve a task in a certain way and we do not imagine that students may have their own forms of solving. We always expect them to solve the way we teach, but their ways are right too and may even be more creative and easier than mine.

> Only in supervised classes did I realize that I was very directive about the students' way of thinking and 'imposed' them to solve the tasks in my own way. But they can solve [the tasks] in different ways that I must accept and be able to understand their ways, and value them.

The attitude of embarrassment and concern was also present in the teachers' reflections. This reaction emerged from some teachers who did not feel comfortable in dealing with the efficient solutions of their students to some particular problems. This confrontation can make the teacher feel vulnerable to his students, and wonder if he/she will be able to meet the challenges of exploring open tasks with the class:

> I confess that for me the episode of the calcium task was very remarkable. I spent so much time solving the problem and trying to simplify its solution to make life easier for my students and after all it seems like the problem was complicated but it was for me (...) On the one hand it's fascinating but on the other hand it's scary. I have to recognize that I am much more limited to problem solving than my students are. And this gives me stress.

The embarrassment and concern may be intensified toward peers, particularly when the situation reveals the teacher's lack of mathematical knowledge.

Another attitude that emerged from analysing the solutions of students from different classes to the same tasks, discussed in the joint reflection in the collaborative sessions, was of awareness of the influence of each teacher's practice on what students learn. Comparing what happens in the classrooms of the peers promotes clear awareness that the characteristics of the practice influence students' learning opportunities. This is an opportunity for teachers to discuss and conclude about the relevance of the options they do in classroom and what are pros and cons of different activities and classroom strategies:

> Discussing the solutions of students from other colleagues who explored the same task was the most interesting to me. It allowed us to see how the different strategies of each teacher influence the way the students correspond to. Basically, the way we teach foster or hinder what students learn and how far they can go.
>
> Analysing how different students in different classes respond to the same tasks helps us understand how important our action in class is and may cause them more or less difficulties.
>
> The reflection in the group sessions was very important because we were also able to learn by seeing how colleagues adapted or modified activities for their classrooms and what their students learned.

For last, an attitude that emerged in some teachers' reflections was the awareness of the importance of taking what students learn as a measure of the success of their practices. These reflections appear to have been determinant for teachers to interrogate their own mathematics teaching practice:

> Our reflection regarding the students (…) was a starting point for the questioning of our own practice.

Conclusion

This study shows that the strategy of discussing and analysing students' mathematical productions with peers and teacher educators, specially in what concerns the processes of solutions of challenging tasks like problems, is a strategy to which teachers recognize potentialities for their professional learning. Nevertheless, this strategy revealed to have complexities for some teachers.

The attitudes teachers revealed to have developed from the discussions of mathematical productions of students are diverse. Some of them developed positive attitudes that contribute to foster professional development oriented to develop teaching practices compatible with students' mathematical learning needs today. This is the case of teachers' openness to students' own solutions of problems, namely for the ones that differ from teachers' strategies. This is of most importance in the scenario of the new curricular trends in mathematics for the 21st century, where challenging mathematical tasks are essential (OECD, 2018). It is also the case of teachers' awareness of the influence of each teacher's practice on what students learn, that calls teacher attention for the necessity of improving its teaching practice based on the appreciation of students' learnings (Sowder, 2007).

Other attitudes revealed by the teachers are not so positive. On the one hand, the surprise with students' mathematical productions in mathematical problems do not necessarily promote better practices of teaching, namely if teachers attribute the good students' performance to particular characteristics of the class — but this can be balanced by teachers getting to know other teachers' student responses to the same problems. On the other hand, the attitude of embarrassment and concern toward students' mathematical performance may inhibit teacher practice. The lack of confidence in mathematics may hinder the development of a rich mathematical experience in classroom with students (Warren, 2008).

So, the analysis of students' mathematical productions can not be seen as an absolute strategy for teachers' professional development. For some teachers, it is a powerful strategy but for others it is a stressful one. To overcome the possible negative effects that this strategy can cause to the ones that feel less confident to deal with the mathematical processes that students may use by their initiative, the articulation with other professional development strategies is of most importance, namely with strategies that evolve the possibility of sharing fears and difficulties with peers or a teacher educator who listen in a climate of complicity (Martins & Santos, 2012; Robutti et al., 2016).

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A COLLABORATIVE INQUIRY MODEL FOR TEACHER PROFESSIONAL LEARNING: WORKING *WITH* TEACHERS RATHER THAN *ON*

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New models for supporting teacher professional learning generate new conceptualizations of teacher learning, afford new designs for studying teacher learning over time, and situate teacher learning in problems of practice relevant to their own circumstances. In this paper, we describe our efforts at engaging with teachers as part of a collaborative-inquiry model for teacher professional learning. We report on our collaborative-inquiry approach to supporting teachers' understanding and use of learning trajectory-based formative assessment, and share examples of the various forms of inquiry we developed, as well as ways in which teachers engaged in these activities as part of our collective inquiry. Finally, we discuss the tensions that emerged from our collaborative inquiry model.

Research documenting different forms of professional learning opportunities indicates that it often takes a variety of learning experiences and considerable time for teachers to change their practices in intended ways. Several recent efforts to design learning opportunities for teachers have focused on supporting teachers in shifting their practice toward pedagogies that in turn support students in engaging with authentic forms of disciplinary practices. These designed learning experiences include inquiry into teaching and learning, learning in contexts that model targeted pedagogical approaches, ongoing reflection on one's own practice and learning, and working with colleagues to translate ideas into their specific contexts (Borko, Jacobs, & Koellner, 2010; Kennedy, 2016). Yet most of these recent efforts, being relatively short in duration, fail to provide the necessary ongoing support for changing practices. The result is that the professional learning opportunities end long before changes in practice are able to take root and become stable.

In addition to the short duration of many professional learning opportunities, the adaptation of these opportunities to different contexts and settings is equally as challenging. Although learning experiences embedded in professional development interventions have brought about changes in teacher practice in the classroom, the adaptation of these interventions to new contexts tends to not be particularly successful (McDonald, Keesler, Kauffman, & Schneider, 2006). In response, some researchers have proposed new models of working with teachers that hold much promise for productively adapting professional learning to new settings and situations. Examples include Network Improvement Communities that draw on principles of improvement science (Bryk, Gomez, Grunow, & LeMahieu, 2015); Design-based Implementation Research (Penuel et al., 2011), and research-practice partnerships (Coburn & Penuel, 2016). Because these new models situate teacher learning in problems of practice that teachers see as relevant to their own circumstances, adaptation of the professional learning experiences naturally occurs.

Common across all of these emerging models is that researchers work *with* teachers as opposed to doing research *on* teachers. Researchers do not deliver interventions to teachers, but instead develop learning experiences with teachers. Teachers and researchers working together towards changing teaching practices offers new conceptualizations of teacher learning, and affords new opportunities and designs for studying teacher learning over time. For example, the models described above all involve iterative cycles of improvement wherein teachers receive feedback on their practice and have opportunities to implement that feedback, and teachers and researchers together examine the underlying design mechanisms that contribute to change over time. Situated in teachers' own problems of practice, these models are inherently adaptive to local teacher needs (Koellner & Jacobs, 2015). These emerging models for working with practicing teachers may not only support the productive adaptation of designed learning experiences to new settings and contexts, but may also provide unique opportunities to study teacher learning over time.

In this paper, we describe our efforts to co-design professional learning experiences that engage practicing teachers in reflecting on and making shifts in their teaching practice. In addition, we relate how researchers and teachers reflect together to build an understanding of the principles underlying what teachers do and how they do it (Bereiter, 2014). In particular, we describe efforts to co-design learning experiences with practicing teachers aimed at developing and enhancing classroom practices around learning trajectory-based formative assessment (Stzajn, Confrey, Wilson & Edgington, 2012), an approach to teaching that places learning trajectories (LTs) at the heart of instructional practice.

Central to our efforts has been a shift in how we have engaged teachers in learning opportunities focused on understanding and implementing LT-based formative assessment. Our efforts to design learning experiences has moved away from delivering professional development *to* teachers and evolved into professional learning *with* teachers as co-researchers and co-designers (Penuel, Roschelle & Schechtman, 2007)—a model of professional learning based on collaborative inquiry. In this paper, we report on the evolution of our collaborative-inquiry approach to supporting teachers' understanding and use of LT-based formative assessment practices in their classrooms. We also share examples of inquiry activities (e.g., analysis of student work and classroom video analysis) and ways we have engaged in these activities as part of our collective-inquiry experiences. We conclude with a discussion of some of the tensions that have emerged from our collaborative-inquiry process that might inform others whose work involves supporting teacher professional learning, in particular, as they consider engaging with teachers around LT-based formative assessment.

Formative Assessment in Mathematics

Formative assessment (FA) can take on a wide variety of forms, calling for an equally wide variety of instructional competencies on the part of both teachers and students. The breadth of what researchers and practitioners understand as "formative assessment" is suggested in Wiliam's (2011) descriptions of the definition—that is, formative assessment is a process where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers to make [more informed] decisions about the next steps in instruction" (p. 43). In practice, however, implementing high quality FA practices can be challenging. Effectively interpreting and responding to students' reasoning requires a set of teaching capacities that are difficult for teachers to adopt into practice. These capacities involve creating a classroom learning environment and set of expectations such that student thinking (and learning) becomes readily visible to both teachers and students, and then identifying how student understanding develops along the conceptual strands of a discipline, such that teachers and students can productively respond to the surfaced student thinking. These capacities may be difficult to establish because of the depth and types of knowledge about math teachers require and the complex changes they need to make in order to interpret student reasoning.

High-quality FA practices require teachers to have a deep understanding of learning goals and of the associated learning trajectories (LTs) students travel while working towards meeting these goals, as well as criteria for making sense of and evaluating students' progress, ways of communicating this information to students, and ways of applying this information to making instructional decisions. Teachers plan for classroom tasks, discussions, and questions such that each of these lesson components will elicit evidence of learning; and based on this evidence, teachers can provide learners with feedback that moves students forward (Wiliam, 2011). Effective use of these practices requires identifying potential pathways for learning the content, as well as recognizing common obstacles that might arise. As such, clearly articulated LTs that enhance teachers' understanding of conceptual landmarks and obstacles to student learning can support the implementation of effective FA practices.

Framing Our Work With Teachers

Our work is situated in the Improving Formative Assessment to Support Teaching (iFAST) in Algebra Project, a multi-year project focused on enhancing middle grades teachers' FA practices in algebra. The two aims of the project were for teachers to develop a deeper understanding of particular algebra LTs, and to explore their implementation of FA practices based on a deep knowledge of the LTs. Drawing from research on high-quality professional development (Borko, Jacobs & Koellner, 2010; Kennedy, 2016), and consistent with the ideas of LT-based instruction, we designed and implemented an adaptive professional development model to meet project goals. During the first three years of the project, a group of ten teachers participated in monthly after-school professional development. Our general approach in Years 1-3 was to enhance teachers' understanding of FA practices more broadly, and to support teachers in analyzing student thinking and making evidence-based claims about understanding. However, through one-on-one interviews related to lesson implementations, the kinds of changes reported in the first three years on the project tended to be more about implementing specific FA techniques and less about engaging students in new or qualitatively different ways.

Thus, we shifted our professional development model in Years 4-5, moving away from providing professional development to teachers and instead, participating in professional learning opportunities with teachers as co-researchers and co-designers. This shift required a new approach where we worked together in teacher/researcher dyads. This more intensive model of professional development could only be realized by reducing group size, and so, moving forward, we focused on three sixth grade teachers. In our whole-group meetings we laid the groundwork for deepening an understanding of LTs, which then could serve as a foundation for enhancing FA practices. In our teacher/researcher dyads, we jointly determined a course of action for individual teachers in terms of FA practices with a focus on 1) developing student agency and 2) making student thinking central to the mathematical activity in the classroom. These then provided the themes for our collaborative inquiry.

In the sections that follow, we describe the co-designed professional learning experiences, and present preliminary analyses of teachers' engagement in the dyads and the resulting shifts in their classroom practice. Data sources include videotaped classroom observations and teacher dyad meetings, researcher notes from the dyads, and fieldnotes from classroom observations.

Collaborative Inquiry Around LT-Based Formative Assessment

During Years 4-5, researchers and teachers engaged in two forms of collaborative inquiry. The first required several days in which teachers co-constructed a micro-LT for unit rate (important mathematical content from the teachers' curriculum). The second involved dyads reflecting on teachers' videotaped lesson implementations.

Co-Constructing a Micro-LT for Understanding Student Thinking

During a two-day summer meeting, teachers co-constructed a micro-trajectory for three lessons the researchers had identified in the teachers' curriculum. Researchers identified these lessons as providing the core experiences for the LT related to the Grade 6 journey from introducing unit rate to introducing rate of change. For each of the three lessons, the teachers considered: 1) Which key mathematical ideas exist in the lesson; 2) Which key ideas seem to carry through from one lesson to the next one in the trajectory; and 3) Which key ideas do students need to make sense of in order to be successful going forward in the trajectory. During the first day, teachers co-constructed a micro-trajectory that illustrated the mathematical ideas students encounter and engage with while expanding their mathematical understandings from unit rate to rate of change.

Once teachers had completed the micro-trajectory, our learning experiences transitioned to applying this micro-trajectory work to their classroom practice—that is, connecting the LT work to work with students. The researchers selected eight papers from the teachers' classes that students had completed during the previous year. These papers presented evidence of student thinking and student strategies related to the micro-trajectory content. Teachers collaboratively compared and sequenced the student papers attending to students' understandings as evidenced in their work and as related to the development of ideas in the micro-trajectory. In sequencing the papers, teachers considered, for example, how the students applied the concept of unit rate, how students were thinking about rates of change in flexible ways, and what understanding about unit rates and/or relationships between quantities the student work revealed. Teachers used the LT language from their micro-trajectory to annotate the student papers and to describe evidence for their claims about student thinking.

During our follow-up meeting a few weeks later, teachers analyzed video clips from the previous year of their students sharing strategies for completing rate tables. As teachers discussed the video clips, they analyzed the type of strategy each student used and identified the evidence that supported the analysis. Finally, teachers discussed how they might engage students in exploring the desired transition from additive to multiplicative reasoning the next time they taught the lessons.

Co-constructing Next Instructional Steps with Teachers: Working Proximal to Practice

After the aforementioned professional learning experiences, we shifted to a focus on applying this LT foundation to increasing and enhancing FA practices. We engaged in what Koellner and Jacobs (2015) refer to as responsive professional development, where we actively generated and adapted our plans for professional learning experiences based on discussions with individual teachers. Although the guiding goals were similar for the three teachers—that is, to distribute mathematical authority for what counts as knowledge, to increase student agency, and to surface and foreground student thinking and ideas—the trajectory of learning experiences varied according to each teacher's particular situation. In the initial phases of building FA practices, we observed algebra lessons in teachers' classrooms and then engaged in collaborative inquiry with teachers around the observed lessons. Because the student-centered FA practices related to our goals require students to share their own strategies and ideas, while additionally engaging as knowledge builders who analyze and synthesize their own and classmates' ideas and explorations, the dyads mutually agreed to investigate how to more regularly and more intentionally foster these activities during the debrief of lessons. During whole class discussion, which was the most accessible part of the lesson (being completely captured on videotape), our initial inquiry efforts attended to who was doing the talking and to whom they were addressing their comments and questions. We found that, in early lesson implementations, most of the discourse during whole-class discussion was student-to-teacher with students responding almost entirely to teacher questions and prompts.

Dyads analyzed classroom video from early lessons exploring what seemed to work and what did not in terms of promoting student ideas and student discourse. Early on, the dyads reached the conclusion that students needed something to talk about—in other words, just reviewing answers or strategies did not motivate discourse among students. Although inviting students to share answers and strategies surfaced more student thinking, it tended to serve only as a conclusion to lesson activities and therefore, not as a meaningful part of the mathematical activity. The student thinking did not provide opportunities for collective reflection (Cobb, Boufi, McClain, & Whitenack, 1997).

Occasionally, instances of student discussion did arise in the classroom video. These moments provided opportunities for dyads to engage in collaborative inquiry around what prompted the discussion. Disagreements about a strategy or an answer, for example, seemed to offer students opportunities and incentive to discuss student ideas. Upon further reflection, teachers noted their processes for small group work might, in fact, interfere with the generation of opportunities for whole class discussion in that, the teachers traveled from group to group using funneling or focusing questions to guide students towards employing particular strategies and scaffolding students' arrival at successful solutions. This realization inspired teachers to remove their influence from the small group discussions. Teachers instead began facilitating student-to-student talk, activating students as resources for each other. This change in practice produced the variability that engaged students in discussion with each other as they could compare, evaluate, and justify their own and others' ideas.

Once teachers identified a productive shift through their video analyses, challenges arose in supporting students in transitioning to new norms and expectations. In the sixth grade classrooms, students had already been players of the game of school for six or seven years. Students at first struggled with the expectation that they should act as resources for each other and as validators of their own and others' ideas. Students initially displayed frustration and a kind of helplessness when the teacher explained she would not provide answers or confirm correctness of answers. Without direct teacher support in their small groups, students did not immediately trust that they could arrive at a successful solution or that they could recognize when they had arrived . As teachers continued to reinforce the new norms—problem solving together and validating each other—students finally began to settle into these norms during their math lessons. As students accepted the norms of working together; acting as resources for each other; and validating, questioning, and justifying their ideas for each other, more student-to-student talk and sharing of ideas during whole class discussion emerged.

The teacher's role changed as students stepped into new roles like facilitator, discussant, and presenter during whole class discussions. The teacher engaged in listening to exactly what processes and understandings students were discussing and debating rather than listening primarily for correct strategies and answers. The information teachers gathered while students worked in small groups provided evidence about what students understood or what was confusing that teachers could then use to make instructional decisions. For example, in a lesson where students matched unlabeled graphs with situations, groups of students presented their assigned situations and the graph they had chosen as a match. Presenting groups of students also facilitated a discussion about their graph choice—soliciting ideas from classmates. Often classmates held different opinions about which graph provided a good match for a situation.

During whole-class discussions, the teacher became an active and intentional listener, attending to the mathematical ideas students were sharing and debating to identify if and when her input would deepen disciplinary opportunities for students. The teacher engaged in a cycle of LT-based FA. During the whole class discussions, the teacher identified mathematical transition points and facilitated students moving through the transitions with carefully chosen questions and prompts. In the example above with matching graphs and situations, the teacher recognized that students seemed confused about the dependent and independent variables on a graph. As the class discussion devolved into students sharing ideas that did not connect back to one another's mathematical reasoning, the teacher invited the group to consider how the axes should be labeled. The teacher then invited students to discuss their new understandings in small groups, and finally, she re-established the presenting group as the facilitator for the whole-class discussion. Armed with new mathematical clarity, the presenting group navigated the class to a conclusion fully warranted by disciplinary ideas. Upon reflecting on this lesson, the teacher hypothesized that, because this was the first time students worked with a negative slope, they might not have recognized a negative slope situation. The teacher speculated that this might have provided a level playing field where no one was sure of the answer, which in turn sparked a lively mathematical debate and discussion.

As is illustrated in the explanations and examples above, LT-based FA practices have evolved throughout the project. Teachers first engaged in professional learning opportunities around analyzing algebra LTs in their curriculum. Then dyads collaboratively reflected on classroom video analyzing what was working or not working for promoting student-to-student discussion and debate. Finally, the dyads immediately debriefed observed lessons to evaluate progress made towards the goals of distributing mathematical authority and increasing student agency. Intermittently, dyads evaluated their progress towards goals to determine next steps for examining instructional practices.

A broadly construed set of steps teachers followed in their development of LT-based FA practices has emerged from this process:

- Teachers recognize when productive student-to-student talk is happening and determine what features are associated with productive student-to-student talk—e.g., they are questioning each other, validating each other's ideas, or explaining ideas to each other.
- Teachers recognize that there are certain contexts that seem to inspire student-to-student talk during whole class discussions—e.g., no one is sure what the right answer is; a presenting student makes a mistake; there are several possible right answers; there are a variety of strategies that yield an analysis of connections.
- Teachers remove themselves from small group discussions allowing more variation among strategies and solutions and making space for students to do the intellectual heavy lifting.
- Teachers recognize that their role evolves to one of listening and determining what mathematical transition points students are navigating. They then foreground these transition points for exploration in whole class discussions.

- Teachers recognize that, when structured space is provided for students, they can and will expose and explore the important mathematical ideas of a lesson.
- Teachers anticipate transition points as where productive discussions might arise. They intentionally structure lessons to privilege a focus on these points during class discussions.

Although there is a general flow to the way teachers progress, the process is not linear. Teachers move among the steps listed above in various ways with a variety of influences possibly affecting a teacher's use of LT-based FA practices.

Reflecting on Our Collaborative Work with Teachers

Several tensions played out over the course of the five-year iFAST Project. First, determining, establishing, and maintaining a balance between the two main strands of the work—deepening knowledge of algebra LTs and expanding and enhancing FA practices presented challenges. We encountered challenges around what LT and FA content was accessible to teachers and under what conditions, how the investigation of one focus area might enhance the exploration of the other, as well as how to establish a process of collaborative-inquiry to engage with the two foci of the project. The dual focus ultimately was fortuitous in that the LT work supported deepening teachers' knowledge about student learning in the context of their curriculum while simultaneously providing the teachers with tools for capitalizing on the student strategies and ideas that were surfacing in the whole-class discussions. We addressed the challenge of establishing and maintaining balance as we organically connected our work to teachers' classroom experiences and student work. When connected to their own lessons, for example, teachers made sense out of the mathematical development in a micro-trajectory. When reflecting on classroom video, teachers could identify and articulate practices that would surface more student thinking, promote student agency, and ultimately provide better opportunities for FA. Teachers drew on their deep understanding of the mathematics and LTs to determine which prompts and questions to use and when to use them.

Second, as the learning experiences evolved into dyads in Year 4, a tension arose around how to facilitate teacher and researcher learning in this context of professional learning experiences proximal to practice and with a collaborative-inquiry approach. Dyads needed to find ways of exploring and traveling individual teacher pathways through a shared field of expected outcomes—enhanced FA practices. As Year 4 began, dyads had mutually negotiated a focus on increasing student agency, which in turn enhanced opportunities for teachers and students to listen to, evaluate, and act on student thinking and performance. Each dyad came with a unique context, developed their work from an individual set of past experiences, and relied on a distinctive set of resources. As our work together evolved, the focus of each dyad's collaborative inquiry followed its own pathway to achieving our shared goals. Dyads reflecting on lesson implementations and lesson observations resulted in crafting pathways to increased student agency that we could analyze together and qualitatively describe. Differences among teachers' pathways have provided new questions about and insights into how teachers learn and what prompts teacher change.

Finally, tensions emerged as the classroom community responded to the teachers' changing practices related to FA, surfacing student thinking, and promoting student ideas as mathematical objects in the classroom. Changes in teacher expectations and class norms opened a new landscape of teacher and student roles that students had to explore and learn to navigate. Researchers learned that each classroom context brought its own particular set of challenges. Dyads confronted these challenges through exploring related questions—where one teacher might ask, how do I get students talking to each other, another teacher might ask, how do I help my students see that they CAN figure out these problems without me giving them answers or scaffolding their thinking?

Through our reframing of professional learning experiences as involving teachers as co-researchers and co-designers, we believe that we are moving in a direction of designing professional learning experiences *with* teachers in ways that hold promise for flexibly adapting to different contexts and situations. Drawing from a larger project focused on LT-based FA, teachers constructed micro-trajectories based on their own curriculum materials, so that as they deepened their knowledge of mathematics and how students learn and understand mathematical ideas, they were able to make connections to their own classrooms—both to their lessons and to how their students engage in those lessons. Researcher-teacher dyads mutually negotiated goals focused on promoting student agency and distributing mathematical authority and then, through reflection on the teacher's classroom video, adaptively navigated dyad-specific pathways to achieving those goals. Although this work has been labor-intensive (one researcher to one teacher in a dyad) and has taken time (two+ years), based on discussions with teachers, we believe the teacher learning is durable—that it will last even in the face of contradictory external pressures (such as mandates for skills testing). We also have anecdotal evidence that, as individual teachers make lasting changes that enhance student experiences, these teachers may influence the practices of other teachers in their schools.

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EXPLORING EFFECTIVE DISCOURSE THREAD OF COLLABORATIVE TEACHERS' EDUCATION: TEACHERS' KNOWLEDGE OF TEACHING CENTROID OF TRIANGLE

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Discourse is an inevitable matter for teachers in collaborative learning. This paper combined content analysis of teachers' reports about a video-taped lesson and time sequential analysis of teachers' discussion process about the lesson, for the purpose of the development process of teachers' knowledge of teaching the centroid of triangle. The results showed that teachers' knowledge of teaching the centroid could be updated after discussion and revealed the discourse thread of teachers' discussion contained requesting information, building on ideas, and providing evidence or reasoning, with the emphasis of challenging ideas or re-focusing talk. In addition, two potential sub-threads existed for teachers requiring more information or challenging.

Teacher education has achieved remarkable results in the form of collaborative learning with different goals related to mathematics content, students' learning experiences, the implementation of new curriculum materials, or daily teaching. The research concluded that Lesson Study led to an increase in teachers' knowledge, perspectives and insights about instruction and subject content, especially geometry (Lawrence & Chong, 2010; Meyer & Wilkerson, 2011). Though there are numerous studies on specific geometry content involved in teacher development, the evidence for producing this development has lacked, especially regarding the collaborative learning process. Therefore, what actually happened and what the characteristics need profoundly explored. Knowledge within groups is often co-constructed as a result of spoken interactions. Teacher professional development in communities revealed collaborative learning impacted the teachers' knowledge, but not to an extent to bring about such an effect. The purpose of this study is to explore the process of teacher collaboration while discussing specific mathematical content such as the centroid of triangle.

Theoretical literature

Discourse in collaborative learning

From the conception proposed by Vygotsky (1978) that people's intramental functioning shaped through social interaction and communication with others, teachers' learning in collaboration can be understood as the process of sharing and building teachers' professional knowledge. Discourse is considered as one of the central tools, enabling people to come to an understanding of others' knowledge and perspectives through interthinking (Mercer, 2000). The interaction, the sharing of ideas, and collaboration with colleagues seem crucial to the learning activities that are likely to lead to positive learning outcomes of teachers. Vrikki and Warwick(2016) reported a reliable coding scheme for teacher learning to identify the dialogic moves.

Knowledge building in collaborative learning

Effective collaborative learning is constituted through interactive, dynamic, and sustained dialogues over time. Knowledge building and creation are generated in discourses of collaborative activities for idea improvement, which must be tested, questioned, criticized and improved in the world (Scardamalia & Bereiter, 2014). To unpack the threads of collaborative learning, the intricate interpersonal interactions that lead to group cognition need to be examined (Stahl, 2006).

One conventional way to analyze interpersonal interactions in collaborative learning is the use of quantifiable online behavioural measures. Such analysis did not take into account the shared knowledge produced by groups, not to mention the dynamic and interactive process of inquiry that contributes to the advancement of such knowledge content. An alternative to quantified measurement is content analysis, which has been broadly employed to understand collaborative learning activities. De Wever and colleagues (2006) reviewed various useful content analysis schemes that allow researchers to unpack the criticality or progression. With such analysis, it is particularly beneficial for assessing the outcome of group knowledge work at a given point of time.

Learning analytics

Learning analytics refers to the analysis and interpretation of data related to learners' behaviors and interactions during the learning process, as well as learners' profiles and the learning contexts they are situated in (Hwang, Chu, & Yin, 2017). One of the popular methods of learning analytics is time sequential analysis that can determine the learning behaviors or interactive content to analyze learning patterns (Hwang et al, 2017). By applying this method, it enables researchers or teachers to have insights into students' interactions with peers by analyzing their behaviors from in-depth aspects, such as raising questions with scientific conceptions, seeking information related to a specific issue, making observations and comparisons, proposing new ideas or submitting a correct answer. Chen and his colleagues (2017) attempted to uncover sequential patterns that distinguish productive threads of knowledge-building discourse, advocating for more analytics tapping into the temporality of learning. However, considerable research focused on the students' collaborative learning, there is still a palpable lack of studies on the development of teachers' collaboration. This study mainly considered the teachers' knowledge development in collaborative discussion in real and sophisticated discussion situations.

Research questions

The major purpose of this study is to explore the effective discourse threads in collaborative learning for developing teachers' knowledge of teaching the centroid of triangle. More specifically, answers to the following research questions are sought in this research:

- What has changed regarding teachers' knowledge of teaching the centroid after collaborative learning?
- Are there any specific discourse threads during the discussion of collaborative learning to evolve their knowledge of teaching the centroid?

Methodology

There is ample evidence that analyzing video lessons has a positive effect on teachers' understanding of the teaching-learning process and on their overall professional development (Alsawaie & Alghazo, 2010). With video-based learning, this study used content analysis and time sequential analysis to delve into the research questions.

Design of the experiment

Participants. Four mathematics teachers with about ten years of teaching experience participated in this study, one is male, the others are female. They are all enrolled in Master's courses in mathematics education and have mathematics teaching experience in middle schools.

Experiment procedure. The experiment was implemented in two parts. In the first part, teachers were provided with a video-taped lesson themed an inquiry on the centroid using a principle inquiry activity, which was a lesson from the Education Research Institute in Gyeonggi-do, Korea in 2010. After watching the video-taped lesson, the teachers were required to write personal reports (pre-report). In the second part, teachers were allowed to have a 40-minute discussion about this video-taped lesson, with the process being recorded both in audio and video formats. Equally, teachers wrote reports after the discussion (post-report). The concrete procedures included: (1)watch the video-taped lesson, (2)write the pre-report, (3)group discussion, and (4)write the post-report. The pre- and post-reports written by teachers before and after the group discussion were about their judgements or opinions on the instructor's teaching in the video.

Instruments

Framework for analyzing teachers' reports. Because the video is a lesson themed as an inquiry on the centroid using a principle inquiry activity, it is required a framework on geometric pedagogy for analyzing teachers' reports contained their ideas or knowledge renewed on this lesson. The Geometry Assessments for Secondary Teachers (GAST) was adopted, which is designed to assess teachers' knowledge for teaching geometry (Margaret, Robert, Susan, Carl, & William, 2017). The blueprint of the framework (Table 1) involved in teaching geometry at the secondary level was implemented and three categories of knowledge for teaching geometry sub-domains were used to analyze the depth of knowledge. Two of the authors in this study who hold certain professional knowledge and experience evaluate the reports by classifying their opinions founded upon the blueprint to analyze teachers' knowledge of teaching the centroid, based on the viewpoints presented in their reports. If there are opinions on the geometry content typically found in secondary geometry curricula and taught in secondary schools, these opinions could be sorted to the knowledge of school geometry; the ideas included the post-secondary geometry could belong to the knowledge of advanced geometry; and the pedagogies and practices in teaching geometry are classified to geometry pedagogical content knowledge.

Coding scheme for analyzing teachers' discourses. Vrikki and Warwick (2016) reported a reliable coding scheme revealing that learning in a group has an impact on teachers' individual learning processes with descriptions and interpretations. This coding scheme was adopted for analyzing the discourse of each teacher. Table 2 presents the scheme, which includes four mutually exclusive categories. Three research assistants (coders) were trained to analyze the teachers' discourse before the experiment. The inter-coder reliability coefficient was .95. The teachers' discussion was divided into five episodes based on the main issues for teaching centroid in the discussion. GSEQ program was used to analyze the whole discourses and each episode, which is a computer program for analyzing sequential observational data for computing varied simple and contingency table statistics including joint frequencies, adjusted residuals, chi-squares, for 2×2 tables, Yule'Q and odds ratios. Adjusted residuals were computed to determine the time sequential patterns of activation (z-score ≥

1.96) of the target behavior (Hou, 2012). Z-scores were computed for each possible event pairing while considering the differences in relative and observed frequencies of target events (Jeong, 2003). A z-score $\geq$ 1.96 was considered to indicate the significant probability of a sequence ($p < .05$).

Table 1: The framework for analyzing teachers' reports. (Margaret et al., 2017)

Sub-domains	Contents
Section 1 (S1): Knowledge of school geometry	a. The teacher recognizes and describes appropriate demonstrations, interpretations, analogies, and justifications to introduce and develop mathematical skills and procedures. b. The teacher recognizes and describes appropriate definitions, representations, examples, distinguishing examples, non-examples, counterexamples, and the necessary and sufficient conditions to introduce and develop mathematical concepts. c. The teacher recognizes and describes meaningful connections (lateral, upward, downward) within and among mathematics content. d. The teacher recognizes and constructs a meaningful mathematical model of real-world situations.
Section 2 (S2): Knowledge of advanced geometry	a. The teacher solves non-routine problems, including real-world applications, in geometry. b. The teacher analyzes and constructs synthetic, transformational, and analytical proof and recognizes valid and invalid arguments (e.g., reasoning by converse, proof by contradiction, negating, arguing the contrapositive, and non-examples). c. The teacher analyzes and justifies geometric formulae.
Section 3 (S3): Geometry pedagogical content knowledge	a. The teacher recognizes and describes strategies and activities that promote student reasoning and problem solving (e.g., questioning, posing a problem, offering conjecture, describing an application). b. The teacher anticipates, recognizes, describes, assesses, and addresses correct and incorrect elements of student responses (e.g., skills, concepts, reasoning). c. The teacher recognizes, describes, and assesses critical student prerequisite knowledge. d. The teacher recognizes and constructs mathematics assessment tasks at different cognitive levels. e. The teacher recognizes and describes the advantages and limitations of using digital technologies (e.g., interactive geometry software, graphing calculators, virtual manipulatives, the other Internet resources) to foster student learning. f. The teacher recognizes and describes advantages and limitations of using physical models (e.g., solids, paper folding) and tools (e.g., compasses, straightedges, and protractors) to foster student learning.

Table 2: The code scheme for analyzing teachers' discourses. (Vrikki et al., 2016)

Codes	Descriptions
[D1]	Requesting information, opinion or clarification
[D2]	Building on ideas
[D3]	Providing evidence or reasoning
[D4]	Challenging ideas or re-focusing talk

Results

Analysis of teachers' reports

To analyze teachers' opinions on the knowledge of teaching geometry, two of the authors evaluated all viewpoints from teachers' pre- and post-reports based on Table 1. Compared with the number of views proposed by the teachers (Table 3), although teachers had many common opinions on the video-taped lesson, two types of updates were indicated.

Table 3: The number of each teacher's updated viewpoints after the discussion.

Teacher		Section 1 (S1)				Section 2 (S2)			Section 3 (S3)					
		a	b	c	d	a	b	c	a	b	c	d	e	f
Teacher 1 (T1)	pre-	1	2	1	0	0	1	0	2	2	1	0	0	0
	post-	0	0	1	0	0	1	0	3	0	0	1	1	1
	updated			1			1		3			1	1	1
Teacher 2 (T2)	pre-	0	4	0	1	0	0	0	1	1	0	0	0	3
	post-	0	1	1	0	0	1	0	1	1	0	1	0	1
	updated		1	1			1		1	1		1		1
Teacher 3 (T3)	pre-	0	0	1	0	0	1	1	7	5	0	0	1	3
	post-	0	0	0	0	0	1	1	2	3	0	0	0	1
	updated						1	1	2	3				1
Teacher 4 (T4)	pre-	0	0	0	0	0	1	0	1	0	0	0	0	6
	post-	0	0	0	0	0	0	0	1	2	0	0	0	0
	updated								1	2				

One type was opinions or comments that existed in both pre- and post- reports in the same section but in different extents. T1's viewpoint about S2-b could be an instance as follows.

> (T1's pre-report) It could not explore the true meaning of centroid. Because students just only knew about that triangle would stand up if put a finger at the intersection of three mid-lines. (S2-b)

> (T1's post-report) … the logic of 'mid-line divided it into two triangles with equal area, so the intersection of the three mid-lines is the centroid.' is not correct. (S2-b)

There were similar opinions on the meaning of the centroid in two reports from T1, which belonged to the recognition of invalid arguments in the knowledge of advanced geometry (S2-b). However, the pre-report focused more on students' behaviours while the post-report illustrated the understanding of the centroid nature related more to the mathematical content, which showed T1's further comprehension of knowledge for analyzing in the view of mathematics and recognizing the invalid arguments after the discussion.

The other type was about opinions noted in the post-report, but nothing proposed in the pre-report. For example, there was no viewpoint in S2-b from T2's pre-report, but after discussion, T2 mentioned that she learned the knowledge about the centroid shown as follows.

> (T2's post-report) I recognize that I did not know about centroids very well when T3 said, I learned it was a not good explanation that the reason of the centroid is defined as the intersection of mid-lines is because one mid-line divides the area into half. (S2-b)

It indicated that T2 had some gains to perceive the invalid argument related to the centroid through the discussion. Also, in S3-d, T1 expanded her knowledge range of centroid in the post-report.

> (T1's post-report) … if setting the speech as for centroid of circle or square at the ending of this lesson, it might be better than setting as homework. (S3-d)

T1's comment above was beyond the triangle level enlarging to circle and quadrilateral level, and she gave the specific proposals of setting the lesson's ending after the discussion. It could be inferred that T1's learning about organizing her own lesson for the centroid was generated during the collaborative process, which affected and motivated T1 to acquired the knowledge of teaching geometry.

Analysis of teachers' discourses

With the coding scheme (Table 2), three trained coders did code work for the whole discourses and each episode. In Figure 1, the transition confirmed the typical logic of discourses with requesting, building on ideas, and providing evidence, shown as the transition route: D1→D2→D3. The route D3→D4 was not visible in this diagram since the *z*-score (=1.67) was not larger than 1.96. However, the route D4→D4 (*z*-score=5.97) was illustrated that challenging ideas or re-focusing talk could be strongly performed during the discussion.

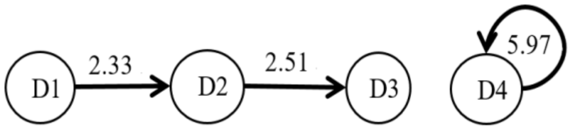

Figure 1: Transfer state diagram of teachers' main discourses.

Focusing on the process of teachers' discussion, the main discourse thread (D1→D2→D3) from time sequential analysis showed that teachers could proffer their own reasonable opinions about the video-taped lesson based upon their background knowledge. From the recorded transcripts, this thread could be found as follows.

T1: Let's talk about the properties! (D1)

T3: It's really awful. It would be better if having tried once, but this one was …actually it could be confirmed simply, but it became like this. (D2, D3)

In addition, in the main thread, D4→D4 displayed teachers' almost debate scenes for challenging ideas or re-focusing talk during the discussion, showing the transcripts as follows.

T4: Wouldn't that be the best if the groups didn't that quite systematically? Actually it would be better to give students more time, but it's impossible obviously. (D4)

T1: Let's talk about this question later. I just thought it would be better if studying deeper about the centroid. (D4)

For each episode, the transition diagrams were plotted in Figure 2. The threads of Episode 1 were fairly consistent with the main threads in Figure 1. Episode 3 and 5 could not be drawn as transfer state diagrams, because there were no *z*-scores larger than 1.96. Some *z*-scores were not prominent and not recorded in the main thread (1.96 for D3→D1 in Episode 2; 2.49 for D3→D4 in Episode 4), which still caught our attention as there might be two kinds of discourse threads after D3.

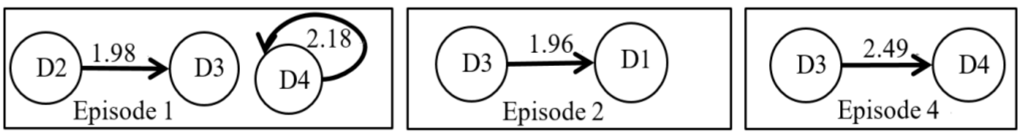

Figure 2: Transfer state diagrams of each episode.

In Episode 2, teachers expressed their understanding about the method and explanation of finding out the triangles' centroid in the video-taped lesson. T3 elaborated own viewpoint about the centroid with agreement on the explanation of the method, but T4 queried the method proposed in the lesson and requested more confirmation, followed by T3's further clarification. It demonstrated the process of sharing and building knowledge from teachers' discourses as detailed as follows.

T3: The explanation could be correct for triangles. However, the intersection of lines that divide the area of a polygon into half is not the centroid of the polygon. The centroid is the point where the rotational force is zero like the fulcrum in the principle of the levers. (D3)

T4: In a polygon, the meaning is not this one, right? (D1)

T3: Yes, the idea of dividing the area into half is a little bit misleading and misunderstanding when you think of a general centroid... (D2)

The excerpted transcripts were exactly conformed to what T2 wrote in the post-report that the invalid argument was obtained from T3's statements in Episode 2 above, which was the evidence of knowledge sharing and constructing clearly emerged in teachers' collaborative learning.

Besides the discourse thread of requesting information after getting the evidence of a certain idea, challenging or re-focusing the topic could also be a productive thread to get satisfying and convincing explanations, as the example below from Episode 4.

T3: …I think they were similar triangles because I would let students write down the specific length and point it out, but the teacher didn't mention that… (D3)

T1: (Interruptting T3) She mentioned it. (D4)

T2: The teacher in the video actually mentioned that there could be certain errors, not necessarily just as 2:1. (D3)

T3 elucidated her views on the teacher's guiding and gave the specific reasons such as she deemed that the teacher did not signalize some details of similar triangles, but T1 directly interrupted T3's speaking and pointed out the inexactness of that evidence provided by T3, which displayed that T1 challenged T3's reasoning. Then, T2 provided the explicit information with quotation of the words from the teacher who was observed in the video-taped lesson. With manifesting the specific evidence of the teacher's awareness of the property of similar triangles, it also demonstrated T2's disagreement to T3's explaining. The thread that the challenges or re-focusing after reasoning could clarify some mistakes of evidence provided, and the direct expression and challenges helped to create a discussion atmosphere and unify the views of topics for knowledge construction.

Discussion and conclusions

The contribution of collaborative learning to teacher professional developments was proposed in an increasing amount of research, but there is not precise enough evidence for generating shared and built knowledge in the collaborative process. In this paper, content analysis and time sequential analysis were used to explore the specific updated knowledge and the effective discourse threads in the collaborative learning for developing teachers' knowledge of teaching the centroid of triangle.

Considering the first research question, we analyzed the teachers' reports with the relevant framework and revealed that teachers' understanding of teaching the centroid was indeed updated after discussion, especially in S2 and S3 related to knowledge of advanced geometry and pedagogical content of geometry. T1's presenting the extended teaching design for ending lesson was evidence of motivating the creation from collaborative learning. T2's direct elaboration about her insufficiency on the advanced geometry knowledge also showed the educational influence from the discussion. Combining the results of content analysis, it is a rather enlightening and clear indication that the consistency of the statement from T2's post-report and the discourses provided by T1 during the discussion. It is acceptable to speculate that teachers' discourses promoted to produce shared knowledge and creative ideas (Mercer, 2000). Previous research used abundant sources of recorded materials and documents involved rich information (Scardamalia & Bereiter, 2014), but this study provided implications of various data by connecting the analysis of reports and discourses for teachers' advancements of specific knowledge content.

In addressing the second research question, teachers' discourses in the discussion involved such features as requesting information, giving reasons, providing evidence, making supportive comments, and challenging ideas. Even though Vrikki and colleagues (2017) proposed these discourse features without noticing the factor of temporality (Chen et al., 2017), the generated threads during the discussion were employed the time sequential analysis in this paper. The main discourse thread D1→D2→D3 illustrated the typical logic of discourses, and D4→D4 indicated the importance of challenging ideas or re-focusing talk during the discussion. This thread seemed to be significant as a particular dialogue move in effecting progress, additionally, we found two potential discourse paths that emerged after providing evidence or reasoning: D3→D1 is requesting more information to make sense; D3→D4 is challenging or re-focusing the talk. The requesting more information could inspire teachers for more knowledge of teaching the centroid, recalling some critical content learned in university to deepen their understandings of the vital concepts for flexible and effective teaching. The challenging others' ideas or re-focusing the talk could construct a debate mood and correct some errors promptly in someone's explanation, helping to build more accurate knowledge.

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ICMI Study 25
TEACHERS OF MATHEMATICS WORKING AND LEARNING IN COLLABORATIVE GROUPS
Theme B
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MATHEMATICS TEACHERS' PROFESSIONAL DEVELOPMENT: PRINCIPLES AND CHALLENGES IN A COLLABORATIVE SETTING

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Collaboration, reflection on practice (one's own and others) and reciprocal learning represent key features of the work of a group of primary teachers and researchers into mathematics education at the University of Huelva (Spain). Concerned for teachers' professional development, and taking a problem-solving approach, the group has, over the course of the last twenty years, carried out various developmental research projects. These have seen a flow of knowledge between the school setting, via the primary teachers belonging to the group, and the university, in terms of theoretical input and the recycling of experiential knowledge constructed within the group in teacher training programmes. We illustrate how these core features have contributed to the professional development of various teachers, and show the complexity and potential of the connections between knowledge, reflection and teachers' professional development.

Introduction

This paper reflects on aspects of a long-standing professional development programme for mathematics teachers which breaks with the traditional dichotomy between practice and research, between what teachers do in the classroom and what researchers produce through their activities. The focus is a research group known as the PIC (from the acronym in Spanish for Collaborative Research Project), now in its twentieth year of bringing researchers and teachers together to generate knowledge (Goos, 2014).

Our understanding of professional development mirrors that expressed in Climent and Carrillo (2003). According to this perspective, teachers" develop their understanding of what they do, and thus amplify their professional knowledge, by honing their capacity to reflect on their own and others" practice, which can be measured by the number and complexity of the elements each individual is able to bring into play at any particular moment. Although this act of reflection depends largely on the individual involved, its effectivity can be enhanced by favourable settings. In the PIC, this setting is structured around bringing together the interests of teacher and researchers within a framework of collaborative research to which each participant brings their experience and knowledge. The variety in terms of training and experience, alongside the different contexts in which each member works, is an important factor for the learning generated within the group. This learning is produced through discussion of classroom scenarios in which theory and practice come together in order to mobilise knowledge about mathematics teaching and learning. The results of this enterprise confirm that joint reflection on practice can be the driver of professional development (Hospesová, Carrillo & Santos, 2018).

The chapter is divided into four parts. Following the introduction, the composition of the group is described, its origins, aims, and key features. The third section provides an illustration of how reflection on practice promotes teachers‟ professional development regarding mathematics teaching through an account of the experience of two primary teachers; this part also narrates some of the milestones in the constitution and development of the group. Finally, one considers the theoretical foundations of the PIC, and assesses its potential, limitations and development. Our publications about the work developed whitin the PIC are the main source of this chapter descriptions and analysis.

Characterisation of the PIC

From its beginnings, the PIC has been formed by students and practitioners from a variety of educational contexts. The original group was constituted by experienced primary teachers and two mathematics primary educator-researchers, whose interests lay in mathematics teachers‟ professional knowledge and development. Since its foundation in 1999, there has been a core membership formed by one of the original primary teachers and the two educator-researchers, while the remaining participants have changed over the years. These have included prospective teachers and doctoral students alongside practising educational professionals of varying degrees of experience from a range of backgrounds, including nursery and primary teachers, primary teacher educators and secondary teachers. The work of the PIC is organised into biannual educational research projects, financed by the education authorities, directed by one of the primary teachers in the group, and recognised by the teachers as in-service training. The actual size and composition of the group varies with each project (there are currently 12 members: 7 teachers from nursery, primary and secondary, and 5 teacher educators, one studying a doctorate and another also working as a secondary teacher).

The heterogeneous nature of the group is one of its strong points, giving rise to a variety of ideological positions and experiences in the meetings. The wealth of perspectives this offers is evident in the role played by each participant and the range of contributions that are generated. The teachers across the different levels contribute specific knowledge about their pupils and how they learn mathematical content, about lessons and the teaching and learning of mathematics in real educational contexts. For their part, the researchers contribute in terms of the theoretical aspects, facilitating methodological tools and the frameworks underlying the development of the projects.

The aim of the PIC has always been closely aligned with professional development through reflection, both on teachers‟ own practice and that of other group members. The group‟s work dynamic takes the form of meetings held every two weeks, in which debate is the vehicle for reflection. The first stage of each project is to decide on the mathematical content to be dealt with (always within the methodological frame of problem-solving), along with the level and particular groups according to the availability of the teachers. The whole group likewise participates in developing the lesson activities and data collection systems. If necessary, a specific instrument for data analysis is designed, or an established, previously validated instrument is adapted. Two types of data are collected: a) video-recordings of lessons, subsequently watched by the rest of the group who take notes for later analysis, and material produced by the pupils and/or the teacher conducting the activity, such as lesson diaries; b) audio recordings of the follow-up discussion in the PIC in which the data are analysed. As a rule, all members enjoy the same authority in the decision-making process, from the aims of each project to the design of the activities and discussion of the results.

In general terms, there can be said to be a reciprocal learning environment between teachers and teacher educators, characterised by a free-flowing exchange of theoretical and practical knowledge, reflected in the project proposals and fostered by the joint reflection in the group discussions. This reflection is the trigger for professional development and accounts for the need for collaborative work. From the point of view of the researchers, it is crucial to learn at first hand the real needs of the teachers, so that their theoretical knowledge of issues touching on the teaching and learning of mathematics finds the most appropriate context for its application. Conversely, for the teachers, they are given the opportunity to work alongside like-minded professionals to find solutions to specific problems concerning mathematics which arise in the course of their teaching.

The first projects undertaken by the PIC grew out of the primary teachers‟ personal interest in improving their skills in managing problem-solving tasks. The following section describes how the group created opportunities for the teachers to develop their teaching in this respect, with particular focus on the role played by reflection as a means for promoting learning, and the collaborative setting of the group. In the years since then, other content areas have been given consideration, such as the role of ICT in mathematics education, spatial orientation, the classification of geometric objects and the definition of a polygon. In the most recent projects the mathematical focus has been on problem posing, a topic which has been studied at all educational levels, from nursery to primary teacher education.

Professional development in the PIC

We noted in the introduction that the PIC embodies several core principles, namely collaboration, reflection and reciprocal learning. To these elements there needs to be added a purpose, specifically, that whilst the PIC is an environment for promoting professional development among its members, this development is, at the same time, considered the object of research. In this regard, it can be said that the PIC is a space dedicated to developmental research (Jaworski, 2005). Other characterising features of the PIC are the interest shown by all members from the group‟s beginnings in problem-solving as an ideal methodological approach to mathematics education, and the willingness to place reflection at the centre of the group dynamic. In the same way, another constant feature has been reflection on our conceptions of the teaching and learning of mathematics, and on the implementation of the activities designed in the group sessions.

In an atmosphere of critical appraisal and collaborative endeavour, there were nevertheless moments when reluctance to cause offence prevented a deeper critical analysis, as a result of which there was little likelihood of suggestions emerging for significant changes to what had been observed in the recordings. This tendency, which was effectively preventing genuine professional development, brought about a change in direction: we decided to focus our analysis on what could be considered good practice. This resulted in a long process of joint reflection on what we understood as „good practice‟. After some varied background reading, lesson analysis and debate, the PIC developed its own framework for „good practice‟. The use of this tool for analysing video recordings of lessons brought to the fore episodes of good practice, and these acted as a catalyst to professional development as teachers recognised areas for improvement in their understanding of teaching (in Carrillo & Climent, 2011, we discuss in detail the improvement in teachers‟ understanding of practice based on a jointly constructed definition of good practice). The shift of focus onto positive aspects had the desired effect: in the absence of face-saving sensitivity to inhibit constructive criticism, debate opened up about how far an episode could be considered an instance of good practice, and exchanges became freer. This focus also enabled participants to make direct links between theoretical constructs and actual classroom events, to shed light on the reasons underlying decisions taken during lessons, to recognise their strengths and weaknesses as practitioners, and to apply a greater degree of precision in articulating their thoughts on situations of interest (Carrillo & Climent, 2011). At the same time, teachers became aware of the need to interrogate their own professional knowledge as a theme for reflection.

Throughout this process, there were milestones and members of the PIC who played particularly significant roles. One such member was Ana, one of the experienced primary teachers who was in at the founding of the group. Ana decided to take control of her professional development through the process of reflecting on her own teaching practices, which led her to attempt to align her teaching more closely with her conceptions of how mathematics teaching and learning should be. In order to do so she made notes on all the mathematics classes she gave in a diary, which became an essential data collection tool for researching her professional development, in a clear example of the developmental research approach mentioned above.

A very different case was that of Julia, a novice primary teacher, who, three years after Ana initiated her personal project, decided to instigate her own professional development within the setting of the PIC. In her case the diary was irrelevant, as she completed it purely out of a sense of commitment to the group, with the result that her entries were largely mechanical and revealed little beyond the different mathematical topics dealt with in class. The diary, considered the instrument par excellence for teacher reflection, was of no use in this instance. In Carrillo and Muñoz-Catalán (2011), the authors note that "Involvement, transparency and the capacity for reflection emerged as useful concepts for describing the fit between instrument (diary) and informant" (p. 91). The close consideration of Julia‟s diary led to a reflection applicable to any data collection instrument.

However, although the diary proved unproductive for Julia and the other members of the PIC, including the researchers, this is not to say that there was no evidence of professional development on her part (which, indeed, could be observed in terms of increased flexibility in her lesson planning, which began to take into account potential learning difficulties; Muñoz-Catalán, Carrillo & Climent, 2010). An essential element in this process was the discussion that took place in the PIC sessions, and in particular the exchanges between Julia and Ana. In this respect Ana acted as a *critical friend* (Jaworski, 2008) or *skilled collaborator* (Day, 1993) to Julia, with whom she empathised. For her part, Julia showed herself open to observations coming from Ana, who presented her with *hard questions* (Jaworski, 1998) challenging her to examine the root cause of how things turned out in her lessons. This could be seen, for example, when, after Julia had rejected suggestions from another teacher on how she might improve her response to a situation arising in class, Ana invited her to identify those events that had been productive for her pupils, those which could have been better and suggestions for achieving this (Muñoz-Catalán, Carrillo & Climent, 2010). In sum, the role of Ana was fundamental in Julia‟s professional development.

One distinguishing feature in the development of the group has been the role accorded to the knowledge required for teaching mathematics. In the very first project, the teachers flatly refused to make their own knowledge of mathematics subject for discussion, and made it a condition of their participation that they would not be called on to do problem-solving themselves, but instead would work on how best to manage a problem-solving approach in class. In broad terms, the teachers‟ work in the PIC constituted the alignment of their practices with their conceptions of how mathematics should be taught. In practical terms it involved devising mathematical scenarios for use in class and inviting pupils to conjecture on possible solutions. As this learning process progressed, Ana became increasingly aware of the limitations of her mathematical knowledge. She admitted that there were situations that "escape me", although she remained adamant in her refusal to tackle this limitation, stating, "*I'm not interested in that* [mathematical content]. *My only interest is in teaching the children better than I was taught, but along the way I've found out that I don't have the maths. (...). I don't have the experience of it* ["doing mathematics"] *from when I was a student myself*" (Ana quoted in the 1999-2001 project; Climent and Carrillo, 2003, p. 399).

Inevitably, the group‟s discussions of scenarios brought up issues of content, albeit indirectly via the teachers‟ interest in how it might best be taught, and their resistance to considering content diminished. Later projects explicitly included the study of how the treatment of content items evolved over the course of the education cycle from nursery to secondary. In this regard we posited the design of *hypothetical learning trajectories* (Simon, 1995) for certain content, such as proportionality, which were then contrasted with student learning observed at different levels. Reflecting on possible antecedents of the idea of proportionality in nursery education (such as the comparison of objects in terms of their shape and size) and primary education (such as the idea of twice or three times something, multiplication relationships, or equivalent fractions), seemed to provide the trigger for the teachers to begin to appreciate the importance of the teacher‟s knowledge. This reflection was accompanied by the reading of various relevant research papers on mathematics teachers‟ knowledge (chiefly MKT – Ball, Thames & Phelps, 2008 – and subsequently MTSK – Carrillo-Yáñez et al., 2018), with the aim of taking this reflection further (for example, with respect to the hypothetical learning trajectories, the notion of *mathematical horizon content knowledge* in MKT was revealing). Once the members of the PIC became aware of the importance of teachers‟ knowledge, one of the group‟s focal points became the knowledge required by mathematics teachers, and the possible connections between this and classroom management.

Another aspect worthy of mention with respect to the development of the PIC concerns the dissemination of the results of the studies carried out under its auspices. In the early projects the teachers had little interest in the publishing and communicating the findings of the work they had participated in. Gradually, and perhaps in proportion with their growing confidence in themselves as teachers and communicators, they became increasingly involved in disseminating the findings of the work of the PIC, with a notable increase, too, in their autonomy in the process. This has included joint presentations at conferences, as well as talks delivered exclusively by the teachers.

Returning to the question of the learning opportunities generated within the PIC, for the researchers it represents an observation post for how professional development is produced, in particular the „how‟ and „what‟ of specialised knowledge mobilised by teachers in their work. For the teachers, the learning takes the form of a growing awareness of their specialised knowledge and its contribution to their professional development, both inside and outside the classroom. For example, inside the classroom, they devise novel activities for their pupils, and reflect on the success of these and how to deal with their pupils‟ difficulties. The process of reflection and debate develops their confidence in deploying their pedagogical knowledge of mathematics in a range of situations. They are able to speak to colleagues with authority, and explain the precise nature of pupils‟ difficulties to parents, suggesting appropriate remedial strategies.

When the researchers step into the role of teacher educators, they recycle the knowledge garnered from their work in the PIC with the prospective teachers they train. In this way, they create a flow from knowledge jointly constructed in an in-service setting to knowledge capable of being constructed in an initial training setting. A key advantage of this knowledge is that it specifically concerns the kind of problems which arise when teaching mathematics and potential solutions to these. In short, a double transfer of knowledge occurs: from the university to society in terms of the professional development of the teachers participating in the PIC, and from the real world to the university in terms of opportunities for learning from actual practices. For this purpose, some of the videos of lessons developed by the PIC to illustrate episodes of good practice have been used in the initial training of primary teachers, and the discussions of good practice have thus been reconvened to the initial training context. This experience has allowed us to verify how this kind of activity encourages future primary teachers to contrast and articulate their ideas about the teaching and learning of mathematics (establishing connections between theoretical principles and actual classroom practice), to observe examples of practice which are not commonly to be seen in primary classroom, to learn to interpret the thinking of primary pupils in an authentic setting, and to value the benefits of reflecting on classroom practice in relation to the teaching and learning of mathematics, especially if this reflection is shared with colleagues (Carrillo & Climent, 2009).

Final reflections

Reflecting on practice can be considered a general-purpose tool for carrying out the kind of professional development we have recounted. It is consistent with Karsenty & Arcavi‟s (2017) description of reflection: "the detailed, analytical and careful observation of „what was done‟ in order to understand intentions, plans, actions and utterances and to consider alternative decisions and their possible implementation" (p. 435). In addition to the potential changes in teachers‟ actions it promotes, it enables experiences to be reconstructed, and knowledge, and the production of knowledge, to be situated in a specific context (Grimmett, Erickson, Mackinnon & Reicken, 1990).

The design and implementation of lesson ideas, with the aim of exploring specific questions about the teaching and learning of mathematics, is the trigger for reflection on practice. The complete process, from planning to execution and analysis, is carried out in the group. In this regard, the experiences we describe here bear certain similarities with the lesson study approach to professional development (Fernández & Yoshida, 2004). In our case, however, there is less emphasis on the redesigning and re-implementation of the proposal (possible improvements are reflected on, but without these being carried out) and the educators intervene to a greater extent in the process (to the same extent as the teachers).

One area in which the educators play a significant role is that of supplying relevant reading material to facilitate and systematise reflection on practice. In this regard, the research experience of the educators has interacted with their work in the PIC. Throughout the various projects we have drawn on theoretical constructs (such as hypothetical learning trajectories; Simon, 1995) and various analytical categories, including those designed for classroom management (Brendefur & Frykholm, 2000), mathematics teachers‟ specialised knowledge (Carrillo-Yáñez et al., 2018), and problem solving and posing (e.g. Singer, Ellerton & Cai, 2015), from the research literature into mathematics education, or adapted from it. The incorporation of such theoretical constructs has only been successful where teachers have been willing to familiarise themselves with the material and to assimilate it, a process undertaken in general on the basis of readings of the relevant research documents and discussion of their application to specific classroom situations. The role of the educators/researchers and doctoral or masters students has been crucial to this process, the latter often carrying out as part of their research work detailed individual analyses of the PIC"s activity, to be later discussed in the group. On the other hand, the results of the work of the PIC and the study of professional development of the teachers involved has influenced the research focus and perspective of the educators.

The configuration of the PIC has parallels with other models of collaboration between university and school. One example is that of *partnering relationships* (Goodlad, 1994), involving practising teachers, prospective teachers and teacher educators, in which the participants study the work of the practitioners. In our case, there is a greater emphasis on in-service than on initial training, although prospective teachers are occasionally involved in the PIC. It should also be noted that the training does have broader ramifications as the educators recycle material from the PIC to their work with prospective primary teachers. To this extent, teacher education can be seen as a continuum, from initial to in-service (Carrillo et al. in press), with an implicit emphasis on practitioners regarding ongoing learning as an essential part of their professional practice, and the prospective teachers seeing themselves as teachers in progress.

The PIC could be described as a *community of inquiry*, in which teachers and researchers "both learn about teaching through inquiring into it" (Potari, Sakonidis, Chatzigoula & Manaridis, 2010, p. 474). Ours is a long-term community of inquiry, in which various challenges have emerged, in response to which solutions have been sought out, discussed and implemented. For example, faced with a reticence to critically analyse individuals, we shifted the focus from the teacher to practice, drawing on the idea of defining and exemplifying good practice in order to overcome the this resistance. Likewise, in the case of the newly-qualified teacher, Julia, her inhibitions regarding the use of critical reflection were overcome by one of the experienced teachers in the role of *critical friend* or *skilled collaborator*. While some challenges find solutions, others arise, not least because of the changing composition of group and its dynamic. At the current time, the increase in the number of members and their different degrees of involvement, has created a new challenge for the equitable distribution of tasks and for maintaining a balance of contributions to the progress of the group. In this regard, issues around the factors affecting legitimate peripheral participation (Lave & Wenger, 1991) in a developmental setting such as we describe need to continue to be researched.

For the teacher educators, the PIC has brought about a change in research perspective, from doing research *about* teachers to doing research *with* teachers. In like fashion, reflection with teachers about their knowledge and practice is the potential departure point for changes of their choosing. Finally, the PIC has allowed us to observe interrelationships between knowledge, reflection and teachers" professional development, illustrating the complexity of these relationships.

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COLLABORATIVE TEACHING IN THE ITALIAN "LICEO MATEMATICO": A CASE STUDY OF CO-PLANNING AND CO-TEACHING

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This contribution is centred on a case study of collaborative teaching, carried out among Upper Secondary School Math teachers. Here we present the context, the forms of the implemented collaboration and the effects that derived from the collaborative teaching. The used investigation tool was the semi-structured interview aimed to induce teachers to gradually reflect on their self and their teaching modus operandi. From the qualitative analysis emerged that collaborative teaching integrated different ways of teaching; it has been a stimulus for a etching "revision" and the possibility to improve the didactic-educational practice. For all teachers this way to define the teaching activities was a valuable "tool" to see mathematics as part of a unified knowledge, creating bridges between scientific and humanistic disciplines, in terms of content and methods.

Introduction

The focus of the paper is refereed to the study of contexts, forms and dynamics that can characterize the collaborative teaching.We are conscious that the variety of these elements are important for the way in which the teaching collaboration takes place. In this sense, it could offer different analysis about the teaching/learning phases in classroom and so differed interpretation about the effective of this teaching modus operandi. For this reason we clearly declare, just at the beginning of our paper, the examined context: it concerns a small group of teachers from various disciplines (specifically, Mathematics, Physics, Literature and English), engaged in co-design and Mathematical co-teaching of a Upper Secondary School. The particular chosen context is related to a classroom that attends the first year of the Scientific Upper Secondary School "Benedetto Croce" in Palermo and that, at the time of enrolment, has chosen to join the experimental project of the Mathematical Upper Secondary School, called "Liceo Matematico" (hereinafter referred to, briefly, as LM). This realty is almost new for the Italian school (but maybe also for other countries), it was promoted by the University of Salerno and in few years it has spread to various parts of Italy, coming to live with about one hundred schools on the national territory and leaving a strong imprint of collaboration between Upper Secondary School teachers (teachers of Mathematics and beyond) and University professors belonging to departments of different areas (for further information, we suggest visiting the website www.liceomatematico.it). Here we present some results related to the terms in which the co-teaching (and the related co-planning of the Mathematics lessons) has been implemented. The research question to which we tried to reply is the following: from the point of view of the Upper Secondary School Mathematics teachers, what kind of possible results can be observed in classroom, defining a Collaborative Teaching approach in this classroom? In order to answer, we decided to conduct a semi-structured and audio-registered interview with almost all teachers attended the LM. According to Egodawatte et al. (2011) there is a strict correlation between the effective teachers collaboration and its effect in the improving of the students learning phase in Mathematics. Referring to their tool (a structured teachers interview), we modified it to better adapt it to the Italian school context and, specifically, to the socio-cultural context in which the classroom was located. In particular, each teacher was semi-structured interviewed; the interview was organised into several parts and related to the personal background (on the educational and work levels), the phase of didactical planning in the LM, the used collaboration approach with other teachers implemented, the possible benefits that emerged from this choicein terms of co-design and/or effective implementation in the classroom of the "interdisciplinary approach". In the following paragraph we briefly explicit the theoretical framework and subsequently, the research methodology adopted is discussed. The structure of interviews, mentioned above, is described. Finally, the results of the qualitative analysis related to the collected teachers replies is analysed and discussed.

Theoretical Framework

In literature the term *collaboration* has been declined in different ways in various areas of knowledge, from social psychology to education, from training in general to politics or sociology, just to name a few. Although different definitions of the term can be found within the various disciplines, in the context of teaching the term is used to describe a way in which two or more people work together to achieve a common purpose. When the people involved are teachers of one or more schools, the working context can be characterized, from a theoretical point of view, as a "cooperative learning Community" (Cooper, Boyd, 1997), and/or a "professional learning Community" (DuFour, 2004), and/or a "Community of practice" (e.g. Wenger, 2007). With reference to the latter one, for example, Wenger (1998) stresses the importance of sharing objectives within the Community of practice where the actors involved have common interests and collaborate in order to support the learning of the entire group and of each member of the same group. The common base of the above defined contexts seems to be the definition of a "Community of teachers" in which the individual teacher no longer acts "in isolation" but within a team made up of other educators (internal to the school or belonging to other educational agencies). Working in a collaborative way implies that everyone brings their own experience, their own professionalism, their own thinking to the work team, with a view to sharing and integrating rather than 'summing up' individual contributions. In this way, collaboration can provide opportunities to reflect on one's teaching practice (Thomas, Pedersen, 2003), to encourage a motivated and responsible attitude (Calderhead, Gates, 1993), to share opinions and criticisms of others' practice and to support change (Clark et al., 1996). In this sense, in order to set a fruitful collaboration, a large number of factors play a significant role. According to Jao& McDougall (2016) one of them is to create a working group whose members share the same teaching contexts. Additional factors include beliefs, among the group members and the sense of community (Dunne, Nave, Lewis, 2000) perceived by them. Trust in colleagues is an essential prerequisite for working with a collaborative approach. After the first working experiences with colleagues, this trust could gradually turn into respect, confidence and esteem, and could pave the way for a collaborative dialogue useful both for teachers and students. Finally, it is precisely through collaborative activities that the sense of community mentioned above can be affirmed (Grossman, Wineburg, Woolworth, 2001). Lieberman (1986, p.6) stated: "Contexts, needs, talents and commitments differ, but one thing appears to be constant: school cannot improve without people working together". His thinking shows the importance of working in groups and of forming groups, within the school environment, to promote the professional development of teachers and the school in general. In another text, he reiterates this important aspect as follow: "each school needs to establish a collaborative culture as a precondition for its own development" (Lieberman, 1990, p. 9). According to this, several empirical studies (e.g. Harris et al. 2006) shown how a collaborative teaching can be an effective approach to achieve educational goals (in relation to a given classroom context), providing educational resources and practices and providing opportunities for professional development such as co-planning and co-teaching (or cooperative teaching, understood in this contribution as a form of teaching in which two teachers come together to teach together in some course). Collaborative teaching also induces teachers to redefine their teaching practices and supports their mutual interactions (Wilson, Berne, 1999). McLaughlin , Talbert (2001) underlined that in a Upper Secondary school this can mainly be done at the level of the disciplinary departments, as a common ground in which to share norms concerning teaching/learning, student evaluation and classroom experiences (Gutierrez, 1996). Other studies have shown that collaborative teaching works in good (effective) way when the trust within the work team is supported by the presence of educational aim shared by all members of the group (Sydow, 2000) by relationships established previously between teachers and by an explicit management aimed to facilitate the collaboration (Muijs et al., 2006). Our research fits into this research area: as we already mentioned before, we in fact want to investigate how the collaboration between teachers of the LM has been implemented and how it influenced the interaction between teachers in the form and the quality of the teaching/learning process in their classroom. In agreement with Schoenfeld (2002) and Dallmer (2004), giving to teachers the opportunity to develop their skills and to increase their knowledge,could allow them to have more positive feedback on the students performance in Mathematics and so to interpret this in different way. We tried to do this in the context of the LM, where teachers from different departments worked together sharing goals and different approaches. It is not the aim of this paper to investigate about the feedback of collaborative teaching on the students learning, our focus is in fact on the teachers Community.

Research methodology

The research moves from the new experimental project of the LM in Palermo, a project that involved first of all in a training course for university teachers and teachers of the Upper Secondary schools of the second degree in which two first mathematics classrooms have been activated. The strong point of this course was to propose lesson and laboratory activities in which different areas of knowledge are integrated with Mathematics and into Mathematics. From these first training ideas, during the 2018/19 school year, a fruitful collaboration between some teachers was born. The forms in which this collaboration was carried out can be divided into two types essentially: workshops, aimed at co-designing the lessons and objectives, and teaching in small groups. For the purposes of the research, it was decided to investigate these forms of collaborative teaching, focusing on the results that teachers have found in their teaching practice and in the students themselves. Specifically, the research to be presented was conducted at a section of LM formed by classroom pupils before a Scientific Upper Secondary School. The classroom council was characterized by the presence of a team of highly qualified teachers who participated with interest and enthusiasm in the design and implementation of 'interdisciplinary lessons' carried out in co-teaching, providing their professionalism and their wealth of knowledge and personal experience. The common goal of the teaching team was to encourage a meeting between mathematics and other disciplines, to support a vision of mathematics as a discipline that integrates with others, supports them and can find in them various and concrete applications.

Collaborative teaching in the form of design and implementation of teaching activities

The team of co-involved teachers actively participated in the co-design of the activities of the LM. The activities were divided into "themes" and for each of them, the classroom council indicated the topics, the laboratory activities, the prerequisites, the objectives to be achieved and the competences and skills that the students would have been acquired. Considering the additional time that some activities required, for logistical reasons or sometimes for inadequacy of teaching technologies in the classroom, during the school year under review (2018/19) was partially implemented only one theme, the theme of the *measure*. Its realization has seen the co-presence in the classroom of teachers from the Department of Mathematics, with the collaboration of the teacher of English in the planning phase. The second part of the *measure* will continue in the next school year and will also see the teacher of Italian, Latin and History with the *agrimensor*.

Survey tool and data collection

The tool used to investigate the results produced by the collaborative teaching under examination was the semi-structured interview, in order to answer the research question. Each teacher was interviewed individually through the use of a recorder; at a later stage, the answers were transcribed and the data processed, as will be better described in the following paragraphs. The interview was designed on the basis of a previous work (Egodawatte et al., 2011) and was formulated in such a way as to adapt to the context of the LM. In defining the parts of the interview, account was also taken of the training that the teachers had received through the university professors of the Department of Mathematics and Informatics of Palermo. This training, in fact, had been an opportunity to present some interdisciplinary contents in which mathematics was linked to the use of artefacts, to the use of ICT (e.g. Geogebra) and in reference to didactic contents that were not exclusively mathematical but, on the contrary, included subjects from other disciplines (e.g. History, Art, Latin). The aim of this training was precisely to introduce the teachers of the various disciplines to create disciplinary links and, consequently, activities in which mathematics could act as a link between the various disciplines. A link between the disciplines would thus have a reflection on the training of the students, on their way of "seeing" mathematics but also on the teachers themselves. No longer a science in its own right, which can be more or less enjoyable than others, but a science that together with the others was born, has evolved and lets itself be discovered in its many facets. Precisely these aspects emerged, to a large extent, in the course of individual interviews. Further aspects that emerged from the qualitative analysis of the interviews conducted will be described in the next paragraph, in which the data collected from interviews with teachers involved in the research will be presented. These are five teachers, one of whom is in Literature, one in English, two in Mathematics and one in Mathematics and Physics. The interview was divided into five parts. The first part, inherent to the teacher's background, aims to collect generic information on the teacher's training, work experience and motivation to teach; the second part aims to induce a reflection on the school context in which he currently teaches, on any changes encountered and on the challenges posed by the school to respond to these changes; the third part concerns the activities of LM specifically (i.e., content, teaching methods, answers from pupils, objectives set by the teacher for the 'interdisciplinary lessons' planned); the fourth concerns the objectives set by the teacher towards the classroom and his opinions on the perception of "success in Mathematics" by the students; the fifth and last part aims to focus on the intra- and interdepartmental collaboration with colleagues, on any effects that may have had on the attitude of students towards mathematics, on the benefits found by such collaborative teaching (in their teaching practice, in professional development, in the sharing of ideas, materials and strategies). Here, reference will be made to the results that emerged from the first four parts, to leave more space for the last.

Qualitative analysis of collected data and discussion

The analysis of the data collected during the interviews was qualitative, given the small sample involved. This was possible starting from the semi-structured form of the interview, which left teachers a certain freedom of expression in their answers and clarified, where necessary, the meaning of the questions asked to them. Each researcher listened several times to the recordings of the interviews to analyse the differences and similarities in the answers given to each question; specifically, each defined the categories of answer (e.g. Students Mathematics perception, particular choices about the didactic practices, example of multidisciplinary approaches), in relation to the variables that have been investigated, compared for each question the similar terminologies used by the interviewed and the meaning with which similar terms were used, compared the occurrences of common terms or terms similar to the same thematic field. After this first data analysis (done by each researcher, authors of the paper, separately), the two researchers met with the aim to compare the work done and find out possible common teachers reply. At this stage, these were categorised. We are working on a subsequent "quali/quanti-tantivave" analsisys based on the implementation of the data by the atlas.ti software.

Results

The results will be discussed in five sections, how many parts the interview is divided into, with particular reference to the most significant answers to some of the questions in the interview. *Teacher's background*: first of all, the teaching of Mathematics concerns teachers who include in their training curriculum studies in different scientific fields. Each of them has been teaching for at least eighteen years and, during the 2018/19 school year, the whole team was involved in the co-programming of the classroom's 'interdisciplinary lessons'. *School context*: the transcript of the various interviews revealed some key words more frequent than others, such as "change" - in a purely negative sense. All teachers stated that in the last ten years they have found a progressive deterioration in the basic knowledge of students entering Upper Secondary School, combined - according to one teacher - with the change in learning styles and the worsening of "skills of expression and communication". Someone else highlighted the fact that the "human material" of the school has changed - referring to the students - not only in the knowledge possessed by the incoming students, but also in their expectations and desires. *LM activities*: in this changing and dynamic context, the school has tried, through the LM, to combine the scientific and humanistic fields, encouraging the development of a "strong literary culture". *Teacher's objectives and perception of "success in Mathematics" in the students*: when asked what do they think the students of the Mathematical Upper Secondary School consider to be "success in Mathematics". For some, it is taking a 'good' grade, in reference to evaluation. For others, such 'success' also consists in understanding the links with the outside world or in being able to connect mathematics with other fields never imagined before. Two teachers, in particular, believe that the students of the LM perceive as a ‘success in Mathematics’ the fact of having knowledge or having acquired additional skills compared to their peers, thanks to the experiences made during the LM. *Intra- and interdepartmental collaboration with colleagues*: regarding collaboration with colleagues, all respondents stated that they had previously collaborated with other colleagues to co-design paths that included multiple disciplines (Mathematics and English, Mathematics and Science, English and Science, Latin and Science). Four of them had also spontaneously implemented co-teaching in the past. Since three out of four teachers teach Mathematics and had previously collaborated with each other, the most interesting reflections emerged regarding the collaboration between colleagues from different departments. It was intended to investigate the influence on teachers through the question *What benefits did you receive from working with colleagues in other departments? (E.g. access to ideas, materials, strategies and skills of others, lesson planning, perception of the quality of one's teaching, professional development, professional gratification, ...)* and about the students through the subsequent question *In your opinion, what benefits have the students received (or will they receive in the future) from such intra- and interdepartmental collaboration?* Among the answers to the first question, there is that of a teacher who refers to having expanded the knowledge of individual teachers (for example, in the part of Mathematics and English, there was a marriage between the English language and the “mathematical language”), having increased the awareness of how much is spendable the teaching action in an interdisciplinary vision, having had the opportunity to intersect their teaching practice with other disciplines from the point of view of content and - even more difficult - from the point of view of method. Continuing, the teacher emphasizes that it is more difficult to share the method in an interdisciplinary collaboration and that, at the same time, this collaboration is even more profitable than a collaboration between colleagues in the same department, because seeing new teaching practices can open new horizons and improve the quality of their teaching. Someone else sees benefits in having experimented with new teaching strategies or in having perfected the times and methodologies previously used. Among the answers to the second question, it is noted that the students were given “the opportunity to see knowledge as unitary and not 'parcelled' in the various disciplines” (it was precisely one of the main objectives of the work team), having “learned to look at the classroom council as a community body” whose members design and work together for the classroom, having acquired a greater awareness of the role played by mathematics in the surrounding world, having learned a part of micro-language (useful for the scientific training of a student), seeing more integrated styles of teaching/learning and last but not least, having contributed to the creation of a certain “openness of mind through these multidisciplinary paths”. In support of this, one teacher, speaking of hopes she has for her students, says she hopes that these paths, or rather, the ‘alternative’ vision they offer of Mathematics and Physics, will help students to “learn to think”. In his case, this is a hope that can only be confirmed in the long term. Going a little deeper on the subject, with the question *In your opinion, how can collaboration between teachers influence students' attitudes towards mathematics?*, the answers were almost unambiguous. Everyone has said that such collaboration can influence this attitude in a positive way and, in particular, they agree that it can contribute to ‘seeing’ Mathematics no longer as a discipline in itself, detached from reality and written in a language incomprehensible to many, but as a discipline that has evolved with others, in history and time, which finds applications in various branches of knowledge. From their own answers:

"I think that in the end the pupils' attitude towards mathematics should be more open, considering it not only as a 'closed' discipline but as a discipline capable of permeating all the others, just as it should be for linguistics, which is a discipline that is absolutely transversal to any teaching."

"I think they can understand that the whole world is mathematical and mathematics enters everywhere, in everyday life, in history, in the history of thought."

"In the meantime it can give an idea of the unity of knowledge and then clearly they can see mathematics from different points of view in areas where it is not generally so easy to 'see' mathematics. It seems strange, but even the fact that the same language has often been used [...] has allowed us to see how, for example, the language of Statistics and the language of Error Theory in Physics are closely linked: the same term has been used in different fields, giving different facets but showing that the concept is always the same. We have noticed this, for example, with the concepts of 'dispersion', 'error' or 'mean value'. We have seen the 'average value' in the statistical field and the meaning of the 'average' as the 'average value' of a physical quantity, as the best estimate of the true value. […]"

Conclusions

The research carried out aimed to examine the results of the qualitative analysis of the interviews conducted with the team of case study teachers who collaborated in the forms of co-planning and co-teaching. Given the small sample of only five teachers, we do not intend to generalize the results discussed here, but only to describe the aspects that emerged from the actors involved in the case study. This research also aims to be a contribution to the existing literature as it refers to collaborative teaching in multidisciplinary paths (e.g. Hurd, Weilbacher, 2018) and in a specific common teaching context (Jao, McDougall, 2016) which is that of the Mathematics Upper Secondary School, a project that is affecting more and more school realities in Italy. The innovative aspect was the application of collaborative teaching in a Mathematics Upper Secondary School: looking at the programming of the Mathematics courses from a multidisciplinary point of view, we wanted to promote a collaborative culture among teachers first and an integration of the disciplines then, to "develop a critical view of reality", as desired in the National Indications (Miur, 2010) of the Italian Ministry of Education. The qualitative analysis of the data collected has made it possible to highlight the benefits that the teachers have derived and the results achieved, both on a professional and human level, during the "interdisciplinary lessons" that have served as a framework for collaborative teaching. Specifically, the interviewees said they had increased, through collaboration with colleagues, their knowledge and skills in several areas related to each other (referring to the study from which we draw inspiration here): the achievement of objectives (first of all, that of the team), the success of the students (influenced by the integration of disciplinary content and methods of teaching-learning), professional development (derived from reflections on their own and others' teaching practice), the opportunities arising from co-planning and co-teaching (eg. improve teaching times and strategies). However, work is still in progress on the data collected through the atlas.ti software; this could lead to further interesting results or confirm what has been described above. Certainly, further questions, possibly more specific, or turning to a larger sample would have brought to light other aspects that are not described here. Although considering the limits of the research, the interview allowed us to respond, limited to the case study, to the research question and to derive food for thought from which to investigate collaborative teaching in wider contexts.

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COLLABORATIVE DESIGN OF RESOURCES FOR ELEMENTARY ALGEBRA TEACHING

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For more than 15 years, we have been working in a collaborative research group (composed of teachers and researchers) to develop and disseminate resources (on the website http://pegame.ens-lyon.fr/) for mathematics teachers and trainers in the teaching of elementary algebra in France (students aged 11 to 15). The aim of this group named SESAMES is to develop activities, which can be carried out in the classroom and disseminated in ordinary classes, with a significant didactic potential to motivate the use of algebraic concepts but also to work in a combined way on the technical aspects, meaning and justifications. In this text we analyze this type of collaborative research, then we highlight some effects for the teachers and for the researcher.

This text analyzes a collaborative work between researchers and teachers carried out within the ICAR[1] laboratory (UMR 5191- French National Centre for Scientific Research, University of Lyon, Ecole Normale Supérieure of Lyon, France) since 2002 and supported by the IFE (Institut Français de l'Education). For more than 15 years, we have been working in a collaborative research group (composed of teachers and a researcher) to develop and disseminate resources for mathematics teachers and trainers in the teaching of algebra (more specifically the beginning of algebra) in France (students aged 11 to 15). The aim of this group (named SESAMES[2]) is to develop activities, which can be carried out in the classroom and disseminated in ordinary classes, with a significant didactic potential to motivate the use of algebraic concepts but also to work in a combined way on the technical aspects, meaning and justifications.

The developed documents are available on the website http://pegame.ens-lyon.fr/. The website offers two entries named Teaching and Training. The first one includes activities to be offered in the classes (classified by level) and the latter presents documents that should enable teachers to better understand our choices in order to allow a more consistent use in the classroom of what we propose (Alves et al., 2013).

The purpose of this text is on one hand, to analyze the functioning of this group of collaborative research between a researcher and mathematics teachers to produce resources on algebra teaching and its evolution and on the other hand, to show some of the effects of collaborative work, particularly on the knowledge of the participants. Thus our research relates to the theme B : Contexts, forms and outcomes of mathematics teacher collaboration.

In the first part, we give some information about the group fonctionning and its brief history. Afterwards, we describe and analyze the type of collaboration using the model of Desgagné (Desgagné, 1997 ; Desgagné, Bednarz, Lebuis, Poirier, & Couture, 2001). Finally, we will seek to determine on what points the teachers' knowledge and skills have evolved.

[1] Interactions, Corpus, Learning and Representation
[2] Scientific Teaching Situations : Modelling, Assessment and Simulation

A brief history and functioning of the group

The group was created in 2002 by the researcher (S. Coppé) and an expert teacher involved in training. For this teacher, it was an opportunity to continue his reflection on the teaching of algebra but also to share activities that he had developed in his classrooms and that he often practiced. For the researcher, it was an opportunity to better understand the difficulties of students in algebra and to test activities from the research, by studying the conditions under which they could be transferred to another setting. At this moment the researcher focused on teaching and learning algebra and not on collaborative work.

Then other teachers joined the group[3] and some of them left after a few years (each year the group consisted of about 5 to 7 teachers). They are volunteers, they join the group either after their initial training (they want to stay in contact with the training or with some trainers), or after in-service training courses organised by the group (they have found the ideas interesting and want to continue), or by people they know (especially in the same school). The primary purpose of this working group is to produce resources for teachers and trainers. It is the contract with the teachers and the researcher. A more distant objective is to change practices of teaching algebra, to allow professional development for the teachers of the group. It is important to note that this is a co-construction work in which everyone contributes their expertise (even if only just beginning) in his domain : for example, concerning the research on elementary algebra, the teaching practices (not everything is possible in the classroom !) and the dissemination (taking into account the practices of other teachers). Another point is that the researcher and the teachers do not always do the same thing : for example, the teachers can use what is done in the group and transfer it to other areas without the help of the researcher and the researcher can take data that will be used for his own research.

To carry out this work, regular meetings (two per month) are organized in which specific topics are discussed (e.g. how to explain a common error, how to introduce the letter x), activities for the class are developed and analyzed, and then documents are written and posted on the website. Depending on the themes and times, we video-recorded classes, collected student productions, conducted interviews or questionnaires with teachers, etc.

Let's now look at the three important periods in the history of this group. A first step in the group's work was to produce and experiment innovative problems with little guidance in which the introduction of the letter was at stake and left entirely under the responsibility of the students (strong influence of the theory of didactic situations of Brousseau (1986)). These activities are experimented in the classes of the designated teachers and then written according to a defined framework (part of the website : Teaching). If at the beginning, the work was limited to this, some new questions appeared when we tried to disseminate them either online or through teacher training. The first concerned the way in which the documents to be distributed to other teachers were written. Very quickly other tasks appeared : developing documents other than the classroom sessions to explain our approach, our choices (part of the website : Training) and to specify

[3] A list of teachers who participate or participated in the group : Christophe Alves, Olivier Arrouch, Véronique Berger, Serge Betton, Maud Chanudet, Anne Sophie Cherpin, Vincent Duval, Stéphane Garapon, Alexandra Goislard, Sylvie Martin-Dametto, Claire Piolti-Lamorthe, Sophie Roubin, Etienne Spaak.

classroom management ; designing the architecture of the site (the sections evolved during the work). So we can note in this first stage a first evolution of the group's work : for the teachers it was a question of moving from the teacher who prepares class sessions and animates them to the teacher who communicates to other teachers and justifies his choices. For the researcher, it was more a question of doing a work of didactic transposition of didactic knowledge from research (Chevallard & Joshua, 1991) or knowledge from practice.

The second evolution concerned the activities that we analyzed as rich and relevant, but the fact that they remained isolated, did not allow good dissemination to other teachers and they did not take sufficient account of questions related to algebraic calculation techniques (Coppé, Grugeon- Allys, & Pilet, 2016). So the second step consisted in developing series of problems according to progressions (in particular on equations and proofs in algebra) and taking into account, in particular, the role and place of the distributivity of multiplication over the addition. However, we have shown through a study of curricula and textbooks that in France since the 1990s, distributivity has had a reduced place in the justification of calculations, which could explain some of the many errors made by students (Assude, Coppé, & Pressiat, 2012). Thus we can note a second evolution from isolated activities towards a series of problems inserted in a planning process. For that we refered to the theoretical framework of the anthropological theory of the didactic developed by Chevallard (1998, 1999). For this purpose we have used recent research in the didactics of algebra and in particular the introduction of computation programs first introduced by Drouhard (1995) and later by Ruiz Monzon (2010). She defines elementary algebra as a process of algebraization of computation programs and proposes mathematical organizations related in particular to functional modeling (Ruiz Monzon, Matheron, Bosch, & Gascon, 2012). Chevallard (2007) indicated in particular that they can be used from primary school onwards, an idea that we share. In our work, we use computation programs to introduce activities of generalization and proof and equations. At the same time, teachers have developed an original classroom management approach that consists of working on certain mathematical themes over the long term with problem cycles of a type of mathematical task (Martin Dametto, Piolti Lamorthe, & Roubin, 2013). To this end, at each session, they propose a problem to look for, a pooling of procedures and, at certain times, an institutionalization of a new mathematical notion. Then they start again with a more complex problem by changing the didactic variables. This class management can be easily performed using the computation programs. This second evolution has enabled teachers to broaden their knowledge of algebra teaching. It has required significant adaptations of their teaching on both content and form. For the researcher, it was an opportunity to participate in the development of new activities for the class and to study their implementation.

The third step was an opportunity to expand the work. First the group participated in a European research project on formative assessment ASSIST-ME (Assess Inquiry in Science, Technology and Mathematics Education. http://assistme.ku.dk). In the last two years, the group has started working with primary school teachers to develop activities related to early algebra (Radford, 2014). It was the teachers who wanted to do this work.

After this brief description of the history and evolution of the group's work, we will now use the three criteria of Desgagné's collaborative research model to show how and where we stand in this model.

The type of collaboration

Rogalski (1994) shows that collective work can be analysed in three ways : collaboration (actors share the same prescribed task and have similar skills), distributed cooperation (actors share the same objective but their tasks are different) and co-action (sharing the same workspace and/or resources). For us, it is indeed a question of distributed cooperation. It is therefore not a top-down transmission work (even if we know that this is never really the case) from the researcher to teachers but rather a joint elaboration to produce resources (Gueudet & Trouche, 2012).

To analyze the type of collaboration, we use the collaborative research model between researchers and teachers developed by Desgagné (1994).

> From an epistemological point of view, this means that, for the supporters of collaborative research, the construction of knowledge related to a given professional practice is not done without considering the real context in which this practice is updated, knowing that the components of this context, in terms of the constraints and resources presented by the practice situations, contribute to its structuring. (Desgagné, 1994, p. 373)

Desgagné (1994) defines three criteria for this type of collaborative research. We will now explain which elements of our work they refer to and how they were implemented. The first one concerns the object of the collaboration : "Collaborative research involves the co-construction of a knowledge object between a researcher and practitioners." (Desgagné, 1994, p. 383). As we have said, for us, the goal is to produce activities that allow algebra learning about both meaning and techniques. At the beginning of the group's work, we looked at the teaching of algebra as it is now practiced in France. The most important findings are : algebra is taught as an object rather than a tool to solve problems, the aspects of generalization/proof tool are not given much prominence in algebra teaching and there are many errors in the calculations because distributivity is not used as a technological element in the calculation techniques. Due to these findings, we wrote together seven principles that have always guided our choices in the preparation of activities (accessible on the website). This includes giving purposes to the teaching of algebra, i.e. finding problems that require as much as possible the use of algebra with the use of the letter, promoting student activity by looking for problems that are part of classroom sessions/sequences, encourage register changes, work on verifications that give meaning to notions, work on formulas to prepare the notion of function, work on proofs in algebra and justify calculations with clearly explained algebraic rules. These principles have allowed us to start the work of co-building resources and then to integrate new teachers more easily.

The second criteria concerns the knowledge production : "Collaborative research combines both knowledge production and professional development activities." (Desgagné, 1994, p. 376). There has been a great deal of didactic work on elementary algebra since the 1980s, which shows the breadth of the field. Many results have been produced but it seems to have difficulties in disseminating them in practice. As we cannot list all of them here, we mention only the most important for us. We used the early works of Chevallard (1985, 1989) or Kieran (1990), those of Booth (1985) on errors and difficulties, those of Vergnaud (1989) on entries into algebra, arithmetic or algebraic procedures, those of Gascon (1993) on the arithmetic/algebra transition (elementary algebra is not a generalized arithmetic), those of Sfard (1991) on the structural and procedural aspects, those of Bednarz and Janvier (1996) on the four main perspectives of introducing algebra to give meaning to new objects (the approach by generalization / recurrence, the approach by problem solving/equation, the approach by modelling and the technological / functional approach). More recently, Kieran (2007) defines three aspects of algebraic activity with the GTG model to conceptualize algebraic activity: generative activity, transformational activity and global activity at the meta level.

The researcher is familiar with these works and can propose them to teachers when the need arises. For example, at the beginning of the work we analyzed common errors and then we tried to broaden the types of problems offered to students. Finally, all group members agreed to use the computation programs ("think of a number, add 3, multiply by 8…") which were introduced to propose progressions on equations and proofs in algebra. This was done at the same time as the change in classroom management proposed by the teachers. The researcher then filmed a class sequences of 18 periods to analyze the students' work. This was done as part of the European research project S-TEAM[4], 2009-2012 (Science-Teacher Education Advanced Methods) (Coppé, 2013).

The third criteria concerns the links between research and practice : "Collaborative research aims to mediate between the research community and the community of practice." (Desgagné, 1994, p. 380). Several research studies (Coulange & Grugeon- Allys, 2008 ; Mangiante Orsola, 2014) have highlighted the difficulty for teachers to set up potentially didactically rich activities in their classrooms. Researchers may then think that these activities are not implemented in a relevant way and that their interest is then reduced. We have hypothesized (Coppé & Grugeon- Allys, 2015) that one of the reasons for this difficulty is that too often isolated activities (not related to a teaching progression) are offered and that teachers do not understand how they can be linked to what they usually do and they have some difficulties to understand the aim of the problem. In conclusion, about collaborative research, we share the position of Bednarz (2013) that emphasizes the need to take into account teaching contexts so that the produced activities can be integrated into "ordinary" practices.

> For us, therefore, it is not only a question of developing rich and relevant teaching situations in terms of learning, contributing to a conceptual construction that is meaningful for students, which a didactic analysis can of course help to clarify, but also of producing situations that are, in addition, viable for teachers in their context. (Bednarz, 2013, p. 13)

Examples of the evolution of practices through collaborative research

Finally, we will now show some examples of evolution in knowledge and practice for teachers and researchers. For this we will use the teacher's knowledge for teaching according to Ball, Thames and Phelps (2008). By taking up Shulman's work (Shulman, 1986), they define two categories of knowledge : subject matter knowledge and pedagogical content knowledge (see figure 1).

We will now characterize the changes in teachers' knowledge with these components. Before then, we can notice that the rapid overview shows us that collaboration is based on a common work that evolves and becomes more complex, which generates new questions. The teachers, becoming more and more experts, are increasingly solicited by the institution, which causes a change in their professional missions (especially the transition from teaching to training). Two of them became trainers in pre service training (they remain part-time teachers) and the others participate in in-

4 https://cordis.europa.eu/project/rcn/91198/reporting/en

service training. Some of them were engaged as supervisors of beginning teachers. We therefore believe that they have increased their knowledge in a global way and certainly in all types.

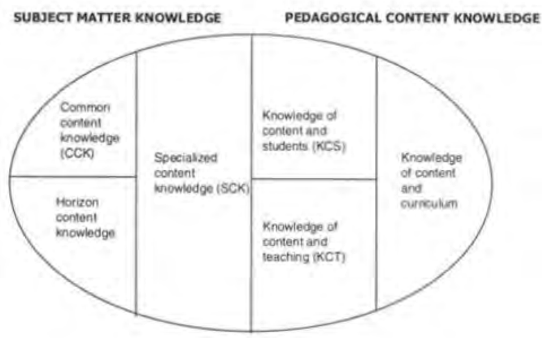

Figure 1. Domain of mathematical content knowledge for teaching (Ball et al., 2008, p. 403)

Concerning the subject matter knowledge, it was difficult to assess teachers' mathematical knowledge, so we don't know exactly, if the common content knowledge has evolved, for each teacher, but we can think so. Due to the work done we can ensure that the horizon and specialized content knowledge has evolved. For example, we have broadened the range of problems proposed by focusing on those leading to proof in algebra, we have better determined the goals and objectives of teaching algebra in middle school, we focused on the justifications for the calculations, etc.

In 2010, we conducted a questionnaire[5] (submitted to all teachers of the SESAMES groups that existed in 2010) in order to determine the effects of participation in such a project on the practices and representations. We received 17 responses, which is the vast majority of participants at that time and before. If there is a large majority who consider the development of design tools to be important, they consider that regular meetings (13/17) and especially exchanges between teachers and between teachers and researchers are essential (16/17). The idea of collaboration is therefore fundamental to the group's work. There is also almost unanimous agreement on the development of sequences associated with classroom tests and their analysis.

The analysis of some answers to this questionnaire provides information on the evolution of their pedagogical content knowledge. The majority of the teachers say that their practice has evolved in terms of how they prepare their teaching lessons, how they are implemented in the classroom, how they are analysed and assessed. The responses show that almost all of them claim to have a different view of students' mistakes (14/17), and also a majority of them believe that their procedures/approaches and responses should be anticipated. 13 out of 17 teachers report changes in their lessons planning : they try to make a better introduction of the concepts in order to motivate students. They take a more critical look at textbooks (9/17).

The answers to the last open-ended question (“How to motivate someone to come and work in the group ?”) confirm the results : the points that are highlighted are once again the role of collaboration for 8/12, to question one's practice (5/12). The development of resources only appears once, which makes it less important ; it appears as a tool and not as the final objective.

Conclusion

5 S-Team project, Work Package 4 : Deliverable 4b (Coppé, S. & Tiberghien, A.)

Through the example of the SESAMES collaborative research group, we have tried to show an example of how such a group works by characterizing it with the components highlighted by Desgagné, then we quickly pointed out how and in what way participation in this group has allowed an evolution of teachers' knowledge and skills. In conclusion, we would like to highlight some of the effects on the researcher.

At the beginning of the work, the role of the researcher consisted both in asking teaching questions that teachers did not necessarily ask themselves and in transforming certain remarks into more general questions. Now the questions come from the teachers. Then experiments were conducted that fed into the discussions and led to a more in-depth analysis particularly, of what the students were doing. As we said before that, this has caused a shift in the teachers' view of the students.

It seems that this work has led to a better understanding of the conditions under which practices can evolve through training. As a result of this work we now believe that too often teachers are given answers to questions they do not yet ask themselves or not in this way. There is therefore work to be done on the teaching questions that are asked before providing answers that are sometimes too far from the context and, therefore, are not recognized by teachers as real answers.

A last point concerns the quality of interactions between teachers (through discussions and sharing of practices), which can also encourage personal development.

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THE COLLABORATIVE WORK OF TEACHERS AND STUDENTS IN MATHEMATICAL ACTIVITIES CONDUCTED IN PUBLIC SCHOOLS

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The collaborative work between higher education teachers, basic school teachers, researchers and future teachers in the Brazilian Federal District is the subject of this paper. This collaborative activity is a project called "Circuito de Vivências em Matemática" (Circuit of Experiences in Mathematics), and one of its main goals is to bring to school students the possibility to experience mathematics in a ludic, practical and useful way which could promote the development of their mathematical learning processes. It is characterized as a project carried out by a multi-faceted group of people engaged and working for a common purpose, contributing to the initial and continuing training of mathematics teachers in that region. The project's scope and challenges are discussed, as well as its importance and current status. Furthermore, a theoretical framework is presented and an outcome for its challenges is addressed. We point out some contributions made by the project, which is now established as a space of professional development, integrating school and academic professionals in the task of planning and mediation of the workshops.

Introduction

A unique experience of a collaborative work between higher education teachers, basic school teachers, researchers and future teachers that has been accomplished since 2004 within the activities of the project called "*Circuito de Vivências em Matemática*" (Circuit of Experiences in Mathematics[1]) will be presented and described in this article.

The initial ideas that led to design and conception of the *Circuit* arose in 2004 as a result of the collaborative and voluntary contributions from the Brazilian Society of Mathematics Education (SBEM - Sociedade Brasileira de Educação Matemática) associated members in the city of Brasília (SBEMDF).

From that period onwards, the actions of the *Circuit* have been developed and implemented by the participants and have been performed in public schools in the Brazilian Federal District (DF). They consist of a set of mathematical workshops offered to a group of students in one day of the week in a previously chosen public school. The activities are conducted by at least two people, who can be either undergraduates or higher education teachers. Each workshop is developed to have a duration of 40 to 50 minutes. After this period, the school participants move to another workshop and do so continuously, so that they can experience until five activities on the same day. In this methodology lays the reason behind the name *Circuit*. The history of the origins of the project has been described in recent publications, namely Dörr, Rabelo, Santos, Silva, & Silva (2019) and Dörr, Silva, & Silva (2019).

The main objectives that guided the development of the project are the following: 1) to promote mathematical thinking and practice in an investigatory and creative way for students from basic

1 The term *Circuit* will be used to refer to the name of the project throughout the text.

public schools from the Federal District in Brazil; 2) to promote the creation of mathematical workshops among undergraduates, graduates, basic school teachers, higher education teachers and researchers working with teaching processes in Mathematics Education; 3) to develop and evaluate the executed mathematical workshops; 4) to establish the collaborative and cooperative research as a tool for the initial and continuing education for future mathematics teachers - as pointed out by Fiorentini (2005) - which is a special expectation for all the stages of this project's process, i.e., creation, planning, evaluation, execution and the workshops' socialization.

In this sense, this report focuses on the description and discussion of the fourth objective, which refers to the related collaborative and cooperative tasks that involve all the main parts of the *Circuit* project. Thus, this is a descriptive study reporting and explaining a project on the collaboration among an educational community. Considering this as the only phenomenon that is detailed on the local context of the investigation in order to describe and through the analysis to gain a better understanding of the different types of interactions and its consequences for the future based activities, it could be characterized as a case study (Ritchie, Lewis, Nicholls & Ormston, 2013; Triviños, 1987).

For gathering the information and construction of the study the authors monitored as coordinators the executions' stages of other research project designed to organize and document the activities of the *Circuit*. This project will be described in the following sections. As a result, a large amount of material was collected by Internet. Consequently, the specific tools used to sort, analyze and report the actions of the *Circuit* was made through the documental analysis.

Therefore, the next section presents the detailed characterization of the collaborative and cooperative activities of the *Circuits*. At the same time, we dialogue with the ideas from some authors to introduce the theoretical framework. Furthermore, we address an outcome of the initiative. Lastly, conclusions regarding the importance of the collaboration, challenges, contributions from this work to the literature on teacher collaboration and the current stage of the *Circuit* workshops are commented.

The performance of the collaboration on the *Circuit* project

All the phases of the described project are characterized as a collaborative and cooperative work carried out by a multi-faceted group of people engaged and working for a common purpose. One of the many goals of the *Circuit* is to bring to school students the possibility to experience mathematics in a ludic, practical and useful way in order to contribute to their mathematical learning processes. Furthermore, as the activities require a certain level of interaction and commitment among the participants, it can be identified as a community of practice, as pointed out by Wenger (2001).

In this sense, Fiorentini (2005; 2009) claims that the mutual engagement shared by the people involved in such gatherings has a practical impact on their related mathematics teaching and learning practices.

The organizing committee of the *Circuit* workshops is formed by higher education teachers and researchers that belong to the public and private academic institutions, as well as teachers from public basic schools. The configuration of the collaboration achieved in the workshops acts as a link between university and society, as described by Cristovão & Fiorentini (2018) and denoted as a *border community* that "it is arranged between the school and the university and is not regulated by any of the two contexts because it has its own regulations" (Cristovão & Fiorentini, 2018, p. 14).

Besides, the same authors explain on page 14 that

> In borderland communities, there is a meeting of institutional cultures, mainly of school and academic cultures. However, there is also the meeting of subjective experiences that occur through the stories of lives, narrated by each of the participants and establishes in different scenarios' practices.

Table 1 below lists the institutions that constitute the group of participants, indicating whether they are public or private.

Table 1: Institutions participating in the *Circuit*

Institution	Type
University of Brasília	Public
Federal Institute of Brasília	Public
Catholic University of Brasília	Private
Projeção Faculty	Private
Estácio de Sá Faculty	Private

Each workshop is led by a coordinator who, in most cases, is an associate member of SBEM-DF. Alongside that, one person belonging to each participating institution is also part of the team. Working together they have the task of conducting and dialoguing with the workshops proponents to organize the events.

The proponent's side consists of undergraduate students from the institutions involved, future mathematics teachers, but also postgraduate students. They are responsible for suggesting the mathematical tasks, constructing new activities or adapting the existing ones. We note that the activities are structured through playful, spontaneous and practical methods to provide to school students a learning environment, not through exercises. This characteristic has elements from the landscapes of investigation developed by Skovsmose (2012).

Our experience with the actions fulfilled by the *Circuit* workshops has shown that assembling such a group of multiple educational backgrounds and professional experiences is a complex issue because it consists of the agreement on a significant number of ideas, beliefs and knowledge (Boavida & Ponte, 2002; Cristovão & Fiorentini, 2018).

This special kind of collaboration performed in the *Circuit*'s activities not only offer mathematical experiences but bring the unusual possibility of getting mathematics educators co-planning and co-acting on a shared goal. The result is that the event has been an educational community with a common effort to contribute to the process of social intervention amid teachers and students from public schools (D'Ambrosio, 2012; Skovsmose & Niss 2004; Skovsmose, 2012).

To define the place for the workshops, the public schools that are interested in taking part in the activities are registered previously by SBEM-DF. The project has attended at least twenty basic schools in the region, from the basic to secondary levels, reaching approximately five thousand students in total since 2004.

The demand for registration has been increasing yearly and the annual calendar is organized in advance. The number of *Circuits* and of people involved would need to triple in order to serve the request from schools.

The documentation of the project

However, the *Circuit of Experiences in Mathematics* of the Federal District encountered big challenges. Firstly, the challenge regarding the lack of formalization as an inter-institutional research project in the area of initial and continuous formation of mathematics educators. This scenario could lead to the lack of scientific validation for the production processes, as well as for the application and evaluation of the experiences produced.

The second challenge was the lack of collecting, formatting and socialization of the countless activities that have been already produced and also of those that are currently in the process of being produced.

Due to these demands and despite all achievements in terms of collaborative production, the number of schools and students assisted, and the number of partner institutions working together, the *Circuit of Experiences in Mathematics* of the Federal District had not organized nor socialized its materials for consultation up to then. This could lead to the loss of experiences which would make it considerably difficult for more people to get access to and benefit from the materials produced.

At the same time as these deficiencies were witnessed, the success of similar experiences was noticed. These initiatives make use of current technological resources, such as blogs, websites, platforms, or specific areas, utilizing the help of the internet, uniting people and communicating their productions to Brazil and to the world.

Hence, it is understood that the compilation, organizing and publishing of the entire production of what was already carried out, as well as of what is currently being produced, on a single website would expand the communication possibilities of the knowledge acquired collaboratively. This website would be designed for teachers and students, being easily accessed from any device and location, utilizing the benefits and convenience provided by the internet. It follows that a research project was initiated in 2016 through a documental study which enabled to meet this demand by cataloguing part of the experiences that took place between 2014 and 2018 and by developing a website.

The website of the *Circuit of Experiences in Mathematics* of the Federal District can be accessed since July 2019 through http://circuitodevivencias.mat.unb.br. In addition to the experiences of the *Circuit*, the website also contains interviews from collaborators who participated and currently still participate in the project since its beginning.

Discussions and some conclusions

This paper uses a descriptive case study to present a collaborative experience that gathers a distinctive combination of mathematics educators: teachers of basic schools, higher education teachers and researchers, school teachers, undergraduate and postgraduate students. The project presented is described as a collaborative and cooperative activity getting the participants to co-planning and co-working in order to bring mathematical alternative methodologies for the classroom that leads and promote the learning processes of school students.

The model of collaboration described verifies the ideas from Fiorentini (2005) in which it is established a collaborative experience as a tool of development of continuous learning processes for teachers, docents and researchers. Further, it brought the opportunity to contribute to their personal and mutual learning processes while investigating, sharing and putting their ideas into practice. Most importantly, the activity has stablished a space of professional development integrating school and academic professionals in the task of planning and mediation of the workshops.

Thus, it could be noticed during the collaboration work the occurrence of professional development, co-planning and co-teaching opportunities and also an increasing of the communication among the participants. These areas are classified by Egodawatte, McDougall & Stoilescu (2011) as some main benefits that effectively impact the participants involved in an educational collaboration.

In the context of diverse mathematical subjects and strategies, the activities have the potential to show and promote Mathematics in an active, attractive and creative way for school teachers and students.

Recently the project has achieved an important outcome through the development and implementation of a website which contains a catalogue with selected mathematical workshops and a historical register from the events. The website is a structured tool that is being used to share and promote the *Circuits* activities specially among teachers that are searching for methodological alternative for their classrooms. Besides, it is an instrument that will permit the access to the contents of the workshops which have been already tested and inspire them to the development of their own materials. It could be also useful to promote discussions and share school experiences.

Furthermore, the website can be used as a way of collect online feedback from the participants. In this sense, it will be possible to approach a limitation of the project, namely its lack of organization and register from impressions of the impact from the activities after the actions.

The pursuit of more engagement of partners for the project will also be more effectively done through the website. In fact, we believe that the more people are engaged in student's education issues at all levels, the better the possibility to enhance learning success.

The reported experience brought together educators belonging to multiple groups leading them to work in an engaged and cooperative manner. This kind of interaction that combines a special partnership between and among a school community and university illustrates that it could be feasible and a way of improving the quality of instruction as well (Egodawatte, McDougall & Stoilescu, 2011).

While considering all the features priorly mentioned, we conclude with hope that the outlined experience of the *Circuit* can be used in the future to promote the development of collaborative teams of mathematics educators that can share their education expectations and mutually work to increase students and teacher's success.

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LEARNING TOGETHER THROUGH CO-TEACHING MATHEMATICS: THE ROLE OF NOTICING IN TEACHERS' COLLABORATIVE INQUIRY.

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Noticing the mathematical thinking of students is a key resource for teachers' learning in the context of their work. This paper explores the role of noticing within teachers' collaborative inquiry activity which was aimed at strengthening the impacts of classroom practice in mathematics. The study used a design-based methodology to explore the affordances of teachers' collaborative inquiry for teacher learning and addresses the questions: what models of teacher collaboration have been developed and what are the affordances and limitations of each form of teacher collaboration? Co-teaching emerged as a key characteristic of the teachers' inquiry activity and noticing student thinking was one of three interconnected fields of practice within which teacher learning occurred. Co-teaching mathematics sharpened teacher noticing and thus expanded access to resources for practice.

Mathematics teaching | learning is unpredictable and contingent. This paper draws on a study that explored the joint activity of a group of teachers in a New Zealand primary (elementary) school as they collaboratively inquired into their practice with the ultimate aim of promoting improved mathematics learning for students identified as at risk of underachievement (Eden, 2019). Co-teaching was a feature of the collaborative inquiry design that emerged and this paper reports on the affordances of co-teaching to catalyse change and open spaces for teacher learning; specifically the role of noticing within teachers' co-taught lessons. The paper explores how co-teaching affords teachers' in the moment reflection and responses to events as they unfold in the lesson, and provides access to diverse perspectives against which to test one another's interpretations of those events.

Teacher noticing

Teacher noticing is central to notions of teaching as inquiry whereby teachers engage in cycles of noticing, interpreting and responding to classroom events as they occur. Noticing generally refers to paying attention to and making sense of specific phenomena, with reference to classroom events (Sherin, Jacobs, & Philipp, 2011). Teacher noticing in mathematics is related to Schön's (1983) notion of reflection-in-action, and involves a "hidden practice of in-the-moment decision-making that is needed to respond to children's … strategy explanations" (Jacobs, Lamb, & Philipp, 2010, p. 197). The professional noticing of children's mathematical thinking is challenging and involves a complex set of skills that must be learned, including attending to the mathematical details in students' strategies; interpreting students' understandings as apparent in their strategies; and reasoning to decide how to respond to students' understandings (Jacobs et al., 2010).

Teacher noticing is an important element of reflective practice, and teacher reflection is key to strengthening the impacts of teaching by making sense of teaching | learning experiences and then using these to inform future practice. Jacobs, Philipp and Sherin (2011) suggest that "noticing is a critical component of mathematics teaching expertise and thus a better understanding of noticing could become a tool for improving mathematics teaching and learning" (p. xxvi). Drawing a connection between noticing and reflection, Jaworski (2003) argues that critical intelligence, a kind of ‘metaknowing’, emerges from an increased awareness of classroom issues. Thus, she suggests, critical reflection *outside* the classroom promotes more explicit awareness of the classroom conditions and events *within* practice which in turn leads to the possibility for expansions in action and shifts in practice. Drawing attention to the role of challenging the taken for granted in teachers’ practice, Larrivee (2000) describes critical reflection as involving:

> a deep exploration process that exposes unexamined beliefs, assumptions, and expectations … reflective practitioners challenge assumptions and question existing practices, thereby continuously accessing new lenses to view their practice and alter their perspectives (p. 296).

Benade (2015) argues that collaboration is a necessary condition of teachers’ work if they are to make their theories of action explicit and open to challenge. Collaborative critical reflection can lead to future noticing in the moment thereby allowing for the possibility of alternative decisions and actions (Mason, 2009). This aligns with a complex view of teaching and suggests an increased capacity for teachers to notice both elements of the teaching activity itself and the impacts of their teaching (Jaworski, 2003). That is, an expanded capacity to notice both the activity and its shifting goals as shifts in student learning outcomes are realised. Jaworski (2008) further suggests that reflecting critically with colleagues on aspects of *past* practice can support the development of enhanced noticing within *future* practice thus influencing classroom actions and potentially changing practice. Such a view provides a compelling rationale for a focus on teacher noticing in the context of teacher learning through inquiry. This paper examines the role of noticing within the activity of co-teaching mathematics in line with Jaworski’s (2003) call for further research to gain improved insights into the associated processes of teacher noticing within inquiry activity.

The research approach

Sociocultural perspectives underpin this study with the aim of capturing the multiple socially and culturally constructed realities of participants’ experiences (Schoen, 2011) whereby teacher learning is presumed to be situated in the social and cultural contexts of teachers’ work. The study explored the affordances of teachers’ collaborative inquiry for teacher learning in the context of primary mathematics teaching | learning (Eden, 2019). It was premised on the assumption that persistent underachievement in mathematics for some groups of learners requires shifts in what teachers know, and that collaboration and inquiry are productive for teacher learning.

Utilising an appreciative, authentic and participatory stance, the study employed a design-based methodology. Design-based research was chosen for its proximity to practice and explicit connection between the enactment of learning designs and outcomes of interest. The research was iterative and cyclical whereby a group of three teachers (Pat, Casey and Kris) in an urban New Zealand primary school and I designed, implemented, and refined an approach to teachers’ collaborative inquiry focused on strengthening mathematics teaching | learning. A range of data were gathered during a 6-month collaboration, including from teacher interviews, classroom observations and three-weekly group meetings. A specific focus of the project was on the generation of practice-based pedagogical knowledge and documentation of the processes involved in knowledge production. Regular group meetings were held to develop ways for the teachers to share their mathematics teaching. Between meetings, the teachers engaged in agreed activities in relation to mathematics teaching|learning including video-recording mathematics lessons and later co-teaching lessons in pairs. The negotiated shared inquiry for the teachers' practice was developing their use of "talk moves" (Chapin, O'Connor, & Anderson, 2009) as a pedagogical approach aimed at strengthening their students' mathematical language and supporting them to engage in mathematical discourse.

The group of teachers met three-weekly on seven occasions usually for an hour or more at the end of the school day. The meetings included reflective conversations about classroom events, the sharing of classroom video, discussions of research-informed articles, and planning for future activities. I participated in and audio-recorded group meetings and observed a mathematics lesson in each classroom followed by a semi-structured interview with each of the teachers at the beginning and end of the study. I transcribed the interviews and group meetings verbatim and listened to the audio recordings repeatedly as the transcripts were analysed. The analysis took a pragmatic and multi-theoretical approach to examine what it meant to design and enact teachers' collaborative inquiry. The transcripts were coded thematically using an open-ended approach (Creswell, 2014) to identify patterns that emerged from data. Cultural historical activity theory (CHAT) was employed to capture the complexity of the teachers' collaborative inquiry activity and to analyse and interpret the contradictions that arose and actions taken to resolve them. As such, it was particularly important to note and account for data that departed from dominant patterns (Braun & Clarke, 2006). This paper draws on data primarily from the group meetings and final teacher interviews.

Findings

Co-teaching expanded teachers' access to resources for their learning, and thus their opportunities to act, within three fields of their shared practice: their enacted practice, their conversations about practice and their noticing of student thinking within practice.

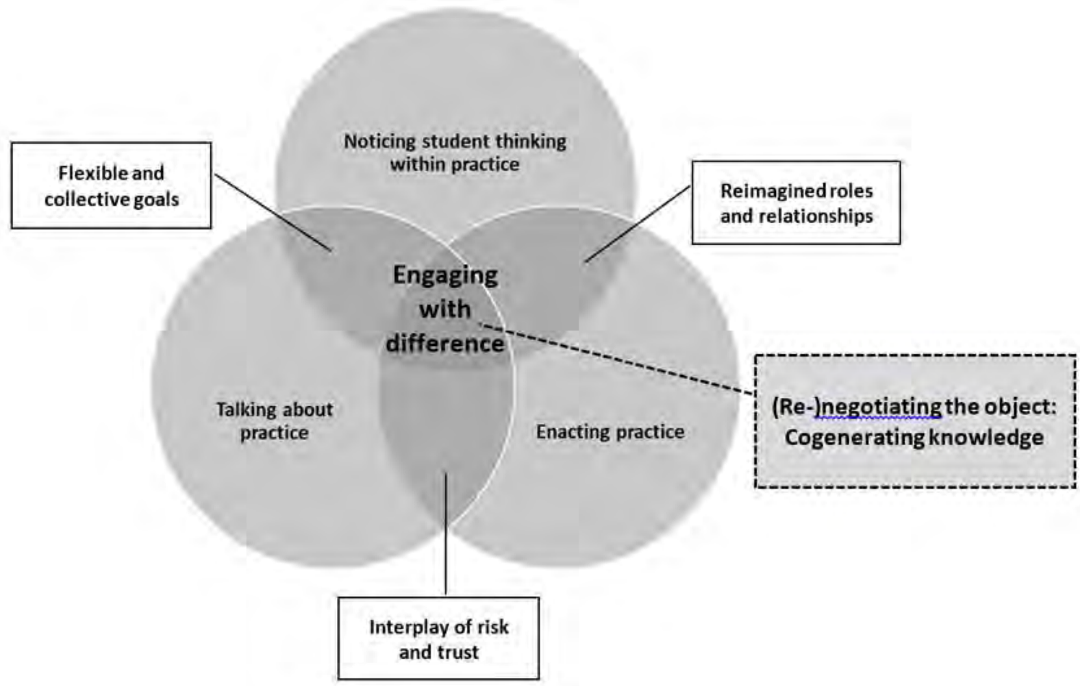

Figure 1. **Restructuring mathematics teaching | learning: Interconnected fields of practice (Eden, 2019).**

Each of these fields is interconnected whereby structures deriving from one field can become resources in others. The findings discussed here focus on the field of noticing student thinking.

Co-teaching sharpened teacher noticing

Co-teaching first emerged as a characteristic of our collaborative inquiry activity when Kris and I co-taught a lesson as a way of trialling decimats, a representation of decimal fractions that was at least somewhat unfamiliar to both of us. We were struck by our heightened noticing of the children's thinking during the shared teaching episode and described the experience of co-teaching as 'powerful'. Alongside our explicit aim of implementing a new pedagogical tool, we discussed how our co-teaching had student learning as the central focus:

> *Kris*: We would have these side conversations as we went … the starting point was the *kids*.… The kids' needs were at the centre which allowed the adults to weave together because there's strengths in both.
>
> *Raewyn*: Doing it together helped to really explicitly focus on what the kids were doing, and getting out of it, and connecting with, and not connecting with, and that's a real strength.
>
> *Kris*: I totally agree … our kind of model puts the child at the centre and what their need is and interestingly that's what's woven over our collaboration today.

Explicitly attending to student learning rather than how we would *teach* together appeared to support us to coordinate our teaching and 'tune in' to one another's teaching (Roth & Tobin, 2004). A focus on student thinking was evident when Pat later described key events from her first co-taught lesson. Her observations were focused on student talk and largely oriented to the ways in which students engaged with the task including some of the challenges they encountered. Pat's noticing appeared greatly enhanced compared to her more general comments at a previous group meeting.

It appeared that sharing responsibility for teaching | learning in a co-teaching context supported teachers to notice the impacts of their teaching on students, as Kris mentioned when reflecting on a lesson she had co-taught with Pat:

> It's the *shared* experience ... the co-teaching aspect put a certain amount of responsibility on *me*, not to be the expert but to be working beside … *[to Pat]* I had to follow *everything* you were doing so that at any point that I needed to support or take it a different [angle] I had to be following.

Casey highlighted the expanded opportunity that co-teaching had afforded Pat and Kris to reflect-*in*-action (Schön, 1983). She noted that the two teachers were able to collaboratively respond 'in the moment' to what they noticed about the impacts of their teaching on student learning:

> It's almost like you've covered the work of two lessons because when you do your lesson, then you reflect and think, "oh the next time I see that group I really need to emphasise this and this and this because I missed out on it" whereas what you're saying [is] you're kind of complimenting each other in that things you *[Pat]* mightn't have emphasised, Kris did and vice-versa.

Co-teaching was adaptive, contingent and emergent: Going "off on a little tangent"

The shared teaching activity was emergent, and teachers' individual actions were contingent on students' responses during the lesson. Teachers saw their shared practice as adaptive whereby co-teaching opened space for them to notice and respond. They described the need to be responsive to the students, each other and the unpredictable nature of lesson as it unfolded. The co-teaching arrangement opened space for teachers to think and respond to students 'in the moment' because they were sharing responsibility for the multiple moment-by-moment classroom events they had to attend to. Teachers explicitly agreed to adjust their teaching in response to what emerged in co-taught lessons. For instance, Pat and Casey explained how during one of their co-taught lessons they took the lesson “off on a little tangent”. This was an agreed action that was jointly taken although apparently not explicitly discussed. The group meeting then provided a further opportunity to deepen both teachers’ reflections on their shared teaching experience and affirm one another’s roles in achieving the lesson’s outcomes.

Through co-teaching, attention was increasingly directed towards the impacts of teaching on student learning. The complexity of the co-teaching activity and the need to be responsive to unpredictable events meant that teachers needed to carefully observe one another’s actions, and the impacts of those actions, in order to respond to events as they unfolded. The co-teaching experiences involved two teachers simultaneously engaging in the activity of teaching whereby elements of the activity were shared including the object of the activity and the role of teacher. At the same time, each teacher could be seen to act individually; that is, individual teachers took their own actions during the co-taught lessons, some of which were explicitly discussed in advance while at other times the teachers “bounced ideas off” one another as the lesson proceeded. Casey evoked the image of co-teaching as a dance in which partners, in unison, step up to occupy and step back to create space. Like unrehearsed dancers, teachers were engaging in a familiar activity with heightened awareness, ready to respond to the unanticipated moves of their co-teaching partner and the unexpected responses of their students. They created space for one another, and at the same time, they needed to occupy their shared space with care to avoid ‘stepping on the toes’ of their partner or tripping them up altogether. In other words, the teachers’ co-constructed practice was emergent and contingent on one another’s actions and the students’ responses as they each made carefully considered moves to keep the lesson on track.

Sharpened noticing expands the resources available for teacher learning

In the context of co-taught lessons, sharpened noticing of students’ mathematical thinking provided teachers with enhanced opportunities to learn from one another’s practice. For instance Casey had an explicit goal of learning to use success criteria more effectively in her teaching. When she and Pat co-taught a lesson together, Casey had an opportunity to observe Pat and the students co-constructing success criteria related to the lesson goals:

> I'd said yes, I'm not great on success criteria it's not something that I put particular focus on but after doing the co-teaching I really saw the benefit of it, it was really worthwhile especially with that particular bit of teaching that we were doing.

Through co-teaching, Casey had access to Pat’s practice as it was enacted in her own teaching context. She had an opportunity to observe Pat’s practice, and notice the impacts on her own students, in the familiar teaching context of her own classroom:

> You were the one that refined the success criteria with them and it was so good because we can come back the next day and say “so what were the things that were important to remember?”

Access to Pat’s teaching expanded the possibilities for Casey’s future practice. As such, the pedagogical shifts that Casey subsequently made align with Guskey’s (2002) model of teacher change whereby changes in classroom practice where students’ learning is positively impacted can be the catalyst for changed teacher beliefs. Casey made changes that were promoted by both seeing the positive impact on her students’ learning of this new practice and her observation that what had seemed novel when described to her was actually strongly aligned with her current practice.

Early in the project, sharing excerpts of classroom video afforded the teachers access to an extended range of pedagogical ideas through opportunities to observe and reflect on aspects of one another's practice. For instance, Pat and Casey had very different approaches to the use of concrete materials to support their students' understanding of new mathematical concepts. Pat's use of equipment in a lesson that I observed at the beginning of the project was limited and unplanned. She initially used no equipment in her lesson which was focused on multiplication, however when she was attempting to explain a concept that a student was struggling to understand she fetched an abacus and attempted to demonstrate. At the time I had reflected that the abacus appeared to be a problematic choice for representing the mathematics idea, and Pat had seemed entirely unprepared for using it. In contrast, in her lesson with a group of learners who were exploring place value in teen numbers, Casey provided a wide range of pre-prepared and highly suitable manipulatives such as tens frames, bundled sticks, place value blocks and counters.

Through the sharing of video from Casey's classroom, Pat had an opportunity to see examples of, and discuss, Casey's use of materials. Pat later commented on Casey's use of materials, resolving to "use a lot more materials and use them differently" in her own teaching. For Pat, Casey's contrasting use of materials served as a potential disruption to her practice in that what was viewed as highly effective teaching appeared very different to what was occurring in Pat's own classroom. In other words, what the students were doing, and using, in each of the classrooms looked very different and prompted Pat to suggest that she might adopt this easily observable aspect of Casey's practice. Nevertheless, like van Es and Sherin (2008) found in the early stages of their video club, Pat's comments about Casey's use of materials tended to be descriptive and non-specific, and largely disconnected from the students' mathematical understandings even when prompted. Such observations on their own are unlikely to deeply engage teachers with their existing practice theories and so Casey's practice of 'using materials' was likely to remain untested and support only superficial changes in Pat's understandings and resulting enactment of this practice.

In contrast, the opportunity to *experience* Casey's use of concrete materials in a co-taught lesson created an opportunity for Pat to experience "actions that were, at the time, not part of [her] own range of possibilities" (Roth & Tobin, 2004, p. 164). Pat commented that:

> Casey was really good with the materials and she really helped me in terms of actually showing how to use those materials really well.... We were working a lot with materials and she was able to feed off the expertise and so I was actually quite happy that she took a little bit more of a role there.

For Pat, the shift from identifying the use of concrete materials as a new possibility for practice, to an enacted experience of doing so, sparked her to notice and ask increasingly sophisticated questions about how concrete materials might support her students to develop understandings of mathematics concepts, in this case subtraction:

> They [students] worked with me another day [and] getting them to really think about "what's the renaming?" and that whole thing of place value … it's opened my world like that day we were using place value blocks but we used decimats or we used different [materials] … I couldn't teach a maths lesson now without materials.

There were enhanced opportunities to notice and respond to student learning as it was taking place:

> When we co-teach together, some of the things that I didn't pick up from my own lessons, others were able to pick up. You notice different things and that's where the improvement comes from [Pat].

Attention, or noticing, appeared to be heightened because teachers needed to be engaged with what was happening in order to actively participate in co-teaching the lesson. Co-teaching comprised co-planning, co-instruction and co-reflection and the teachers felt that the conversations they had prior to teaching together were important preparation. Developing a shared plan prior to teaching appeared to provide a reflexive object for the two co-teachers that promoted their noticing of their adaptive practice. In other words, it was because they had an *intended* direction that they were able to notice that their lesson had diverted from it. This represented both a sharpening of the teachers' noticing of what happened during their co-taught lesson and a comparative case, in the form of a set of anticipated events, in light of which to make sense of what did occur in the lesson. Co-planning appeared to set teachers up to be responsive during the lesson by supporting them to anticipate how students might respond to a task. Teacher learning was afforded by the teachers' heightened attention to important aspects of practice as they acted to make sense of and reconcile the contradiction between what was anticipated and what actually occurred in the lesson.

Discussion

Sharing the teaching task as co-teachers expanded teachers' opportunities to attend to important details in the complex classroom environment. There appeared to be a dynamic and mutually generative relationship between the teachers' reflections-*in*-action and their later reflections-*on*-action. Their sharpened noticing during a co-taught lesson served to expand the breadth and depth of their reflective thinking after the lesson. Carambo and Stickney (2009) argue that reflection after the fact, or reflection *on* action, "cannot replicate the depth of understanding that emerges when two teachers cohabit the classroom and cooperate on the variety of actions needed to successfully teach [a lesson together]" (p. 438). In this study, co-teaching experiences where teachers took different approaches, including those that were not necessarily agreed upon prior to the lesson, catalysed conversations that allowed the teachers to surface their different understandings and goals. Such conversations prompted teachers to make otherwise tacit ideas about practice explicit. Teachers' interactions *within* practice are central to co-teaching; it is through such interactions that meanings are shared, and change is created (Gallo-Fox, 2010).

Teachers' sharpened noticing promoted enhanced opportunities for their reflection-in-action (Schön, 1983) and thus afforded co-teachers access to enhanced resources for generating understandings of mathematics teaching | learning and expanding the possibilities for their future classroom practice. Co-teaching pressed teachers to engage with their existing practice theories and reconcile new and sometimes dissonant understandings with those they currently held, a process that Timperley and colleagues (2007) identified as necessary for substantive teacher change. Co-teaching pressed teachers to make sense of the intended and actual impacts of their co-teaching partner's moves. When a co-teaching partner's moves were experienced as non-routine, they served as mini interruptions to practice, provoking unanticipated responses which teachers then needed to make sense of. The unfamiliar moves of co-teaching partners and unanticipated responses of students introduced novel practices (teacher moves) and tools (students' responses) into the activity system constituting new possibilities for others and leading to expanded action possibilities; that is learning (Roth & Lee, 2007).

This paper adds nuanced understandings of the interrelated roles of collaboration and inquiry in improving teaching. Teacher learning was afforded whereby possibilities for teachers' moment-by-moment actions in the context of the co-taught lesson were expanded. CHAT provided an analytical and theoretical framework to address the research question whereby contradictions in the co-teaching inquiry activity could be seen to expand (or constrain) teachers' access to resources and thus opened space for an expanded set of understandings and classroom practices. Opportunities to engage deeply with one another's practice opened space for an expanded set of actions for each of the teachers in their own practice. What teachers noticed within co-taught lessons became a resource for future actions within other interrelated fields of their shared practice. In other words, what is noticed can be talked about and enacted, and what is talked about and enacted promotes teachers' future noticing.

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PROFESSIONAL DEVELOPMENT OF MATHEMATICS AND SCIENCE TEACHERS WITH CULTURALLY RESPONSIVE INTERDISCIPLINARY MODELING PRACTICES

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Despite the need, underrepresented students struggle to identify themselves with science, technology, engineering, and mathematics (STEM) professions. Faculty at the University of the Virgin Islands developed a model research program for isolated communities to support the development of culturally responsive STEM education training and curriculum for secondary mathematics and science teachers. Based in St. Croix, U.S.V.I., the program brings together 40-50 mathematics and science teachers, five STEM and education faculty, and community partners to work in collaborative groups and form interdisciplinary professional learning communities (PLCs) that focus on the development and implementation of locally relevant integrated STEM projects using mathematical and scientific modeling practices. These PLCs serve as a sustainable and inclusive professional development model and are supported through ongoing training in collaborative action research. The trajectory of a model PLC focused on local environmental issues is described, and observations, recommendations, and observed student impacts are given.

We strongly believe that approaches to science, technology, engineering and mathematics (STEM) education in the Caribbean need to situate STEM learning events in their historical and cultural contexts, allowing students to create, recreate, and shape their meanings (Lave, 1988). Since 2014, with funding from the National Science Foundation, the University of the Virgin Islands (UVI) has provided year-long training and support for in-service STEM teachers through the Virgin Islands Institute for STEM Education, Research and Practice (VI-ISERP). VI-ISERP aims to provide a more equitable access to STEM learning within a Caribbean context, offering professional development and long-term support for teachers as they present mathematical and scientific inquiries relevant to the socio-cultural history and context of the Virgin Islands. Adopting Professional Learning Communities (PLC) approach as described by Fulton and Britton (2010), VI-ISERP taps into the funds of knowledge of teachers, students, university professionals, and STEM community partners, either formally or informally involved in STEM education, by designing and implementing integrated STEM projects with an inclusive approach for student and community access and engagement for locally relevant STEM learning both inside and outside of the classroom. This paper reflects the professional development efforts of VI-ISERP on St. Croix only (although a similar program exists on St. Thomas) where the first author directed the efforts of VI-ISERP from 2014-2017, with the second author assuming this role in 2017. The goal of this research program is to provide equitable access to quality STEM instruction for all students using culturally responsive practices, supporting and disseminating local best practices.

Background and Approach

There is a scarcity of educational research in STEM education practices for this region. Researchers have noted that there has been passive reliance and compliance to external authority for learning new ideas, rather than active learning pedagogies that build on inquiry and agency among students. The cultural image of a good student in the USVI is a student who attends regularly and listens without questioning and active engagement (Greenstein and Ekici, 2017). Ladson-Billings (1994) defined student-teacher relationships as dynamic and equitable, extending beyond the classroom, and demonstrating connectedness with students and among students. According to Foster (1997), the classroom community needs to expand individual classrooms to be inclusive of the entire school community through collaborations with colleagues and the community.

As such, there is a need to develop supported professional learning communities (PLCs) among the community, teachers and students, to improve student STEM identity and engagement. PLCs are known to enhance authentic pedagogy with integrated STEM projects through collegial interactions and by socially motivating student achievement with integrated projects from multidisciplinary standpoints (Louis and Mark, 1998). Further, supported PLCs can facilitate the adoption and implementation of interdisciplinary STEM projects (Krajcik et al.,1994; Fulton & Britton, 2011). A qualitative meta-synthesis of earlier research has suggested that PBL is effective in promoting long-term retention of knowledge and skills (Strobel & van Barneveld, 2009). In addition, directed professional development on integrated STEM education can enhance both pedagogy and STEM content knowledge, can increase the understanding of interdisciplinary practices (such as modeling), and can increase teacher self-efficacy in planning and implementing integrated STEM projects (National Research Council, 2012; Lambert et al. 2018).

Processes of Collaboration

Building on Fulton and Britton (2011), VI-ISERP worked on establishing PLCs with collaborative groups of mathematics and science teachers and practitioners. Our approach to building culturally responsive STEM education begins with bringing local integrated STEM problems into schools to foster teacher and student interest, to serve as a catalyst for locally relevant curricula development, and to help build a school environment that nurtures a researcher identity in both students and teachers. We utilize a bottom-up curriculum and project design approach, with different local STEM phenomena (such as lionfish population dynamics, water/soil quality) acting as the driving theme. It is important that these themes are driven from students' and teachers' lives and practices, and are connected to the people, marine environment, plants and animals that impact the livelihoods in Caribbean. Participants then take part in year-long PLC professional development activities designed to foster and provide sustained support for the development and implementation of their integrated STEM projects, in and out of school, with critical feedback from other PLCs. Central to this development is enrollment in ongoing collaborative action research training at UVI, where participants learn that the interventions with students continually evolve and cycle through assessment and culturally responsive revisions through subsequent years. Through this course, teachers identities are transformed into researchers conducting research on their practices with STEM teaching and learning implementing their projects in and out of classrooms. Within the context of each project-centered PLC, there is a reciprocal learning exchange between "mentor teachers" that lead the project and the other educational stakeholders. With this interdisciplinary project-based approach, the extension of the classroom community to the wider community become more natural and expected.

Assuming a constructive critical stance and recognizing the importance of social and cultural context of learning (Lave 1988), we took the charge of explicating and understanding the cultural impediments and socio-cultural habits hindering progress into higher STEM learning for the isolated communities as in the Caribbean. Through this process, we sought a collaborative transformation of local STEM practices and culture and built on resource pedagogies inclusively, giving more agency to students and teachers with culturally responsive pedagogies for STEM learning (Aronson & Laughter, 2016; Civil 2016; Kern, Howard, Brasch, Fiedler, & Cadwell,2015; Ladson-Billings, 1995; Gay 2010; Moll et al. 1992; Powell, 2004).

Professional Development for Culturally Responsive In service STEM Teacher Education

From our experience since 2014 interdisciplinary mathematics and science teacher collaboration for in service teachers, we offer our emerging guidelines, which are not-necessarily linear. The shifts of focus for teacher collaboration and professional development are described in these stages, which may be useful for other practitioners in isolated communities planning to develop culturally responsive interdisciplinary STEM practices. VI-ISERP facilitated participants' progress along these stages, with the goal of developing culturally responsive STEM education practices, culturally responsive classroom and assessment resources, teacher leadership, and supported PLCs, with coordinated activities during the summer and academic year. First stage is to *build a vision* by identifying a local STEM issue or a phenomenon that a group of educators and students feel passionate about, and form an interdisciplinary team of teachers. Second stage is to *build partnerships* among mathematics and science faculty and other community partners and search for community funds of knowledge and resources. *Include students* or former students as partners and peer leaders in learning communities during summer STEM academies. Third is to *build a plan* by forming short and long-term project goals, set expectations for partners, and map learning outcomes and trajectories across disciplines. Recognize that plans and goals are dynamic and change due to time, resource, and environmental constraints. Fourth stage is the *supported action* for implementing interdisciplinary learning modules with PLC support on interdisciplinary content. As a part of PLC, participants support each other and supported by the professional learning community members including faculty and broader community members with relevant expertise. With this support, participants build advanced perspectives and interdisciplinary connections with mathematical and scientific modeling applied to a local context. Ongoing assessment and support of relevant teacher content knowledge to implement STEM projects. Fifth is the *reflective action* after/during the implementation where participants reflect and build pedagogical content knowledge by attending cross-cutting ideas and practices, building coherence across interdisciplinary modules. During each cycle, revisit culturally responsive aspects of the interdisciplinary modules and revise as needed with feedback from teachers, students, and community members. *Collaborative synthesis* is the sixth stage where participants revisit the vision and form connections through shared experiences with different culturally responsive interdisciplinary projects across curricula. Dissemination of *scholarship of teaching and learning* is the last stage where the results of collaborative action research on local best practices with culturally responsive interdisciplinary STEM learning shared through local and national conferences, and local teacher professional development events.

Trajectory of a Model Culturally Responsive Integrated PLC

Below we present a list of the professional development activities and dissemination products belonging to a model culturally responsive integrated PLC focusing on the mathematical and scientific modeling of water and soil quality issues impacting the U.S. Virgin Islands. Table 1 and Table 2 lists the alignment of professional development events and resulting dissemination activities by the PLC community members focused on Water Quality, which is only one of six major themes. This PLC synthesized multiple integrated STEM projects; for example, Dr. Steve Lawrence, a high school chemistry teacher, developed curriculum on soil testing as he engaged his students in studying heavy metal presence in local waste bin sites and modeling their environmental impact. Physical sciences high school teacher Andre Pompey developed student research activities investigating the presence of contaminants at mulch sites in St. Croix, and whether they satisfied the recommended federal guidelines. High school mathematics & physics teacher Dr. Michael Henry and biology teacher A. M. Gibbs tested the water quality and runoff in local watersheds and guts. Throughout these implementations, mathematical and statistical modeling and data analysis support was given to teachers by mathematics faculty from VI-ISERP. After individual implementations of their developed curricula, these teams synthesized their efforts by sharing emerging student capacity in conducting experiments with soil and water quality testing in local sites during the school year. Soil and water samples were collected from different locations on St. Croix, and experienced students facilitated and supported running experiments in subsequent years with newer groups of students. This allowed for the emergence of authentic research for students and teachers, with critical findings, such as the identification of consistently high levels of mercury in the soil near bin sites. This finding further probed students and teachers to investigate the reasons behind this, leading to a discovery of volcanic rock deposits in the soil contributed to high mercury levels. Teachers action research reported that high volcanic activity is a significant natural source of mercury, which is the case for the Caribbean.

Table 1: Professional Development Trajectory of Water Quality Modeling PLC as an Integrated STEM Learning Theme

Professional Development Event	Dates	Facilitators
VI-ISERP Summer Training Institute on Culturally Responsive STEM Learning	June 2015	CE +fm
Collaborative Action Research Course for Secondary Math & Science Teachers	January - May 2016	CE
VI-ISERP Workshop on Culturally Responsive Practices with Interdisciplinary PBL STEM Learning	June 2016	CE, SG+fm &AP +fe
VI-ISERP Workshop on Mathematical Modeling Perspectives for Interdisciplinary STEM Learning	June 2016	CE & CP+fm
Collaborative Action Research Course for Secondary Math & Science Teachers	August 2016 - May 2017	CE
VI ISERP Workshop on Assessing Mathematical Modeling and Integrated STEM Learning Outcomes with PBL	June 2017	CE & CA+fe
Collaborative Action Research Course for Secondary Math & Science Teachers	August 2017 - May 2018	CP
VI ISERP Workshop on Water Quality and Soil Testing in support of Interdisciplinary PBL	June 2018	SL+tc & AP+ts.
VI-ISERP 2018 Summer Workshops - Mathematical Modeling - Advanced Perspectives	June 2018	CP & CE
Collaborative Action Research Course for Secondary Math & Science Teachers	August 2018 - May 2019	CP
Other VI-ISERP Organized STEM Professional Learning Community Meetings (> twice per semester)	Each Semester 2015- 2019	CE & CP

Interdisciplinary membership of facilitators; +fm: faculty of mathematics; +fe: faculty of education;+tc :Teacher of chemistry; +ts. Teacher of Science. PBL: Project based Learning CE: Celil Ekici, CP: Chris Plyley, CA: Cigdem Alagoz Ekici, AP: Andre Pompey, AMG: Ann Marie Gibbs

Table 2: Dissemination* of Water Quality Modeling STEM Learning Theme

Dissemination Event	Date	Presenters
Local Conference Presentation: 2016 VI-EPSCoR Annual Research Conference.	April 2016	CE+fm, AMG+tb, MH+tp, D. Theophilus+tm, & CA+fe.
Local Conference Presentation: American Federation of Teachers (AFT)-TEACH Conference.	October 2016	MH+tp/+fs, AMG+tb, DT+tm, SL+tc, CE, TH+spl, & KJ+spl.
International(Int.) Conf. Presentation: Int. STEM Education Association (ISEA) Meeting	October 2016	MH, AMG, DT, & CE
Int. Conf. Presentation: 13th Meeting of the Int. Society of Mathematics Education.	July 2016	CE & CA

Local Conference Presentation: 2017 VI-EPSCoR Annual Research Conference.	April 2017	KH$_{+spl}$, BD$_{+spl}$, DM$_{+spl}$, AMG, MH, CE
National Conference Presentation: AMTE	Feb 2018	CA, CE
Int. Conference Presentation: International STEM Educ. Assoc.(ISEA) Meeting	October 2018	SL$_{+tc}$, MH$_{+tp}$, AP$_{+ts}$, AMG, CE
Int. Conference Paper: Int.Conference on Education in Math, Science, & Technology	April 2018	CE, CP, CA, RG$_{+ts}$, NS$_{+ts}$
Int. Conf. Presentation: Association for the Sciences of Limnology and Oceanography (ASLO) 2019 Aquatic Sciences Meeting	March 2019	MH$_{+tp/+fs}$, AP$_{+tb}$, SL$_{+tc}$, AMG, CK$_{+tm}$, SG$_{+ts}$, JW$_{+tm}$.
National Conf. Presentation: 2019 National Science Teachers Assoc. (NSTA) Meeting.	April 2019	SL, MH, AP, AMG

*Full references are included at the end. Interdisciplinary membership of presenters from STEM PLCs denoted by the following symbols- +ts: teacher of science, +tm: teacher of math, +spl: Student peer leader, +tp:Teacher of physics, +fe: Faculty of education; +fm: faculty of math;+tc: Teacher of chemistry; +fp: Faculty of physics. CE: Celil Ekici, CA: Cigdem Alagoz, CP: Christopher Plyley, MH.: Michael Henry, AP: Andre Pompey, SL: Steve Lawrence, DT:Diane Theophilus, RG: Resa Gordon, NS: Nora Santana,

Student Impact and Observations

Student Impact

We observed that students that participated in STEM projects developed by these VI-ISERP supported teachers demonstrated improved attitudes towards STEM learning, a heightened interest in attending college, and higher degrees of engagement, as they studied problems and concerns situated within their community. The Virgin Islands Department of Education selected three VI-ISERP supported teachers involved in the project trajectory above to participate in a 5 week Summer STEM Academies in 2018; each teacher was given a group of 10-15 students to work on investigating a local phenomena. Student participants in the STEM academies were administered a pre and post survey measuring their attitudes and interest towards STEM using Student Attitudes toward STEM Survey(S-STEM) (Friday Institute for Educational Innovation, 2012). A total of $n=35$ students underwent the project with the VI-ISERP teachers, an independent-samples t-test was conducted to measure the student attitudes towards each of math, science, technology, and 21st century skills. There was a significant difference in the scores for math attitudes in the pre-test ($M=3.2$) and post test ($M=3.7$) conditions; $t(59)=-2.22$, $p = 0.015$; and a significant difference in the scores for attitudes towards 21st Century learning in the pre-test ($M=3.6$) and post test ($M=4.0$) conditions; $t(59)=-2.07$, $p = 0.021$. These students also showed gains in attitudes towards science on the pretest ($M=3.3$) and the posttest ($M=3.7$) and in attitudes towards engineering and technology on the pretest ($M=3.2$) and the posttest ($M=3.4$), although not statistically significant. These results suggest that the locally relevant curricula development through VI-ISERP can have a positive impact on student attitudes towards STEM and 21st century skills. Teachers involved in the summer STEM academies that did not develop their curricula through VI-ISERP ($n=49$) showed no gains or statistically insignificant gains in student attitudes towards each of math, science, technology, and 21st century skills.

In addition, evidence that these VI-ISERP teachers critically improved their student attitudes and fostered student STEM identity can be seen in their responses to the question: Do you plan to go to college? The percentage of students responding Yes to this question went from 85% on the pretest to 97% on the posttest. The students taking part in the research academies with non-VI-ISERP trained teachers did not see such gains; on the pretest 78% of students responded Yes compared to 75% on the posttest.

Observations

Our observations on successful practices indicated the importance in interdisciplinary team collaboration of the careful support and planning with the school and district administrators and the STEM educators at the local university. It is often not possible for interdisciplinary STEM teachers to meet face-to-face, observe each other, and team teach to develop, implement and support the integrated STEM PBL modules. We learned that a culture of interdisciplinary STEM learning can best be supported when interdisciplinary teachers have common planning periods, or have pre-arranged meeting times (such as the action research course). This worked well in a middle school with monthly interdisciplinary planning times for each grade level allowing teachers to interact and plan together. The PLCs and sustained involvement and leadership in an interdisciplinary project with community environment began to show its positive effects on teacher identity, as they grew professionally through conference presentations and leading workshop seminars on their project. Their involvement in these supported PLCs helped them develop their capacity as researchers and mentors, as evidenced by the active facilitator roles they undertook.

Since 2014, there has been a steady increase in the number of active interdisciplinary STEM projects being implemented by VI-ISERP supported teachers. Imperative in achieving this is a shared common vision with the Virgin Islands Department of Education, who is actively seeking to develop a positive learning culture for STEM in schools. In this sense, the work of the participants was in alignment with the Territorial STEM plan, providing other in-service teachers with working examples of locally relevant STEM project-based learning curriculum. Professional development sessions provided opportunities for teachers and student peer leaders share the transformative experiences with culturally responsive STEM projects allowing them to engage in their communities as agents of change while learning STEM in their local context. The increased collaboration among teachers consequently improved teacher relations and communication, which resulted in more consistent cross-curricular instruction. Dr. Henry, as a high school math teacher, reported that "this project helped students see the interconnectivity of science, including math, through research. They were accustomed to subjects being taught separately so they think of subjects being totally independent of each other. The way these projects developed and merged as analysis of the data dictated was a tremendous experience for the students and faculty."

By adopting a culturally responsive approach, teachers let their students know that individually and collectively their voices are heard, their choices matter, and their presence and contributions are valued. This agency is fostered throughout the PBL units from problem posing to creative problem solving. Giving students voice and choice was instrumental for students who started to develop their STEM identities as they pursue their mathematical and scientific inquiries in locally and culturally relevant contexts. We observed more student interests in developing mathematical and scientific arguments when they are given more opportunities to directly experience science and mathematics in a local context. Students posed more STEM related questions and discussed among themselves. They showed more ownership of their STEM learning. Each semester, teachers brought the cases of students that are usually not engaged in teacher-centered instruction yet demonstrated leadership among their classmates with this interdisciplinary project-based learning approach. This indicated the link between equitable access and STEM identity in successful PBL projects.

Discussion and Conclusion

The model of culturally responsive STEM education described above provided an inclusive, non-threatening, engaging professional learning community consisting of teachers and faculty from different disciplines. Centered around locally meaningful integrative STEM learning further engaged their students and communities and tapped into local funds and resources of knowledge for STEM learning. Interdisciplinary mathematics and science learning also proved to provide a pathway of excellence and service for teachers working in isolated communities where they ended up having to develop different disciplinary perspectives along the course of their professional lives. Mathematical and scientific modeling of local STEM issues allowed the community to engage and support students and teachers working on these communities. As evidenced by the trajectory of professional learning community activities around a local STEM theme among others, this interdisciplinary professional development model for isolated communities is found to be empowering teachers to become researchers, mentors, and leaders with a joint purpose to build collaborative action research and to provide inclusive mathematics and science education practices for students and the community.

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USING SPIRALS OF INQUIRY TO COLLABORATIVELY IMPROVE TEACHING AND LEARNING IN MATHEMATICS

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Teachers conducting collaborative inquiries into practice is a way of tailoring professional learning to local contexts and students' needs. Two primary schools that were part of a professional learning initiative centered on using inquiry to address student learning needs, chose mathematics as the context for their inquiry work. These two schools made substantial changes to organizational, leadership and teacher practices, and to student learning in mathematics over a two-year period. Comparison of the schools' journeys suggest that gathering information about students, from both conversations with students about their learning and specific diagnostic tools, led to greater curiosity and enthusiasm from the teachers, as well as developing a shared language which was critical to seeking help and further learning. Alongside this, the process of 'checking out hunches' was significant in building effective collaborations that made an impact on teacher practice and student learning. Results suggest that changes in teacher practice and improvements in student learning can be made through the application of a Spiral of Inquiry approach to teaching and learning in mathematics.

Professional learning for teachers is complex (Opfer & Pedder, 2011), occurring within a web of interconnected factors and contexts in a way that makes cause and effect models unhelpful for understanding how teachers learn or what the outcomes of professional learning experiences might be. Over time, evidence has accumulated that teachers working in collaboration with each other can make changes to their practice (Dogan & Adams, 2018; Vangrieken et al., 2017; Vescio et al., 2008) and can improve student learning outcomes (Hattie, 2009). Just getting teachers together to discuss teaching and learning may not lead to changes in student outcomes, however (Timperley, Wilson, Barrar & Fung, 2007). Timperley (2008) offers ten principles for effective professional learning that impacts student outcomes. In her synthesis of the literature, opportunities to process new learning with others did contribute to better outcomes for learners, provided the interaction among the teachers was focused on becoming responsive to students (Timperley, 2008). There have been a range of initiatives that use structures, tools and protocols to help teachers collaborate around the idea of being responsive to students, for example, Japanese Lesson Study (Akiba, Murata, Howard & Wilkinson, 2019; Doig & Groves, 2011) or teaching rich mathematical tasks and then sharing student work (for example, Kazemi & Franke, 2004). These examples are of mathematics-specific tools and protocols, which centre on the mathematics of the lessons and student responses, explicitly developing teachers' understanding of students' mathematical learning and the mathematics of the tasks. Many teachers are involved, however, in more generic professional learning in collaborative groups. These groups of teachers, sometimes known as professional learning communities or professional learning teams, work together on their practice, and take a range of forms, including cross-school collaborative groups. They occur in jurisdictions around the world. Many authors offer lists of factors that make these groups effective (Bolam, McMahon, Stoll, Thomas, & Wallace, 2005; Chen, Lee, Liu &Zhang, 2016; Dogan & Adams, 2018; DuFour, 2004; Katz & Earl, 2010; Prenger et al., 2017; Vangrieken et al., 2017; Vescio, Ross, &Adams, 2008), with some agreement that goals and vision, focus, collaboration, leadership and relationships/trust are central to change in practice. Professional learning communities sometimes use the idea of 'teaching as inquiry' (Aitken & Sinnema, 2008) to provide them with a tool or process that guides their improvement work. A version of the 'teaching as inquiry' process that has been widely used in New Zealand, Australia, Canada and Scandinavia is 'spirals of inquiry' (Timperley, Kaser & Halbert, 2014). Spirals of inquiry always begin from a student learning challenge, but they are not necessarily based on mathematics. This study looks at the use of spirals of inquiry in collaborative teacher groups in primary schools in Australia, which focused on mathematics as their learning challenge, and asks: How can learning to use a collaborative inquiry approach to improving student outcomes, with mathematics as a context, result in improvements in mathematics teaching and learning? In other words, how does a 'generic' professional learning initiative that does not grow from the concerns of mathematics, but does provide a structure for collaboration, impact mathematics teaching and learning?

The Spiral of Inquiry (Timperley, Kaser & Halbert, 2014) is a six-part inquiry process that is designed to be a continuous learning process for those involved. Findings from one journey through the six stages feed into the next journey, leading to an ongoing spiral of activity and learning. The spiral begins with students, their learning aspirations, strengths, needs and wellbeing. In a *scanning* phase, the collaborating teachers explore the data they have about students, and seek more information if they need it. Then they ask themselves where they should put their energy to begin with; of the things they have discovered, what should they start with. This is the *focusing* phase. Once a focus is determined the team works together to *develop hunches* about why things are as they are. In this phase people's assumptions are made explicit and are questioned collegially. The aim is to work together to uncover implicit or intuitive ideas and check them out with the evidence. This phase often opens up space for new ideas about what might be leading to the situation for the students. In the *learning* phase the team identifies what they need to know in order to address the focus area as they now see it, and seek professional learning that is tailored to their needs. *Taking action* based on this learning is next, with teams supporting each other to make change. The sixth phase, which feeds into the next inquiry, is *checking* – asking 'have we made enough of a difference?'. The spiral has much in common with other approaches to teacher inquiry, with its key tenets being starting from student experience, the use of evidence, understanding the role of teachers and leaders in contributing to the situation and checking that adequate progress has been made (Timperley, Kaser & Halbert, 2014). The Spiral of Inquiry is a generic tool, in that it can be used to work with any focus in any context. This study explored what happened when it was used to look closely at student experience and achievement in mathematics.

Methodology

This qualitative study used a complexity theory framework (Cochran-Smith et al, 2014; Opfer & Pedder, 2011) to understand the learning of students, teacher, leaders and schools during and after a two-year professional learning initiative focused on the Spiral of Inquiry. The complexity theory framework led to the adoption of an extended case study method (Hetherington, 2013), where artefacts of schools' participation in the initiative, interviews with school leaders, inquiry team leaders and teachers who were not in the inquiry team and evidence of student learning and school climate were combined into rich cases and compared to look for mechanisms that might underpin learning through participation in the inquiry process. Mechanisms are combinations of conditions, catalysts and participant factors that intersect in unpredictable ways in a complex system (Opfer & Pedder, 2011). Rather than considering cause and effect, this study sought patterns in change within and across school sites, and looked for the emergence of student, teacher, leader and school learning.

Participants

Two primary schools (with students from 5-12 years old) that were part of a Spiral of Inquiry professional learning initiative and chose mathematics as their focus, participated in this study. The schools had been selected for the professional learning initiative because they had the lowest results in the school district on standardized tests of literacy and numeracy. Informed and written ethical consent was sought and obtained from the body that governed the schools, then from the school principals and from each of the leader and teacher participants. The principals gave consent for access to their schools' national student achievement data (NAPLAN; collected through a national, standardized test at Year 3 and Year 5) as well as to any videos or documents created during the professional learning initiative and stored on the initiative's website. In each participating school the principal, the leader in charge of the inquiry project, and two teachers were interviewed.

Structure of the professional learning initiative

Spirals of Inquiry (Timperley, Kaser & Halbert, 2014) were introduced to a district of schools in Australia over a period of four years. The first of three phases involved eight schools, and ran for two years. The two schools in this study are drawn from these eight schools. Data collection took place a year after the first phase schools finished their two years of professional learning. The initiative consisted of termly two-day meetings held at a central venue. These sessions were led by two facilitators from outside the district, one of whom was the researcher. A condition of participating in the professional learning was that principals attend with a team from their school, which was to comprise school and curriculum leaders who could lead change back in the schools. These teams became known as 'inquiry teams'. The school inquiry teams were joined by district facilitators who were available as resource people for the schools if they wanted input or support. In between the two-day sessions, the school teams and the district facilitators worked in their schools on using the Spiral of Inquiry to try to improve student outcomes in their chosen focus area.

The two day sessions comprised input, processing activities and discussion aimed at developing a deep understanding of the Spiral of Inquiry process and why it can work to improve outcomes for learners. The school teams worked through the Spiral stages with the two outside facilitators, and the district facilitators, receiving both generalized input and tools, and tailored conversations and advice about their particular situations. Over the two-year timespan of the initiative, the schools worked through the whole spiral. In addition, the outside facilitators, who were leading the professional learning, visited the schools during their involvement with the initiative, and provided guidance and ideas in context.

Data collection

For the study, the researcher visited each volunteer school and conducted 30-45 minute interviews with the principal, the person who had led the initiative in the school (a deputy principal or curriculum leader) and two teachers, one who had attended the professional learning and one who had not. The purpose of the interview was to gather evidence of any change in organizational, leadership and/or teaching practice that might have occurred in relation to the student learning focus during and after the professional learning experience. Interviewees were asked about their personal experience of the project, what they thought about the process as a way to improve learner outcomes and whether they could give specific 'before and after' examples of practices that had changed, as well as what they thought the catalysts for any change had been in their context, and what the key conditions for that change were. All interviews were transcribed in full by a professional transcriber. The researcher collated the schools' NAPLAN results (standardized tests of mathematics in Year 3 and Year 5) over the years of involvement with the professional learning initiative and the years that followed. The researcher also collated any shared products from the professional learning initiative. Some schools had made presentations that had been videoed, posters that had been photographed and plans that had been uploaded to the shared website.

Data Analysis

NVivo was used to collate and store the materials relating to each school. The schools formed cases comprising data from interviews, NAPLAN results, and artefacts from participation in the professional learning initiative. Three key codes were used to analyze the transcripts and artefacts: catalysts, conditions and outcomes. Interviewee contributions and artefacts were read and re-read to identify factors that were mentioned as catalysts for change (coded as 'catalysts') and as conditions that supported the change or proved to be a barrier to change (coded as 'conditions'). Any outcomes or changes that were mentioned with evidence to support them were coded as 'outcomes'. The artefacts and transcripts within a case were then compared, to look for themes or discontinuities between the different evidence sources. A chart of catalysts, conditions and outcomes was compiled for each school and a case narrative summary of each chart was prepared, using quotes and evidence from the data sources. The charts were then compared among schools, looking for similarities and differences in catalysts, conditions and outcomes. A comparative chart was developed, summarizing the presence and absence of important factors that were emerging from the comparison analysis.

Study Limitations

This study is limited by the number of participants, the role of the researcher and the interpretive nature of the analysis. Data in this paper comes from only two schools, and any emergent patterns may be unique to these places or to this school district. The researcher had been part of the facilitation team, and although that provided insights and relationships for deep conversations during the interviews it may have shaped the participants' responses. The analysis, while guided by a coding framework, is interpretive and other interpretations are possible.

Findings

Echidna School

Catalysts. At Echidna school there were four elements that combined to be a catalyst for changes in practice and new learning: the Spiral of Inquiry stages, student voice collected as part of the scanning phase of the spiral, a visit from the facilitation team to the school, and the vision and enthusiasm of a key staff member (the mathematics leader).

Conditions. The key conditions for Echidna's learning and change process were: the tool they used to look at student learning, their work with a mathematics facilitator, the shared understandings they developed through their work together and the level of trust and willingness to improve in the staff. Surrounding this was the alignment of the Principal with the ideas in the Spiral of Inquiry, the availability of useful data and the skills to use it, the use of school structures such as Professional Learning Communities to facilitate the professional learning, focusing on one thing and 'clearing the decks' and the extended timeframe of the professional learning experience. Echidna school's district facilitator introduced the school to a diagnostic interview test of problem solving in mathematics, which closely matched their inquiry question. This test and the data that came from it were central to the teachers' developing understandings of the students' learning, the problem solving process and each other. The test results disconfirmed the teachers' hunches, raised new questions and provoked curiosity.

Outcomes. In 2016, immediately after the project, both Y3 and Y5 NAPLAN results in numeracy were still significantly below the state mean, although the progress profile of the school showed that learners were making more rapid progress than expected. By 2017, both Y3 and Y5 groups were within range of the state mean in NAPLAN numeracy, with 47% of the students showing faster than average progress. This pattern continued in 2018. Teachers learned a lot of content knowledge about mathematics, and associated knowledge about student learning in mathematics and pedagogical approaches. Alongside this they learned new assessment techniques and increased their ability to use evidence to make instructional decisions, learning more about the diagnostic uses of data sets. Teachers now use more open-ended tasks to teach mathematics, and use more manipulatives to help children establish conceptual understanding. There is now clear alignment between the school focus, staff meetings and PLC work. This is indicative of increased clarity of purpose. Evidence is used to inform decisions and evaluate actions at all levels of the school, from the Principal to the classrooms, and there are protocols and resources in place to make this happen (available data sets, time allocated to discussion of evidence, requiring evidence for decisions and actions).

Wallaby School

Catalysts. At Wallaby School, five factors combined to provide the catalyst for change and new learning. A new leadership team came in to the school and decided to use the Spiral of Inquiry as their key means for making change. Collating and looking together at their data was an important motivator that also built shared language and knowledge. The support and questioning of both district and outside facilitators catalyzed change, as did the Spiral of Inquiry ideas themselves, in particular the idea of 'hunches'. The teachers were all eager to make change, as the school was rapidly losing students and staff and had been struggling for a number of years.

Conditions. Student losses meant a redundancy process had to occur. Being at 'rock bottom' had awakened a desire to improve in staff. For the first time, they all looked at the data together, and worked through their hunches. Being able to have hunches, and to explore and discuss these was a powerful condition for change at Wallaby School. Laying their hunches on the table led to rich discussion and increased shared understanding and cohesion. The whole staff worked as one 'professional learning team' with strong leadership from the Deputy Principal. The school focused on the Spiral of Inquiry as its sole professional learning focus and focus for the school. The structure and language of the Spiral of Inquiry gave the staff tools to work together and share ideas.

A key condition for change at Wallaby School was the use of a diagnostic task that the school developed themselves (in the task students explained the meaning of a multi-digit number; younger children were given two digit numbers, older children decimal numbers). The task could be used at all year levels, and was targeted at the focus area, providing rich information about student understandings. In addition each teacher videoed some target students talking about how they did the task, and these videos were shared amongst the staff. In general, the students' answers were more sophisticated than the teachers had imagined, opening up the possibility that they could do more and that teacher expectations should be adjusted. The students' responses to the tasks provided teachers with rich information on which to begin their inquiries, and led to the teachers questioning how much they themselves knew about the concept they were testing. The staff worked together to develop a continuum that described progression in the key concept. This task meant that they all had to delve in to what the concept really meant at different levels of the school, and what progress might consist of. The test and the continuum were key tools that supported the development of an inquiry mindset in the staff, because they provided the evidence of what students knew, a sense of the complexity of the concept and the means of assessing progress. The expertise to develop the task and the continuum was present in the staff, but previously had not been utilised.

Outcomes. Evidence of student learning on the large-scale NAPLAN data is inconsistent over the years since the Inquiry Spiral intervention. Y3 and Y5 students moved to within range of the state mean immediately following the professional learning initiative, from being substantially below the mean prior to this. The following year they fell back again, but in 2018 Y3 results are above the state mean for the first time. Y5 remain below. Student engagement increased and referrals to senior staff for behavioural issues reduced during the two years of the project. Teachers learned both knowledge and skills, and made observable changes to their practice. Pedagogical content knowledge about the key concept and how to teach it was central to teacher learning, alongside increased knowledge of the mathematics curriculum, progressions in early mathematics learning and the design of diagnostic mathematics tasks. Teachers gained skills in collaborating with each other to improve learning outcomes, how to talk and listen to each other, how to identify and deal with hunches and how to work with data. Teachers' practice changed to include pre and post testing to evaluate their impact. They undertook more explicit teaching, worked with small focus groups as well as the whole class and used more hands-on activities, rather than worksheets and chalk and talk that had dominated previously. Using data to check hunches and to inform planning and decision making became a habit. In particular, they became aware of their hunches and assumptions and worked actively to explore them using data. Shared vocabulary about the Spiral of Inquiry and about the key mathematical ideas aided teachers' communication. Teachers' meetings as professional learning teams became focused on students' needs and how to change teaching to help students progress. Several structural changes also occurred. Professional learning teams (PLT) were established and used to discuss student learning. This structure is valued by all levels of staff and cited as a major vehicle for focusing on data, student learning and the impact of teaching. The school now collects, analyses and disseminates data and other forms of evidence, which previously were filed and not shared. School time is given for PLT groups to meet and plan, in order to encourage professional discussion and collaboration. Providing time to continue the collaboration begun with the Spiral of Inquiry has been important in maintaining this way of working.

Discussion

Both Echidna and Wallaby Schools made substantial changes to the way mathematics was taught in their schools as a result of participating in the Spiral of Inquiry. In both schools there was a lift in student performance on NAPLAN, although only Echidna School has maintained this lift. There is evidence in both schools of teacher learning about mathematics concepts, student learning in mathematics, assessment and mathematics teaching, and evidence of organizational change in meetings and resourcing. Despite being a generic inquiry process, in these schools the Spiral of Inquiry was able to support change in mathematics teaching and learning.

Central to the changes in these two schools was the way that the Spiral of Inquiry built collaboration. The Spiral provided a structure for groups of teachers to talk about student learning and their own learning and to support each other to make change. A key element of this was the shared language the spiral provided. The names of the stages in the spiral became shared terms that were frequently used amongst staff to talk about what they were doing and why. The ‘hunches’ stage was particularly significant in both these schools. Exploring hunches gave the teachers a chance to explore what they thought and to examine their assumptions. This process tightened the collaboration amongst the teachers as the honesty required by the process and the shared sense of agency it developed were mechanisms for building trust and motivation. Inquiry processes often move from data patterns to action without considering the hidden ideas and assumptions that underpin the way things are and may act to keep them that way. The ‘hunches’ phase was an important step for building collaborative action, and the term ‘hunches’ and the idea of checking them persisted in both schools two years after the professional learning initiative.

At the heart of change in both these schools was a diagnostic test/task that yielded relevant, fine-grained information about students. In this way the Spiral of Inquiry was similar to specific mathematics professional learning opportunities (eg: Kazemi & Franke, 2004), but rather than all schools using one diagnostic test/task, in this context the tools were tailored to the school’s inquiry focus. The nature of these tools (diagnostic interview tasks) and the information they yielded (rich information about student thinking) formed the basis for the collaborative work of the teachers. The teachers were fascinated by the results, which in both cases challenged their assumptions about students’ capability and caused the teachers to re-frame the ‘problem’ they were addressing. Collecting student voice (through both the test/tasks and talking to students about their mathematics learning) was a powerful condition for building collaboration in these schools.

As the professional learning progressed, both schools delved deeply into mathematics concepts (place value and multiplication) in order to address what they found in their data. As with the spiral terms, the shared language of mathematics and mathematics learning built through this process enabled more effective communication among teachers, who also became more willing to admit when they did not know something and to seek help. In this way exploring the data together opened up a space for new, collaborative learning about mathematics and mathematics teaching.

The Spiral of Inquiry may be a helpful structure for schools wanting to improve mathematics teaching and learning. This study suggests that having rich, targeted data is important as a basis for collaboration and that spending time unpacking hunches and assumptions about a situation can build trust in collaborations. Both the data and the hunches engaged teachers, sparking curiosity and raising questions for them to purse

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TEACHERS COLLABORATING IN COMMUNITIES OF MATHEMATICS IMMERSION

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Beyond initial college preparation, secondary teachers in the United States have few professional opportunities to do and learn challenging mathematics, especially in collaboration with colleagues. The Mathematics Immersion for Secondary Teachers at Scale program engages sets of teachers in local school sites, connected synchronously and asynchronously to colleagues in other sites, in doing mathematics designed to promote experiences of mathematical immersion, community, and connection to the work of teaching. This study of two groups of sites over one year examines fidelity to the program as a model for systematically providing these opportunities, and the extent to which teacher participants experienced immersion, community, and connection in their collaborative work with the course facilitator and their local and distant colleagues.

Research Questions

Our research team is studying a professional development (PD) model that fosters a connected community of mathematical practice. The Mathematics Immersion for Secondary Teachers at Scale (MIST)[1] program engages secondary mathematics teachers in professional collaborations using a blended approach that combines local face-to-face meetings and online communication with teachers in other locales. In MIST, teachers do challenging mathematics together to experience immersion in mathematics, to develop community, and to connect what they are learning to their instructional practice. We address Theme B by describing the MIST model, identifying design features and goals of the model, and investigating collaboration when the model is implemented. This work is situated within a larger experimental study of fourteen groups from across the United States. We examined two selected groups to address two research questions:

> Research Question 1: How well can facilitators use the provided tools and resources to enact the MIST blended professional development model with fidelity?
>
> Research Question 2: How well do teachers' experiences reflect the intended collaborative work in mathematical immersion, development of community, and connection to teaching practice?

Background to the Problem

Secondary mathematics teachers in the US generally come to the profession with experiences learning abstract, college level mathematics as part of their preparation for teaching. Over half of high school teachers have a degree in mathematics, and approximately three-fourths have taken at least 5 courses at the calculus level or higher. Middle school teachers tend to have less extensive

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mathematics experience, but over 80 percent have taken at least 2 or 3 college-level mathematics courses such as statistics, calculus, and courses for prospective teachers (Banilower et al., 2018).

In their classrooms, secondary teachers are expected to teach abstract content—although at a more fundamental level than their college mathematics experiences—as well as disciplinary practices for doing mathematics. States provide content standards for each grade level, typically addressing ideas from the areas of algebra, functions, geometry, number and operations, and probability and statistics. Over the past decade, states have added standards related to disciplinary practices, such as the Standards for Mathematical Practice (National Governors Association, 2010).

Teachers' continuing education does not generally reinforce connections between learning mathematical content and using disciplinary practices by engaging teachers in advanced mathematical work. In 2018, 90 percent of secondary teachers had experienced PD focused on mathematics or mathematics teaching in the previous 3 years, but only about half reported that it gave heavy emphasis to deepening their understanding of how mathematics is done, and less than half reported a heavy emphasis on deepening their own content knowledge (Banilower et al., 2018). By engaging practicing teachers in collaborative study of mathematics, MIST helps them connect learning content and using disciplinary practices.

Theoretical Framing and Description of the MIST Model

Mathematics Knowledge for Teaching

Scholars have devoted significant attention to defining and measuring the mathematical knowledge teachers use in their practice, and to designing and evaluating experiences that develop this knowledge. Much of this work focuses on elementary school teachers. Less attention has been given to secondary teachers' knowledge of mathematics, particularly of advanced mathematics and practices of the discipline. Cuoco (2003) identified four ways that secondary teachers should know mathematics, namely, as a: scholar, mathematician, philosopher, and teacher. Knowing mathematics as a scholar and as a teacher relates to what Ball and colleagues (Ball, Thames, & Phelps, 2008) have termed common and specialized content knowledge, knowledge of content and students, and knowledge of content and teaching. Knowing mathematics as a mathematician and philosopher more aptly fit into what Ball and colleagues term horizon content knowledge, which has been scantly studied. Cuoco (2003, p. 16) describes this knowledge as understanding that "Mathematics is about solving problems, building abstractions, and developing theories. Mathematical expertise lies as much in what one can figure out as it does in what one knows." Cuoco argues that immersing teachers in mathematical investigation can develop such knowledge, because teachers experience the nature of the discipline in their own mathematical work, revealing connections among ideas that strengthen their learning. This puts them in a position similar to their students' of grappling with challenging ideas, supporting connections to their teaching practice.

Mathematics Immersion as Collaborative Teacher Professional Development

Describing the design features of mathematics immersion (Sztajn, Borko, & Smith, 2017) provides a basis for analyzing both how these features are carried out in various contexts, and the extent to which they promote intended learning experiences. Mathematics immersion is conceived as a collaborative endeavor; its focus on mathematical work demands key elements of professional collaboration (Boavida & Ponte, 2002; Day, 1999): negotiation of definitions and meanings, shared decision making about approaches, and precise communication about results. Consistent with Activity Theory (e.g., Engeström, 2001), teachers are meant to experience the nature of the discipline and develop new understandings of mathematics as authentic learners and consider implications for their students' learning experiences as well implications for their own instruction that can promote these experiences. Previous studies of mathematics immersion have demonstrated impacts on teachers' sense of what defines mathematical expertise (Kraus et al., 2008), their own mathematical habits of mind, and their sense of professional community (Gates et al., 2016). We are expanding the investigation of teacher learning and teaching practice outcomes. The present study addresses the teachers' learning experience itself as a means to understand the design of the program and its enactment, and to explain outcomes that result. We focus on three constructs characteristic of the intended experience: immersion in mathematical work, development of mathematical community, and connection to the work of teaching (Gates et al., 2016).

The MIST Professional Development Model

The goal of the MIST model is to engage teachers in mathematics immersion as part of a mathematical community during synchronous, online sessions and to connect their immersion experiences to the work of teaching during synchronous and asynchronous discussions. In MIST, 4–7 teacher participants gather at each local site together with a table leader who may be an administrator or another teacher. Participants at each site are able to share ideas, mull over conjectures, learn from, and laugh with each other—all benefits of face-to-face collaboration. Each site provides high-speed internet, a device for videoconferencing, and a physical location for the sessions, and we provide a tablet device for taking photographs of participant work. A MIST group consists of several sites and a facilitator meeting through synchronous technology for nine 2.5 hour sessions. Facilitators use online video-conferencing to connect the sites, share examples of participant work, and lead cross-site discussions of the mathematics and classroom connections. Groups also use a collaboration app for asynchronous discussions and to enhance communication between leaders and facilitators during sessions.

MIST course materials are problem sets that are designed for facilitators to use a hands-off approach, allowing the participants to work collaboratively (at each site) through the carefully crafted sets of questions. The materials also include norms for participants to work together. The primary activity in each session is exploring mathematics within sites, with the video connection to the facilitator and other sites open but not the focus of attention. Typically, once or twice during each session the facilitator calls the sites together to share one or more pieces of participant work, providing opportunities for sites to discuss their mathematical insights. To keep the facilitator apprised of progress during the session, every 15 minutes table leaders upload a photograph of a participant's mathematical work and write a brief update that the facilitator and other table leaders can access about their site's mathematical progress. For research purposes, course facilitators video record each MIST session using the recording tool in the video conferencing platform.

Study Design and Context

The research for this paper is part of a larger study examining the MIST PD experience and its impacts on teachers' knowledge of mathematics, beliefs about teaching and learning mathematics, and instruction. The larger study is a two-group, semi-randomized block, delayed participation design over two years. Participants in Cohort A (Course groups A1-A7) received the MIST PD during the 2018-2019 school year and participants in Cohort B (Course groups B1-B7) will receive it during the 2019-2020 school year. The design enables across-group, participant-control comparisons between Cohorts A and B, and within-group comparisons over time to examine pre- to post-participation effects for Cohort B and sustained participation effects for Cohort A.

This investigation focuses on the experiences of two groups in Cohort A. Course group A3 and A6 were purposely selected from Cohort A for having high fidelity to the MIST model, but showing evidence of differences in facilitators' actions that we hypothesized might impact participants' experiences in their MIST courses. Both course facilitators were experienced mathematics teachers who were familiar with the mathematics immersion approach. The A3 facilitator, a female middle school teacher with over 5 years of teaching experience, participated in a pilot of the MIST course as a table leader. The A6 facilitator, a male high school mathematics department chair with over 10 years of teaching experience, had participated in an in-person mathematics immersion program.

Group A3 was composed of four sites with 3 or 4 participants each, all located in the central US. The A3 table leaders were two female high school mathematics teachers and two female school personnel not currently teaching mathematics. A6 comprised two sites with 5 or 6 participants each, both in the eastern US. One female and one male high school mathematics teacher served as table leaders. Additional information about the teacher participants in each group is shown in Table 1.

Table 1: Participant Gender, Teaching Assignment, and Teaching Experience, by Group

Course Group	Female Participants	Male Participants	Middle School Teachers	High School Teachers	Years of Teaching [Range (Mean)]
A3	8	6	9	4	1–24 (7.29)
A6	7	4	2	9	0–29 (15.82)

Data collection

Data collected for this study included digital artifacts (video recordings, photographs, and spreadsheet updates) from the sessions and interviews with table leaders and course facilitators. Researchers randomly selected three session videos per group for analysis, drawing one each from the beginning (Sessions 1-3), middle (Sessions 4-6), and end (Sessions 7-9) of the course. The analytic sample included Sessions 3, 5, and 9 for group A3 and Sessions 2, 6, and 9 for group A6. Following the final course session, we interviewed table leaders about how the teachers at their site worked together. In addition, we interviewed the course facilitators about how they facilitated the course and their perceptions of their group's experience with the course.

Data analysis

To measure logistical fidelity to the MIST model, using data from all sessions, we examined the extent to which: sites attended the sessions, facilitators adhered to the timing and structure of the sessions, and table leaders updated their groups' progress in writing and with photographs. For site attendance, we documented the frequency with which any sites did not attend a session. For adherence to timing and structure, we considered the number of sessions and length of each session. We also examined whether the facilitator included a 10-minute break and two whole-group conversations in each session. Facilitators were asked to have sites read the MIST norms aloud in early sessions (Sessions 1-3) and address connections to classroom instruction in whole group discussions in later sessions (Sessions 6-9), and we tracked whether these activities occurred.

To measure fidelity to expectations for table leader updates, we took different approaches for the written updates and photo uploads. For the written updates, we calculated the average number of written updates across all sessions and all sites within each group and recorded the minimum and maximum number of written updates within each session for each site. For the photos, we divided sessions into 15 minute intervals and counted the number of intervals in which either (a) at least one photo was taken or (b) at least two photos were taken in the interval immediately before or after that interval. To understand regularity of photo uploads, we recorded the second-largest time interval between two photo uploads, then calculated the average of this interval across all sessions and sites.

To investigate variations in the course enactment and how they related to teachers' experiences in the course, we analyzed session video recordings and interviews with course facilitators and table leaders. Session video recordings were coded using a time sample approach, segmenting each recording into 3-minute intervals. The coding scheme used indicators to identify when teachers were immersed in mathematics (Immersion), engaged with one another as members of a mathematical community (Community), and connected to the work of teaching (Connection). We developed the indicators using an expert panel methodology in an earlier phase of our research (Gordon, et. al., 2019). In addition, the coding scheme includes indicators adapted from Reeve (2006) for observable autonomy supporting practices (Autonomy Support). We also coded instances that appeared contrary to the autonomy supporting practices (Autonomy Inhibition).

Before coding the sampled videos, we completed three rounds of practice coding on short sections of videos that were not part of the sample. By the third round, coders had reached pairwise inter-rater agreement greater than or equal to 70 percent for each category and for all but one of the indicators (Immersion: Seek mathematical connections), for which agreement was 60 percent.

We also summarized the table leader and course facilitator interviews, and identified commonalities and differences in the participant experience of the MIST course.

Results

Fidelity to the Model

Both groups enacted the MIST course with generally high fidelity to the model. One A3 site missed a single session due to inclement weather. Otherwise sites attended the sessions regularly, groups adhered to the expected timing and structure, and table leaders provided updates as intended. However, A6 did not take a 10-minute break each session or read the norms aloud in Session 2. Table leaders in both groups updated the progress spreadsheet and uploaded photos periodically as expected. However, for both groups, the time intervals between photo uploads included some spans of 30 minutes or more. Two sites in A3 encountered difficulty uploading photos and instead sent photos to the facilitator by text message. Also, whole group discussions about connections to classroom instruction in both groups, although present, were lacking in depth.

Teachers' Collaborative Experiences of Mathematics Immersion

The video analysis indicated that participants in both A3 and A6 engaged in the intended immersive mathematical experiences regularly, although some differences were evident. Interviews with table leaders and course facilitators were largely consistent with the results of the video analysis. Table 2 shows the percent of video segments in the sampled sessions exhibiting evidence for each code.

Table 2: Percent of Coded Segments Receiving Codes and Indicators, by Group

Codes	**Percent of Segments**	
Indicators	**Group A3**	**Group A6**
Immersion, any indicator	**80.3**	**87.5**
Abstract and generalize	12.9	21.3
Develop and Support Conjectures	13.6	11.0
Solve Problems	77.6	84.6
Ask Mathematical Questions	17.7	21.3
Use Experimentation and Reasoning	32.7	30.9
Develop and Apply Structures and Definitions	29.3	39.0
Seek Mathematical Connections	21.1	31.6
Community, any indicator	**90.5**	**94.1**
Share Products of Mathematical Work	71.4	73.0
Work on Mathematics Together	76.9	78.8
Discuss Off-task Topics	40.8	38.6
Discuss Norms	5.4	4.0
Connection, any indicator	**14.3**	**8.1**
Discuss Pedagogical Strategies	3.4	3.0
Discuss Student Thinking and Learning	11.6	8.6
Discuss Curriculum and Content Seeking	5.4	4.9
Discuss Applying MIST Takeaways in the Classroom	5.4	4.0
Autonomy Support, any indicator	**34.7**	**21.3**
Non-judgment	19.7	8.1
Provide opportunities for engagement	19.0	11.0
Acknowledge and Respond to Participants	24.5	17.6
Praise and Encourage Effort, Progress, and Mastery	2.0	0.7
Autonomy Inhibition	**6.1**	**4.4**

The results suggest that the groups were immersed in mathematics for much of the sessions. For both groups, over 80 percent of segments contained at least one indicator of Immersion. In both groups, Solve Problems was most prevalent. Both “Use Experimentation and Reasoning” and “Develop and Apply Structures and Definitions” were among the more common indicators for each group. In interviews, members of both groups described their experience using terms similar to the indicators of Immersion, including solving problems and seeking mathematical connections. In the words of the course facilitator from A3:

> Ultimately both [sites] really came away with … some good connections between the isometric dot paper and the square paper and the numerical problems that they were working on. They were able to find a connection to find the Pythagorean triples, and then finding, ultimately, Eisenstein triples.

The groups worked as mathematical communities for much of the time. At least one indicator of engaging in Community occurred in around 90 percent of the video segments for both groups, and the portion of segments for each individual indicator were similar between groups. The most prevalent indicators of Community were mathematics related, and Discuss Norms was the least prevalent. In interviews, members of both groups described working in a way consistent with indicators of Community, including working on mathematics together, sharing work with one another, and engaging in non-mathematical conversations. A table leader from A3 shared:

> We'd … probably all together get through the opener, and then…. it seemed like we kind of pair off and then the pairs would change as we move through. And then somebody would have a question … for somebody that had already finished something, and just kind of flow back and forth, people moving back and forth who they were working with.

Also, most table leaders reported that the norms went smoothly with little intervention, which may explain the low frequency of the Discuss Norms code. In contrast, interviewees reported little cross-site discussion during the sessions, and some described the whole group discussions as “stilted” and conversations of “show and tell.” One table leader from A6 reported:

> It was a show and tell versus a meaningful collaboration with the other [site]. It was maybe once throughout the whole two hours that we kind of came together, and it was just “Site A, you share something that you thought was cool, and then Site B, you share something,” There was no counterpoint, or “here’s two ways that we did it,” or “here’s an extension.” It was just sort of like we were both there in a room, but we never had a dialogue.

In both groups, making explicit connections to practice during the sessions was relatively infrequent. Only 14.3 percent of segments in A3 and 8.1 percent in A6 were coded as Connecting. Table leaders’ descriptions of discussing connections to teaching were aligned with the video coding results. Table leaders in A3 reported brief conversations about connections to teaching during sessions, and table leaders from A6 reported little to none. Furthermore, interviewees from A3 mentioned that conversations about connections to teaching took place primarily in the asynchronous platform, while those from A6 reported little activity on the asynchronous platform.

A notably higher portion of segments in A3 (34.7) compared to A6 (21.3) received at least one Autonomy Support code. For both groups, Provide Opportunities for Engagement and Acknowledge and Respond to Participants were coded relatively frequently, and Praise and Encourage Effort, Progress, and Mastery was coded relatively rarely. Autonomy Inhibition was rare in both groups. Interviewees indicated that the course facilitators communicated primarily through the spreadsheet, which may explain the relatively low frequency of video codes for Autonomy Support. Their descriptions of the support facilitators provided in this manner were consistent with Autonomy Support. For both groups, they indicated that facilitators gave progress-enabling hints, responded to their questions, and provided opportunities for participants to share their work.

Conclusions and Next Steps

MIST scales up an existing in-person approach to teachers working collaboratively in mathematical immersion through a blended model (in-person and distance, synchronous and asynchronous). This model shares several design features with its precursor: the instructional materials, norms for collaboration, goals for teacher interaction, and intended outcomes for teachers’ understandings of and about mathematics, and related changes in their instruction. The primary focus is on teachers collaborating and doing mathematics as mathematicians do by exploring unfamiliar mathematical ideas. Novel design features include meeting online with other table groups and the facilitator, having a table leader at each site to facilitate communication with the facilitator, and using digital tools for communication among the sites and facilitator.

For both analyzed groups, the MIST model was implemented with ample fidelity, generally adhering to expected timing and activities. Participants persistently engaged with the mathematics problem sets and discussions in ways characteristic of immersion. Their work together provided evidence of engaging in mathematical community. The MIST model structures less frequent dedicated discussions of teaching practice. Accordingly, indications of explicit connection to practice occurred less often. Of some concern, they appeared to lack depth for both groups.

We are continuing to examine both the collaborations among teachers during MIST sessions and the impacts of these collaborations. In addition to the two groups described in this paper, we are analyzing data from five other groups in our first cohort and seven new groups in a second cohort who are participating in the program in 2019-20. In addition to digital data from the course implementations, we are collecting data related to teachers' beliefs about teaching and learning mathematics, their content knowledge, and their instruction. We will examine changes in these data over time, using comparisons between the two cohorts, to better understand the impacts of mathematics immersion experiences on secondary mathematics teachers and their teaching.

Our findings suggest that MIST offers a scalable model for teacher collaboration with local and geographically distant colleagues focused on learning mathematics through immersion. Teachers' collaborative learning experiences reflect intended aspects of mathematics immersion, working in a mathematical community, and connecting to teaching practice. Following tenets of activity theory, further investigation will trace impacts of these experiences on beliefs, knowledge, and practice, and explore participants' ongoing experiences of collaboration.

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LEA RMG: A 5 YEAR-STUDY WITHIN A COLLABORATIVE PROJECT BETWEEN TEACHERS AND RESEARCHERS.

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The LéA RMG is a collaborative work group of secondary school teachers and researchers that focuses on the teaching, learning and assessment of school algebra. This contribution is proposing analytic tools to characterize the work inside the LéA. It draws a set of collaborative work principles and conditions that favour a long lasting partnership and promote positive outcomes on teachers' practices and postures, and students' learning. Some research and methodology questions are raised, to problematize the specificity of this type of collaboration.

This study focuses on a collaborative work, which has been going on for 5 years now, in a *LéA* (*Lieu d'Education Associée*, meaning a place where education takes place, and which is associated to the research group). The LéAs have been promoted by the IFÉ[1] (French Institute for Education) as places with educational questions and challenges, bringing together practitioners, with the support of the local educational institution, and a research team, in order to build a common project over time, through a joint action between the researchers and the actors in the field. The LéA system also aims to disseminate the knowledge and results of this research and make them available for pre and in-service teachers, educators and researchers. One of the expected outcome of this type of collaborative work is the design of *resources* (Gueudet et al., 2009). In this proposal, we question the ways of working inside such a collaborative project, and the conditions that made this collaboration possible over 5 years. We also question the effect of our action in this collaboration, both on the development of teachers' practices and their students' learning, and on the knowledge generated for the research in math education (as in Jaworski et al., 2017).

The aims and actions of the LéA RMG

Presentation of the LéA RMG : where teachers' and researchers' questions possibly meet

The LéA RMG[2] (named after the high school where the involved teachers work, Roger Martin du Gard[3]), started in 2014 (and will last until 2020), with the aim of designing resources for the teaching and learning of school algebra, for *cycle 4* students (secondary school, 12-15 year-old students), but with different personal goals for its participants. For the 4 teachers who initially joined the LéA RMG, the aim was to enrich their professional practices to best support students' learning and promote every student's success in the field of school algebra. For the math education researchers from the LDAR (Laboratoire de Didactique André Revuz, Université de Paris), another aim was to study the effects of this collaborative work, both on the teachers and their students, in order to enrich the math education research knowledge about the development of teachers' practices, and the learning of

1 http://ife.ens-lyon.fr/lea
2 http://ife.ens-lyon.fr/lea/le-reseau/les-differents-lea/reseau-de-colleges-martin-du-gard
3 This is a secondary school where the socio-cultural background is officially considered as rather disadvantaged, and where the students are usually assessed as low-achieving.

algebra. The researchers of the LéA team also wanted to explore the questions of assessment and regulation of teaching for students' learning, with regard to classroom interactions. The first challenge is to help the teachers take better account of the specific needs of the students and encourage their students to understand the gap between their production and the expectations of the teacher or of the institution, by developing an awareness, in moments of collective interactions in class, of the nature of the mathematical reasoning and argumentation used to formulate and validate mathematical proposals. Another challenge is for researchers to succeed in enriching teachers' practices, without going so far as to intervene in their classes. We thus consider the teacher as a professional in development and work *with* rather than *on* his practices (Bednarz, 2013, p7).

Over the years, the LéA RMG team has grown (from 4 to 12 teachers) and expanded geographically, as voluntary colleagues from neighboring secondary schools became involved. Other mathematical topics, prior or subsequent to the teaching of algebra, were added to the collective reflection, to take into account possible difficulties met in the transition between primary and secondary school, or secondary school and high school. The LéA gradually adopted an iterative approach, through cycles between design and implementation, based on the joint analysis of real practices (through video and transcription from real sessions in class, undertaken by the teachers of the LéA) and epistemological and didactic contributions of the researchers on the mathematical concepts at stake. Time has been dedicated in the working sessions of the LéA, to draft resources that could be disseminated for teacher education, and to enable the voluntary LéA teachers to participate to teacher education as educators. Our ways of working are close to the *Lesson study* (as in Huang et al., 2018), but with the use of video instead of classroom observation (as in Stigler et al., 1999, Jacob et al., 2009, or Cole et al., 2018).

This description shows that the work inside the LéA intricates different aims, tasks and roles, evolving as the collaboration goes by. How can we describe the ways of working and interactions of the participants of the LéA, and the evolution of their roles throughout time? How does the balance between training and research reveal sustainable collaborative work principles for the different participants of the LéA? We wonder about the constraints and possibilities resulting from the fact that different objectives co-exist in LéA to meet the dual challenge of studying didactic phenomena and providing tools to improve teaching.

A collaborative research?

At the beginning of the project, the researchers had to assume a role as educators in algebra teaching, while collecting data for research on teachers' usual practices. These usual practices, combined with research inputs, have been the starting point for the co-constructed resources: viable mathematical activities and classroom progress (Coppé & Grugeon, 2015). Some of these resources are both working tools for the training of LéA teachers, but also data for the researcher, to observe practices and their evolution, through the content of these documents and the discussions that accompanied their construction. The research question initially focused on the evolution of the practices, particularly the assessment practices, which was not a concern of LéA teachers, who, as a consequence, did not immediately endorsed the researchers' questions. Teachers' expectations and researchers' questions had then to be adjusted: it took, for the researcher, to answer some of the teachers' questions about the teaching of algebra, in order for the project to go on, on the topic of assessment. This also allowed the researchers, at the beginning of the project, to collect data on *ordinary* assessment practices.

The progressive participation of teachers in questioning and measuring the effects of newly introduced algebra lessons in their classrooms, has gradually promoted their interest for the study of classroom interactions and made it possible to put formative assessment back at the center of discussions. The question of assessment was then studied by all the participants, through the choice of relevant tasks, the identification and interpretation of students' procedures and the use of the information collected on the students, to promote learning.

Even though the teachers were more or less involved in the research, the ways of working between researchers and teachers has always been based on taking into account the teachers' work, their questions, their constraints and needs in terms of renewing their own resources, which have made this collaboration possible over a long period of time.

Principles for collaborative work and conditions for their implementation in the LéA RMG

Educational research involving researchers and practitioners - working together to design innovative practices for the improvement of teaching - is numerous and falls under different streams: action research, collaborative research (Bednarz, 2013), Lesson study (Takeshi & Winslow, 2009) or development engineering (Perrin-Glorian, 2011). They all agree on the need to take into account the teaching contexts and habits of teachers in order to develop mechanisms to promote the development of professional practices and student learning. We question the ways of working of a collective of researchers and teachers within the LéA RMG in an attempt to identify the principles of collaborative work that underlie it. We develop how, in our opinion, these principles make it possible to combine two strong issues of a LéA: the professional development of teachers and the production of scientific knowledge. In this section, we present the four principles that structure our collaborative work and specify the conditions to be put in place in the group, in order for these principles to be implemented.

Co-construction of a common issue

The first principle is the co-construction of a common issue to investigate together, with each participant bringing his or her own needs and expertise to the table. It seems necessary to us to start from the problems felt by teachers and the questions they raise (Butlen and Masselot, 2018) and transform them into research issues for the researcher and for the whole collective. This passage is accompanied by a step back to analyze what is happening in the classroom. Based on the difficulties reported by teachers and their observations of their students' poor performance, and without losing sight of our questions about assessment, we have established a common issue. We investigate how, through the choice of a relevant teaching scenario, we can improve both the algebra learning and the information gathered on this learning, therefore refocusing the research on practices and assessment. The didactic tools developed by the researchers around the issue of assessment, during the first years of the LéA, enabled us to put into words what we meant by assessment practices and collectively explain the conditions of their development (Pilet and Horoks, 2016).

Taking into account the teachers’ practices and context

The second principle is to take into account the teachers’ ordinary practices and the context in which they appear. In order to start from the practices "*almost already there*" (Robert et al., 2008), we first study those practices and the reasons for the choices behind them. For the same reason, we chose to offer to the teachers not methods to be applied but rather alternatives for teaching (Robert & Horoks, 2007) that can be used to enrich practices, and rely on collective work to share and analyze the actual classroom processes (Rogalski 2008). Within the exchanges taking place in the LéA meetings, we gather information on the teachers' goals and on what justifies their choices, through the analytical activities on objects for - or from – their classes. For example, studying, with teachers, their students' productions, allows us to better document how they analyze them, and identify the mathematical and didactic knowledge on which they rely. This information is data for research as well as support for training and the construction of common resources. Starting from the usual practices of teachers also allows us to minimize the cost of their investment so that it remains reasonable, which seems necessary to us, so that teachers remain invested in the LéA over a long period of time. Relying on data from their class and highlighting their work, both contribute to the involvement of the teachers in the study about assessment practices. The fact that many LéA teachers generally work in the same schools seems to us to be a favourable condition for a reasonable ratio between the cost and benefit of their investment (Goigoux, 2017), and also for their students, who can perceive continuity in the implementation of innovative practices. Despite this constitution of a *professional community of practice* (Wenger, 1998), the fact that the LéA meetings take place outside of the school context, with teachers from neighbouring schools, might encourage a reflexive and formative step back for the teachers.

Designing and implementing resources over time

We can link the third principle with the previous one, by stating that the involvement of the teachers in the LéA over a long period of time promotes duration, which is a necessary condition for the construction of a common resource and for the research, particularly with regard to analyzing the effects of this collaboration on practices and students' learning. We adopt an iterative design principle in which resources are designed and tested. The results of the collective analyses lead to a stabilized and shared version, after several cycles. We start from the teachers' documents for their class, analyzed by the researchers' tools and enriched by their proposals, which are negotiated to remain possible in the teaching contexts of the teachers, with the contributions of each of the LéA participants symmetrically taken into consideration in the negotiation. Resources are thus built, tested in class by several teachers, sometimes in several classes for the same teacher, supplemented by tangible evidence by bringing traces of this implementation to the collective discussion of the LéA[4], then tested again by the teachers, including some of the teachers who join the LéA in the following years. This long time, combined with the generally growing number of LéA members, allows both the resources built and the possibility of their dissemination to other teachers to be put to the test.

Sharing tasks and responsibilities

Our fourth and last principle states that all members should be involved in the various tasks and responsibilities of the LéA. The management and organizational tasks (emails, administrative documents to be completed, group's reports, agendas) are divided between the two leaders of the LéA (one teacher and one researcher). All members can be involved in the collection and analysis of data, which also proves to be valuable elements to illustrate the co-constructed resources. Teachers film

[4] Such as photographs of the board, student productions, videos of the session, testimonies.

themselves, scan student productions and make class documents available. Researchers and teachers are involved in didactic transposition processes (Chevallard, 1985) to take into consideration research findings and usual practices.

The dissemination of research results and resources is also to be shared: we have presented our scientific work in conferences and organized co-intervention training courses. This sharing of responsibilities also calls for conviviality, taking into account that a collective plays a social role for the participants, through cooperation and mutual, as values shared by all. This user-friendly workspace is built around the common definition of agendas for the group meetings, the respect of everyone's commitments and schedules but also by friendly moments between participants.

All these principles lead, in our opinion, to the establishment of a lasting trust between researchers and teachers, that leads to a fruitful collaboration over time. In this collaboration, the teachers become aware that the researchers do not have an immediate solution to their problems, and commit to the development of research. The researchers take into account that not everything is possible for every teacher in any class. They do not limit the participation of the teachers to the collection of data but also engage the teachers in reflective analysis and in the construction of hypotheses and scientific results. Responsibilities, and acknowledgement of each other's goals and actions, are therefore shared in what is experienced in the LéA.

The outcomes of the LéA's collaborative work: effects on practices and learning

A progressive evolution of teachers' assessment practices

In studies based on our collaborative work in the LéA (Horoks & Pilet, 2016), both *with* the teachers and *on* their practices, we have shown that the effects of this work on teachers' practices as well as students learning are visible over a long period of time. Based on De Ketele & al's definition of assessment (De Ketele & al, 1997) and on our hypothesis on the role of students' mathematical activities in their learning (Vandebrouck, 2018), we consider that assessment involves, for the teacher, taking information on the students' mathematical activities, interpreting this information, and using it, with a focus on a possible use for a teaching that promote students' learning. In order to characterize the teachers' assessment practices and their evolution, we therefore have a close look at how teachers take information (through which tasks), what kind of information (about the result, the procedure, or the mathematical knowledge behind the procedure), and how they use this information during collective moments in class (when sharing solutions on an exercise) or individual interactions (through summative assessment), by making a more or less explicit use of the students' productions (which productions, showing or not a certain variety and certain errors), and a more or less explicit link with the mathematical knowledge underlying them.

After one year of collaborative work with the teachers, we noticed (Pilet & Horoks, 2017) a change in the tasks chosen for their classes, widening the range of the type of tasks they proposed, especially the tasks giving a meaning to the use of algebra, as a better tool than the sole calculus to solve problems. But it took a longer time before they also changed the way they interacted with their students, giving them more responsibilities in the assessment processes, to justify and validate their productions, or to participate in the definition of criteria for their own assessment. In the (in)validation process in particular, the explicit highlighting of the mathematical properties at stake behind the algebraic transformations, was not easy to foster among the teachers, pleading for a less decontextualized way of teaching, to take into account the difficulties and socio-cultural background of their students. This shows the importance of taking into account the context of teachers' everyday work, including the always changing curricula and other institutional constraints, even though some of the resources that we designed, evolved towards a more challenging offer for the students, despite or within this context.

A difficulty to assess the effect on students' algebraic learning

To identify some effects of the collaborative work on students' learning (Pilet & Horoks, to be published), we used a diagnostic test of student algebraic knowledge and skills (Grugeon-Allys et al., 2012). This assessment, based on an epistemological analysis of school algebra, automatically profiles each student on a mastery scale, according to three components: Use of algebra, Translation from one representation register to another and Algebraic transformations. The students were supposed to be tested every year of cycle 4, but only if their teacher was a participant of the LéA. But because not all the teachers of the RMG secondary school were part of the LéA, and due to technical difficulties linked to the lack of digital technology, the number of students that were actually tested each year is rather small (only 5 students, among the approximately 200 students that were concerned at least one year by this study!). We can still draw conclusions from the results of all the students tested each year, without any real follow-up for each student through the 3 years cycle.

Over the years, fewer students are using arithmetical procedures and more use algebra in algebraic problems, which can be linked to the fact that the moment of discovery of algebra is more organized by the LéA teachers around algebra as a tool. However, this use of algebra is limited because students choose to use letters but do not always know how to use them afterwards. The level of the students stagnates in algebraic transformation and in translation between registers throughout the two last years of the cycle, which can be explained by the fact that algebraic expressions are becoming more and more complex in the test. The small number of students tested each of the 3 years limits the interpretation that we can make of these results. But the general perception of the LéA's teachers, validated by the school's improving results in the national assessment, is that their students are less afraid of the use of a letter than their students used to be, and more prompt to use algebra when needed.

A change in the teachers' posture

At the beginning of each school year, the LéA RMG welcomes new teachers from the same or neighbouring secondary schools, to participate in the reflection on the construction of teaching resources. The integration of these teachers raises new questions about the relationship between researchers and teachers and about their roles (as in van Es, 2009; Jaworsky, 2005). It seems to us that the teachers that were present from the first years of the LéA's existence can play a role in passing on and justifying objects already built together. This role is, in our opinion, both a lever for the professional development of the newly arrived teacher and even for the former ones, but also a way to collect data on how the teachers who have been present in the LéA for a few years have appropriated these objects.

Another evolution of the posture of the LéA's teachers concerns their involvment in teacher education. Some of the LéA's teachers participate, as educators, in in-service training programs for secondary school teachers (Grugeon-Allys et al., to be published). The aim of this training is to share and disseminate the resources and results obtained in the LéA, to teachers who will also, in turn, have to transmit these resources to their colleagues. The role played by the LéA's collaborative work in promoting a change in posture, from teacher to educator, or even researcher, should also be questioned. We started developing some indicators of these postures' changes, for the teachers as well as the researchers, through the analysis of the interactions between the participants during the LéA meetings. These indicators focus on the reasons given to justify the choices for teaching, whether they concern elements of the context (the level of the students, the time spent on preparing or managing the class, the programs) rather than epistemological or didactical elements linked to the mathematics at stake. This is still a work in progress.

Conclusion

The collaborative work undertaken in the LéA RMG started in 2014, as a study of ordinary assessment practices as well as a reflexive group on the teaching of algebra in secondary school. Since then, it has developed into a team which aim is to design, in an iterative process, resources to teach algebra, to assess the students and to train other teachers on these questions.

The LéA's teachers, by practicing the observation of the learners, in order to gather relevant information, have gradually adapted their teaching to situate the students as closely as possible to their learning. They also have equipped themselves with observation tools on their own practice, based on the researcher's research tools. Finally, the researchers are witnesses of all these phenomena (the teacher teaching, justifying his teaching, or training other teachers) that are data for research, and raise new questions for research on such collaborative methods and their outcomes. Several open questions remain to be debated on this last aspect: what specific methodology should be put in place to meet the challenge of analyzing a system that is changing both in terms of teaching practices and in terms of the status of teachers in relation to research? How does this type of collaborative work produce research results that researchers would not otherwise be able to produce?

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TEACHERS' LEARNING THROUGH PARTICIPATION IN CONFERENCES AND NETWORK MEETINGS – THE NORDIC AND BALTIC GEOGEBRA NETWORK

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GeoGebra, www.geogebra.org is a software for teaching and learning mathematics at all school levels. It is open source and has been translated to many languages including Nordic and Baltic languages. In this paper I describe the activities of the Nordic and Baltic GeoGebra Network that was founded to facilitate collaboration and sharing, among teachers and researchers, on the use of ICT in the teaching and learning of mathematics. The network has organized yearly conferences for the last 10 years and several smaller meetings to work with groups of teachers interested in particular issues related to the use of ICT. During these 10 years several interesting questions have risen concerning the value and usefulness of such actitivites. The participants have filled in several surveys that have aided the organizers in developing our activities and some participants have been interviewed by the author on the value of such activities.

The software GeoGebra is used for teaching and learning mathematics at all school levels (Hohenwarter, Hohenwarter & Lavicza, 2009). It has an extensive user community with local GeoGebra Institutes in many countries and an International GeoGebra Institute that maintains a website www.geogebra.org (Hohenwarter & Lavicza, 2007). In each of the Nordic and Baltic countries there have been efforts to promote the use of GeoGebra and other software in the teaching and learning of mathematics. The Nordic GeoGebra Network (NGGN) was founded in 2010 by 7 mathematics teachers (4 from university (teacher education), 2 from upper secondary school and 1 from lower secondary school). They are from 7 Nordic and Baltic countries; Iceland (Freyja Hreinsdóttir), Norway (Anders Sanne), Sweden (Thomas Lingefjärd), Denmark (Mette Andresen), Lithuania (Rokas Tamoiunas), Finland (Mikko Rahikka) and Estonia (Kristi Kreutzberg). Many of us are translators of the software GeoGebra to our own languages and we had all worked locally in giving GeoGebra courses to preservice and/or inservice mathematics teachers. The original aims of the network were:

- To create opportunities for mathematics teachers to learn from each other about the use of ICT/GeoGebra in the teaching of mathematics.
- To create possibilities for research collaboration for teachers and researchers, across the Nordic and Baltic countries, on the use of ICT in the teaching and learning of mathematics.

The first aim has much in common with the goals of the International GeoGebra Institute and of our local GeoGebra Institutes so we are extending the local activities to the Nordic/Baltic level. These countries have similar school systems and in some cases similar languages and there is a tradition of cooperation among them. The use of GeoGebra and ICT in general is however not necessary similar in those countries e.g. in Denmark GeoGebra is used quite a lot in primary and lower secondary school but in Norway it is used very much in secondary school. There is therefore knowledge to be shared. For the second aim we hoped, for instance, to create opportunities for researchers to disseminate their results to teachers and for teachers to learn from researchers and possibly participate in research projects. We also hoped for research projects to grow from ideas initiated at our conferences. In other words we wanted to bring those two groups, teachers and researchers, closer together.

The main activitiy of the network is organising one conference every year (80 – 100 participants) and one network meeting (10 – 20 participants). Talks and workshops at the conferences are given by mathematics teachers at all school levels. Anyone can submit an abstract for a talk or a workshop and usually the selection process is very inclusive in the sense that we welcome talks/workshops from almost any teacher willing to share their use of GeoGebra (or other ICT) in their teaching.

During the last 10 years new members have joined the network and our activities have changed based on knowledge that we have gained at our conferences and meetings. The network has the website http://nordic.geogebra.no/. It received grants from Nordplus for the years 2010 – 2013, 2014 – 2016, 2017, 2018 and 2019.

Professional development of teachers and communities of practice

Professsional development of teachers occurs in many different settings. Borko (2004) states:

> For teachers, learning occurs in many different aspects of practice, including their classrooms, their school communities, and professional development courses or workshops. It can occur in a brief hallway conversation with a colleague, or after school when counseling a troubled child. To understand teacher learning, we must study it within these multiple contexts, taking into account both the individual teacher-learners and the social systems in which they are participants. (p. 4)

The activities of NGGN do not directly fit into any of the above since they are organized over a long time and a teacher can partipate in many different ways. Some teachers are educators of others, a teacher can also attend a conference and just listen to talks and perhaps participate in a workshop to learn a specific technique or a teacher can choose to take a more active role by giving a workshop and teaching something to others. Some teachers only attend one conference but others come almost every year. In that sense it is more like loose-knit a community of practice as defined by Wenger although there have not been the same teachers participating every year (Wenger,1999).

In traditional professional developement courses the interest of participant tends to wary, in part based on their reasons for attending. Liljedahl (2018) gives 5 categories of wants (what the teachers wants from the professional development); Resistance, Do not disturb/Willing to Reorganize, Willing to Rethink, Out with the Old and Inquiry. He gives a Pseudo-Hierarchy of his taxonomy of wants and suggests that each category (apart from Resistance) requires as slightly greater openness to change than the previous one (p. 10). In his study the wants of the teachers tended to change in time towards more openness to change.

In NGGN’s activities there are no resistant participants but all other categories occur and can effect each other. Teachers with wants in the Do not disturb/Willing to Reorganize category are mainly interested in getting knowledge on material that they can use directly in their teaching such that it fits with their syllabus.

For a teacher to decide to give a talk or a workshop in English at a conference is a big step and sometimes they need some encouragement and support to do this the first time. However, this most often is a very rewarding experience and creates new contacts between teachers. Tseng and Kuo (2014) study knowledge sharing behavior of members in an online community of practice. Among their results is that performance expectation and self-efficacy belief play an essential role in knowledge sharing participation (p. 44). They find that knowledge giving is predicted by prosocial commitment but knowledge receiving is predicted by performance expectations.

Questions

The network was founded out of pure interest in the use of GeoGebra and ICT and was not as such considered a research activity on the behalf of the founding members. We wanted to promote good software use through collaboration of teachers in our countries. Since most of us do not have such developemental work as a part of our working duties this has largely been volunteer work.

Even though our goal was not to do research on the activities of the network we have continuously investigated the use and value of our activities and made changes to better achieve our goals, thus following a design based research model. Our research question may therefore be considered as:

- How to create opportunities for mathematics teachers to learn from each other about the use of ICT/GeoGebra in the teaching of mathematics?
- How to create possibilities for research collaboration for teachers and researchers, across the Nordic and Baltic countries, on the use of ICT in the teaching and learning of mathematics?

Among some related questions that have risen during the last 10 years, are:

- What do teachers learn from attending such a conference?
- How does the knowledge gained affect the teachers teaching?
- How does it affect a teacher to be chosen as speakers at an international conference?

Data collection

At each of our conferences the participants (80 – 100 people) have filled in surveys on different aspects of the program. They are specifically asked what they liked the most/least and what they would like to see more of in the future. In a more extensive survey at our conference in 2012 participants were asked questions on the benefit of attending such a conference (learning new material, sharing own material, making new contacts, etc), if they were interested in research collaboration and what we need more research on in connection with the use of ICT in the teaching of mathematics.

So far 9 teachers, who have all participated in many conferences have been interviewed. They were asked how useful the conferences were as well as the group meetings, if GeoGebra and participation in NGGN’s activities had change their teaching, if they thought that participating was useful for teachers in general, how they viewed collaboration with researchers and what kind of research is needed.

The author of this paper has been the chair of the Nordic GeoGebra Network from the beginning and has also been the main applicant in applications for support from Nordplus in the years 2010 – 2017. I have attended and been a member of the program committee of all our conferences and have attended almost every meeting. During this time I have had many informal discussions with teachers on the benefit of participating in this kind of activity.

NGGN’s first years 2010 – 2012, sharing and learning

During the first 3 years the network focused on organizing yearly conferences that were held in Iceland (2010), Lithuania (2011) and Estonia (2012). Our main effort was to encourage teachers to share their experiences with each other through workshops and talks they gave. Network members have been very involved in the promotion of GeoGebra and other ICT in their home countries and thus have contact with a large number of teachers using some software in their teaching of mathematics. Since we have been granted by Nordplus we have been able to support financially at least 2 teachers from each country. The Network members themselves also gave talks and workshops as needed.

At the conferences we had plenary speakers that were researchers and regular teachers from secondary and primary school as well as one plenary speaker from the GeoGebra Team. The conferences each had 80 – 100 participants. These participants filled out typical surveys concerning the program at the conferences and were generally pleased with their participation. Our own evaluation was that we were successful in fulfilling the first of our aims but not the second, mostly because we have not been successful in attracting researchers to our conferences.

Much has been written on the gap between researcher and teachers (Vanderlinde & Braak, 2010) so it was therefore not surprising that we were not successful with this. However, the network is still very interested in fulfilling this aim.

Survey in 2012

In an effort to evaluate the use of our activities and to help with the planning of future activities a thorough survey was given in 2012 at the NGGNs third conference. We asked the participants questions about the usefulness of such conferences, if they were interested in research collaboration and what we need more research on in connection with the use of ICT in the teaching of mathematics. The survey was answered by most participants so a total of 72 answers were acquired. Among the questions asked was one on the benefit of such conferences.

Table 1: Answers to a question on the benefit of such conferences

The benefit of such a conference for me is:	Number of participants crossing (out of 72)
Learning how to do things in GeoGebra	56
Learning what works in teaching mathematics and what does not work	47
Telling others how I use GeoGebra	26
Getting to know teachers in other countries with similar interests	42
It is inspiring in general	48

When answering the question “If you are a teacher: Has GeoGebra helped you to change your teaching during the last few years?” 61 crossed yes and 2 crossed no. When asked about their (teachers) interest in collaborating with researchers almost everyone responded positively. Only 5 researchers were present at the conference and they all clained to be interested in collaborating with teachers. There was an open question asking what we need research on and this was answered by many of the participants. The following issues emerged: classroom based “hands-on” research is needed, proof that using ICT is actually useful is needed and how to use ICT for the following: improving mathematics learning, for increasing students interest in mathematics, for changing teaching practices and for increasing collaboration among students.

The years 2013 – 2016, key-topic groups and experiments

In 2013 we enlarged the network to include one university teacher/researcher and one schoolteacher from each country. The goal of this was to establish better contact with both mathematics education research community as well as the teaching community in each of our countries. Two teachers from Latvia also joined us so now the network had members from all the Nordic and Baltic countries. We continued with the conferences which were held in Denmark (2013), Finland (2014), Sweden (2015) and Norway (2016).

In January 2013 this enlarged network of 16 persons met to discuss NGGN's future and based on that discussion and the survey in the fall of 2012 we decided to add some possibilities for organized experimental cooperation among teachers to the conferences in the hope that this would lead to joint research activities. Rather than deciding on specific material or teaching methods for teachers to experiment with we chose several key-topics that those interested would work with. This was implemented in such a way that there is a key-topic for each conference and some lectures related to that key-topic, 2 slots are put in the conference schedule to form a group (open to everyone interested) to work with the key-topic for one year, a meeting is organized between conferences for the group and the group gives a plenary lecture on its findings at the next conference. The people in the group decided themselves what they wanted to do but a network member chaired the group. The key topics chosen were: Learning mathematics through screencast technology and video (2013/2014), development and use of resources (2014/2015), teacher knowledge and classroom management in a technological environment (2015/2016) and assessment of mathematical competencies in a technological situation (2016). The choice of these topics was made after brain storming at our meeting.

Silent screencasts

The first key-topic group with 20 participants was formed at our conference in September 2013 in Copenhagen. It was open to everyone interested and a question posed to participants was "how can we use screencast technology in the teaching and learning of mathematics?". After some discussion and brain storming the group decided to form 4 subgroups (with 5-10 people each) related to the following: cultural differences (use screencast technology to do experiments in our countries in such a way that our different languages would not be a barrier), gifted children (how to use screencast technology to create meaningful tasks for gifted children), complete learning modules (made by using screencast technology) and a technical group who would assist the other groups. Each subgroup created a plan for the next months, until a meeting was held in Iceland in March 2014.

In spite of the original enthusiasm not much was done in the coming months except for the work of the cultural group. The original plan to have regular online meetings for each subgroup did not really work since the participants were busy with their teaching and perhaps a bit unsure of the purpose of this work. In the cultural group the idea eventually developed into making short, silent screencasts displaying some mathematical phenomena in GeoGebra and test this in our countries. After some (online) discussion on how to use silent screencasts and how to evaluate the benefit of their use the idea developed into the following: simple, short screencasts showing some mathematical phenomena were given to students in different countries and the students then recorded a voice-over, describing in their own words what was going on in the screencast.

Many of the people who joined the key-topic group at the conference dropped out but others replaced them since participants in each country managed to recruit many others who were interested in this particular topic. A total of approximately 200 groups/pairs of students participated in the project through their teachers in Estonia, Iceland, Latvia, Lithuania and Sweden (Hreinsdóttir & Kristinsdóttir, 2016). The group gave a plenary lecture on its finding at the V Nordic GeoGebra Conference in Finland. http://hylblog.edu.hel.fi/wpmu/ggylojarvi14/programme/

In general the teachers who tried this out noticed that they were able to gain insight into the students' way of thinking that they would not have gained otherwise. They also experienced that working on this task stimulated discussions on mathematics among the students. One of these teachers became very interested in exploring this further and is currently a Ph.d. student who is doing research on teachers' work when using this technique (Kristinsdóttir, Hreinsdóttir & Lavicza, 2018).

Another teacher in Iceland has used this for formative assessment (see Hreinsdóttir & Kristinsdóttir, 2016) and I use this in a course for preservice teachers as a first step in making a screencast explaining some mathematical phenomena. My students get a 2 min. long screencast on the area of a triangle and they do a voice-over explaning what is happening in the screencast.

From 2017

The network has continued to work in a similar manner. Our conferences have continued to attract 80 – 100 participants but unfortunately our work with other key-topics have not been as successful as the work on the screencasts which actually became an inspiration to a research project and created a new tool to use in mathematics teaching.

It is however very important to us to continue our work at the grass root level so teachers at all school levels participate in the decisions on the main focus of the network.

Surveys and interviews

As mentioned earlier, each regular conference partipant has filled in a tradional conference survey asking what is the most useful item on the program, what they would like to see more of and what they would like to see less of etc. In general, what they like the most is the talk given by someone from the International GeoGebra Institute. These talks usually describe new features of the GeoGebra software and are more technical than pedagogical. Workshops that teach how to do something in particular are usually popular but activites such as the key-topic groups have not been rated very highly.

Interviews with 9 teachers (mostly from upper secondary school and teacher education) who have participated in many NGGN's activities and given talks/workshops themselves, indicate that they find the conferences very useful since they have connected with teachers from other Nordic and Baltic countries and been able to exchange ideas on the use of ICT in the teaching of mathematics and learn about each others curricula, assessment and didactical approaches. In their first years of participation they learned a lot on they use of GeoGebra but gradually, as they became experts on GeoGebra, the learning was more on didactics. They find it useful to give talks on their work since it helps them structure their thoughts, they get feedbacks and suggestions from others and they generally feel good about doing something useful for other teachers. By giving a talk or a workshop at an international conference they in some cases create a name for themselves and are asked to give talks in their own country.

There were mixed feelings on the work of the key-topic groups, some found them to be very inspiring, they had done experiments connected with the key-topic and been very happy to have a forum or a group where they can discuss the outcomes with other teachers doing similar experiments. Others mentioned that the organization and work was not focused enough.

The interviewed teachers were asked to give their view on research collaboration, how we should try to establish that, if it was important and what we need more research on. Most of them think that this has not worked since the network has not really been able to attract researchers to come to our conferences. A general view was that the research needs should be classroom oriented.

The teachers who have been interviewed have all attended many conferences and it can therefore be assumed that they find them useful. To get a more complete picture it is necessary to interview newcomers as well since they might give a different view.

Discussion

The work of NGGN is not strictly research oriented but we try to develope our work to be more successful in fulfilling our aims.

Some of the teachers come to our conferences primarily looking for knowhow, i.e. there is something in their syllabus that they need GeoGebra for and they want to know how to do it. These teachers wants could be considered in Liljedahl's categorizations as Do not disturb/Willing to Reorganize. These teachers' wants sometimes change over time and we have seen example of teachers developing from being relatively inactive receivers to perhaps making a poster on their teaching at the next conference and eventually giving a talk or a workshop.

Some teachers like to attend and give talks/workshops since they like to tell others about their work and possibly get some discussion around that. In the survey in 2012 there were 26 out of 80 who considered "Telling others how I use GeoGebra" as a benefit. This was almost everyone who gave a talk or a workshop at that conference. One reason for this might be their prosocial commitment (Tseng & Kuo, 2014) i.e. these are people who have a genuine interest in helping others but also interest in having their work acknowledged by others and discussed by others. These teachers' wants would fit into Liljedahl's category "Out with the Old and Inquiry". Interviews indicate that giving a talk/workshop is seen by these teachers as a possibility to make contacts with others and attending such a conference is inspiring and gives them new ideas to use in their teaching. They also value getting to know teachers in other countries with similar interests. This is similar to results in the 2012 survey where "Getting to know others with similar interests" was chosen by 42 participants of 72 and "It is inspiring in general" was crossed by 48 participants.

We have generally not been very successful in creating opportunities for research collaboration. An exception to this has been the work on silent screencasts, this was an idea originating in a key-topic group, then an experiment done by several teachers in the Nordic and Baltic which later developed into a research project. In our other key-topic groups it was hard to create as much interest and commitment in testing things out. It seems that a more efficient way would be to try to recruit researchers who would be interested in attending our conferences and meeting teachers to work with.

Last but not least it is interesting that in the survey in 2012, many participants (47 of 72) considered it a benefit to "Learning what works in teaching mathematics and what does not work". Even though the focus of the conferences has been the use of GeoGebra and ICT in general the mathematics teaching itself is very important.

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DEVELOPING QUESTIONS AND PROMPTS: ENGLISH PRIMARY TEACHERS' LEARNING ABOUT VARIATION THROUGH LESSON STUDY

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This paper reports doctoral work that seeks to reveal the professional learning of 12 primary teachers in England working collaboratively when developing their practices within a school-based professional development initiative. The common goal was to develop questions and prompts associated with promoting learning from variation during three iterative lesson studies. These cycles were observed through the lens of quarternary analysis to capture individuals' accounts of practice. Thematic analysis of the accounts of teachers' practices revealed four patterns of questions and prompts for promoting learning from variation: describing relationships, comparing, explaining and noticing. The findings reveal how the community's awareness of questions and prompts for pupils to generalize increased while awareness of questions and prompts that promoted 'explanation' reduced. The community's awareness of describing relationships and comparing, on aggregate, remained similar across all three lesson studies but in different proportions.

Introduction

This paper responds to the invitation to contribute to the ICMI Study 25 theme B. It provides a context for teachers working in collaboration and a methodological approach to understand teacher learning in a community of inquiry (Jaworski, 2006). The data resulted from the first author's doctoral work (who is also a primary teacher and primary teacher educator), that seeks to reveal primary teachers' professional learning in England as they develop a pedagogical approach for teaching mathematics in England that is associated with centrally-driven pedagogical reform (Department for Education, 2016). Using a modified form of Chinese lesson study (Yang & Ricks, 2012a), the common goal was to develop teachers' use of questions and prompts with their pupils to promote learning from variation (F. Gu et al., 2017). This paper reports the outcome of the first application of this methodology.

We offer a theoretical overview for how professional learning is conceptualised in the context of this study as well as an observational framework to collect teachers' accounts of other teachers' practices (Simon & Tzur, 1999). It then presents findings on how the community's awareness of questions and prompts that promote pupils to learn from variation changed over the period of three lesson studies. Finally, other possibilities to learn from the outcomes of this trial are discussed, alongside conclusions on how the study might contribute to knowledge on capturing outcomes of teachers' collaborative work.

Theoretical Overview

This section offers a theoretical perspective for teacher professional learning with respect to a particular pedagogical approach for teaching mathematics and how a collaborative community might serve as a catalyst for this professional learning to take place.

Professional learning of mathematics teachers

The notion of teacher professional learning remains a nebulous concept where little agreement within the field resides (Lerman, 2013). Some studies have situated professional learning in terms of identity (Hodgen & Askew, 2011; Potari, 2013) and others in terms of participation in a social setting (Lave & Wenger, 1991); (Skott, 2013). Using Clarke and Hollingsworth's (2002) interconnected model for professional growth as their conceptual framework, Goldsmith, Doerr, and Lewis (2014) conducted a review of literature of 106 studies over the period 1985-2008 that related to mathematics teachers' learning. They identified 10 characteristics of professional learning, one of which was changes in teachers' instructional practice. Simon and Tzur (1999) define teachers' practice as,

> "not only everything teachers do that contributes to their teaching (planning, assessing, interacting with students) but also everything teachers think about, know, and believe about what they do. In addition, teachers' intuitions, skills, values, and feelings about what they do are part of their practice. Thus, we see a teacher's practice as a conglomerate that cannot be understood by looking at parts split off from the whole" (pp. 253-254)

This definition is used in Simon & Tzur's (1999) *accounts of teachers' practice* ethnographic methodology to illuminate professional learning where teachers' practices are analysed from the perspective of the researcher. Simon and Tzur (1999) argue that such an approach offers an alternative lens for research on teacher learning in contrast to evidence that reports teachers' beliefs about their own practices. These accounts can shed light on the nature of pedagogical problems or critical incidents (Lerman, 2013) encountered by teachers. However, Simon & Tzur (1999) only report on individual teachers. They do not comment on whether their sample worked collaboratively with other teachers during their period of the study. However, the *accounts of teachers' practice* methodology (Simon & Tzur, 1999) may also be useful to consider in the context of communities of inquiry. In which case, teachers' accounts of other teachers' practices could be interpreted by the researcher to determine professional learning for that community of teachers. The next section sets out how this might be achieved using Chinese lesson study as a context for professional learning.

Teachers working as a community of inquiry in adapted Chinese lesson study

Chinese lesson study (CLS) is an iterative form of lesson study that is widely used in the collaborative work of teacher research groups (TRGs) in China to design exemplary lessons. Teachers in the group identify a common goal, usually a mathematical topic for a particular year group of pupils and collectively design a lesson which is then taught by one of the teachers in the group and watched by the others. Inter-cycle discussions between the teacher and the observers, which take place immediately after the lesson, contribute to a second iteration of the lesson plan. This lesson is then taught by another member of the group, usually a couple of weeks after the first iteration. In CLS, planning and observation of the lessons are informed by *quarternary analysis* (Yang & Ricks, 2012b). There are four features of quarternary analysis: the *key point*: identifying what is to be learned; the *difficult point*: potential misconceptions; the *critical point:* the means by which the learning of the key point is accomplished; and *learning effect*: the observer's interpretation of the pupils' understanding as a result of experiencing the lesson. Hence, this lens provides a fine-grained framework with which to capture the teachers' and researcher's *accounts of teachers' practice*.

Variation Pedagogy

Variation pedagogy is concerned with how teachers bring into focus for their learners, what is to be learned, by helping them to notice what is varied against a background of invariance (Marton & Booth, 2013) or by experiencing invariance against a background of variation (L. Gu, Huang, & Marton, 2004). Such a focus on variation has been observed as an indigenous practice (Sun, 2011) in mathematics teaching in Shanghai, known as *Bianshi* (F. Gu et al., 2017) or 'teaching with variation'. One form of Bianshi is *procedural* variation (L. Gu et al., 2004; Sun, 2013) which seeks to promote pupils' deductions of generalisations from sequences of calculations where one feature is varied at a time. E.g., by working on the calculations in the sequence: 16 + 9 = ( ); 17 + 9 = ( ); 18 + 9 = ( ) where 9 is kept invariant, the students should experience that the value of the digit in the 'ones' position of the sum of the two numbers is always 1 less than the value of the digit in the ones position of the first addend. It can be deduced that the invariant feature, 9, can be the cause of this phenomena. This type of carefully considered sequence for the purpose of deductive reasoning and generalization is not commonly used by primary teachers in England but features as part of a wider pedagogical reform happening there at the time of writing (Department for Education, 2016). The relevance of its use in England is discussed in an earlier paper (Jacques, 2018). Little, however, is reported in the literature about the explicit role the teacher plays in supporting pupils to learn from such sequences, particularly in classrooms where variation pedagogy is not an indigenous practice.

Methodology

A group of 12 primary (Year 1-6) teachers (11 teachers and 1 researcher/teacher - first author) chose to participate in a professional development (PD) initiative (workgroup) that was advertised by a regional network for mathematics PD. The advert made explicit the common goal of the workgroup: developing classroom discussion to promote learning from variation. The intention was to develop teachers' skills to orchestrate pupil/teacher discussion about the variation sequences. These skills include using questions and prompts for pupils when pupils are working on variation tasks. The teachers did not know one another prior to involvement in the programme and none had previously used procedural variation in their teaching.

A modified CLS was used by designing a lesson 'extract' that would take approximately 20 mins to teach, this was called a *learning episode*. Three iterations of the learning episode took place in one gathering. A different teacher hosted each lesson study at his/her school. The host teacher identified an area of arithmetic that his/her pupils were working on and prior to the first iterative cycle, presented some examples of pupils' work to help the group to decide on the key point, difficult and critical point (Yang & Ricks, 2012b) that would then influence the decision about the design of sequence of examples (the task). Each iterative cycle took approximately 50 minutes (planning, teaching, reflecting, redesigning) and each involved three different groups of six to eight children from the host teacher's class.

The researcher taught cycle 1, another teacher from the group taught cycle 2 and the host teacher taught cycle 3. The observing group of teachers captured accounts of practice (Simon & Tzur, 1999) using a common proforma based on the quaternary analysis framework (Yang & Ricks, 2012) referred to as a 'learning episode notes sheet' or LENS (Table 1). After each learning episode the teachers shared and compared their accounts of practice with one another and used this to redesign the sequence and teaching. The researcher acted as facilitator and contributor to the discussion each time and also completed a LENS.

Table 1: Transcript from one teacher's LENS

Task	Examples from accounts of practice	
12 x 9 □12 x 8 13 x 8 □12 x 8 96 ÷ 6 □ 96 ÷ 8 96 ÷ 12 □96 ÷ 26	What pupils say/ do	What teacher says/ does
Key Point *Compare by thinking about the number of groups.*		What do you notice? Discuss what you can see
Difficult Point *Pupils wanting to perform the calculation*	Working out each calculation.	What do you notice? What's different? Any patterns? Arrays.
Critical Point *Representing relationship between two multiplication or two division calculations.*	Puts them into a word problem. I know that 12 x8 is smaller than 12 x 9.	Does is matter what the answer was? What is the same? It's one extra lot of 8.

Data Analysis & Results

An overview of the available data for analysis is described in Table 2. Not all teachers were present for all lesson studies.

Table 2: Attendance and data sources for each lesson study.

Lesson Study	Lesson Study 1 (LS1)				Lesson Study 2 (LS2)			Lesson Study 3 (LS3)		
Cycle Number	Cycle 1 LS1C 1	Cycle 2 LS1C 2	Cycle 3 LS1C 3	Cycle 4 LS1C 4	Cycle 1 LS2C 1	Cycle 2 LS2C 2	Cycle 3 LS2C 3	Cycle 1 LS2C 1	Cycle 2 LS2C 2	Cycle 3 LS2C 3
Participants Present	9	10	10	10	8	8	8	7	7	7
No. of LENSs	39				24			21		

These data was interrogated using a mix of thematic analysis (Braun & Clarke, 2006) and frequency analysis. The objective was to identify patterns of teachers' awareness of the questions and prompts used when promoting learning from variation during the lesson studies.

Thematic Analysis

The set of 39 LENSs (see table 1) were interrogated by hand. Notes were made of any similarities between what had been recorded by the teachers. After all of the LENSs had been read through once, this process was repeated to check the emerging patterns in teachers' accounts. Repeating this process meant that similarities and differences that were noticed later on could become the focus of attention in the LENSs that had been looked at prior to when those patterns emerged. After two rounds of the familiarisation exercise, possible codes for what the teachers had noticed were defined.

Six codes categorised the phrases the observing teachers had noticed being used in the lesson studies. The term 'phrases' includes comments, prompts or questions. This process of codifying was based on the first author's interpretation and classified according to the *purpose* of the phrases (i.e. what the teacher might have been drawing from the children. Each documented phrase was assigned uniquely to one code for the purpose of this analysis. Table 3 provides some examples of the kinds of recorded phrases documented by the observing teachers and their classification. The first four categories, describing relationship, comparing, explaining and generalising were grouped into a single theme: *navigational questions and prompts for promoting learning from variation*.

Table 3: Sample of phrases illustrating the codes

Code	Example
Describing Relationship	Are there any other links? How did this answer help you to find this answer? What happens to the answer when…?
Comparing	What's the same about…? What's different…?
Explaining	What did you notice? Can you explain…?
Generalising	Can you see a pattern? Is there another similar one you could create?
Meta-cognition	Can anyone help him? How did that help you?
Other	What does this symbol "<" mean? Can you speak a little louder?

Frequency Analysis

Lerman (2013) claims that teacher learning may occur when they experience a disequilibrium and is most likely to be evident when teachers interact or reflect on critical incidents leading to a new experience which must then connect with the the teachers existing network of knowledge, a theoretical construct Simon (2013) defines as "major assimilatory structures". He claims that these structures are difficult to change because the network is complex. Yang and Ricks' (2012) quaternary analysis illuminates disequilibrium by assuming that what the observing teachers choose to record in their accounts of practice during the learning episode, are moments of disequilibrium experienced by the observers themselves or by the teachers and pupils they are observing. However, a single experienced moment of disequilibrium may not in itself be an indicator that learning has happened. A chronological sequence of the disequilibriums experienced over the course of the PD programme is assumed to constitute a learning journey for the community of teachers.

The thematic analysis exercise revealed that some codes were accounted for by teachers more frequently than others. Therefore, the next step was to interrogate the data for the frequencies of each of the types of navigational questions and prompts associated with promoting learning from variation for each lesson study (Table 4). These aggregated data provide an account of the community of teachers' patterns of awareness of question types unique to navigating variation problems as well as other questions and prompts (e.g. meta-cognition and other).

Table 4: Frequency Distribution of question/prompt type in each lesson study (LS)

Theme	Navigational questions and prompts for PLV						
Code	Describing Relationship	Comparing	Explaining	Generalising	Meta-cognition	Other	Total phrases noticed
LS1	53	38	95	9	3	66	264
LS2	35	16	50	5	7	30	143
LS3	12	28	17	10	7	31	105

Discussion

The aggregated data for each lesson study revealed the proportion of different questions and prompts that the teachers were aware of over the course of the three lesson studies each with a different mathematical focus (Figure 1).

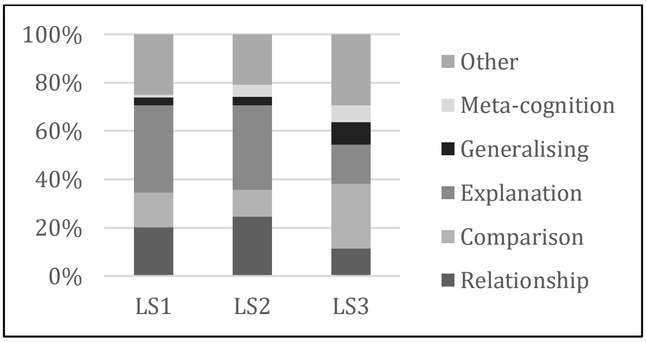

Figure 1: Proportion of question/ prompt type per lesson study

The accounts of practices generated similar profiles in the first two lesson studies. In contrast, in the third lesson study some examples of potential changes in professional learning within the community could be interpreted. The goal of promoting learning from variation is to recognise generalised features of an object of learning. The data shows that in LS1 teachers were most aware of questions and prompts to encourage the pupils to *explain* whereas they were least aware of questions and prompts associated with encouraging pupils to *generalise*. Although generalising was the category with the smallest proportion in each lesson study, by the third, the teachers were recording more instances of questions and prompts to promote generalisations and noting fewer instances of questions and prompts to promote pupils' explanations. One possibility is that the community's inter-cycle evaluation and redesign discussions drew out the need for the teachers to use more questions and prompts to promote pupils to generalise, therefore the community became sensitised to its prominence. An alternative suggestion is that the teachers used a greater number of phrases associated with promoting generalisation over the course of the lesson studies and so increased the occurrence of these phrases. Another explanation might be a combination of the two, where the discussions not only inspired the teachers to increase the use of these types of questions and prompts but also increased awareness of them at the same time.

Similarly, the decrease in the use of questions and prompts associated with promoting pupils to use explanations might be explained in multiple ways. For instance, the high frequency in LS1 may have suggested that teachers were most sensitised to this category when observing teaching (from their existing construct of teaching). Over time the inter-cycle discussions may have shifted the community's attention to other purposes of questions and prompts. Alternatively, it may have been that the teachers used fewer questions and prompts to promote explanation in favour of other types associated with promoting learning from variation.

Before it is possible to generalize, relationships between features of a mathematical object must first be noticed. When using a task designed with procedural variation, this can be accomplished by prompting pupils to make comparisons between deliberately varied examples. Together, the two categories of describing relationships and comparing were noticed in similar proportion across all three lesson studies. Awareness of phrases associated with promoting pupils to describe relationships was highest in LS2 and lowest in the LS3 whereas the phrases associated with comparing were lowest in LS2 but highest in LS3. A transcript of the lesson might reveal possible explanations such as the content of what was being taught required the teachers to spend more time prompting pupils to compare examples in LS3 than in LS2.

Conclusion

This analysis has revealed changes in teachers' awareness of questions and prompts associated with promoting learning from variation when participating in three iterative lesson studies where the focus was on developing the use of procedural variation tasks. The teachers' accounts of teachers' practices recorded an increase in the use of questions and prompts that require pupils to make generalisations. This innovative methodology has also revealed additional data that would enhance the reliability and validity of the study which may help to understand how the collaborative work of the teachers contributed to these changes. In the next stage of the research methodology, video recordings of the lesson studies and inter-cycle discussions will be used to enable further triangulation between accounts of practice, revealing more of the actual work of the community, and supporting the researcher's interpretations and analysis.

There are also further grain-sizes of data to interrogate. For example, by using quaternary analysis, it may be possible to notice patterns in the use of the questions and prompts for the key point, difficult point and critical point. Similarly any changes noted within the community of inquiry as a whole do not take account of the contributions that individual teachers make and how that may skew the outcomes of the work of the community of inquiry. Therefore, the data should also provide a window on individual teachers' awareness of questions and prompts associated with promoting learning from variation.

The analysis was guided by accounts of practices, but not only from the researcher's perspective as suggested by Simon and Tzur (1999) but also from the collective perspectives of the teachers in a community of inquiry (Jaworski, 2006). The researcher's LENS was treated equally alongside the teachers. By collating and interpreting these accounts, it is possible to interpret outcomes of the work of the community. It may also be useful to include pupils' accounts of practices as an additional lens, whereby the community of teachers are then able to take into account their interpretations of the pupils' experiences in the lesson study cycle.

This analysis has responded to the invitation to contribute to the ICMI Study 25 theme B. It has illuminated outcomes from a context of mathematics teacher collaboration which in turn has revealed further lines of enquiry for this study and other studies wishing to reveal professional learning of mathematics teachers working collaboratively.

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MATHEMATICS TEACHING PROJECTS: AN IME-USP'S COLLABORATIVE EXPERIENCE IN PRE AND IN-SERVICE TEACHER EDUCATION

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In the Institute of Mathematics and Statistics of the University of São Paulo (IME-USP, Brazil) has been created a Program that promotes the articulation of the curricular internship supervised of its undergraduate students with the in-service training of São Paulo public's schools teachers through the formation of collaborative groups of students and teachers guided by university professors. This Program is active since 2009 within the scope of the pre-service teacher's education in mathematics for elementary and high school. In this article we describe the context that gave rise to such a Program and its system of operation over time. Finally, it is analyzed some of the results obtained regarding the quality of learning developed by students and teachers participating in the Program with respect to theoretical and practical knowledge for the exercise of the teaching-math profession.

Introduction

This article aims to present the main thoughts on the foundations and main results of collaborative work among researchers, teachers, educators and students within the scope of a discipline of the IME-USP's pre-service teacher education course, developed in conjunction with a in-service teacher training course. This interaction was established in order to organically integrate the compulsory curricular internship of the students with the professional practice of public school's teachers. It is a project that emphasizes the collaboration between the different actors mentioned, through activities that involve conception, implementation and discussion of classroom practices, in the search for the construction, by students, of knowledge for teaching, as well as the improvement of in-service teachers' professional development.

Project Context

In 1988, Brazil's new Constitution was promulgated, which reestablished the democratic state in the country. After a long discussion between associations and professionals related to Basic Education (elementary and high schools education), a new Law of Guidelines and Bases of National Education was approved which establishes, in its articles 61 and 63:

> Article 61. The pre-service teacher education, in order to meet the specificities of the exercise of their activities, as well as the objectives of the different stages and modalities of basic education, will be based on: [...] II – the association between theories and practices, through supervised internships and in-service training;
>
> Article 63. Higher institutes of education will maintain [...] III – continuing education programs for education professionals at various levels. (LDB n. 9394, of December 20, 1996, http://www.planalto.gov.br/ccivil_03/Leis/L9394.htm)

Following this, the National Council of Education, in its Resolution 2/2002 (CNE-CB 2/2002), established National Curriculum Guidelines for Initial and Continuing Education at the Higher Level of Teaching Professionals for Basic Education. The fifth paragraph of this resolution sets out eleven principles to be followed in the training of professionals in the teaching of basic education, of which we highlight the following four:

> V – the articulation between theory and practice in the process of teacher education, founded on the domain of scientific and didactics knowledge, contemplating the inseparability between teaching, research and extension;
>
> VI – the recognition of basic education institutions as necessary spaces for the training of teaching professionals; […]
>
> IX – the articulation between initial and continuing formation, as well as between the different levels and modalities of education. (CNE-CB 2/2002 http://portal.mec.gov.br/docman/agosto-2017-pdf/70431-res-cne-cp-002-03072015-pdf/file)

From this new federal legislation, recognizing the importance of the movement of renewal of Basic Education and Teacher Education triggered by LDB/1996, the University of São Paulo discussed structural changes in its teacher education undergraduate courses – in Brazil, called *Licenciaturas* – to adapt them to the new guidelines. From this discussion, in 2004 the University Council approved the USP Teacher's Training Program (PFPUSP), establishing principles, objectives and common curriculum organization, as guiding axes for the restructuring of all USP's *Licenciaturas*. We highlight here part of the fourth principle:

> 4. The training project must foresee the inseparability between teaching, research and extension, so as to guarantee the quality of initial education, introducing the graduates in the investigative processes in their specific area and in the teaching practice, making them professionals capable of promoting their continuous formation.
>
> The pre-service teacher education will constitute a stage of professional formation, the basis of the permanent and necessary process of continuous education, in which the teacher will stay diagnosing and proposing appropriate alternatives to the challenges of his professional action, as well as participating in University extension projects.
>
> The relations between teaching, research and extension require articulation between theories, practices, disciplines and intervention projects, in view of the understanding that the educational and school reality is not a specific subject of a discipline, but allows a plurality of approaches.

Particularly in relation to supervised curricular internship, which is a compulsory component of undergraduate courses, PFPUSP has established that:

> The "supervised internships" will preferably be done in schools and institutions previously determined and linked to a work project prepared by a team of teachers involved with *Licenciatura*'s courses.
>
> From this perspective, the "supervised internship" should play an integrative role in teacher education, offering the undergraduate student opportunities to expand and employ the skills and knowledge acquired in the course to respond to the needs and challenges of the school reality. [...]
>
> The planned activities are: • participation of USP students in pedagogical activities [...], in order to ensure the integration of the pedagogical project of undergraduate programs and the pedagogical proposals of schools; • participation of field school teachers in internship organization projects, research projects, disciplines involving teachers' formation and extension courses offered by USP. (PFPUSP: http://www4.fe.usp.br/wp-content/uploads/professor-training-program/professor-training-program.pdf)

Thus, in a comprehensive synthesis, it can be stated that the PFPUSP has as its main objective the promotion of a pre-service teacher education that takes into account the realities of public schools in the sense that the internship activities contemplate a strong articulation with the continuing education of in-service teachers.

In 2004, the Institute of Mathematics and Statistics (IME-USP) began to discuss a new curriculum structure for its Teacher Training course, appropriate to the new standards, to be initiated in 2006. The new curriculum included, among other changes, the existence of an IME-USP's Supervised Internship Program, embodied in the creation of a mandatory annual undergraduate discipline – "Internship Projects" – and the creation of an annual university extension course for public school teachers, called "Internship Projects: Learning Mathematics with Projects". Both courses are offered to different public, but happen simultaneously, under the responsibility of a professor from the IME-USP's Department of Mathematics, with the support of educators linked to the Program. The objective of the Program is to promote activities of IME-USP students' with elementary and high school Education's teachers, aiming to promote the joint elaboration of projects or didactics sequences to be developed by student-trainees in classroom's teachers, under their supervision. In fact, the program scope of both, undergraduate and extension courses, is to develop the studies of pedagogical foundations of learning through projects and foment a collaborative discussion, in mixed groups composed by students and school teachers, in order to elaborate projects to be applied in the classroom of the teachers. At the end, the direct actors also make the assessment of these activities in conjoint discussion of all collaborative groups together with the educator and the professor.

To enable the implementation of the Program, we managed to establish a partnership with several public schools that made a commitment to welcome our students as interns and facilitate the participation of teachers interested in the extension course offered to teachers in service. This process began in 2009. This collaborative strategy between teachers, educators, students and in-service teachers establishes an organic articulation – which has been proving very productive ever since – between pre-service and in-service teacher education.

Collaboration Development

The collaborative work, which will be called here *team*, formed by researchers (university professors), public school teachers (internship supervisors), undergraduate students (interns) and graduate students (educators), lasts nine months and foresees at least eighteen hundred-minute face-to-face meetings with the team.

At the first meeting the professor presents the program and at the next the public school teachers introduce themselves, as well as expose their schedules and expectations regarding the project to be developed. After this the undergraduate students choose the teacher(s) with whom they will train during the year.

From the second month on students visit the schools where they will intern, sometimes accompanied by the educator, but always under the supervision of their internship supervisor's teachers. These visits allow them to observe the school environment, the teacher's practice, the classroom dynamics, the students' behavior, and are made with the main objective of promoting the integration of the trainee in the school, with their insertion in the classes in which they will work. At this point of the course meetings of the groups with an educator begin to take place to address specifically the peculiarities of each project. Meanwhile, during the meetings at the University and guided by the responsible professor, the team discusses articles or chapters of books on teaching projects and about issues and problems of mathematics didactics, as learning assessment and diagnostic activities, among others.

Around the fifth and sixth meeting, all the team present and discuss the diagnostic activities prepared and applied by each group of students in their Basic Education class to better guide the construction and progress of the project they are developing. At the end of the first semester, the pre-projects of each group are presented to and discussed in the collective of students, teachers, educators and professor. At this moment, it is evident not only the requirement for specialized knowledge, as defined in Ball, Thames, and Phelps (2008), but also the importance of practice in training of future mathematics teachers. Such evidences appear during the participation of the whole team in the discussion, since they promote the groups' reflection on various aspects ranging from pedagogical or specific training to professional practice and leading to the improvement of the project they are constructing.

At the last meeting of the first semester, in June, after the project adjustments, the groups deliver a complete version of the teaching projects, which is commented by the professor together with the educators, in order to support the improvement of the proposals to be implemented in the school classroom in second semester, generally about August.

In the second half of the scholar year, the projects are already more “mature” and are carried out in schools by the groups, accompanied, in part by the activities, by the educator in order to verify the progress and implementation conditions of the planned activities. Meanwhile, in the August's meetings at the University are held workshops on project issues, especially on discussing activities involving specialized knowledge for elementary math teaching. Presentations about the progress of projects at the schools happen during September and October.

In November, with the projects mostly completed, the groups present and deliver a final report which must contain a detailed description of the activities developed, and reflections on: the collaborative work carried out; the experiences lived in group work; the internship performed and its supervision; the goals achieved or not; the reality witnessed in schools; and about the changes or improvement that the collaborative experience had brought on their formation and perspectives of career. Once again, in the final reports, students and teachers should interact in order to emphasize how much collaborative educational work had promote specific theoretical knowledge of: mathematics; pedagogical approaches; specialized knowledge; professional practice; supervised internship developed during the year; and how important they had been for their initial or continuing teacher education, as suggested by the multidimensional model of Ball et al (2008, p.403).

In the last meeting of the year, the team promotes an evaluation discussion of the collaborative work done during the scholar year, with the intention to improve future offerings of discipline and training course as well as to highlight its importance in the education of undergraduates and teachers.

In an attempt to analyze the contributions of this training program to students and teachers, the study used questionnaires, informal interviews and written productions to collect data – specially the projects and their respective final reports from some groups of classes under responsibility of the authors from 2013 to 2019. Some elements of this analysis are presented below.

Some Results

According to Zeichner (2010), research has clearly shown that field experiences are important occasions for teacher learning to take place, rather than merely when future teachers can demonstrate or apply knowledge previously learned at the University. It further states that in the USA the problem of the lack of connection between teacher education courses at the University and the field of practice in schools is perennial. We can say that in Brazil is no different and, as already mentioned, a guiding aspect of the proposal of MAT1500 aimed to tackle this problem, betting on collaboration between students and teachers, supported by educators and university trainers. Our propose is to seek together ideas and strategies aimed to impulse a rich and more productive environment for everyone's learning. In particular, prioritizing the approach of students to the school reality that, according to Pimenta and Lima (2006, p.14), "only makes sense when there is connotation of intentional involvement, because most bureaucratized stages, loaded with index cards observation, is in a near-sighted approach to reality". Moreover this is expressed in the statements of students from different classes of the discipline when characterizing their stages, for example: "It's a real experience with students in a project that runs over a period of several months" (2014 Class Student); "Unlike the other subjects that contain internship, MAT1500 made us actually experience school. It brought us the idea of project, pertinent discussions about the school and gave us the feeling of being a teacher" (2013 Class Student).

We also mention other aspects that are associated with the theoretical-methodological approach of the discipline, such as curricular component practice and project work.

Some evidence of this necessary articulation appears in the interactions between teachers and students discussing the production of groups in the elaboration: the group proposes to structure its project involving the theme of quantities and measures (length, mass, capacity and time), in the form of workshops and making use of narratives as a problematizing element and elaboration of portfolios as an evaluation instrument, including self-evaluation activities. Quoting a 2017's students group:

> In general, math classes follow a pattern of content exposure and exercise resolution. In particular, for the chosen theme, we want to explore other didactic possibilities, considering that it is of utmost importance to vary the approach of content, aiming a more actively students learning. […] They [children] have to experience comparing, measuring, using non-standard units, and solving problems, so that these ideas related to measurements can be effectively assimilated. Therefore, we will prioritize activities that enable this experimentation. […] We also want the students to evaluate themselves, and at the end of the activities that each one is able to write down or draw on a sheet of paper what they learned, what they liked best, what they didn't like, what attracted the most attention (Group 1, Class 2017).

In this context, the tutor teacher intervenes and makes his reflection explicit from the students' proposal:

> From the diagnostic activity, I realized that the interns have very well captured the class profile and that this experimental component is essential. On the other hand, I have doubts about self-assessment activities for children in this age group, due to their immaturity. I never used it systematically. (Tutor Group 1, Class 2017)

Given this scenario, a fruitful interaction was developed for the discussion of these ideas, how is highlighted by Boavida and Ponte (2009, p.3): "Bringing together diverse people with diverse experiences, skills and perspectives, more resources are gathered to successfully complete a certain amount of work, thereby adding security to change and initiate innovation".

In another project, on the subject *Geometry and Art*, which aimed to work on geometric concepts, associated with patterns in indigenous and African art, the teacher made the following synthesis:

> Participation in MAT1500 allowed me to put into practice something that I wanted at the beginning of the year. Combining the teaching of Mathematics with Art, two languages that express a lot and that are powerful translations of the look we have for ourselves as historical beings and linked by symbols. Perhaps without the support of the students, I don't know if I could have done it. […] It was very rich to be together in the whole process and feel free to give your opinion and listen to the children. (Tutor of Group 2, Class 2017)

In her considerations after participating in the course, this teacher states:

> The key words to take in account in the teacher's school practice are: 1) Paulo Freire, in exchanges and in the creation of living situations; 2) collaborative process, in the planning and management of workshops; 3) Attentive ears of students for the theme and for the children. I learned a lot and convinced me that I taught them too! (Tutor of Group 2, Class 2017)

Regarding aspects related to collaboration between the actors involved, it should be noted that researchers seek to help establish trusting relationships among the various team members, valuing the environments for collaboration and ensuring that students do not remain solely in a cooperative posture, as observed in some situations of group academic work.

The methodology used in the educational process involved a systematic articulation between mathematical content and pedagogical strategies, through the sharing of experiences in the two contexts – the University (with the working groups in the face-to-face meetings under the supervising of the professor and also in the guiding meetings on the project's development of the educators with the groups) and that of schools. These experiences contributed to the students being able to play new roles: the teacher – the main objective of the internship – and, occasionally, been himself a trainer which helps the school teacher in deepening questions related to mathematical content. This was particularly true for non-specialist teachers of elementary schools. So the teachers of the schools acted as coach, but eventually also in the role of apprentices.

Thus, in our experience we observed a situation similar to that reported by Lobo da Costa and Prado (2012),

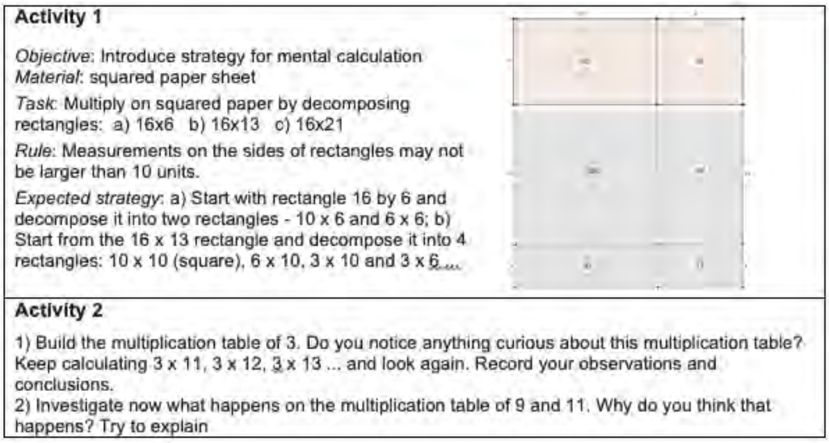
> This type of interaction, involving the sharing of experiences, knowledge, reflections and queries, helps build a collaborative learning-reflection space among the teachers. This form of learning, in turn, makes every participant able to experience simultaneously being a learner and a teacher to the others, and moving towards the sustainability of learning throughout life. (p.155)

Considering the set of students' productions, in particular the project and its final report, one aspect can be highlighted: most of them clearly stress the purpose of presenting differential activities and, in methodological terms, an explicit concern is identified to adopt different strategies and varied teaching resources from the perspective of focusing on active learning:

> In each class, we will use problem situations, concrete materials that students can handle, a game, and record sheets. Some narratives will also be used to trigger a problem that raises the need for resolution with a view to mobilizing knowledge and engaging students. (Group 2, Class 2017)

In general, with the support of trainers, educators, and especially school teachers, the students have been able to carry out these goals in the developed activities. Some examples of tasks from this perspective, drawn from the final group reports, are briefly described bellow.

In the first activity, a figural representation is proposed to support the decomposition of numbers and to explain a strategy of mental calculation in multiplication operations for 10-year-old. This option was thought after interaction with the teacher who reported that this rectangular representation appeared in textbooks and could be familiar to them. In the second one, it was emphasized more open questions to search for regularities, strongly inspired by activities developed with students by the trainers in face-to-face meetings.



Activity 1

Objective: Introduce strategy for mental calculation
Material: squared paper sheet
Task: Multiply on squared paper by decomposing rectangles: a) 16x6 b) 16x13 c) 16x21
Rule: Measurements on the sides of rectangles may not be larger than 10 units.
Expected strategy: a) Start with rectangle 16 by 6 and decompose it into two rectangles - 10 x 6 and 6 x 6; b) Start from the 16 x 13 rectangle and decompose it into 4 rectangles: 10 x 10 (square), 6 x 10, 3 x 10 and 3 x 6 ...

Activity 2

1) Build the multiplication table of 3. Do you notice anything curious about this multiplication table? Keep calculating 3 x 11, 3 x 12, 3 x 13 ... and look again. Record your observations and conclusions.
2) Investigate now what happens on the multiplication table of 9 and 11. Why do you think that happens? Try to explain

Figure 1: Multiplication Activity (Group 3, Class 2015)

In the activity related to Figure 2, the focus was on the active participation of 7-8 year-old students in all stages of the graphic construction, connecting with other concepts under study, like counting and measuring length. In this case, it was a student proposal based on statistics course experiences. It mentions the tutor statement: “The activity was very productive, with great commitment from the children, and for me it was also very important because I had no such experience.”

Figure 2: Graphing activity by 7-8 year old students (Group 5, Class 2019)

One aspect that was revealed in the analysis of the reports refers to the fact of learning how to deal with heterogeneous classes, as one group points out: “It was very important to be able to know the different student profiles from the teacher's point of view and try to work with different questions, receptivity and involvement. A challenge!” (Group 1, Class 2015).

Conclusion

The need to meet the requirements of the PFPUSP led to the creation of the *Internship Projects* discipline, as previously reported. Its conception was deeply discussed, and elaborated and implemented. Its operation is not merely bureaucratic, as some students refer to the internship supervised. The conceptual perspective of the discipline supposed a collaborative context with the participation of public school teachers. Thus, it has been associated with an in-service teacher-training course.

The analysis of the students' productions, especially the didactic sequences and the reports of the activities developed in the school, shows how these subjects conceive the teacher's role in their complex task of teaching and how they modify these perceptions during the formation. This kind of experience that shares the different knowledge mobilized in such a context may be considered infrequent in teacher training courses, as mentioned earlier. In fact, the reports of teachers and students in formation described here prove that an environment in which everyone interact, discuss and represent professional practices, in specific pedagogical and specialized knowledge for teaching, ends up promoting an effective and fruitful approximation with the public school (managers, teachers and students) and their communities in general.

Finally, it is important to emphasize that collaborative work in our program classes is not immediately established among participants. Considering the students, it can be related to various factors, for instance they are strongly influenced by institutional aspects linked to the learning evaluation. On the other hand, some teachers consider the researchers as specialists, in which their theoretical knowledge is overestimated comparing the knowledge derived from teaching practices. In this case, it should working on an equal basis for an effective collaboration, eliminating hierarchical relationships, as mentioned by Boavida and Ponte (2002).

Acknowledgement

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TEACHERS' VOICES FROM TWO COMMUNITIES OF INQUIRY ENGAGED IN PRACTICES OF MATHEMATICS EDUCATION RESEARCH

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This paper presents voices of high-school teachers who collaborated with mathematics education researchers in two communities of inquiry designed in accordance with a theoretical-organizational framework called Teacher-Researcher Alliance for Investigating Learning or TRAIL. In TRAIL communities, teachers and researchers study pedagogical questions of importance and mutual interest by going through the stages of a research cycle as partners. We address the following research question: What do mathematics teachers choose to reflect on when openly asked about their experiences in the TRAIL communities over one year? The data were collected from 16 videotaped meetings of the communities and from the teachers' written essays. Five themes emerged from the data: (1) novelty, (2) community and interesting participants, (3) implications for students and teaching, (4) research practices, and (5) tensions between being a teacher and being a researcher. The teachers' voices reveal a complex but generally encouraging picture of the collaboration.

Focus of the Study

This paper concerns collaboration between high-school mathematics teachers and mathematics education researchers who joined forces in order to explore pedagogical questions of importance and mutual interest. The chosen questions have not yet obtained clear-cut answers from academic research or from the teacher-participants' experiences. The collaboration occurred in two *communities of inquiry* (Jaworski, 2006) organized in accordance with a theoretical-organizational framework called *Teacher-Researcher Alliance for Investigating Learning* or *TRAIL* (Koichu & Pinto, 2018, in press). The TRAIL framework (presented below) has been developed as part of ongoing effort to situate teachers as stakeholders in mathematics education research, either for the sake of their professional learning or for the sake of scaling up mathematics education research and enhancing its implementability in practice (e.g., Krainer, 2014; Kieran, Krainer & Shaughnessy, 2012).

TRAIL communities have been active in Israel since 2017 in yearly cycles of 60-hour professional development (PD). In each PD cycle, mathematics teachers and mathematics education researchers jointly formulate research questions, design classroom activities, develop appropriate methods of data collection in the teacher-participant classrooms, create a database of documented classroom episodes, and then collaboratively work on analyzing the episodes and drawing conclusions. This iterative process is rich with learning opportunities for both sides, but also entails tensions, negotiation of expectations, developing common language and refining agendas.

This paper is part of a series of reports on the TRAIL communities (see Koichu & Pinto, 2018, in press). Our goal in the current paper is to contribute to fulfilling the need identified in the ICMI-13 survey on mathematics teachers working and learning through collaboration (Jaworski et al., 2017), which is reiterated in the ICMI-25 Discussion Document. Namely, we aim at presenting teachers' voices in accounts of their learning in collaborative settings. Specifically, we address the following research question: What do mathematics teachers engaged in TRAIL communities of inquiry choose to reflect upon when openly asked about their experiences in the community during a year?

The TRAIL Framework[1]

It is broadly agreed that *teacher inquiry* should be an inseparable part of teaching as a professional occupation. Following Menter et al. (2011), we refer to *teacher inquiry* as a systematic effort to develop new knowledge or understanding in an educational setting carried out by someone working in that setting, in collaboration with practitioners working in similar settings. Teacher inquiry includes many practices that are also pertinent to research or *disciplined inquiry* (Kilpatrick, 1981), that is, inquiry conducted in scientific communities. These include: listening to students, examining their work, assessing what students know and how they think, examining teaching materials, designing tasks and observing how students engage with them, comparing different teaching approaches, talking to colleagues about educational issues, sharing experiences in professional forums and more.

In addition to acknowledging similarities, it is important to point out that teacher inquiry and disciplined inquiry are essentially different. Labaree (2003) points out that teacher inquiry differs from disciplined inquiry in that it is normative rather than analytical, personal rather than intellectual, particular rather than universal, and experiential rather than theoretical. Researchers (Kontorovich & Rouleau, 2018; Labaree, 2003) assert that these differences might hinder collaboration between teachers and researchers. Furthermore, there are scholars (e.g., Kieran, Krainer & Shaughnessy, 2012) who acknowledge tensions between teaching and researching and discuss ways to make teacher-researcher cooperation and collaboration feasible and productive despite these tensions.

The TRAIL framework attempts to be sensitive to the abovementioned similarities and differences. In line with some other frameworks, TRAIL adopts a stance whereby partially overlapping practices and essential differences between teaching inquiry and disciplined inquiry can trigger not only tensions but also various learning opportunities within a specially organized setting, in which both types of inquiry are practiced. To this end, TRAIL draws on theoretical constructs and ideas developed in several bodies of the professional literature, including the literature on modes of research-practice partnership (Wagner, 1997), teaching experimentation in collaboration with teachers (Cobb, 2000), and communities of inquiry involving teachers and didacticians (Jaworski, 2006). Wagner's (1997) construct of *co-learning partnership* is central in the TRAIL framework. In co-learning partnerships, researchers and practitioners join forces to inquire together and aid one another in order to learn something new and worthwhile about their worlds and themselves. The goals, methods and principles of inquiry are negotiated openly to maximize the learning and professional growth of both sides.

The TRAIL framework consists of a system of theoretical premises and heuristics for guiding the design and conduct of research-practice co-learning partnerships. Three premises and three heuristics are particularly relevant to the concerns of this article (see Koichu & Pinto, 2018, for the full picture).

[1] This section is a modified and abridged version of descriptions that appear in Koichu and Pinto (2018, in press).

Professional Growth through Involvement in Research premise: Active involvement in the various stages of educational research generates opportunities for teachers to enhance their abilities to engage effectively in inquiry, noticing and reflection as part of the day-to-day practice.

Authenticity premise: Teachers' engagement in research is more likely to produce positive effects if conducted in the context of authentic research rather than an exercise in doing research.

Shared Agency premise: Collaboration of teachers and researchers can be stable and productive if the opportunity to share the agency over the partnership is available for both parties. This means that individual members of each community are to be involved in the partnership in ways that can advance their peculiar goals and needs, including the needs to contribute, develop professionally and have room for expressing personal creativity.

A relevant subset of TRAIL design heuristics is as follows.

- The research goals and questions that underlie TRAIL partnerships are openly negotiated and deal with issues that have the potential to resonate with mathematical and pedagogical dilemmas that teachers encounter in their daily work at the level of a class, a small group or an individual student.

- TRAIL partnerships must have "clear utility" for practitioners that can be convincingly communicated without heavily relying on the scientific literature in which the research is situated. In a similar vein, a TRAIL partnership must have "clear utility" for researchers, that is, have the potential to yield insights of importance to the education research community at large.

- TRAIL partnerships employ accessible data-collection and data-analysis procedures. We call a research procedure accessible if it can be mastered by an interested individual with no background in educational research after a brief training period, and if its use requires reasonable time and effort. Examples are: conducting a questionnaire in a classroom or writing a reflective summary of a lesson.

The above premises and heuristics constitute the guideline of the project. That is, we seriously try to base our decisions on them, but we cannot be sure that the resulting activity is indeed fully compatible with these premises and heuristics. To this end, the current study's focus on the teacher voices helps us to scrutinize our theory-based intentions when confronted with practice.

The Conduct of the TRAIL Communities

Approximately 35 high-school mathematics teachers voluntarily responded to call to join one of four TRAIL communities in the 2018-2019 school year. One community (hereafter Community R, nine teachers) was located in the central part of Israel, and another (hereafter Community M, seven teachers) – in the northern part of Israel. The communities were led by the authors of this paper (R and B for Community R; M and B for Community M), along with two of our colleagues. Systematic meetings of the TRAIL team (10 members, including teacher-consultants and researcher-consultants) supported the project. The organizational setting of each community consisted of 10 face-to-face four-hour meetings, three online meetings and a final conference for all the communities. During the year, the teachers were required to submit three intermediate assignments and a final essay.

Based on the TRAIL framework and on our experience, each face-to-face meeting was designed bearing in mind the need to balance three working dimensions: "mathematics", "community development" and "research practices". The "mathematics" dimension was ongoing. Every meeting included instances of dealing with mathematical problems, which were brought to the meetings either by the researcher-participants or by the teacher-participants as relevant or interesting. The "community development" dimension was supported by means of 15-30 minute activities aimed at familiarizing with each other's schools, teaching practices, and at sharing feelings and expectations.

The "research practices" dimension consisted of discussions of past studies and short lectures, by us and by invited experts, on stages of a research cycle and on research methodology. These discussions and lectures were precursors for actual engagement of the teacher-participants in developing task sequences and data-collection procedures. During the first meetings, the main activities within the "research practices" dimension were related to formulating research questions. The questions were eventually decided by the teacher-participants; providing examples from past studies was an influential tool for the researcher-participants in consolidating the teachers' ideas. Then the focus was shifted to purposeful task design in service of the research questions and to data-collection procedures, and later on – to the data analysis and preparation of reports and oral presentations.

The process of selecting research questions in each community was facilitated by the following request at the first meeting: "*Suppose your school hires a professional mathematics education researcher in order to help you improve your practice, what questions about your teaching or your students' learning would you like to ask him or her to explore?*" The teacher responses were very diverse (see Koichu & Pinto, in press, for examples). Then the researchers introduced a system of filters: the questions should be authentic (see the *Authenticity Premise*), researchable by means of relatively simple procedures, within the field of the expertise of the researcher-participants, and more. The filters were applied to the initial lists of questions, which generated short lists.

After several negotiation-refinement cycles, Community M decided upon the following question: *How can thinking flexibility in mathematical problem solving in high-school be facilitated*? The community interpreted *thinking flexibility* as one's capacity either to produce more than one solution to a problem or to find a new solution path when a previous approach led to a dead-end.

The process of choosing a worthwhile question in Community R was even less straightforward. This is because the teacher-participants had many different ideas and because they quickly connected these ideas to their favorite types of tasks (e.g., let's explore problem-posing tasks, or sorting tasks, or typical mistakes). The negotiation revealed common features of the proposed ideas. In particular, all teachers were interested in tasks that evoke classroom discussions and provide opportunities for achieving predefined pedagogical goals. In this situation, we proposed to accommodate all the ideas under the overarching question based on a sequence of three ad-hoc definitions, as follows.

(1) *Talking task* is a mathematical task embedded in a teaching situation that involves opportunities for problem solving, discussion and reflection. (2) *Pathway of talking tasks* is a sequence of talking tasks designed to achieve a chosen goal, such as solving a target problem, developing a thinking skill, fostering self-efficacy, capitalizing upon an anticipated mistake as a learning opportunity. (3) A pathway of talking tasks is considered *successful* if it is enacted by some of the community members in their classes and there is empirical evidence supporting the conclusion that the goal of the pathway has been achieved. In these terms, the research question of Community R was formulated as follows: *How can successful pathways of talking tasks be designed and enacted?*

After formulating the questions, the teacher-participants collaboratively designed and then conducted 2-3 lessons for the community research. The lessons were documented by means of audio- or video-recording and transcribing. Some teachers also used questionnaires and interviews, which as a rule were discussed in the communities. The documented episodes were analyzed either collectively or individually. During the analysis, the teachers were encouraged to look not only for evidence related to the research questions, but to pay attention to anything that they deemed pedagogically or mathematically important. For example, one teacher recognized an unusual extent of engagement of his students when working on a pathway of talking tasks. Another teacher noticed student independence, but was disturbed to see that her students did not perceive π as a number. An additional teacher was surprised by what her students considered as "different solutions to the same problem". A systematic set of findings obtained in two communities will be reported elsewhere.

Data Collection and Data Analysis

The teachers had many opportunities to reflect on their TRAIL experience during the year. Most of the teacher voices presented below were heard at the 10th meeting. A reflection-evoking question asked in that meeting was "*Share what happened to you in this community during the year* ". This question was repeated in a guideline for writing a final essay in both communities. We formulated this question to be inviting yet not leading.

Videotapes of the 10th meeting in both communities served as a main data source. Complementary data sources consisted of the teacher essays and videotapes of the additional meetings. The data analysis was conducted in accordance with the principles of grounded theory (Dey, 1999). Namely, after reading and re-reading the written material and watching the videotapes, we reduced the data by selecting the most suggestive, with respect to the research question, teacher assertions. These assertions were transcribed (about 6000 words). In an iterative process of discussion and refinement, five major themes of the teacher reflection were isolated and theoretically validated by comparing with the constructs included in the TRAIL framework.

Findings

The identified themes of the teacher reflection on their TRAIL experience are as follows: (1) the novelty of the TRAIL experience; (2) community and interesting participants; (3) implications, with two subcategories: (3a) benefits for (current) students; (3b) benefits for (future) teaching; (4) research practices; (5) tensions between being a teacher and being a researcher.

The novelty of the TRAIL experience

Three teachers in Community R and three teachers in Community M chose to reflect on the novelty of their experience in TRAIL. Examples from the transcript of the 10th meeting follow.

> Teacher G. (Community R): This is completely different [from the other PDs]; this changes your thinking style. This PD puts objectives that I don't know how to achieve. […]. On one hand, this is depressing, and on the other hand attractive. I would leave if this were not so.

> Teacher N. (Community M): This was refreshing, given what happens in the other PDs that I've attended.

> Teacher P. (Community R): We are exposed to something that in fact causes us to look differently at what we do, and this is novel.

As can be seen in Teacher G's response, the feeling of novelty was sometimes accompanied by mixed feelings. This is particularly evident in the following voice.

Teacher I. (Community R): You took us outside our comfort zone, and in my opinion we took you out of your comfort zone. It was reciprocal.

Community and interesting participants

Three teachers in Community M and five teachers in Community R chose to reflect on how special the participants in the community were. For example:

Teacher S. (Community R): Conducting research taught me a lot, mainly because of the community. I felt that I am among very intelligent people who are willing to contribute from their experience… I sat next to I. in the first lesson, and she solved [a problem] with vectors, and this was beyond me, and I loved this group.

Teacher E. (Community M): First of all, I met here very interesting people. We always discussed interesting [mathematical] problems, even before the meetings had begun and during the breaks. All the time there were interesting problems that I could learn something from.

Teacher V. (Community M): I learned something from everyone here. It was fun… I was looking forward to our meetings and did not miss a single one of them.

Of note is that S. and E. emphasized in their reflections "interesting people" in relation to the "mathematics" dimension, and V. meant a stimulating atmosphere of the community meetings.

Implications for students and for teaching

Benefits for (current) students. Two teachers in Community M and two teachers in Community R noted that the very fact of their participation in community research was beneficial for their students, because their students experienced "special" lessons that they conducted.

Teacher B (Community M): [Engagement in research] gives a lot, a lot. My students, they also felt this. They challenged themselves with our question [about flexibility]. {Speaks in the student voice} "Aha, today you will give us again something special, something tricky". They felt so because it was clear to them that there would be something special, something that would require flexibility.

Teacher S (Community R): There is an added value for a class in which [the research] was conducted, because [the students] received special attention, and this made them feel good, as the lessons were non-standard.

Benefits for (future) teaching. This theme was reflected upon by three teachers in Community M and by two teacher in Community R. Within the space constraints, we present two voices from Community M, which are characterized by different levels of specificity.

Teacher A. (Community M): I do use questions that require thinking flexibility in my classes. Now I see that I should use them intentionally, and not just in passing. I should use such questions not spontaneously but in the right moments and with the right emphasis… [participation in the research on flexibility] gives me perspective on how I can enhance flexibility. That is, not only to give a problem [that requires flexibility] but create a situation with the problem that would have consequences.

Teacher N. (Community M): I am exposed to research occasionally. But here, every research paper that I read or am exposed to, it causes a switch in my teaching. That is, it influences me, my teaching, and it influences my students as well.

The above assertion of N. made us think about the role of context, in which reading research papers can occur. Namely, reading research can be more impactful when the reader knows that the reading is for a specific and immediate use, and not just for enriching pedagogical knowledge.

Research practices

Six and four teachers in Community R and Community M, respectively, reflected on the research cycle stages. We believe that three voices below speak for themselves.

> Teacher V. (Community M): I thought at the beginning that [doing research] is straightforward: we choose research question, go for it and that's it. I discovered that it is not so. It is possible to change a research question, it is normal to change questions. Everything here is a about flexibility. There is a lot of flexibility in the research process.

> Teacher G. (Community R): Then I realized that I don't know how to interview. This is not simple, this is an art... There were several places on the tape where I really wanted [the student] to explain, but I cut him off!

> Teacher P. (Community R, in the final essay): Data analysis turned out to be thrilling. I can compare it to looking at a white sheet of paper in which something is written in hidden ink, and then you pour on the sheet a special substance, and suddenly you see what is written. When I succeeded to extract relevant information from the data, the information that was possible "to share with others", I felt tremendous satisfaction. I can unequivocally say that I went through a learning process in this PD and acquired tools that will serve me in the future.

Tensions between being a teacher and being a researcher

Four teachers in Community M and three teachers in Community R reflected on this theme. Even at the expense of diminishing the variety of the teacher voices, we choose to present in this sub-section a voice of one teacher (Teacher P., Community R) as heard on different occasions. This is in order to highlight the dynamic nature of the *Tensions* theme.

A reflection-evoking question used at four (out of 10) meetings in Community R was, "*Express in one word or in one sentence at most, what you feel about your experience today*". At the first meeting, P.'s response was "I feel challenged; I don't think the same way as you [the researchers] do". At the seventh meeting, after conducting a lesson for the research, P. said: "I try to milk my lesson, to take something out of it. When I listen to the student conversations, I think that what you [the researchers] have said now – knowledge construction – is perhaps going on there." Of note is that at that stage P. began thinking about her lessons in terms offered by the researchers-participants. From her further reflection (see the previous subsection), we deduce that P. had eventually developed agency over the data analysis.

In her final essay, P. wrote: "The course of the PD contained many situations where I felt how different my thinking in the teacher hat was, and how difficult for me it was to wear the researcher hat. Moreover, there were moments when the researcher thinking seemed to me unnecessary or artificial. For example, I finished a lesson with a good feeling. The lesson was successful in my view, the students worked nicely, participated and the lesson was flowing. It appeared later to me that all that I could say about that lesson gave zero information from the research perspective. Later I conducted that lesson once more [in another class], audiotaped it and also administered questionnaires at the end, and thus I really had material to work with as a researcher." Interestingly, P.'s final assertion was, "Still, I enjoy my teaching hat more, but the researcher hat looks now more relevant and useful for me, and an opportunity to wear it, from time to time, may improve my teaching."

Concluding remarks

The voice of P. was aligned with some other voices, in which the teacher role as a person who wants her students to succeed was contrasted with the researcher role as a person who wants to identify "interesting phenomena" without necessarily being committed to the students' success. The tension between the aspiration to help students succeed "here and now" and the aspiration to understand how to better teach future students was noticeable. In Labaree's (2003) terms, this means that at least some of the described experience was particular rather than universal for the teacher-participants.

Even so, the TRAIL premise - to pursue authentic research questions - seems to add a special flavor to the collaboration. The teachers were invited to participate in research not only for personal learning, but also for the joy of being a part in producing new (universal) knowledge. As a result, TRAIL communities seem to actually adopt practices of both teacher inquiry and disciplined inquiry. This suggestion is supported by the quest for research questions, by P.'s joy of discovering something worthwhile "to share with others" and by the teachers' dedicated work with transcripts of their lessons. In conclusion, we find that the teacher voices reveal a complex yet encouraging picture of collaboration guided by the TRAIL framework. Accordingly, we plan to continue experimenting with further implementation of the principles of the community of inquiry and co-learning partnerships.

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COLLABORATIVELY DEVELOPING CLASSROOM DISCOURSE

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We report on a collaboration of five Dutch mathematics teachers and a researcher who developed discourse-based lessons that foster students to think, to articulate their thinking, and to discuss each other's ideas. Collaborative reflection on video recordings of the teachers' orchestration of classroom discourse was an important design feature and combines our basic premises for the collaboration. The first premise is that the teachers' learning should be closely linked to their practice. The second premise is that the teachers' learning should be analogue to the student learning they aim for. The discussions during collaborative reflection in nine meetings were analyzed to investigate the learning of the group with regard to the teaching of mathematics through classroom discourse. The findings reveal increased and deeper discussion of students' thinking and increased linking of teacher actions to students' thinking.

Introduction and theoretical framework

In this paper we report on a collaboration of five Dutch mathematics teachers and one researcher who, during one school year, developed discourse-based mathematics lessons.

Classroom discourse

Teaching mathematics involves getting students to think, figuring out what they think, and supporting them in developing their thinking. An essential part of student-centered teaching is figuring out "students' mathematics" (Steffe & Thompson, 2000), or the "models that children develop (explicitly, not just implicitly) to construct, describe, or explain mathematically significant systems that they encounter", in the words of Lesh and Doerr (2003, p. 9). Inevitably, in a class full of different students, variation exists among students' mathematics. Taking Sfard's (2008) perspective of thinking as communicating, helps to overcome the difficulty of trying to 'look inside peoples heads'. Classroom discourse is a way to elicit students' thinking and make that thinking public, and to build on the variation among students by getting them to publicly discuss their different ideas. Mathematics classroom discourse has been studied increasingly during the last decennia, as shown in several reviews (e.g., Herbel-Eisenmann, Meany, Pierson Bishop, & Heyd-Metzuyanim, 2017). Still, how teachers can orchestrate classroom discourse and be supported in the development of this teaching practice remains an important question. What makes the orchestration of classroom discourse particularly complex is the combination of building on student ideas and steering the discussion toward disciplinary ideas (e.g., Sherin, 2002).

Learning in a collaborative group

In this study, we investigate a group of teachers and a researcher, collaboratively learning to develop and orchestrate classroom discourse. The set-up of our collaboration shares design features with established models for teacher collaboration, such as lesson study (Verhoef, Tall, Coenders, & van Smaalen, 2014), cooperative intervention research (Krainer, 2003), design-based research (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003), and multitier teaching experiments (Clark & Lesh, 2003). These models all make effort to narrow the existing gap between research and practice in mathematics education, and lead us to the first premise of our collaboration: If teachers are learning to change their practice, this learning should be situated close to their practice. This implies taking the collaboration into practice, as is done in lesson study, or taking the practice into the collaboration, for example by video recordings of the teachers' teaching.

Studies such as ours, were researchers and teachers collaborate to change the teachers' teaching practice, involve several layers. Lesh and colleagues (e.g., Clark & Lesh, 2003) describe multitier teaching experiment in which the three tiers are those of students, teachers, and researchers. These multitier teaching experiments are based on the assertion that people understand the world in their own way and construct their own conceptual models for sense-making, describing, explaining, and justifying. This implies that teachers cannot teach mathematics as it is modeled in their own perception, they have to try to figure out students' mathematics and guide students in developing their mathematics. Similarly, teachers cannot be told what their students think or what they should do with that (Males, Otten, & Herbel-Eisenmann, 2010). If researchers and teachers collaborate to change the teachers' teaching of mathematics, teacher development and student development should be analogue and intertwined (Clark & Lesh, 2003). This leads us to the second premise of our collaboration: If teachers are learning to teach in a certain way, they should learn it in a similar way.

Collaborative learning of mathematics teachers has gotten increased attention of researchers, but it remains difficult to make explicit what teachers learn from these collaborations and how this learning relates to the collaboration (Robutti et al., 2016). In this paper we adhere a situated view of learning that combines the perspectives of thinking as communicating (Sfard, 2008) and learning as boundary crossing (Akkerman & Bakker, 2011). In this view, learning mathematics or learning mathematics teaching involves developing and taking part in a discourse at the intersection of existing discourses. For the students, these are their existing discourse of school mathematics and the discourse of disciplinary mathematics. For the teachers these are the discourses of their classroom practice, disciplinary mathematics, and scholarly discourse on classroom discourse and mathematics teaching.

Research question

In this study we zoom in on the learning of the teacher-researcher collaboration as situated in the discourse during collaborative reflection of video-recordings of the teachers' teaching, and we aim to answer the following question: How can we characterize changes in the teachers' discourse during collaborative reflection on videos of classroom discourse?

Methodology

Context of the collaboration

Five Dutch upper secondary school mathematics teachers and one researcher collaborated in the project 'Classroom discourse in mathematics lessons' ('*Klassengesprekken in de wiskundeles*'). The researcher combined his job as a teacher of secondary school mathematics with his PhD research on classroom discourse in mathematics teaching. To find Dutch mathematics teachers who were interested in developing the practice of discourse-based mathematics teaching, he organized a workshop on classroom discourse to initiate a collaboration. Seven people from different schools across the Netherlands attended voluntarily, and during the workshop the researcher shared his plan for the project. The teachers were all enthusiastic and eventually, five of the teachers and the researcher collaborated during the 2018-2019 school year. There were nine four-hour group-meetings, which took place at the researcher's university.

Goals of the collaboration

The teachers' main goal was to learn how to prepare and orchestrate classroom discourse about different student ideas. The shared goal of the group was to develop guidelines for mathematics teachers to prepare and orchestrate classroom discourse. The researcher's main goal was to investigate the group's learning from different perspectives, in particular the learning of the group as situated in the discourse of the group, and the learning of the teachers as situated in the discourse with their students. In addition, the researcher's goal was to facilitate and foster the learning of the group.

Design features of the collaboration

To keep the development of classroom discourse close to the teachers' practice, it was structured in iterative design cycles of approximately one month, and each teacher monthly choose one lesson out of their program to make into a discourse-based lesson. One of such cycles is illustrated below.

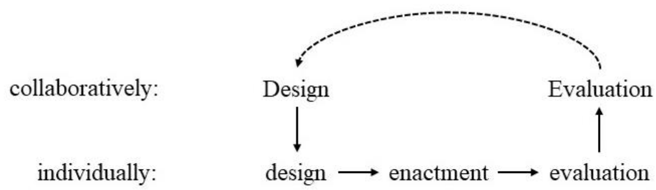

Figure 1: One cycle of development

At the end of a meeting, there was time for collaborative design of subsequent lessons (Design in Figure 1). In between meetings, each teacher individually designed, enacted, and evaluated one discourse-based lesson. In the subsequent meeting, the group collaboratively evaluated the lessons (Evaluation in Figure 1), before starting a new design cycle.

Collaborative reflection

During each meeting, the group evaluated lessons by collaboratively reflecting on video recordings of classroom discourse. In this collaborative reflection, our two basic premises are met: First, the teachers are watching and discussing video recordings of classroom discourse from their own classrooms, so their teaching practice is brought into the collaboration. Second, by letting the teachers reflect on the video recording and share and discuss their thoughts, the researcher facilitated learning in the same way the teachers facilitated learning in their classrooms, namely through discourse. The set-up of collaborative reflection changed halfway through the year.

Set-up of collaborative reflection in the first half year. In the first four meetings, the set-up was structured based on Case Stories (Hughes, Smith, Boston, & Hogel, 2008), foremost to establish a safe climate for reflection, and consisted of five phases:

1. The presenting teacher shares the story of the designt and enactment of the lesson and the group watches a part of the video recording of classroom discourse. During watching, the teachers make notes in three categories: "noticings", "wonderings", and guidelines
2. The teachers share their noticings, starting each sentence with "I notice…"
3. The teachers share their wonderings, starting each sentence with "I wonder…"
4. The presenting teacher reflects on the lesson and the video and may respond to the noticings and wonderings of the group
5. The group reflects on what they learn from this collaborative reflection with regard the guidelines for classroom discourse they develop.

Set-up of collaborative reflection in the second half year. After four meetings, the researcher decided that a safe climate for discussion was established (in consultation with the teachers) and proposed a new structure for collaborative reflection to bring focus on students' mathematics and elicit normative statements (with argumentation) about the teacher's role in classroom discourse. In the new set-up, the presenting teacher also started with sharing the story of design and enactment of the lesson. Then the group watched a part of the video recording of classroom discourse, and the researcher paused the video after students' utterances, asking, "What are possible scenarios for what could happen now, what scenario would you prefer, and why?". In this set up, direct attention is paid to what a student says and what the teacher (or students) could do with that

Outcomes of the collaboration

Many outcomes of collaboration can be mentioned here, such as lesson plans, some of which have undergone several cycles of development by different teachers, and a new, ongoing, community of teachers and researchers working together. The first main outcome that we mention here are the guidelines that were developed for preparing and orchestrating classroom discourse. These guidelines were presented to the participating teachers' colleagues during a one-day workshop in which the group shared their experiences and findings and discussed ways to integrate the work of the group in each school. The second main outcome we mention here is the learning of the group. In the remainder of this paper, we report on the learning of the group as situated in their discourse during collaborative reflection.

Data collection and interpretation

To investigate the learning of the group with regard to classroom discourse, we analyze the discursive activity of the teachers during collaborative reflection of enacted classroom discourse. Data are audio recordings of collaborative reflection. More specifically, the parts of the audio recordings that regard teachers' discourse during or after watching a video recording. These parts last, on average, 40 minutes per meeting, over nine meetings in total. Analysis was done in four consecutive steps. First, the audio recordings were transcribed verbatim. Second, the transcripts were exploratory coded on the level of utterances. Third, the exploratory coding was used to identify major shifts. Fourth, a qualitative analysis was done to investigate and characterize these shifts. In the next section we describe and illustrate the results of our analysis.

Findings

We identified three main shifts in the discussions during collaborative reflection. First, the focus of collaborative reflection shifts from the teacher's thinking and actions towards the students' thinking. In fact, a focus on the teacher is sustained throughout the project, but the focus on students' thinking increases and the discussion of teachers' thoughts and actions becomes linked to students' thinking in later meetings. Second, the discussion of guidelines for the teacher's work shifts from being mainly about work outside the classroom towards being mainly on the teacher's work inside the classroom. In the beginning of the project, teachers articulate guidelines about teacher's work in preparation of the lesson, whereas in later meetings, the teachers articulate more specific suggestions for teacher actions during classroom discourse. Third, the discussion of students' thinking shifts from very general towards more content-specific and from individual students' thinking towards the thinking of the group. In the beginning the teachers wonder "what are they thinking?", and later the teachers go into specific utterances of students to try to figure out what that student was thinking. We illlustrate these shifts using examples from meetings throughout the year.

The first half of the project

In the beginning of the project, teachers were new to the variation that exists in student thinking and in solution methods and they struggled especially with anticipating students' thinking and responding to unanticipated student ideas. The focus of reflection was the teacher's thoughts and the teacher's work in preparation:

Sigrid	I notice that you already need to be quite flexible here with students' contributions. It seems to me that you, kind of, have to switch all the time.
José	It was very difficult for me because I had only thought of one solution method, so I did not anticipate that Sid would use two variables. Therefore I was very happy when Sam said "Isn't it better to take y as the time?" Then I thought "Yes, y is the time".

When teachers wondered about students thinking, it was in a very general sense:

Ward	I wonder what those two girls are thinking.

When discussing what was learned from the reflection of José's video, the teachers emphasized the preparation that is needed to be able to anticipate and react to students' ideas:

Anna	The preparation of the problem is essential. You need different solution methods.
Jakka	And you need to be able to map out those different solution methods.
Anna	We need to get rid of our one track mind. For years we have been doing it in one way.

During the first months of the school year the teachers became more comfortable with preparing for different solution methods and supporting students in sharing their own solution methods. In the collaborative reflection of the fourth meeting, they openly wondered what the teacher can do to let students articulate their thinking. However, the wonderings about students thinking are still very general:

Sigrid	I wonder what students are thinking and why this doesn't get articulated generally.
Ward	When you say "look here" I wonder how you could let that student say more. I wonder how, as the teacher, you could make students' learning public without it being a fill-in exercise.

The teacher whose video is being reflected on used these wonderings to identify possible moments with possibilities of getting students to think and talk:

Jesse	I wanted to see how I could limit my steering and now I see that for example when I say "Look here" or reply to a student contribution by explaining it, these are moments when I let the students do less thinking.

He articulated his dilemma in steering for a smooth lesson but also building on student thinking and realized that he was working for different kinds of goals. The other teachers recognized his dilemma and agreed with his clarification:

Jesse:	I was having a big dilemma in wanting a smooth lesson but also wanting to let students talk, and I just realized that my short-term goal is to have a smooth lesson but my long-term goal is having student talk more. So I really need to invest in that now. This means that some lessons will be a bit rough.
Everyone:	Yes!
Sigrid:	This is something for the guidelines because more of us are experiencing this.

The second half of the project

In the second half of the school year, the discussions of the teacher's work narrowed down to being mainly about the teacher's work during classroom discourse. In addition, the focus on students' thinking largely increased and got more specific, and teacher thinking and teacher actions were often linked to student thinking:

José:	Maybe if you ask "Why doesn't it go under the x-axis?" it becomes clear that he actually means left from the y-axis.
Jesse:	That's what you need to enlighten. The student is reasoning based upon the shape of the graph and predicting the rest of the graph.
Chris:	Wat could happen after this student's contribution?
Jesse:	I think the teacher should ask why it is relevant that the graph does not go under the x-axis.
Ward:	First of all: What does this student mean?

The teachers realized that students' talk can often be interpreted in many different ways and that it is hard to know what they mean:

Ward:	I like this because this happens all the time. That we think we know what a student says, but actually we have no idea.
Sigrid:	So in fact often we do not really listen to what the student says. You have an expectation of what they will say and that is what you react to.

Towards the end of the project, the teachers were still struggling with getting students to talk and understanding what students say. However, they had a broader selection of suggestions for teacher actions to get students to talk. In addition, instead of just talking about how the teacher can understand what a student is saying and get that particular student to explain his thinking, there focus changed to getting the class as a community to understand and they gave more suggestions to get other students to react.

Anna: And then I hope that the teacher will ask “why?”.

Ward: It may be more open if you ask “could you react?” because then you get a story, and if you get a story then the rest of the students can think along.

Conclusion and discussion

Learning as situated in discourse

From a communicational perspective, learning of a collaborative group entails a development in the discourse of that group (Sfard, 2008). As elaborated above, we identified three main shifts in the discourse during collaborative reflection. In particular, increased attention is being paid to students’ mathematics, and teacher actions are linked to student thinking. So the group collaboratively learned in the sense of participating in a changing discourse. Similar results were found in previous studies on teacher learning in groups of mathematics teachers that are structured around collaborative reflection of video of the teachers’ teaching (e.g., Sherin & Han, 2004). The changes in the discourse could partly be linked to the change in the set-up of collaborative reflection. In the set-up from the fifth meeting on, the researcher stopped the video after student utterances and asked the teachers about possible scenarios and teacher actions. Although this question is focused on teacher actions, the fact that it is asked after a student utterance helped the group to focus on student thinking and link possible teacher actions to student thinking.

Learning as boundary crossing

In this study we investigated the learning of a group of teachers as situated in their discussions during collaborative reflection of their videos of classroom discourse. This collaborative learning takes place in a new space at the intersection of similar but distinct communities of practice (Goos, 2014), namely the communities of the teachers’ classroom practice, and the community of scholarly discourse on mathematics teaching. Adhering Akkerman and Bakker’s (2011) perspective of “boundary crossing” we can frame the learning of this group as reflection and transformation. We might say that the guidelines that were developed are a boundary object and a crystallization of the learning of the group. However, to ensure transformation the teachers would need to commit to the guidelines (Akkerman & Bakker, 2011). The teachers may commit to the guidelines during their discourse-based lessons, but it is hard to say to what extend they commit to the guidelines during ‘regular’ lessons. In other words, during the collaboration, the space of the group meetings transformed, as can be seen in the findings of this study, and the space of the teachers’ discourse-based lessons also transformed, which can be seen in the teachers’ videos (and will be thoroughly investigated and reported in a forthcoming study), but, we do not know if the space of ‘regular’ lessons also transformed. In fact, the discourse-based lessons that started as part of a boundary space, may have been transformed to a whole new space, apart from the teachers’ regular lessons.The teachers themselves started to wonder during the year what the effects of their discourse-based lessons were on their regular lessons, and vice versa.

We do suspect that the teachers' collaboration in this study has an influence on their teaching in general. For example, because they develop their "professional vision" in that they "learned to attend to particular kinds of events that happen in a classroom and they learned to reason about these events in particular ways" (Sherin & Han, 2004, p. 180). We join the field of research on teacher noticing (e.g. Stockero, Rupnow, & Pascoe, 2017) and conjecture that the development of this kind of professional vision has an influence on the way that the teachers interact with their students.

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COLLABORATIONS BETWEEN TEACHERS AND WITH ACADEMICS IN AN IREM GROUP ON A COLLABORATIVE PROBLEM-SOLVING DEVICE

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The IREMs are a French experience of sustainable collaborative work between teachers and with academics started 50 years ago. We investigate this experience through the study of the collaborative work developed in an IREM group in Montpellier (ResCo), which provides a device for collaborative problem solving between classes, and facilitates collaboration between teachers.

Introduction: context and questions

The IREMs (Instituts de Recherche sur l'Enseignement des Mathématiques) are a well-known structure in the history of mathematics education (see Cortella & Arnoux, 2019). Created in France in the 1960's to facilitate the modern mathematics reform (the IREMs' network turned 50 years in 2019), they remain a specific mathematics-education model. Existing in many universities in France, IREMs are places where mathematics teachers work together and with researchers or university teachers in mathematics, and nowadays also researchers in mathematics education and teacher trainers (all named *academics* in the following), with a key role of collaboration:

> Since [their creation], the IREM have contrived to be one of the most active and unavoidable actors in mathematics education in the country, by responding to the three missions entrusted to them:
> - research to improve the teaching of mathematics in non-hierarchical groups of university researchers and teachers of the first or second degree school or in higher education;
> - Train teachers, especially using the results of network research
> - Disseminate the results of research in mathematics, mathematics education or history. (Cortella & Arnoux, 2019)

The IREMs and their network have played a strong role in the development of the didactics of mathematics in France (Brousseau, 2004, Artigue & al, 2019) and they contribute now in the interaction between ground and research, by contributing to the dissemination of research results and foster relationships between research and action (Artigue, 2017).

In the IREM of Montpellier, the ResCo group (for *Résolution Collaborative de problèmes*, Collaborative Problem-Solving) started in the 2000's. In addition to the collaborative work inside the group between teachers and with academics, the ResCo group supports a device of collaborative Problem-Solving between classes and provides every year an original problem that is studied inside this device (ResCo, 2014). The device consists in providing every year a collaborative problem solving session (Collaborative PS session) starting with an initial problem given to the many classes involved (up to 120 classes *i.e.* more than 3000 students some years), about which the classes, associated by groups of three, exchange questions and answers, followed by the sending of a "relaunched" version of the problem, on which the classes search and exchange. The communication between classes, through the teachers, is is made on an online forum. This device also includes a teacher training device, on two or three days, oriented towards fostering problem solving in the classroom and preparing teachers to participate in the collaborative PS session.

Currently, the aim of the ResCo group is to foster problem solving and modeling in the classrooms and in teachers' practices (for more details about ResCo see ResCo, 2014, Modeste & Yvain, 2018).

For the ICMI study, we will not detail the collaborative work between classes or students promoted by the device, but we will focus on the collaborative work between the teachers whose classes are involved in the collaborative PS session and between members (teachers and academics) of the ResCo group, in supporting the device and in its others activities. To this end, we will question the history and the dynamic of the group and examine its evolution, and the influence of the involved academics on the group trajectory. Let us consider the following questions:

- What have been the evolution of the ResCo group and its device since its beginning, leading to the current organization? How the academics involved in the collaboration have influenced the trajectory of the group? This will be discussed in the first section (Dynamics and evolution).
- In the ResCo group and device, what are the collaborations at stake? How do they work? This will be discussed in the second section (Nature of the collaborations at stake).
- What are the effects of this collaborative work for the teachers involved in the device? For the members of the ResCo group? For the device itself, and the work of the group? This will be discussed in the conclusion.

Methodology

The results presented are based on the analysis of data of various types: the problems given in the collaborative PS sessions, the contents of the messages exchanged by the teachers through the online forum (including students' productions exchanged by the teachers), the results of a satisfaction questionnaire given every year to the teachers, the notes of the work of the IREM group exchanges during the elaboration of the problem (and the versions of the problem that are discussed), and videos of two teachers implementing the PS session with their students. The analysis of this data have been made in previous work, quoted in the paper. It has mostly been a qualitative analysis, but questions and answers have been analyzed quantitatively, a large part of these analyses have been developed by Yvain-Prébiski (2018).

Dynamics and evolution of the group and influence of the academics

The idea of collaboration between teachers was present from the beginning of the ResCo group. It arose from a project called SFODEM[1] developed in the IREM of Montpellier between 2000 and 2006 (Guin & Trouche, 2005). In this context, in 2001, a "problem-solving" group has been created within the SFODEM (with teachers already involved in this topic) to disseminate problem-solving in teachers' practice based on collaboration between teachers. It was supported and motivated by the development of digital tools (messaging services, chat rooms, forum). The objectives were 1) to foster mathematics problem solving in classrooms, 2) to conceive and disseminate resources, 3) to develop and facilitate a community of practices (Wenger, 1998), and 4) to support teachers with in-service teacher training. We will see that these four aspects are invariant in the history of the group.

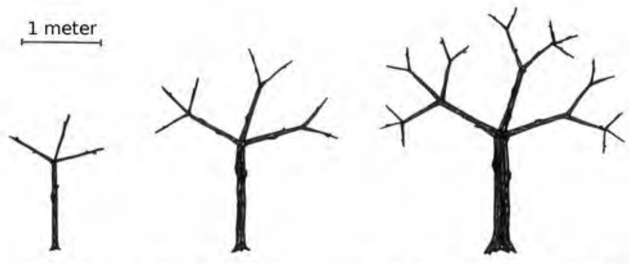
We analyzed the evolution of the group focusing on the objectives of the group, the points of view adopted and the research work associated to the group, the academics involved (as he has, in general, the responsibility of the group, and at least, the responsibility of the quality of the

1 Suivi de Formation à Distance pour les Enseignants de Mathématiques (Monitoring of distance training for mathematics teachers): http://www.math.univ-montp2.fr/sfodem/GuideSfodem

mathematical contents), and, as it is central in the topic of the group, the type of problem proposed to the collaborative PS session. We synthesize in table 1 the different periods of the group.

Table 1. Evolution of the group: particularities, and emblematic problems.

Period, context	Objectives / points of view / research	Academics (initials)	Types of problems
2001 to 2002	To develop PS in the practices of teachers, strong anchoring in the idea of exchanges on practices between teachers, use of new digital tools.	DT, university teacher, mathematician.	Classical problems taken from the mathematical folklore, discrete problems.
SFODEM	Emblematic problem: The Art Gallery Problem (see https://en.wikipedia.org/wiki/Art_gallery_problem)		
2006 to 2009 ResCo group	Context of IBL and PS. Search for realistic problems, concrete situations. Questioning on the link between the realistic problem and the mathematical problem. implementation of a Q-A phase in the session (still existing now).	JD, university teacher, mathematician.	Classical problems from applied mathematics, probabilities.
	Emblematic problems: - Evolution of a lemming population, - Prevision of loss and gain in a gambling problem (roulette-like).		
2009 to 2011 ResCo group	Focus on PS and experimental dimension (heuristic) in PS. Mathematical problems contextualized “out of the mathematical frame”, “intentionally vague” that has to be “mathematized”. Also, solutions of many problems are becoming easily available on internet. Creation of the “relaunched” problem, stabilizing the mathematized problem (still existing now).	VDG, teacher trainer, didactician, university teacher. Collaborations with another IREM group in Lyon working on IBL and PS.	Mathematical problems identified for their mathematical interest for students’ research and experimental activity (“open problems”, “problems for searching”…) in “a context that is fictional but realistic”.
	Emblematic problem: “The Artist. A modern artist wants to create a work of art on a round support, by putting in nails on the rim and stretching strings between the nails. He intends to paint each zone with a different color. How many colors does he need?”		
2012 to 2014 ResCo group	Same context. Formalization of the concept of “Realistic Fictions” (Ray, 2013, and described in Modeste &Yvain, 2018) as “situations that are *a priori* not mathematical, with a fictional but realistic context, which need a modeling phase to be efficiently handled, and the modeling phase can lead to various mathematical problems according to the choices made”.	EM, university teacher, mathematician.	Problems from the mathematical folklore (combinatorics, games) or classical issues in mathematics (random walks), contextualized according to “realistic fictions”.
	Emblematic problems: - Anubis Stones: Chomp (https://en.wikipedia.org/wiki/Chomp) in a archeological context. - Leaks at Fukushima (see Gardes & Yvain, 2014; Yvain-Prébiski, 2018): “Under the cooling pound of the reactor 4 of the nuclear power station of Fukushima, TEPCO engineers leaded by Toshio Nishizawa have founded micro-leaks contaminated by highly radioactive Cesium 137. They decided to send a robot to aspirate the radioactive droplets. Sadly, its guidance device has been damaged by the radioactivity, and it moves by step of 10 cm in the directions north, east, south and west randomly selected. The engineers only know that the robots starts on its charging base and is programmed to make a finite number of moves. Worried, they wonder how many ways the robot has to get to a droplet. Help them!”		
2014 to today	Same context with a focus on modeling and making students aware of the necessity of making choices in modeling a situation. Modeling motivated by a goal: predicting, optimizing…	SM, teacher trainer, didactician, university teacher, and	Realistic fictions with two more aspects (Yvain, 2018): the problem is an adaptation of a professional modeling

<table>
<tr><td rowspan="2">ResCo group</td><td>Grasping of the notions of horizontal and vertical mathematization (Treffers, 1978) adapted by Yvain-Prébiski (2018).</td><td>SYP, teacher trainer, didactician.</td><td>situation; the values of the didactic variables (Brousseau & Warfield, 2014) are chosen to foster modeling activity.</td></tr>
<tr><td colspan="3">Emblematic problem:
“The tree. Botanists from Botanical Garden have discovered an exotic tree. To study this new species, the botanists have sketched the tree every year since 2013. The botanists want to build a greenhouse to protect it. They believe it will have reached its full size by 2023. To help them, predict how the tree will be in 2023.”


Sketches of the tree in November 2013, 2014 and 2015.</td></tr>
</table>

Notes. PS: Problem Solving, IBL: Inquiry-Based Learning, Q-A: Questions-and-Answers.

Since 2014, the focus has been on Modeling and Solving the mathematized problem. The initial problem given to the classes is called “Realistic Fiction”. It is design collaboratively in the ResCo group, taking into account epistemological considerations on epistemology (Yvain-Prébiski & Modeste, in press), the previous experiences of the group, and the expertise of the teachers. This problem lead to a Q-A phase between classes, followed by a “relaunched realistic fiction” that stabilizes a mathematized version of problem with common choices in order to go on working collaboratively between classes. The latter is conceive according to our *a priori* analysis of the produced problem (taking into account the levels of the classes involved) and the exchanges observed during the Q-A phase, in order to remain as close as possible to the students perception of the problem (see figure 1 for a sketch of the ResCo device).

Discussion on the collaborative work

We can notice many aspects and effects of the collaborative work between teachers and with academics. First, we can see invariant global goals in the history of the group: fostering PS in teachers practices, produce and disseminate resources, develop a community of practice, and developing in-service teacher training (the device has always gone along with a teacher-training device). Second, the influence of the academics involved in the group (and its status) strongly impacts the direction of the group (see table 1). Third, beyond that, there is a sort of collaboration over time that can be seen as a dialogue between the different collaborators of the group, because the teachers-members change independently of the academics, but also because the memory of the group is kept up. This continuity and legacy over time permits perpetual improvements over the one-year cycle of the collaborative PS session, foster reflexive perspectives and formalization on the objects involved, and help new directions to develop. Forth, it brings to light a specific model of collaborative work between teachers and with academics in a central core, and with a larger group of teachers involved in a collaborative RP session every year. The next section will examine the nature of the collaborations at stake in the current situation.

Nature of the collaborations at stake

We distinguish three levels of collaboration allowed by the ResCo device: collaboration between teachers and members of the ResCo group, then collaboration between academics and teachers within the ResCo group, and collaboration between teachers. The online forum plays a major role in these levels of collaboration by allowing: Inter-class exchanges, Exchanges between teachers, whether at the level of practical, organizational, pedagogical and didactic aspects, Exchanges between teachers and the ResCo group, Access for teachers to all the productions of the classes posted online that can serve as resources to teach classes but also as a support for professional reflection (didactic and pedagogical), Having a memory of the activities that took place each year can thus serve as a resource for teacher-training.

In order to characterize the different types of interactions involving teachers, we will rely on the following criteria for each moment of the collaborative PS session: Which actors? At which stages of the ResCo device? About what? For what purpose? What role have the collaboration? The collaborations analyzed in this section are synthesized in figure 1.

Collaboration between the members of the ResCo group and the teachers

Before the collaborative PS session. The members of the ResCo group provide to the teachers various documents explaining the objective of the collaborative session, the organization of the collaborative PS session and its schedule. This happens either during workshops or communication at conferences and professional meetings or during the training device. This is essential to enable teachers to appropriate the organization of the device and to include it in their syllabus. In these documents, there are also examples of student's productions on the previous collaborative session with analysis elements and some elements of *a priori* analysis of the realistic fiction that will be proposed for the new session. Actually this is not yet the final version of the realistic fiction: the ResCo group presents a draft version of the realistic fiction and relies on feedback from teachers during the training device to develop the final version.

During the collaborative PS session. The exchanges between teachers and the ResCo group are most often related to the organization or functioning of the platform. The ResCo group deposits on the platform the statement of the realistic fiction, then the *relaunched realistic fiction*. In order to support teachers in the implementation of the collaborative PS session, the ResCo group sends to teachers a document to support them in the work following the relaunched realistic fiction and documents to enable them to finalize the collaborative session in their classrooms (expert solution, synthesis of students' productions, mathematical knowledge and skills at stake).

After the collaborative PS session. The ResCo group gives teachers the opportunity to share their analysis of the work done, by sending them an online questionnaire. These exchanges enable the group to improve conditions for the device to work better and also, to adjust the content of the documents given before the collaborative PS session.

Collaboration between teachers and with academics within the ResCo group

Before the collaborative PS session. At meetings of the ResCo group, the academics propose to the group a professional modeling problem. From this, all the group members will collaborate to elaborate the future problem of the session. Based on the expertise of the teachers, the group conceive a first statement, whose collaborative *a priori* analysis will identify its potential for learning mathematics. Several meetings are necessary to reach the final version used in the collaborative PS session. Collaborations between academics and teachers of the ResCo group are also very strong in the preparation of the training courses. Teachers and academics assume the role of trainer and co-construct the content, relying heavily on the experiments of the group's teachers.

During the collaborative PS session. Each member of the group analyses the productions submitted to the platform during the question-and-answer phase in order to co-construct the relaunched realistic fiction. This collaboration is one of the most important as the group's teachers themselves involve some of their classes in the session.

After the collaborative PS session. Similarly, for the preparation of the documents to finalize the session, the group relies heavily on feedback from ResCo teachers and also on all the productions uploaded on the platform. The ResCo academic writes an expert solution of the problem and the ResCo teachers synthesize and analyze the various students' productions from the point of view of the mathematics involved. Each document is discussed between the members of the group and validated by each one. This collaboration, which is based on everyone's expertise, is very fruitful and necessary to think out the favorable conditions to implement the collaborative PS session.

Collaboration between the teachers involved in the ResCo device

Before and after the collaborative session. Sometimes, teachers who participated in a previous collaborative PS session interact with their colleagues to introduce them to the ResCo device.

During the collaborative session. Teachers use the forum to deposit their students' work and also to communicate with each other, for example:

- Share any difficulties they may have in implementing the system: "The students have answered your questions succinctly, they have difficulty getting into the response phase, they tell me they can't know." (Excerpt from a message submitted during the question-and-answer phase)
- Sometimes support each other, as illustrated by these exchanges between two colleagues:
  P1 to P2: “These are the questions from the 9th graders of middle school xxx. Surprisingly, for my part, many strategies came up when I read the statement. Few questions emerged. I have the feeling that they will arise in the following sessions. Have a good reading and good research.”
  P2 to P1: “I am in a bit the same situation as you, few questions were asked, they especially wanted to start measuring or looking for relationships between lengths. I think answering your questions can’t do any harm to them! Have a good day, and good research.”

In addition, via the forum, the system offers teachers the opportunity to consult all of the students' work, to share and exchange on their practices. Similarly, when the ResCo group sends the documents to finalize the collaborative PS session, teachers have at their disposal elements for *a posteriori* analysis of the work carried out, which allows them to reflect on the proposed activity.

Conclusion: Effects and outcomes of the collaborations

The collaborative work between teachers in collaborative PS sessions, and inside the ResCo group between teachers and with academics, has been developing for almost 20 years through the ResCo device. This model of multiple-collaboration has enabled many effects and outcomes.

On teachers practices. A specific relation develops between the IREM group and the teachers. Indeed, the ResCo group is responsible for the choice of the problem and its organization. Teachers can choose to step back by informing students that they are not decision-makers, neither of the choice of the problem nor of the different phases of work proposed. The work of Yvain-Prébiski (2018) and Yvain-Prébiski & Chesnais (in press), shows that this negotiation of the *didactic contract* of the class is mainly manifested in the exchanges with the class, which each teacher manages in a singular way with an objective that seems common: to encourage the *devolution* to the students of the work requested, based on the collaborative dimension of the system. The support provided by the ResCo group throughout the implementation of the ResCo device thus reduces any hesitation on the part of some teachers to participate in the device. This can lead these teachers to dare to conduct a PS activity in the classroom, and change their practices over time, to develop PS activities in the classroom, anticipate modeling issues, or simply give students more autonomy.

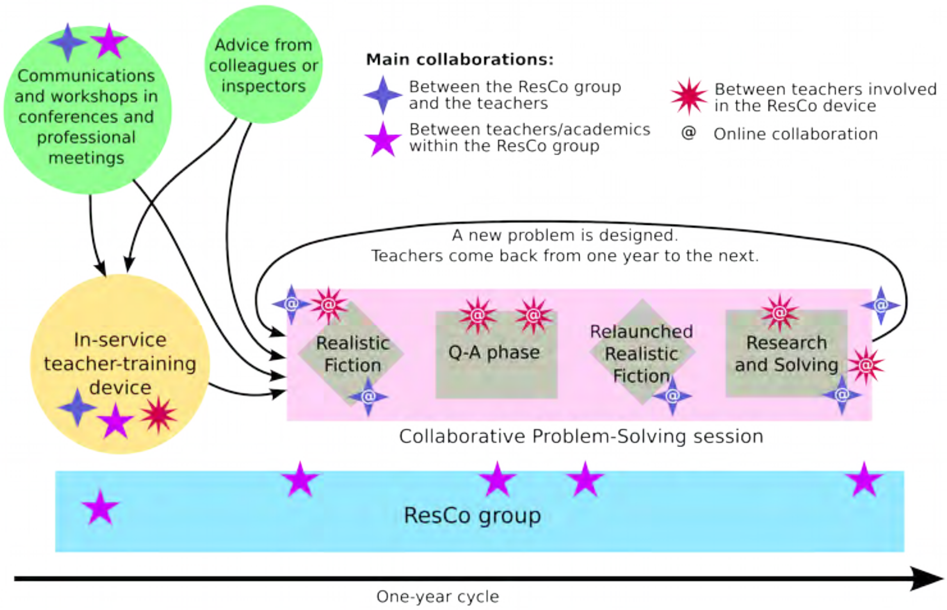


Figure 1: The ResCo device and the various collaborations at stake.

On teachers professional development. Collaborations also impact on the teachers' trajectory. By co-option, teachers who have participated for several years in the device can join the ResCo group and participate in its activities. This contributes to the training and professional development of these teachers, which can lead them to become teacher-trainers or to be initiated into research. We illustrate these professional developments, with the trajectories of four teachers (figure 2). Teacher T1 participated in the ResCo group, and this led him to complete a master's degree in didactics of sciences. He later moved and got pedagogical and administrative responsibilities in the national education system. Teacher T2 participated in the ResCo device, then became a member of the ResCo group. This led her to complete a master's degree in didactics of sciences and to become a teacher trainer. Then T1 prepared and defended a PhD whose field of experimentation was the ResCo device. Teachers T3 and T4 participated in the ResCo device, then became members of the ResCo group. T3 plans to prepare a master's degree in didactics of sciences.

On the work of the ResCo group. As explained previously, the collaboration in the IREM group is non-heriarchical and based on the diversity of the expertise of the group. The cyclic organization of the collaborative RP session permits to improve sequentially the organization of the session, the support to the teachers, and the work on the design the annual problem, with collaborative analyses of the problem and its implementation before, during and after the session. The central role of the teachers of the group having students involved in the collaborative RP session is fundamental to this end. Finally, the dynamic of the group is also influenced by its past, some transmission of the memory of the group is done with newcommers, and enables continuity and progress over years.

Perspectives. It would be interesting to compare this model of collaboration and this type of device with other experiences over the world in mathematics education and teacher training and professional development in order to characterize the criteria that makes work such collaborations between teachers, but also to understand how it can be scaled up or replicated.

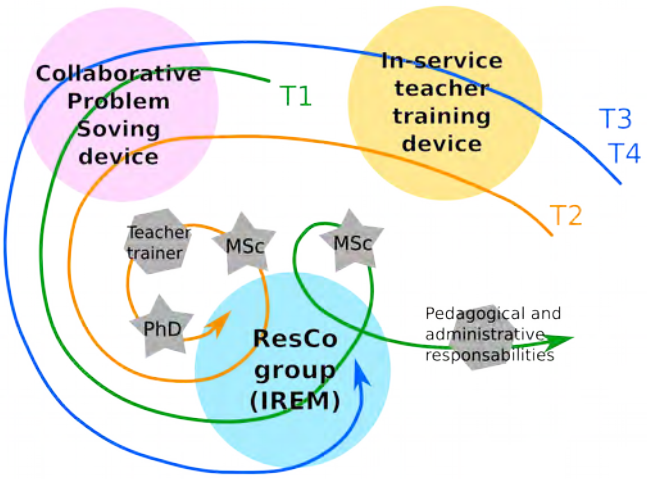


Figure 2: Trajectories and professional development of teachers.

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COLLECTIVE DISCUSSIONS FOR THE DEVELOPMENT OF INTERPRETATIVE KNOWLEDGE IN MATHEMATICS TEACHER EDUCATION

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We start from the assumption that teachers need a deep and broad mathematical knowledge —called Interpretative Knowledge (IK)—that allows them to support students in building their mathematical knowledge from their own reasoning and productions. In the present study, we aimed to ascertain how collective discussions focusing on the interpretation of students' productions engage Prospective Teachers (PTs) and impact their IK development. In particular, we observe how this form of collaborative work among PTs allows for the emergence of novel insights into the mathematical aspects of students' productions that were not considered during previous individual work, and produce changes in PTs' attitudes towards students' productions.

Introduction

During class work, mathematics teachers often have to interpret and give meaning to their students' productions. Empirical evidence indicates that the effectiveness of their practice is strongly linked to the quality of these interpretation processes. Indeed, teachers' ability to correctly interpret students' productions can support them in developing class mathematics activities that are based on what students actually know and perceive about mathematics. The "interpretation activity" of students' productions is, therefore, a crucial task for teachers (e.g., Hallman-Thrasher, 2017; Ribeiro, Mellone, & Jakobsen, 2016). It is important to design professional development (PD) program activities around students' productions in order to allow prospective (and in-service) teachers to explore the diversity of students' reasoning in a more reflective context, allowing them to be better prepared to respond to a large (and sometimes unexpected) variety of students' ideas in real and highly interactive classroom settings (e.g., Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Putnam & Borko, 2000).

In previous research, we have been using tasks containing students' productions to analyze and develop in prospective teachers the so-called *Interpretative Knowledge* (IK, see, for example, Di Martino, Mellone, & Ribeiro, 2019). Following the design research approach (Cobb, Zhao, & Dean, 2009), and adopting the process of continuous revision, we have been exploring the ways of designing and implementing these tasks as a part of training activities included in both initial and continuous PD programs. Indeed, gradually, after several cycles of design and redesign, we chose to focus on developing particular implementations of what we call *interpretative tasks*. As a part of the task, the participants are required to solve a mathematical problem and then, individually and collectively, give meaning and feedback to some students' productions pertaining to the same problem. This initiative has revealed the central role that the collective mathematical discussion among PTs, led by the educators (EDs), can play during and after the implementation of these tasks.

In this study, we focus on the role played by the collective discussions among teachers on their different interpretations of students' productions, aiming to elucidate how these discussions can affect their ability to recognize mathematical aspects and potentialities embodied in students' productions they were unable to see previously. The aim is to support the development of PTs' specialized knowledge, IK in particular. So, our research question is: *How do collective discussions on the interpretation of students' production engage PTs and impact their IK development?*

Theoretical Background

In recent years, taking cue from Shulman's (1986) perspective, the role played by teachers' knowledge in the process of teaching mathematics has been the focus of numerous studies in the teacher education field. For the purpose of meeting the present study aims, we refer to Mathematics Teacher Specialized Knowledge (MTSK, Carrillo et al., 2018) that places teachers' beliefs at the core of the model and considers all the mathematical knowledge for teaching as specialized.

We believe that teachers should possess mathematical knowledge that allows them to support students in building mathematical knowledge from their own productions—including those containing mathematical ambiguities, errors, and non-standard reasoning—assuming that they can be used in the class practice as learning opportunities (Borasi, 1996). Therefore, we state that mathematics teachers should have positive beliefs regarding students' productions and should be able to explore the mathematical potentialities embedded in them. However, in order to achieve this goal, teachers not only need to have a positive attitude toward students' reasoning, but also a broad knowledge of the possible students' strategies, representations, and errors related to a mathematical problem, as this would enable them to make sense of unanticipated solving processes adopted by their students. In this frame, we have been developing the notion of IK (Jakobsen, Ribeiro, & Mellone, 2014), referring to a deep and wide mathematical knowledge that enables teachers to support students in building their mathematical knowledge by starting from their own reasoning and productions (Di Martino et al., 2019). IK includes the ability to expand one's own space of solutions by looking at situations from a wide range of different points of view. Moreover, it includes the capacity for developing specific feedback based on the meaning ascribed to individual students' reasoning (Jakobsen et al., 2014).

During the course of our research, we have studied and measured the level of IK in PTs. We have designed and experimented on different interpretative tasks as a part of various studies, aiming to create effective activities for the development of IK in teacher education (e.g., Jakobsen et al., 2014; Ribeiro et al., 2016). The analysis of the PTs' written answers to the interpretative task allowed us to identify the crucial role that the content knowledge plays in the IK level. However, our findings also revealed that, in isolation, this cannot guarantee a high level of IK. Moreover, our research has indicated that the PTs' individual work on the interpretative is not enough for teachers to develop the IK. As a consequence, following the design study methodology defined as a "family of methodological approaches in which instructional design and research are interdependent" (Cobb et al., 2009, p.169), we designed an Individual−Collective−Individual (ICI) cycle. Thus, after measuring the initial level of PTs' IK through their individual work, all participating PTs were involved in a collective mathematical discussion about the mathematical problem and the students' productions. We used the collective mathematical discussion as a collaborative and knowledge-generating activity, in which students' productions are placed at the center of interpretations and feedback construction (see Cobb et al., 2009; Franke, Carpenter, Levi, & Fennema, 2001).

In mathematics education research, collective discussion is seen as an important tool for constructing knowledge in a social way (e.g., Bartolini Bussi, 1996) and its important role for developing new awareness about errors and nonstandard strategies is also well known in teacher education (Levin, 1995). Spillane (2005) observed that, in mathematics education PD, knowledge is usually gained by attending lectures given by an expert ED, whereby teachers are merely passive listeners, and "when they do talk they ask clarifying questions or acknowledge that they agree or understand" (p. 394). Thus, a further aim of the design and the implementation of the interpretative task (using an ICI cycle) is to disrupt this vicious cycle of teachers being passive listeners, prompting them to assume an active role in their learning—a strategy they should implement when working with their students. Moreover, it is widely established that, when teachers work and learn through collaboration, this can have a crucial positive effects on their practices (Jaworski et al., 2017; Robutti et al., 2016).

In our approach, focusing on the PTs' collective discussion on students' productions related to a specific mathematical problem, the aim is to develop PTs' IK based on social interactions among peers under the guidance of the ED. The basic idea is that the interpretative task involving students' productions, used to measure the PTs' IK level, can stimulate subsequent peer discussions in the teacher group. Owing to its nature and structure, the task should prompt the PTs to develop novel insights into the mathematical aspects of students' productions. Consequently, IK development is transformed from an individual to a collective activity within the teacher group—a transformation characterized by the evolution of community's norms. This evolution is facilitated by the social setting, which is seen as crucial for the development of PTs' IK. The diversity of reasoning, reflecting, and participating in collective discussions of each individual PT represents a resource for the ED who orchestrates collective discussions, in order to identify mathematical and pedagogical issues and develop the IK. The ultimate goal of this process is an evolution of a teacher group into a professional teaching community (Cobb et al., 2009). Cobb et al. (2009) identified four types of norms pertaining to the evolution of a teacher group into a professional teaching community: (a) *norms of general participation*, (b) *norms of pedagogical reasoning,* (c) *norms of mathematical reasoning,* and (d) *norms of institutional reasoning*. The authors further argued that the evolution of norms of one type creates conditions within the teacher group for the evolution of norms of another type:

> For example, shifts that occurred in norms of mathematical reasoning appeared to make possible subsequent developments in general norms of participation. In particular, the norm of challenging others' thinking in mathematics discussions did not emerge until it had become normative for the teachers to develop more sophisticated arguments for justifying their mathematical reasoning. (Cobb at al., 2009, p. 185)

Methodology

Sample

The sample for this study consisted of 34 Master of Mathematics students at an Italian University, who were supposed to have a strong mathematical knowledge, having already completed a Bachelor Degree in Mathematics. All participants were enrolled at a Mathematics Education course, held by one of the authors, which covered several topics, such as number sense, symbol sense, algebraic symbols, etc. As the course is not compulsory, it is typically chosen by students who want to become secondary school teachers. Hence, all study participants are considered PTs.

Task and Activities

The *Interpretative Task*, proposed to this group of PTs by the ED at the beginning of her course, consisted of two parts. Part 1 (Figure 1) involved providing a solution to a problem adapted from the annual Italian National Assessment (2010–2011) for Grade 10 students, released by INVALSI (Istituto Nazionale per la VALutazione del Sistema educativo di Istruzione e di formazione).

Part I

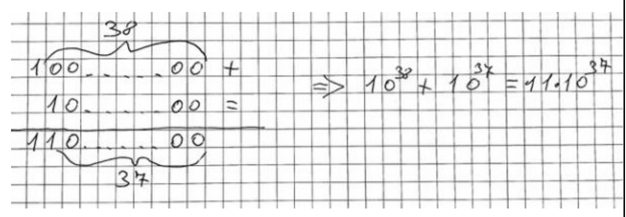
1. Consider the following item taken from the Annual Italian National Assessment (INVALSI) for Grade 10.
The expression. $10^{37}+10^{38}$ is equal to
   A. 20^{75}
   B. 10^{75}
   C. $11 \cdot 10^{37}$
   D. $10^{37 \cdot 38}$
2. What is the right answer? Explain why, moreover, discuss why the other ones are not acceptable.

Figure 1: Part I of the Interpretative Task

In Part II (Figure 2), the PTs were asked to interpret seven student productions, chosen by the researchers, as they are deemed suitable for stimulating interesting collective mathematical discussion among the PTs and potentially supporting the development of their IK. Due to the lack of space, only Ciro's production is shown here (for more information, see Mellone, Romano, & Tortora, 2013).

Part II

2. Piera wants to explore with her 9th grade students the above item and she gave the item during a classwork.
   i) For each of the students' productions, indicate if you think it is mathematically correct or not.
   ii) Provide constructive feedback to students, in particular, build possible questions to help them grow in their mathematical knowledge.



Ciro's production

Figure 2: Part II of the Interpretative Task and one of the seven students' productions

For this study, the Interpretative Task involved three activities and was implemented in the *Individual−Collective−Individual* (ICI) cycle format. The first activity lasted 60 minutes and comprised of *Individual Work* by the PTs, who were asked to solve the mathematical problem as well as interpret student productions. The second activity lasted about 90 minutes and involved a *Collective Mathematical Discussion* on the Interpretative Task. It took place in the same class immediately after the individual work and required the PTs to discuss the Interpretative Task, as well as their individual answers to the problem. It was facilitated by the ED, whose aim was to let PTs' different interpretations and perspectives on the students' productions emerge, as well as to prompt the participants to compare their individual productions with those of others in order to identify some mathematical aspects not previously considered. The main phases are: *Opening:* the ED focuses on the topic of discussion with questions such as *What about his production?*; *Dialectic:* the PTs discuss about their interpretations of the students productions, linking them to their own productions, with few interventions of the ED; *Processes Explicitation:* the ED underlines similarities/divergences of solutions/interpretations, and by mirroring techniques (repeating, summarizing or integrating) focus on the error as a resource for building new mathematical knowledge; *Additional Aspects Explicitation*: the ED highlights the emerging mathematical additional aspects. The third activity consisted of requesting that PTs provided an *Individual Extensive Report*, as a part of which they were instructed to describe their experience with the preceding two activities and to give a "final" interpretation of the students' productions. This task took place four weeks after the collective discussion, in order to allow the PTs to reflect on their experience, as well as on the mathematical/pedagogical value of this type of exploration.

Data Collection

The data for the present study was collected in different forms. Specifically, after the individual work, PTs' written answers were collected and were subjected to analysis supporting the design of the collective discussion. The collective discussion was audio-recorded and transcribed, while the PTs' written individual extensive reports were collected, and both sets of data were thematically analyzed.

Analysis

As is often the case in a design research (Cobb et al., 2009), the analysis of data pertaining to each activity allowed us to plan the subsequent activities. In this study, our goal was to understand whether an ICI cycle can be adopted as a methodological approach to support the evolution of the PTs' group in a professional teaching community and the PTs' IK development. In particular, our analyses focused on the effects of norms of general participation, pedagogical reasoning, and mathematical reasoning, and whether these can evolve to support the development of the PTs' IK via the transition from an individual activity to a collective one within the teacher group.

First Activity: Individual Work

Due to limited space, we give only a very brief overview of PTs' individual work. Our analysis revealed that all 34 PTs correctly solved the mathematical problem. However, although the teachers included in our study sample are supposed to have strong mathematical knowledge, the Interpretative Task revealed some gaps in their IK. Indeed, three PTs failed to provide any interpretation of Ciro's production, two PTs considered it incorrect, while 29 PTs' considered it correct. The PTs who considered Ciro's production as incorrect justified this evaluation by stating that a correct production has to incorporate the powers rule, as exemplified by Ornella's comment: "Even if the result is correct the reasoning is wrong because he didn't use the powers rule." Several PTs raised doubts about the range of applicability of Ciro's reasoning. For example, Anna observed: "It is ok, but [what would happen] if the difference between the exponents is larger?".

Moreover, only four PTs provided written feedback. However, even in these cases, their comments were largely limited to a request for a solution by means of the powers rule or algebraic symbols.

Second Activity: Collective Discussion

The analysis pertaining to the second activity focused on the diversity of the teacher group members, of their reasoning and reflections, and of the ways of participating in the collective discussion, orchestrated by the ED, about Ciro's production. In particular, analysis of the discussion transcript revealed presence of steps through which new mathematical awareness is developed as a part of such group interactions. Below, we provide an excerpt exemplifying how PTs' individual interpretations emerge and how these are compared with those of others.

Francesco: He is very visual. He writes each number with the relevant zeros and then he adds them; he doesn't use the properties of the powers.

Antonella: But with other bases different from ten, he can't do that. If for example we have $2_{37}+2_{38}$, he couldn't consider 2 and then a string of zeros. So, it works only in this case.

Samuele: But the request is to argue about this case, not to find a general rule, and he did it. Then, obviously one can prompt him to reflect on the generality or not of his proceeding.

Roberto: Moreover, whatever is the base I can write the number in, this base of representation and the number would appear as "one" followed of a series of many "zeros" [zeri in Italian], so the proceeding doesn't depend on the base.

Educator: What is going on here seems really interesting. Antonella is trying to reflect about the range of applicability of Ciro's proceeding, whereas we are reflecting on the suitability of the chosen representation according to this particular task. In this case, [the question should be] is it better to write the number as decimal alignment or as power? Roberto is prompting us to think that the base of the power is not a limit for Ciro's reasoning because if, for example, we have two powers of 2, if we represent them in base 2 we have the same writings as decimal alignment, and we can use Ciro's strategy. But if we reflect on the range of applicability and the limits of Ciro's proceeding, what can we comment on?

Anna: If the difference between the exponents is very large, it is difficult to use his symbolism and it would be difficult to use it with bases that are not whole.

Educator: Interesting. We are trying to establish the limits of this writing in column with the ellipses in place of the zeros. It has to have some limits in comparison with the compactness and the reification power of the algebraic writing [the ED writes on the board $x_n + x_{n+1} = x_n + x_n x = x_n(x + 1)$]

Francesco: For me it is brilliant because Ciro doesn't apply any rule but he proceeds with his own logic. His uncomfortable, so to speak, representation seems to offer a window into the rules to which we are trying to give sense.

In this excerpt, it is important to highlight the step in which Roberto, who has played a crucial role in the discussion, responds to Antonella's comment regarding the crucial mathematical issue about the generality of Ciro's procedure, by noticing that Ciro's process is also valid when applied to other power bases. This comment reveals that he possesses flexible mathematical knowledge derived from the topic introduced by Antonella. In this sense, we can appreciate the effectiveness of a collective discussion that gives participants the space and time to explore the emerging of issues such as the one that arose in this case, whenever such discussion is designed with such an aim. The norm of challenging others' thinking, put in practice by Roberto, can emerge, as it has become normative for PTs' group to develop more sophisticated arguments to justify their mathematical reasoning. At this point, Anna raises the issue of applicability of Ciro's proceeding when there is a very large difference between the exponents or in case of non-whole bases. However, the ED does not give space to these reflections, because her intention is to lead the discussion toward more meta-mathematical issues about the use of arithmetic representations, in particular the decimal alignment vs. the algebraic ones. The comment following the ED's intervention seems really interesting: Francesco describes Ciro's production as "a window into the rules," pointing toward the possible use of Ciro's production in the class work.

Third Activity: Individual Extensive Report

The analysis at this stage of the study focused on identifying a higher level of PTs' IK, with respect to the first activity, evolution supported by the previous collective discussion. When examining the PTs' extensive reports, we first focused on their "new" interpretations, striving to identify any shifts in their focus of attention. Afterwards, a joint analysis was conducted in order to perceive the changes in PTs' interpretations. Here, we present only a case study regarding the interpretations given by Gennaro. By triangulating information obtained in the course of the three activities, we recognized a radical change in Gennaro's interpretation of Ciro's production. In the following excerpts, denoted as Before (BD) and After (AD) the Discussion, we provide evidence of Gennaro's IK development.

Gennaro (BF):	Ciro reached the correct answer by a more practical method than those employed by his peers. In addition, the formalism seems original. He appears to have a strong expertise in the calculations with powers of 10, which highlights their significance and the importance of handling them correctly. Still, his method seems limited to powers of 10. It would be interesting to see how Ciro would proceed if presented with a different base.
Gennaro (AF):	Ciro's argument is of an arithmetic character. Nonetheless, it allows us to appreciate some deep algebraic insights. Moreover, although it seems confined to powers of 10, it can actually be generalized to any base, if one represents the number in the base of the power. Hence, from Ciro's production going further, it would be possible to study the tables of operations in different bases, or even the divisibility rules in bases other than 10.

In the BD interpretation, Gennaro acknowledges the originality of Ciro's method, while noting that it is limited to powers with base 10. In the AD comment, Gennaro shifts his focus, whereby his arguments reveal his awareness that Ciro's method can be applied to other bases (indeed, 100…0 always represents the *n*-th power of the base). As indicated in the analysis of the second activity, this fact was noted by Roberto during the collective discussion. For Gennaro, this discovery was so important that it became an essential part of his individual extensive report. Consequently, Gennaro's IK benefitted from the mathematical discussion on Ciro's production. He reconceived the systems of representation of numbers on different bases, motivating him to explore the true meaning of digits, as well as strings of numbers.

Discussion and conclusions

In this study, we aimed to elucidate how an ICI cycle can support the evolution of the PTs' group in a professional teaching community and aid in the development of their IK. Our joint analysis revealed that the ICI cycle adopted in this case promoted an evolution of norms of general participation, as well as pedagogical/mathematical reasoning, supporting the development of the PTs' IK via a transition from an individual activity to collective one within the teacher group.

With respect to the *first activity*, analysis of the individual interpretative task revealed a low level of IK. Indeed, the PTs, in carrying out their individual activities, adopted the norms of pedagogical/mathematical reasoning (Cobb et al., 2009) based on the "assessment" and not on the "interpretation" of the students' production. They focused on building new mathematical knowledge, asking primarily for a production that is more adherent to their vision of how the task has to be solved, rather than proposing feedback starting from the students' productions. In the context of the *second activity*, we can see a collective development of the PTs' IK. It prompts an evolution of norms of general participation as well as norms of pedagogical/mathematical reasoning in an interdependent way (Cobb et al., 2009). We can also observe in each PT an evolution, with respect to the first activity, of the norms of mathematical/pedagogical reasoning based on what other group members were saying. These changes seem to support an evolution of norms of general participation. Moreover, this collective discussion not only stimulates a deeper reflection on the different students' productions, but also prompts the evolution of norms of general participation, which in turn leads to the evolution of norms of mathematical/pedagogical reasoning, transforming it from evaluative to interpretative. The *third activity* involved institutionalization, as PTs were required to summarize their experience of both individual work and the collective discussion in an Individual Extensive Report, as exemplified by Gennaro's case. In his BD interpretation, we observe the presence of norms of mathematical/pedagogical reasoning based on an evaluative vision about the limits of applicability of Ciro's procedure. The AD interpretation is different, signifying that the evolution of the norms of mathematical/pedagogical reasoning, correlated with the evolution of the norms of general participation (Cobb et al., 2009), has occurred during the collective discussion. The evolution of the norms allowed and supported the development of IK, constructed in a social way through the interactions facilitated by the second activity, based in turn on the first activity.

As future perspective we would like to conduct a longitudinal study evaluating the impact of these PD activities designed around interpretative tasks, on the participating PTs future teaching practices.

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A COLLABORATION MODEL FOR THE TRAINING OF IN-SERVICE SECONDARY MATHEMATICS TEACHERS

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In this document, we describe a collaboration model used in a training program for in-service teachers of secondary mathematics. We briefly describe the curricular design of the program and focus on the methodological aspects that promote interdependent learning. We found that the program contributes to the learning of in-service teachers in relation to their planning practices in, for example, the use of curricular documents and their prediction of student errors.

Mathematics teachers do not work or learn alone. Teaching and learning are social practices and collaborative enterprises (Secada & Adajian, 1997). For these reasons, research in teacher training has been concerned with teacher professional development programs that are based on sociocultural views of learning (Lerman, 2001; Llinares, 1998). In particular, Wenger's social theory of learning (1998) and his notions of community of practice and interdependent learning provide a conceptual framework for the investigation of the learning processes that take place when teachers work together. However, although education promotes the learning of teachers and informs what teachers learn in social terms, very little is known about how these contexts allow learning (Graven & Aurbaugh, 2003). In addition, there has been little research that examines the specific interactions and dynamics that occur in these contexts. Hence, "one analytic task, therefore, is to show how teachers, in and through their interactions with one another and with the material environment, convey and construct particular representations of practice" (Little, 2002). As Krainer (2003) has stated, there is much to be asked in relation to the role of social learning in teacher training: "To what extent can an approach like 'community of practice' be applied to learning at schools and universities? What can we learn from 'learning enterprises'? What implication for research in teacher education has an approach that builds on 'community of practice'? (p. 96).

The work that we present in this document relates to the theme of "Contexts, forms and results of collaboration between teachers of mathematics" which is a focus of the ICMI Study Conference 2020. It aims to contribute to the reflection on interdependent learning processes in the context of secondary mathematics teacher training programs. We address the following research question: What effect does the program have on teacher planning practices? We firstly present a postgraduate in-service training program for secondary mathematics teachers that is conceptually based on the social theory of learning. Next, we describe the curricular design of the program and focus on the methodological aspects that promote interdependent learning. Finally, we reflect on the contribution of the program in the learning of in-service teachers in relation to their planning practices.

Training program

The Master's Degree in Mathematics Education at the Universidad de los Andes (Colombia) is designed to deepen the pedagogical content knowledge of in-service secondary mathematics teachers. Its purpose is to contribute to those teachers' ability to design and implement learning opportunities that lead to the improvement of the mathematics performance of students. The program provides opportunities for teachers to develop competence in the analysis of topics taught in school mathematics, to predict students' learning processes of those topics and use that information to design and develop curriculum (Gómez, 2018).

The structure of the training program is based on the model of didactic analysis, as a conceptualization of the activities that the teacher performs in order to plan, carry out and evaluate their mathematics lessons (Gómez, 2002, 2007). The didactic analysis is configured around a cycle that comprises four stages of analysis: subject matter, cognitive, instructional and performance. Each stage of analysis brings into play pedagogical concepts on the basis of which in-service teachers can: (a) identify and organize the multiple meanings of a specific mathematical topic (subject matter analysis); (b) select the relevant meanings for instruction and predict the performance of schoolchildren when tackling tasks (cognitive analysis); (c) select the tasks that can contribute to the achievement of the learning objectives (instruction analysis); and (d) evaluate their lesson planning with the purpose of producing information that is relevant for subsequent cycles (performance analysis).

In summary, the program provides teachers with the opportunity to deepen their mathematical and pedagogical knowledge for planning, implementing and evaluating their mathematics lessons.

Organization of academic activities

The program is delivered in blended mode and consists of eight consecutive modules that span over four semesters. The students are organized in groups of three or four people. Each group works on a topic of school mathematics. Each group analyses the topic it has chosen, produces a curricular design, implements it in the classroom, evaluates the implementation and then revises the design. Each group is accompanied throughout the two years of the program by a mentor. The role of the mentor is to comment on the work of the group and to guide the group in each of its activities. The groups also receive the ongoing support of the coordinator who is in charge of the management of the program.

At the end of the first five modules, each group produces a design of a series of lessons on its topic. The last three modules focus on the implementation of that series of lessons in one of the group member's schools. The group collects and analyses the information that emerges from the implementation and produces a report that includes the final version of their curricular design (see Figure 1).

Each module lasts for nine weeks and is composed of four activities of two weeks duration that share the same methodological structure. For example, in the second activity of the module on cognitive analysis the groups are asked to predict the difficulties and errors that students may make when they engage in the group's chosen topic .

The responsibilities of groups and teachers are the same in every activity (see Figure 1). Groups submit a draft of their work at the end of the first week, after having solved their doubts, if needed, with the professor. This meeting takes place on Skype. Four days later they receive their mentors' revision of the draft. At the end of the second week, groups can meet on Skype with the coordinator in order to resolve any doubts concerning their presentation. The next day, the groups, the professor and the coordinator meet at the university for a four hour session. In the first part of this session, each group has nine-minutes to present its work. After the presentation, the in-service teachers can make comments and ask questions about each group's presentation. In the third part of the session, each group reacts to those comments and questions. The session ends when the professor and the coordinator summarize the main issues of the discussion and comment on the presentations. Four days later, each student submits his comments and criticisms to the work of a group assigned to him.

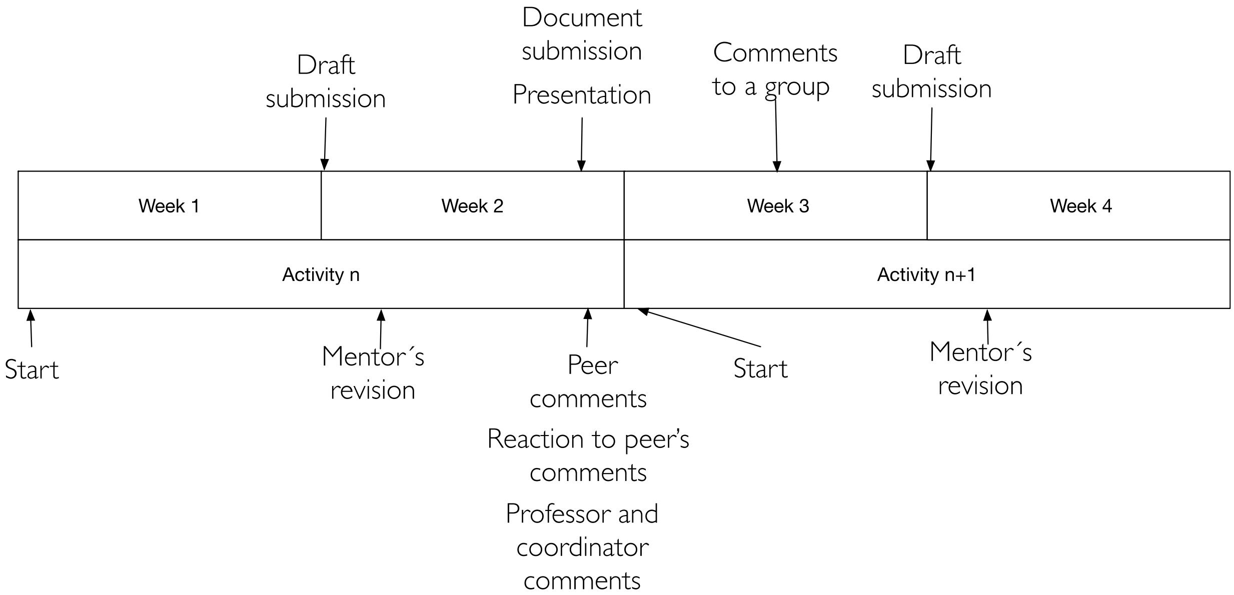

Figure 1. Methodological structure of an activity

Interdependent learning in the program

In this program, the in-service teachers learn by working in a virtual or face-to-face manner, interacting with the professor and mentors and each other to compare and discuss their work with that of other groups. Therefore, the learning of the in-service teachers is fostered through group work and group interaction, meetings with the professor, interaction with their mentor, other students and other groups, and through autonomous work.

Team work. The in-service teachers work in groups throughout the program. The members of each group work collaboratively to prepare the draft and the final document with the results of each activity. The collaboration among the members of a group is stimulated by the fact that they have to submit a draft, a presentation and a final document with their work by specific dates and times. This collaborative work implies inquiring about the requirements of the activity in relation to their school mathematics topic. They have to compare their points of view and reach agreement on the presentation of the result of their work. Through these processes, a collaborative environment is fostered in which each member of the group learns and contributes to the learning of their colleagues. The coordinator of the program contributes to the functioning of the group and promotes interdependent learning among the members.

Meetings with the professors. During the first week of each module, the in-service teachers attend a classroom session in which the professor introduces the curricular design of the module and presents the activities that the groups must carry out. During the following weeks, groups meet with the professor every fortnight in order to allay concerns that may arise during the completion of the draft document. They also interact with the professor during the face-to-face session.

Interacction with colleagues. In addition to promoting interdependent learning among members of a group, the program encourages interaction among students through the constructive comments and criticisms that each student makes to the final work of another group. Each group receives the comments of at least three colleagues from other groups and can react to these individual comments in the following face-to-face session.

Another interaction with colleagues arises from the discussions that are generated as a result of the presentations of each activity. Once the groups finalize their presentations, their colleagues make comments and critique their work. Each group reacts immediately to resolve the concerns that may arise at that time or that have been identified in the individual comments.

Interaction with the mentor. Each group has a mentor who comments on the work of the group. The mentor makes comments to the draft document and the group can react to those comments by making adjustments for their final document and presentation. Subsequently, when the results of each activity are presented in the final document, the mentor comments again on the work of the group. The professor takes the mentor's comments into account in the assessment of the group's work.

Autonomous work. Each in-service teacher is involved in individual work. This includes reading the module notes, watching videos of the professor, and engaging in his own inquiries in order to comment on the final document of another group. Each teacher, while contributing to the work of his group, has to reach agreements with his peers on the outcomes of the submitted group's work. In this way, each in-service teacher builds his own identity within the group.

Coordinator counseling. Every Friday, prior to the face-to-face session, the groups have the option to meet with the coordinator to resolve concerns relating to their presentation. The coordinator closes the face-to-face session with an evaluation of each group's team-work.

In short, the in-service teachers learn by putting pedagogical concepts into practice. They do this by analysing a specific mathematical topic, for which they produce, implement and assess a series of lessons. They work in groups and have to reach agreements in order to present the results of their work to their colleagues. They also have to contribute individually to the work of the group and to react systematically and periodically to the mentor's comments on the drafts and final versions of their work. They also have to observe the work of other groups on different mathematical topics, and comment on and critique the work of other groups. In Figure 2, we summarize the different products and interactions that take place in the training program that favour interdependent learning.

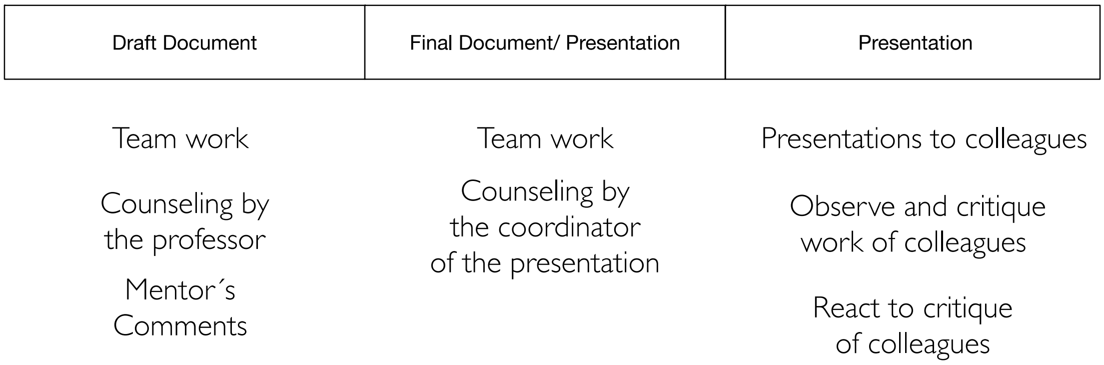

Figure 2: Interdependent learning in the training program

Inquiry into the learning of the in-service teachers

Below, we present preliminary results of the pilot study of the learning of a group of in-service teachers who participated in the program.

Sample and information gathering

The sample for this study corresponds to a group of 23 in-service teachers of the Master's Degree in Mathematics Education of the Universidad de los Andes, of whom 12 were women. The majority (90%) taught in public schools, 75% had undergraduate teacher training and the majority (89%) had more than 5 years of teaching experience. We identified the curricular planning practices of these teachers through a survey that each teacher answered twice: when entering the program and on completion of the program. Although the program is designed based on the idea of community of practice, the questionnaire is based on the learning expectations of the program in relation to the teachers' planning practices. The survey asks in-service teachers to describe one of their most recent lessons. In so doing, the survey focuses on facts relating to the in-service teacher's current practice, not about the his opinions about ideal curricular practices.

The survey is organized into three sections: planning, implementation and evaluation. The questions include open ended and multiple-choice questions. In this paper, we focus on the planning questions. These were organized into five categories: use of curricular documents, conceptual dimension, cognitive dimension, formative dimension and social dimension. The cognitive dimension, in turn, was divided into expectations, errors and forecasts. The formative dimension was divided into the selection of tasks, student performances, teacher performance and sequence of tasks.

For each question, we formulated codes to allow us to classify the answers of the teachers. In some cases, we also formulated categories for groups of codes. The in-service teachers' responses were coded by text segments. Therefore, an answer to a question could be coded with more than one code. For example, a teacher claimed, "It is common for students to confuse the properties of addition with the properties of multiplication. So, I propose activities to differentiate them in each case". This teacher has anticipated a specific difficulty and, at the same time, he has proposed a way to address it. Each question's codes were weighted according to the expected response of an "ideal" teacher. An "ideal" teacher is one who had successfully satisfied the program's learning expectations.

We verified the normality of the data through the Shapiro-Wilk test with a confidence level of 95%, P value= 0,997. Finally, given that the sample is composed of the same subjects who completed the entry and exit survey, we used the t-Student test for paired data to compare the responses of the teachers and establish whether there are statistically significant differences.

Results in planning practices

For each category, we calculated the value of the t-statistic, the P value of a tail, and the rejection or not of the hypothesis on equality of means (α= 0.05). If the hypothesis of equality of the means was rejected, the effect size (Cohen's d) was calculated. There are three categories in which we found statistically significant differences. In the use of curricular documents (basic competency standards, guidelines and institutional syllabus), we found a positive difference (P= 0.025) and a medium effect (d= 0.55). This is the case of teachers who, before starting the training program, did not use the curricular documents or used them only to select content. Once the program has concluded, these teachers use those documents to address more curricular issues (e.g., learning objectives or assessment).

Another positive difference (P= 0.03), and with a little more effect than in the previous category (d= 0.61), concerns the student error forecast category. This is evidenced in the fact that the teachers at the end of the program take into account, in greater proportion, the possible errors the students may make when solving the tasks.

In the third category, related to the prediction of student performance (strategies not foreseen in the solution of tasks), the difference is negative (P= 0.03). This implies an opportunity for improvement of the program. For the other categories considered in the planning, we did not find statistically significant differences.

Regarding the dimensions of the curriculum, the cognitive dimension shows a statistically significant difference (P= 0.04) and a positive median effect (d= 0.52). Finally, when assessing the whole section of planning practices, we found that there is a statistically significant difference (P= 0.02) and a positive median effect (d= 0.57). Therefore, the training program seems to have a positive effect on the planning practices of its graduates.

Discussion

In this study, we have described the design, objectives and methodological framework of an in-service teacher training program that promotes interdependent learning. In this program, teachers are expected to develop a sufficiently in-depth pedagogical content knowledge of a subject that enables them to support the decisions they make in their lesson planning (Gómez, 2018). We showed that the in-service teachers work in groups on a single topic throughout the program and that group-work systematically promotes the processes of negotiating meaning between the members of a group. The comments of the mentor and critique of their colleagues (both written and oral) encourage these processes of meaning negotiation to generate doubts, raise differences of opinion and require a solution to the problems that are posed by the mentor and their colleagues. By solving these problems and reaching agreements, each group builds a shared repertoire of concepts, procedures and techniques that materialize their learning. Additionally, individuals build their own identity (and develop their knowledge) by contributing to the group's work, and by commenting and critiquing the work of a group different from their own (Pinzón, Gómez, & Acebedo, 2015).

The methodology used to measure the learning of the teachers participating in the training program was the comparison of the characteristics of their planning practices, before and after passing through the program, by means of a questionnaire. The use of this questionnaire allowed us to formulate and verify hypotheses about the extent to which they put into play what they learned in their classrooms.

Although the results for this sample of teachers does not seem to account for the total achievement of the program's goals, this pilot study provides us with relevant information on opportunities for improvement, in the short and medium term. Interactions between and within groups can be affected by factors such as pre-program training (graduates and non-graduates) and classroom experience (more or less than five years). Likewise, this kind of study can be complemented with other studies that use class observations.

The results of the study show us that participation in the training program contributes to the improvement of teacher planning practices. This is evidenced primarily in the use of curricular documents and in the prediction of errors that students may make when solving tasks. These results may be useful for those who are interested in the evaluation of mathematics teacher training programs whose focus is on classroom practices.

Our findings align with other studies that show that teachers plan more from intuition than from academic learning (Miller et al., 2014) and that they are more concerned about the tasks to be assigned to students than what they aim to achieve (Akyuz et al., 2013; Strangis, Pringle, & Knopf, 2006).

Unlike other teacher training programs, this program does not teach theory, nor does it aim for teachers to become researchers. Teachers collaborate in groups by addressing problems related to their practice and, in this way, construct meanings about concepts and curricular techniques that, when used in practice, can contribute to the learning of their students.

We have described a mathematics teachers' training program that promotes teachers' collaboration in order to contribute to their learning. We have shown that teachers improve their planning practices. How this type of collaboration enhance teachers' learning and contribute to teachers' practice is still an open question.

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LESSON STUDY: A MODEL FOR TEACHER COLLABORATION IN IRAN

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Main purpose of this study is adopt a lesson study model for Iran cultural context. In this study six secondary school mathematics teachers work together for developing a new modified research lesson on mathematical trigonometric ratios. They participate in fifteen sessions during 8 months and developed eight research lessons. Results of this study show that modified lesson study cycle work effectively in Iran context and teachers teaching was changed during this lesson study. Analysis of data also show that Joint attention, Sharing experiences, Observing teaching challenges and feeling need for change are mechanism of changes which lead to useful changes in research lesson . Teachers noted that lesson study raised teachers' consciousness about needs of students, group working improve teachers ability in teaching, and life long learning occurred for teachers, as affordances of using lesson study approach. They revealed constraints in time and administrative structure as big challenges and limitation for conducting lesson study in their actual classroom practice.

Introduction

In recent decades, many scholars, such as Stigler and Hibert (1999), have emphasized the influence of teachers' knowledge on quality of mathematics learning. According to Gooya (2001), a good and suitable curriculum succeeds on condition that implemented by powerful and educated teachers. But, curriculum changes are not much interest for mathematics teachers because they think the curriculum changes are inadequate for favorable change at school level (Clements and Elerton, 1996). This unwillingness of teachers of mathematics, both to the developmental plans derived from academic research findings and to the programs announced by the policy maker, can indicate a low relationship between theory and action. Most of teachers didn't see the relation between their classroom experiences with theories which they learnt during their pre-service education at university. Specially most of teachers reported that until they began to teach themselves, they have not learned anything valuable about teaching.

There are some approaches in literature review in order to improve teacher collaboration for supporting curriculum change. For example through lesson study - jugyokenkyu- which is a method of continual professional development that used in Japan for the first time, teachers can collaborate for improving their teaching method upon latest research finding. This method was introduced by Yoshida (1999) and then was distributed with a book with title "Teaching Gap" (Stigler and Hiebert, 1999) to English speaking world. They (1999, pp. 112-116) summarize lesson study through eight steps: defining the problem (Step1); planning the lesson (Step2); teaching the lesson (Step3); reflecting and evaluating the lesson (Step4); revising the lesson (Step5); teaching (Step6); the revised lesson (Step7); reflecting and evaluating, and sharing results (Step8).

Lesson study and action research introduced in Iranian schools by ministry of education for several years, but there are a few evidence for improving teaching method of teachers. Although some research papers in Farsi (official language of Iran) report results of research about outcome of ministry of education program for distributing action research in schools all over the country (e.g. Chaichi, Gooya, Mehrabani, and Saki, 2006). But, the question that arises here is why lesson study does not work properly in Iranian teacher professional development like Japanese school? I find some reasons for this question during my visiting Japanese school when Japanese teachers doing actual lesson study. Actually the context and climate of schools in Iran and Japan are completely different. So, modified model of lesson study should be developed which consistence with Iranian culture.

In this study a local (modified) model for lesson study will be presented which is suitable for Iranian teachers' professional development. Specifically, following research questions were guided present study.

- Whether or not Japanese lesson study successful in Iran contexts? Whether Japanese lesson study does work in the context of Iran?
- What are the mechanisms of change in teaching math?
- Which forms of math teacher collaboration/ model for lesson study (lesson study cycle) is appropriate in Iran context?
- What are the affordances and limitations of this form of math teacher collaboration?

Literature Review

The Lesson Study approach is a form of professional development which practiced in the Japanese schools. The Japanese teachers conduct lesson study in school through in-service teacher training program. Japanese name for lesson study is, the "konaikenshu" (校内研修), to develop, research, and share as well as to critique new teaching practices. The Lesson Study model has been started in Japan in the 1880s under the Meiji government (Isoda, 2007). Lesson study first introduced to worldwide through Yoshida's (1999) doctoral dissertation, and after publishing the book, "The Teaching Gap" by Stigler and Hiebert (1999) lesson study introduced to international educators. They introduced lesson study as part of TIMSS video study. A Japanese LS typically includes four steps (Lewis, 2002) as follows:

- **Goal Setting**: consider students' current characteristics, their long-term goals and development. Identify gaps between these long-term goals and current reality.
- **Planning**: Based on studies of teaching materials (textbooks, curriculum and teacher guides, relevant research), collaboratively plan a "research lesson" to address the identified goals.
- **Research Lesson**: One team member teaches the research lesson and other members of the team observe and collect data.
- **Reflection**: Share and discuss the data collected from research lesson, draw implications for redesign, and write a report.

Nowadays, lesson study practiced in many countries around the world. For example Doig and Groves (2011) reported that lesson study approach has spread to Western countries and Southeast Asian countries, such as Thailand, Indonesia and Malaysia. Specifically, Lewis et al. (2006) report that lesson study were taking place in over 335 schools across 32 states in USA. In UK a large-scale adoption of lesson study launched (e.g. Department for Children, Schools and Families, 2008). In the Netherlands (Kaskens and Goei 2016), Ontario, Canada (e.g. Miller 2010), and Australia (Pierce and Stacey 2009) modified model of lesson study has been implemented.

Lesson study introduced to Iranian educator through publishing several papers in national journals. One of the pioneers paper in this field was Gooya (1991) in quaterly journal of education. After that "The Teaching Gap" traslated into the Farsi (official languge of Iran) and then several master thesis in the field of mathematics education conducted around the countries. For example, Khakbaz (2007) used modified lesson study model for middle school mathematics teachers' professional develop. She finds that modified lesson study model can be useful for teacher professional development.

A number of studies (e.g. Chen and Yang 2013; Lewis and Tsuchida 1998; Stigler and Hiebert 1999), shown that lesson study can transform teaching. Some other studies (e.g. Lewis et al., 2009; Lewis et al., 2006; Lewis et al., 2011; Lewis et al., 2013; Murata et al., 2012; Puchner and Taylor, 2006) demonstrate that lesson study can promote teachers knowledge. Specifically, Lewis et al. (2009) discussed that lesson study can improve teachers' knowledge and beliefs.

Research Method

In this study six secondary school math teachers participated voluntary. All of teachers were female. Their teaching experiences varied from 10-20 years at secondary school in one the south east province in Iran. They engaged in the process of lesson study for 8 months and they repeated lesson study cycle for two time. Totally 15 sessions devoted to group discussion and lesson plane meeting during this lesson study and finally 8 research lessons was implemented. Mathematical content which in this lesson study investigated was trigonometric relations.

For data collection, several tools were employed such as interview with individual teachers regarding their learning, observation of teaching research lesson which implemented by one of teachers in group, fields notes of all 6 teachers which participated in the study, voice and video recording of the research lessons and students' worksheets. For data analysis two sessions of research lesson were compared by the constant comparison method (Glaser and Strauss, 1967) and mechanism of changes was identified. There are several models for lesson study cycle, such as below cycle (figure 1) which based on Lewis et al. (2006). But in current study, modified lesson study cycle (figure 3) was used.

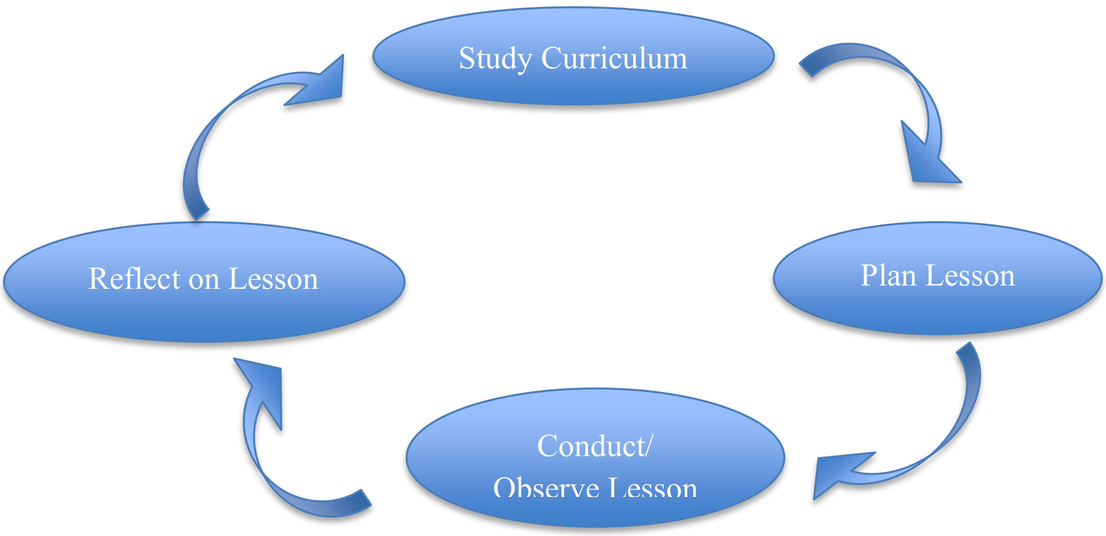

Figure 1: Lesson Study Cycle based on Lewis et al. (2006)

Results

Results of this study show that Japanese lesson study work in the cultural context of Iran. Changes which occurred in teaching of six teachers who participant in this study revealed efficiency of lesson study. For example one of teachers says that from now on, she start her lesson with diagnostic test for finding weaknesses and strengths of students.

Analyses of data show that there are at least four significant changes in research lesson in comparison with initial lesson plan as below.

- Changes in lesson structure;
- Changes in whole class discussion;
- Changes in mathematical tasks that introduced during teaching;
- Changes in question which teachers ask from students during problem solving.

Mechanisms of changes which lead to above changes were:

- Joint attention;
- Sharing experiences;
- Observing teaching challenges and feeling need for change;
- Brain Storming.

When participant teachers in this study was asked for talking about affordances and challenges of using this model of lesson study in their actual practice, they reveal below affordances:

- Lesson study raised teachers' consciousness about needs of students;
- Group working improve teachers ability in teaching;
- Lifelong learning for teachers through learning from each other.

They also mentioned constraints in time and administrative sutures as big challenges for conducting lesson study in their actual classroom practice. For example in teaching of trigonometry ratios, group of teachers firstly define overall goals as below.

- Familiarity with trigonometry ratios;
- Familiarity with trigonometry formulas;
- Finding trigonometry ratios with using a right triangular;
- Familiarity with application of trigonometry;

Secondly, group of teachers start to discuss about their usual teaching method in regular math class. Teacher c said that I start my teaching with this question "how we can calculate the height of a building with using math?" and then I teach trigonometry ratios trough drawing a right triangular. But, teachers B and C said we start with activity which proposed by math textbook. We ask student to do this activity by themselves and then we introduced trigonometry ratios. Teacher D said, I start with figure 2 for introducing trigonometry ratios.

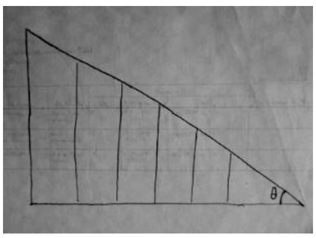

Figure 2: Starting point for introducing trigonometry ratios

She said in each triangle definition of each trigonometry ratios give us same results. Indeed by keeping the angles constant, the sides become larger as the ratio of the opposite side to the chord and the side adjacent to the chord remain constant.

After this initial discussion, design for plan lesson was started. First issue was finding a suitable question for diagnostic assessment of students. Group of teachers start to discuss about drawing a triangle with given data. Teacher A believed that students have to able to draw a right triangle which one of its angle is 26 degrees. Main purpose of this activity was check students' ability to work with protractor/goniometer. So, below two questions designed as diagnostic test.

- Draw a right triangular and measure its angels;
- Draw a right triangular with an angle with 30 degrees.

After reviewing literature about lesson study cycle, below model was created and used in this study (see Figure 3).

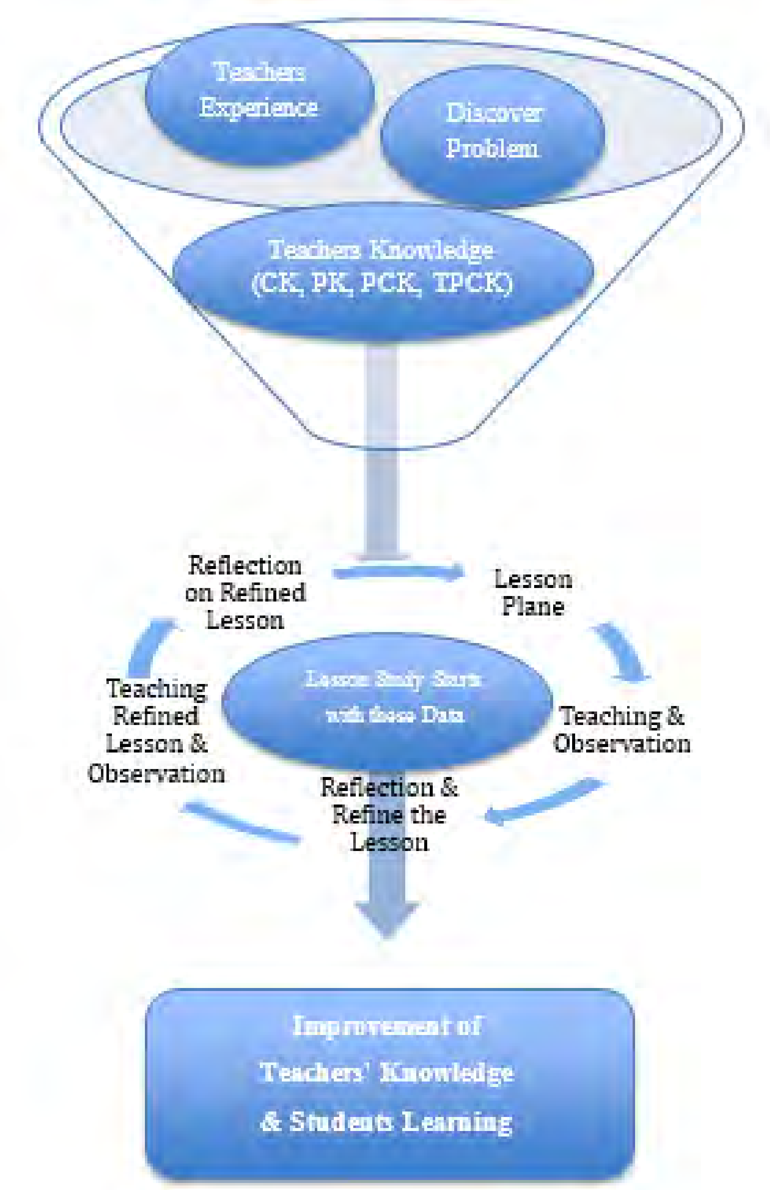

Figure 3: Lesson Study Cycle used in this study

Discussion & Conclusion

According to Yoshida (2008) although the process of implementing lesson study seems simple on the surface level, but implement lesson study effectively is not an easy task. Several studies report on early efforts to import lesson study into new cultural contexts. But in some cases lesson study in new context was not successful. For example Stigler and Hibert (2016) explain the situation in United States as below

> In the United States, we tend to cycle through ideas quickly. We brainstorm easily, and we think of ourselves as very creative. When we learn about a new idea—such as lesson study—we are interested in “trying it out” to “see if it works.” Usually it doesn’t—not because the idea isn’t a good one, but because we underestimate the time and effort it will take to actually implement the idea in an effective way. Because of this, because of our impatience, we often dismiss ideas prematurely, moving on to the next idea before we have done the hard work of maximizing what we can get from an idea well implemented (p.586).

Yoshida (2012) pointed out that some modifications of lesson study in the United States may negatively impact on effectiveness of lesson study. For examples in some cases:

- lesson study focused on developing exemplary lessons rather than developing teachers’ knowledge and expertise in teaching;
- Lesson study in which others watch videotaped research lessons rather than observe live lessons.

There have been various adaptations of Japanese lesson study in different countries. However, some adaptations of lesson study outside Japan (Fujii, 2014; Yoshida, 2012) include misunderstanding of core components and/or ideas of Japanese lesson study. For example, Fujii (2014) identified four types of inappropriateness when adapting lesson study as below:

- observing lessons without any questions to address;
- sticking to lesson plans without using evidence of students’ learning;
- teaching for solving problems rather than teaching through problem solving;
- focusing debriefing on the teacher rather than on the teaching;

However, as Stigler and Hiebert (1999) point out, teaching is a cultural activity, so for improving teachers teaching we have to consider gradual change and we have to mentioned to the cultural assumptions underpinning teaching and learning. So, the question about to what extent we can replicate lesson study in different cultural context still remain open (Lewis et al., 2009). In current study repeatability of lesson study approach in Iran context was examined. Results show that if adequate time spent on implementing lesson study approach, it can be useful and efficient for Iranian professional development. Through this lesson study a lesson study cycle developed and used by teacher during this study.

Finally according to Ponte (2017) which after reviewing research studies about lesson study, called for further research with a critical view in exploring affordances and challenges in conducting lesson study in new context; In the current study affordances and challenges for conducting lesson study in the cultural context of Iran has been investigate.

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THE DEVELOPMENT OF ELEMENTARY SCHOOL TEACHERS' COLLABORATIVE PRACTICE IN A LESSON STUDY

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This paper addresses the process of construction of professional collaboration in a lesson study that involved six elementary school teachers of Brazilian public schools, concerning length measure at grade 4. The participants worked collaboratively planning a lesson about this topic. The methodology is qualitative, with data collected by records and field notes, conversations and interviews. The results show that the professional collaboration that developed during the lesson study was facilitated by the dynamics of the different steps and, especially, by the relation among participants and teacher educators. Collaboration emerged from activities such as planning, classroom teaching, observation and reflection, especially given the encouragement and strengthening of the group insofar as teachers felt confident, encouraged and welcomed in the group.

Introduction

Professional collaboration, in the context of lesson study, constitutes a theme increasingly investigated by researchers from different countries (Lewis, 2009; Burroughs & Luebeck, 2010; Quaresma & Ponte, 2019; Richit & Ponte, 2017). Lesson study is a reflexive and collaborative teacher professional development process centered in teaching practice (Lewis et al., 2009; Huang & Shimizu, 2016). In lesson study, teachers work collaboratively through five main steps: *goal setting,* for a lesson focused on students' learning difficulties on a topic chosen by teachers; *lesson planning*, studying curriculum materials, producing tasks to promote students' learning and planning a lesson in detail; *teaching the research lesson*, by a teacher of the team, whereas the others observe and take notes about the students' actions and talk; *reflection*, analyzing and discussing aspects registered by the observers about the students' actions, questions, conjectures, strategies, and conclusions; and the dissemination of the *experience*. Lesson study gained much attention in Occidental countries as far as it begun to be developed in the United States and its results were disseminated in English language (Stiegler & Hiebert, 1999). From this perspective, we consider relevant to analyze and discuss the professional collaboration in lesson study, assumed as a professional development process, signaling key practices and situations of this process, and striving to understand how teachers experience collaborative professional practices. We are particularly interested in examining the promotion of professional collaboration in a lesson study carried out with six elementary school teachers, involved in a formative activity and a classroom practice quite different from what they are used to.

Professional Collaboration and Lesson Study

Professional collaboration, as a fundamental dimension of professional culture, constitutes a theme increasingly investigated, as it supports teacher professional development promoting professional learning and changes in professional practice (Xu & Pedder, 2015). We regard professional culture as the ways by which teachers relate to one another and act. That is, professional culture is represented especially in the "practice that they undertake, in the beliefs that underpin their way to understand the work that they did and they do, as well as the routines that motivate them in their actions in the school, with other teachers and, naturally, with their students" (Fialho & Sarroeira, 2012, p. 4).

Professional cultures reflect, on the one hand, the socialization process of teachers in the profession (Dubar, 1997) and, on the other hand, the sense of belonging to this profession and also to a specific school. The entry in the professional career, as well as the teacher professional development process, is strongly influenced by the prevalent professional culture in the school or in the subject group to which they belong (Marcelo, 2009). Fialho and Sarroeira (2012) add that the prevalent ways of professional culture in the school, the norms, the beliefs, the ways to act are (or no) assimilated by the teachers, and become reflected in their professional identity.

Hargreaves (1998) differentiates four ways of professional cultures by which the teacher work is undertaken and legitimized: individualism, collaboration, artificial collegiality and balkanization. In his view, collaboration characterizes the interaction that teachers establish when they are involved in a joint activity, sharing professional goals. Boavida and Ponte (2002) add that collaboration involves a "careful negotiation, joint decision making, effective communication and mutual learning in an enterprise that focus in the promotion of professional dialog" (p. 4). In this perspective, collaboration requires equality and mutual assistance among participants in undertaking a work that promotes collective development, which is different from cooperation which just focus in carrying out a collective work to reach a specific aim. Borges (2007) considers that "a professional collaboration culture is that one in which everything is shared, is discussed, in which support and aid required to learn is sought" (pp. 370-371). Hargreaves (1998) underlines that the confidence that emerges from sharing and from collegial support among teachers "leads to a greater disposition to carry out experiences and take risks, and, with these, to a commitment of teachers in continual growth, as an integral part of professional obligations" (p. 209). And, finally, Teixeira (199) considers that the teacher is "mainly a professional of relationship" (p. 161), because in the educational organizations, professional well-being and development of innovative practice and change depends, fundamentally, from professional relations established among teachers.

In a study that examines professional collaboration in a lesson study, Robinson and Leikin (2012) analyze two classes conducted by a mathematics teacher in an Israeli school. The research shows that the lesson study provided the teacher the opportunity to develop collaborative observation, collaborative consciousness and collaborative discussion/reflection, which the authors regard as fundamental mechanisms to promote teacher's professional development. In a similar perspective, Wake, Swan and Foster (2016) examine lesson studies developed by English teams, aiming to show teacher's professional learning through the development of problem solving processes, using the lenses of historical-cultural activity theory. The research shows that the development of tasks for research lessons, undertaken in a collaborative way, constitutes an essential mechanism to promote teacher professional development.

Based on teacher learning theory, Lewis et al. (2009), in a study with teachers who taught in North American elementary and high schools, provided a framework for interpreting why lesson study contributes to improving teaching and student learning from changes in teacher knowledge and collaborative learning. The authors highlight that a lesson study is a system of collaborative learning from live instruction, which involves investigation, planning, research lessons, and reflection, to create changes in teachers' knowledge and beliefs, professional community, and teaching resources.

Finally, they indicate that "shared tools (such as curriculum and assessment) and individual and community characteristics (such as beliefs about students and structures to support collaborative work) mediate teacher learning" (Lewis et al., 2009, p. 399). Xu and Pedder (2015), based on a systematic literature review of lesson study and learning study with both pre-service and in-service teachers, identified benefits of implementing lesson into four categories: teacher collaboration and development of a professional learning community, development of professional knowledge, practice and professionalism, more explicit focus on pupil learning, and improved quality of classroom teaching and learning. Finally, Huang and Shimizu (2016) highlight that teacher learning contributes to improve teaching and student learning, implement curriculum changes, share instructional products and resources, and improve the dynamic between theory and practice. They add that a supportive and collaborative context in a lesson study can develop teachers' confidence in experimenting with innovative practices and ideas, with an increase in their self-efficacy making a positive impact on students' learning. In addition, Burroughs and Luebeck (2010) stress that the lesson study allows teachers to carry out collaborative work in their professional practice, characterized by sharing goals, by discussion of ideas and the collaborative development of teaching resources.

Method

This qualitative research follows an interpretative perspective, with data collection by interviews, conversations, field notes and gathering records (Denzin & Lincoln, 2000). The participants are six teachers of public elementary schools of the state education network of Rio Grande do Sul, Brazil, belonging to the 15.ª Coordenadoria Regional de Educação (15.ª CRE). All names indicated in this paper are pseudonyms. The lesson study took place from August to November 2018, conducted by a team of the Group of Studies and Research in Mathematics Education and Technology (GEPEM@T), from the Universidade Federal da Fronteira Sul – UFFS. The activities were undertaken along ten meetings with three hours each, carried out two time a month in the facilities of 15.ª CRE. During the lesson study meetings the teachers planned a research lesson, aiming to stimulate the understanding of the mathematical concept of 'meter' of a grade 4 class. After each meeting we made a detailed description of activities and of work done by the group. In addition, we analyzed the teachers' written productions, which were produced in the lesson study, especially the reflections about the meetings (teachers' logbook), as well as the plan for the research lesson. The teacher's reflections and their evaluation of the formative experience in the reflection session were also transcribed. The empirical material was analyzed, interpreted and discussed (Erickson, 1986) seeking to depict the process which enabled the promotion of professional collaboration.

Promoting collaboration in a lesson study

The professional routine of participant teachers, which was marked by isolation and individualism (as the teachers indicated), was shaken when group was challenged to become involved in a lesson study. They found very strange all aspects of the dynamics of the lesson study. They were very surprised by working collectively and through dialogue in the planning of the research lesson, observing classroom teaching and, especially, supporting and encouraging each other in overcoming professional challenges and conflicts. The construction of professional collaboration in this experience begun with the planning of the lesson and was intensified with the research lesson and the post-lesson reflection.

Emerging of collaboration

The isolation and individualism that prevail in teachers' professional culture, especially regarding classroom practice, was voiced in several interventions. Erika highlights the possibilities of collaborative work for professional planning, indicating that this practice is not part of her routine:

> As we finish our meetings I became very happy with that I learned through planning of classes. I think that we should meet other times. I feel the lack of collective planning in the everyday routine. At first I was a bit confused about the real aim of the lesson study, but during sessions I could understand the importance of lesson planning and, especially, the observations and the evaluation of class enactment. (Erika, Nov 2018, teacher logbook).

In the same direction, Ivy contrasts the collaboration to the prevalent individualism in the teaching profession, stressing importance of collective work for the quality of professional planning:

> [Everyday] we are alone, we are lonely, we think alone, we analyses alone and we find some way out. But what we made here in the group [lesson study] is think together and reflect together. There is many people analyzing the same question, thinking in the same class. And this made a difference. Things can happen as such, but it is different to plan, think, apply and analyze alone. It is different to have a group. And because of that our group will be different at our school. (Ivy, Nov 2018, audio record).

Ivy confirms the prevalent individualism in schools, mainly concerning lesson planning, and concludes that the experience in the lesson study provided them the opportunity to constitute a different group at their school. We must note that, when the teacher educators perceived that the group was not accustomed to work collaboratively, they aimed to promote dynamics to stimulate interaction, dialogue and collective work:

> The group was unfocussed and confused about the task to realize. There is some difficulty in working as a collective. We suggested them the teachers to search materials and activities related to the topic, to discuss with the other teachers. The outcome of this search was shared and discussed in group so that each teacher could make comments about what had been presented. [...]. In addition, we stimulated the group to plan mathematical activities, designing and modifying them at the face to face meetings. (Field notes, Aug 2018).

The collective study was well evaluated by teachers, as Ranya indicates. The collective planning, based in study sessions, is a need that she had already pointed to education managing bodies, because change in teaching is a process enacted at school, by teachers.

> I have already suggested it several times to the Education Department. I said them: "you don't need to bring here lecturers to say us what have to do". Organize study groups at schools. The process takes place [in the group]. It doesn't need to come ready […]. We need time to meet, study and plan: collective planning. […] If we now go back to school we cannot meet. […] When to meet to plan? For this to succeed we need time. (Ranya, Nov 2018, audio record).

Another important aspect was stressed by teachers in the planning meetings for the research lesson. As the meetings took place twice a month, the systematization of activities for the research lesson which made up the lesson plan, only took place at face to face meetings. The time was short to recall everything that had been discussed and defined in last meeting and to systematize . Aiming to aid the teachers, we oriented the group to use a resource for sharing documents, Google Drive, teaching the teachers how use it:

> Initially, the teachers had difficulty in using communication resources, as well as in storing and editing documents used in the lesson study (especially Moodle). However, in planning the research lesson, the group began to use the Google Drive resource, by which [the teachers] developed a different

> collaboration perspective insofar as they began to work in a shared manner in the tasks for the research lesson, which were available in this resource. According to the teachers, this experience was excellent because they had the possibility to undertake the lesson study activities in a shared way, even if they were in different places. A teacher said: "This resource helped us a lot because we could work in the lesson plan and monitor what each one had done". (Sep 2018, Field notes).

This way of work provided the teachers the opportunity to expand their collaboration experiences during the process of constructing the lesson plan, in collaborative writing, which is a difficult and complex activity, especially for teachers immersed in individualist professional cultures.

Therefore, professional planning was marked by dialogue, sharing, collective study and shared edition of the lesson plan, in a context framed by the respect for participants' ideas regarding activities which were designed by them, as well as for their professional anguishes:

> [In several sections we dedicated to] planning for the research lesson, we sat down and dialogued. Initially, the regular teachers of the class gave us some descriptions of the student group and the school, such as the number of students and space available to the research lesson. We discussed some ideas for the activities. (Yasmin, Oct 2018, teacher logbook).

In summary, the professional planning, enabled by the lesson study dynamic, was a context in which collaboration emerged, providing the teachers the opportunity to learn from the experiences of each other, as well as to think and plan a lesson which was analyzed and discussed by the group and improved at each session. Also, it allowed the teachers to undertake professional activities such as to study about the students and about their social and family contexts, and, especially, the collaborative writing in a way very different of the individualism that they were used to. This process was mainly influenced by the way in which the teacher educators conducted the lesson study and how they worked to overcome hierarchies among teacher educators and teachers.

Expanding the collaboration

The lesson study dynamic provided the teachers the possibility to expand their collaborative experiences to classroom teaching. For the teachers, the fact they were involved in enacting classroom teaching, through the research lesson, was a real difference in the lesson study. They highlight that the group collaborated in thinking and planning the research lesson and, mainly, in enacting it, in a way such that allowed all of them to assume important roles in different steps of this process:

> Different from lesson studies that took place recently, this was carried out in two days. The activities begun with the participation of people from the school community, with interviews aiming to find out about the units of measure which were used in the past. (Janaína, Oct 2018, teacher logbook).

In planning the research lesson all teachers were involved in an intense and lasting way. First, the group analyzed the student and family contexts, aiming to plan the initial activity of the research lesson, which was a survey of measuring strategies and instruments used by their family. Then, it was made a detailed study about external and internal space of the school, aiming to plan a "treasure hunt" activity, for which the students would use measure units of length:

> A thing that marked me was that all of us went to the classroom and all of us were involved in something. When students made the treasure hunt, all of us participated, even the school assistants and school direction. After, when the students made the classroom activities, we were very involved. It was as if all of us were teaching the class. (Ivy, Nov 2018, audio record).

Carrying out the treasure hunt, in the research lesson, also involved all the teachers and the school pedagogical staff in helping the students. The school pedagogical staff also participated in the research lesson, observing students' actions. In addition, another aspect which contributed to the expansion of collaboration was the opportunity of learning from the observation of colleagues:

> As we learn listening, as we learn with the colleagues, watching our colleague teaching! And it has to come somebody from [outside] to show us the importance of this that we know, but never do. Just the fact that we listen to each other, report [what we did] already is relevant learning. (Ranya, Nov 2018, audio record).

Finally, the collaboration also took place in the development of the research lesson. In a moment when she felt challenged in role assumed, Janaína, the teacher in that was leading the lesson, sought support from the educators:

> I felt safe, not because I am a so confident person, but because you provided this tranquility. And I knew that when I could not say the [lesson] words I could say, "teacher, help me" as it happened when I began speaking of the history of the units of measurement and I said "teacher, will you lead the lesson now?" And you went there and led the class. Because sometimes we miss the words and it is good to have someone there, a support, a safety net, someone we may count on. And I knew that you were not there to evaluate, because you said very clearly that the aim of the research lesson was not to evaluate the student, not to evaluate the teacher. It would be a study about what the students would do. And we learn with that. (Janaína, Nov 2018, audio record).

Therefore, with the research lesson, the collaboration, went beyond the physical and time boundaries of the school, because besides the six teachers, also the school pedagogical staff helped in different moments, as well as students' families (the class involved in the research lesson) participated in a direct way, answering interviews about measuring procedures and instruments used in the past. It also involved the teacher educators, that were called by the research lesson teacher to lead the class. This confidence among teacher educators and participants, favored by the way we conducted the process, was key for promoting collaboration. In this perspective, the collaboration expanded to classroom teaching, transposing hierarchies and roles held by the different participants involved in the educational process at the school and in the lesson study context. Therefore, the research lesson constituted a context in which collaboration was expanded to classroom teaching, showing the possibilities of professional development from overcoming isolation regarding classroom work.

Valuing collaboration

The teachers valued the process of promoting collaboration insofar as it pervaded personal and professional relations. The teachers stressed that the fact that teacher educators worked in equality grounds with the other participants, listening to their anguishes and professional needs, and especially, encouraging them to collaborate as a way to face professional challenges. This attitude from the teacher educators established a high level of confidence in the group. The teachers began to mutually support and encourage each other. The fact that the collaborative experiences expanded to the dimension of personal relations, contributed to Janaína, who faced a stressful professional situation, not giving up to participate in the lesson study and accepting to teach a research lesson with great commitment, doing very good work. About this, Janaína emphasized that from the beginning she felt very comfortable with teaching the research lesson, which is considered a major challenge in lesson study:

> You as a group provided me this confidence […]. Because at beginning I was very unsure. Because you know that I already was evaluated, I was criticized by my colleagues [referring a critique made by a member of the school managing]. And you showed up and proposed a lesson study. I felt afraid. But you explained, what we are going to do is a lesson study, it will be great. This safety that you

> provide to everyone, make us feel cool. I felt at home. I felt as no body else was in the classroom. I felt very safe. At the beginning, it was scaring, but I felt safe. (Janaína, Nov 2018, audio record).

Erika confirms this aspect stressing that, besides overcoming the challenge of teaching in front of a group, Janaína carried out an excellent work with the students:

> You saw as the universe conspires with us to favor good things. You were in a fragile moment. And the this challenge arrived and you went there and made that excellent work with the students. (Erika, Nov 2018, audio record).

Finally, Janaína evaluates this experience as very, mostly because of this aspect, and she is very thankful to the group for providing her this opportunity for personal improvement:

> And even if I learned nothing in this process, if it had added nothing to my professional career, just the fact that we are here talking now, we are listening our colleagues, and you exposing your opinion, which is other reality, which is from outside, from another experience, that already adds much, already is worthwhile. I only have to thank you and the group for this. (Janaína, Nov 2018, audio record).

Therefore, the lesson study dynamic provided the teachers the opportunity to overcome the individualism and professional isolation, insofar as the process was oriented to promote professional collaboration. In the several lesson study steps, especially in planning and undertaking the research lesson, professional relations were established and activities stimulating professional collaboration were promoted. This process begun in professional planning, went through classroom teaching and consolidated itself in the personal dimension, facilitated for the way how the teacher educators conducted the process, as well as by the lesson study structure and dynamic.

Discussion and conclusion

Professional collaboration emerged in the context of the *planning* sessions for the research lesson, with teachers experiencing a teacher education approach geared to professional development (Dubar, 1997; Flores, 2004). This was a different process because it involved teachers in discussing ideas and collectively developing teaching resources (Burroughs & Lubeck, 2010). It also enabled collective work about the research lesson and discussion about the students' characteristics and the school context. The teachers had a chance to experience collaborative planning, which contributed to the growth of the group, bringing them closer to each other (Borges, 2007). An aspect repeatedly stressed by teachers was the discussion about the activities for the research lesson, the collaborative study and writing, which enabled the elaboration of the lesson plan. This experience was enhanced by the way how the educators conducted the lesson study steps and activities.

The collaboration extended to *classroom teaching* as all teachers involved in the research lesson took part in it, in shared teaching (Richit & Ponte, 2019). In the research lesson, which was organized in two consecutive days participated the students' families, all teachers involved in the lesson study, and the school pedagogical staff, undertaking a collaboration in different times, spaces and segments of the school community. In addition, the way in which the process was conducted established a higher level of confidence through sharing and collegial support, which resulted in the disposition of participants to take risks and experience a new practice (Hargraves, 1998).

Finally, the experiences of *personal encouragement and empowerment* constituted a context for the valorization of collaboration, providing the teachers with opportunities to share uncertainties, conjectures, anguishes, and goals, as well as to discuss different topics related to teaching and to feel welcomed in the group (Borges, 2007). The stimulus and confidence manifested by colleagues and by teacher educators empowered the teachers (Hargreaves, 1998), just as encouraged Janaína to develop a research lesson in a critical professional moment (Borges, 2007). Thus, the confidence that was developed through the respect and care with the needs, interests and anguishes of all participants in the lesson study crossed the personal and professional levels (Hargreaves, 1998), facilitating the participants' personal growth. The promotion of collaboration was fostered by the teacher educators, who conducted this process priorizing respect, dialog and confidence. In fact, the promotion of collaboration in a lesson study requires care, attention and appropriate intervention of those who lead it, so that the group feels welcomed, respected and valued in the collective.

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I DON'T KNOW! WHAT DO YOU THINK? WHY? COLLABORATIVE WORK BETWEEN PRIMARY AND SECONDARY SCHOOL TEACHERS

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Teaching is an activity characterized by a constant decision-making process that is linked to the specificity of the contents and contexts in which teaching takes place. It requires the teacher to be able to function comfortably in scenes of an uncertain or disconcerting nature which may also be recognized as new learning opportunities. Mathematical knowledge for teaching is a tool for the teaching task that helps teachers cope with classroom uncertainty. Such knowledge may also be in tension with those derived from one's own professional practice and sometimes even as opposed to the prescribed curriculum. In this paper we present a case study of a community of practice in which educators work together with primary and secondary mathematics teachers to identify bridges between content in order to help students move from primary to secondary school and to help teachers and educators to reflect on professional tensions and practices.

Introduction

Teaching is characterized by the constant decision making about what, why and how to teach, involving the specificity of content, the contexts in which teaching takes place and the characteristics of the learning subjects. The analysis of teaching practices requires understanding the conditions and contexts in which they operate, valuing them, transforming them and providing them with a moral sense (Litwin, 2008). Cultural changes and technological advances in the current phase of modernity have complicated the demands on teaching practices (Artigue, 2012); it requires being able to function comfortably in uncertain and disconcerting areas, without being paralyzed by recognizing these scenes as new learning opportunities (Alliaud, 2014). Mathematical Knowledge for Teaching (MKT) (Ball, Thames and Phelps, 2008) is a training tool that helps teachers to face the uncertainty of classrooms and, it is in constant tension with the knowledge emerging from the professional practice, from now on practical knowledge (PK). Often, PK is in a position of exteriority regarding the prescribed pedagogical and disciplinary decisions (Tardiff, 2004). Thus, it becomes necessary to study how these knowledges interact with one another in order to put them to the service of practice, having teachers as (co)developers of this knowledge in order to be sustainable over time (Goodchild, 2008). While it is true that understanding educational and mathematical theories are necessary to better teaching practices (Ball, 2017), they are learnt through practice and an informed reflection on practice (Goodchild, 2008). This learning must necessarily involve researchers because otherwise the advances made on theories on mathematical education, do not have a desirable impact on the corresponding practice (Bosch, Álvarez Díaz, Correa, Druck & McEachin, 2010).

Communities of practice then emerge as natural environments for collective reflection about the tension between the MKT and the PK. The practice itself becomes a source of coherence and cohesion that facilitates and sustains the Communities of Practice. They are also necessary so that the social and academic values of the teaching profession converge and thus transform the knowledge (co)created by the community into a tool for professional empowerment that accompanies the training of mathematics teachers (Chevallard, 2003). Ensuring its continuity is fundamental because the reasons and themes that mobilize the educational community to formalize spaces of formation, are correlated with the transitory nature to these questions. This is so because the answers are always conditioned by the spatial-temporal context in which the question took place (Putnam & Borko, 2000). As pointed out by Cambell (2009)

> Sustained communities of practice are examples of professional development experiences valued by teachers, important to characterize the work of teachers and to identify what influences teacher learning and aid in the design of effective professional development tools (p. 956).

However the outcomes of the learning process in communities is difficult to address because they did not have consistent clarity on the specific mathematics knowledge and pedagogy that were learned, the ways in which learning occurred, or the relationship between learning and collaboration (Jaworski et al, 2017).

In this paper we describe the community of practice in which educators, primary and secondary school in-service teachers discuss mathematics problems and their potential use as activities in the classroom, and present instances in which they collectively reflect on their mathematical background and their own practice.

Description of the community

At the beginning of 2017, a professional development activity was created with the objective of finding ways that would help students transition from primary level mathematics to secondary level mathematics. This initiative was supported by the Ministry of Education of the Province of Chubut and the Universidad Nacional de la Patagonia San Juan Bosco, in the city of Comodoro Rivadavia (Patagonia Argentina). It involved primary and secondary in-service mathematics teachers from public primary and secondary schools of the city as well as mathematics educators from the university. The community met eight times each year and even though the attendance is voluntary, we have had about twelve teachers that have actively participated in this community. During this activity, teachers were asked to solve problems, to discuss possible classroom implementations for those problems?, to implement them in their classrooms and then to present back the results of such implementation. The problems proposed were used to test problematic situations that allow to explain the characteristics of the mathematical transition from primary school to secondary school (Freudenthal, 2002). The cyclical nature of the work almost immediately transformed the group enrolled in the professional activity into a community of practice, in which the heterogeneity of the members forced to base all the discussions in terms of the practice itself. Such a fact, was fundamental for the negotiation of the meanings of the tasks designed (Fiorentini, 2013). This community is one example among the few cases of collaborative communities between primary and secondary schools as it is reported in Robutti et al (2016). Once the problems were solved, the focus of the discussions was placed on the problems of the ubiquitous problems of the teaching task from multiple perspectives with an iterative characteristic, of collective design and of constant interpellation of the teaching theories (Sensevy, Forest, Quilio & Morales, 2013), which in turn allow teachers to reflect on the relation between the underlying mathematical knowledge and the knowledge produced in the professional practice (Moreira & David, 2003). In order to collect and organize the data produced in the community we created and *observation diary,* in which together with the problem, purposes and goals of the activity, mathematical knowledge involved and possible scenarios are included that we could face during the implementation of the problem. Together with non-participatory observations (Yuni and Urbano, 2014), the *observation diary* was used as a narrative document (Suarez, 2007) as a tool for interpreting the ways in which collaboration triggers reflection about our own teaching knowledge, goals and beliefs.

Some results

Primary and secondary school teachers have different problem-solving skills. The glass case

The heterogeneous collaborative problem-solving environment in which teachers work in this community "helped them notice critical aspects in the learning environment that promoted their own learning and were motivated to change facets of their own practice" (Olson & Kirtley, 2005, p.25). One of the problems presented to teachers to solve was the following. It was adapted from Tahan (1993):

> Three men would receive as a payment for a service 21 equal glasses, 7 being full, 7 half full and 7 empty. They now want to divide the 21 glasses so that each one receives the same quantity of glasses and the same quantity of wine.

During the resolution of this problem, it took place the following dialog between a shy primary school teacher (Mr A) and a very confident secondary school teacher (Mrs B)

Mr A: We obtained the following graphic solution: Since they had 21 glasses and 3 people, each man should get 7 glasses: two men should get 3 full glasses, 1 half full glass and 3 empty glasses and the remaining should get 1 full glass, 5 half full glasses and 1 empty glass.

Mrs B: I do not think this is right. The answer is 3.5.

Mr A: Ok. So, my answer must be wrong.

The confidence with which Mrs B stated her answer led to believe to Mr A that his answer was erroneous, because it was a graphic solutions, whereas Mrs B's solutions must be right because it was obtained arithmetically. The interesting thing about working collaboratively with peers with different expertise, is critical to help teachers to "re-examine their own content knowledge and their identities" (Olson and Kirtley, 2005, p. 31): pictorial representations and solutions are valid, and not always a problem is solved by performing arithmetic operations.

Learning inside a community changes beliefs and professional profiles. The case of Miss "E"

Miss "E" is a sixth-grade teacher for a public primary school and has participated in the community since 2017. She says

> "in the beginning the work proposed with peers proved to be a challenge ... we were involved and supported in the resolution of problems, analysis of possible answers or obstacles, reflection on underlying content in order to establish a culture of collaboration between peers that led to the transformation through collaborative work of teaching practices."

At first, she recognized that, like others, she felt a certain fear of being exposed to colleagues of different levels, which quickly dissipated when she noticed that when faced with a real problem, no one has the answer per se. This is built from interaction and collaborative work with colleagues who share the objective of reflection on their practices and follow the principle of symmetry in a cooperative learning environment (Sensevy et al, 2013). Miss "E" set out to work along with a secondary school teacher and they decided to implement activities based on the puzzle Tangram in their own classrooms. Beyond the similar results of the implementation, considering what was done in secondary or primary school, Miss "E" expressed that her own voice was heard and her contributions, opinions, experience were recognized. She felt valued by the community which allowed her to feel as a part of it, encouraging her more and more to present proposals to their peers to work collectively. Her enthusiasm and experience had a multiplier effect in the places where she works, encouraging her colleagues to appropriate of this way of working: "... My colleagues tell me I am different… and I feel different… Now, I am confident in what I do… "

Mathematical knowledge develops in a community of practice. The case of the remainders

One of the objectives of this community is to give opportunities to its members to revise and reflect on what they do, so that their own content and pedagogical knowledge can grow. One key aspect of understanding and knowledge growth is what Putnam and Borko (2000) identified as "the nature of discourse communities for teacher learning" (p. 8), for instance the ability to listen carefully to what others are saying, and to ask questions and elaborate on someone else's ideas (Chapin, O'Connor & Cavanan Anderson, 2003). Even though such specific growths is difficult to account for (Campbell, 2009), we present an instance in which we were able to identify growth in the mathematical knowledge from the discussion of the solution of the following problem

> Find a number that divided by 2 has remainder 1, that divided by 3 has remainder 2, that divided by 4 has remainder 3, that divided by 5 has remainder 4 and that divided by 6 has remainder 5.

During the resolution of the problem either they looked for solutions conducting an exhaustive search by trying on different numbers or they looked for a formula that allowed them to find at least one number that satisfied all the given conditions. Teachers had resorted on divisibility criteria to rule out numbers when looking for an answer and it was a good opportunity to revise such mathematical contents

Mr S: Why do you say you are using the divisibility criteria?

Mr C: Because we use it to make the division exact.

Mr S: So why did 59 work? Divisibility criteria are associated with exact division, zero remainder! But here, why does it work with the divisibility criteria if none appears with zero remainder?

Mr C: Because we added the remainder.

Mr S: What characteristics do numbers have that when I divide them by 5 they have remainder 4?

Mr C: Their unit digit has to be 4 or 9.

Mr S: Then in our case...

Mr C: It has to end in 9, because the number must be odd!

The mathematical potential of this problem (Barreiro, Leonian, Marino, Pochulu & Rodriguez, 2016) permitted to reflect on the meaning of the division in the ring of integers and its relationship with divisibility criteria making, in this way, a clear link between school mathematics and professional mathematics (Klein, 2016). This problem was then generalized to include negative integers and teachers were able to solve non routine problems like the one below

Solve the following divisions in the ring of integers:

25:6	-25:6	25:(-6)	-25:(-6)

One of the main objectives of teacher training is to determine the balance between theoretical and practical knowledge and skills. The basic question concerns the content and extent of knowledge required from future teachers (Adler, Ball, Krainer, Lin & Novotna, 2005) in particular problems with mathematical potential helped modify the existing belief among teachers participating in this community from mathematics being a set of interesting topics for teachers, professors, trainers or educators onto coherent disciplinary field distinguished by the analysis of concepts, analysis of problems and the connections and generalizations thereof (Usiskin, 2000).

Discussion and future work

Teaching is characterized by a constant decision-making process that involves the specificity of content and the contexts in which teaching takes place. Teachers need support if the goal of mathematical proficiency for all is to be reached. The demands this makes on teacher educators and the enterprise of teacher education are substantial, and often under-appreciated (Adler et al, 2005, p. 371). The mathematical knowledge for teaching (MKT) is a professional development training tool that helps teachers to face the uncertainty of the classrooms and it is in constant tension with the knowledge arising from the practice itself (PK).

Under the joint initiative of the Ministry of Education of the Province of Chubut and the Universidad Nacional de la Patagonia San Juan Bosco (Comodoro Rivadavia, Argentina), we created a space in which teacher educators, primary and secondary in-service mathematics teachers from public schools, solve problems, discuss possible classroom implementations and present the results obtained, as a way to identify tasks that would help students transition from primary to secondary school. The cyclic and symmetric nature of the work transform this space into a community of practice in which the emerging knowledge is built and modified in the action, members of this community are in a permanent state of learning challenging the status quo both in school and in training (Fiorentini, 2013). Several *observation diaries* were presented as community products to become part of a database of good transition problems from primary to secondary school. The selection of the problems was done collaborative by teachers, among those previously worked in the community. The goal is to create a database of good transition problems that can be used by peers with the hope that collaborative discussions can also emerge outside the community.

We observed how teacher's identities were being constantly challenged and in some cases, as in the Mrs. E case, teachers were able to (re)define the social and academic values of their teaching profession. Following Campbell's learning stage model for communities of practice (Campbell, 2009) it was clear that the community reached the Collaboration Stage, in which members of a community share beliefs on collaboration, shared values and goals, and shared role. The cases of the problems about remainder and glasses and others instances (not reported here) in which MKT is confronted to PK, indicates that the community is reaching the Teacher Learning Stage (collective inquiry, assessment of practice on the basis of results, and focus on student learning), however we still need further analysis in order to see whether our community reaches the Specialized Growth (content focus, use of artifacts of practice, and planning and implementing reform-inspired instructional practices).

This community gave us a teacher/teacher-educator collaboration model designed to articulate primary and secondary school mathematics with a specific goal of identifying bridges that would help students transition from primary to secondary school and deepen mathematical knowledge for teaching. This represented an opportunity to investigate, within the framework of developmental research (Hans Freudenthal, 2002) what teachers learn from experience, whether teachers learn from experience, and what supports learning from experience (Adler et al, 2005). We started in the development cycle (thought and practical experiment) of the developmental research cycle (Goodchild, 2008) and we are moving into the research cycle in which, from global theories we want to construct local theories that would help us answer questions like: how effective are various models for promoting different outcomes? and which forms of collaboration are appropriate in different contexts? The cyclic nature of the title of this paper is a reflection of the work we set out to do since 2017. Our preliminary findings allowed us to begin to characterize professional profiles of primary and secondary in-service mathematics teachers and have the potential to shed light on to how collaborative work and communities of practice should be explicit in professional development for preservice teachers.

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SEMINAR ON RE-THINKING MATHEMATICS: A COLLABORATIVE ENVIRONMENT WHICH OFFERS RESOURCES FOR MATHEMATICS TEACHERS AND RESEARCHERS

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This document analyses the Seminar on Re-thinking Mathematics (SRM) as an environment of collaboration, which offers resources to enhance different mathematics-teaching approaches for teachers and researchers. It is based on the idea of establishing communication between researchers and teachers through a dialogue on a specific research product. It describes how the SRM works through its parties and their practices. It is focused on three of the key processes taking place: the dialogue between researchers and teachers, the shared reading, and an example of collaboration among Mexican and Colombian institutions. Finally, it describes how the SRM model has been transferred to other areas of expertise.

Seminar on Re-thinking Mathematics

This Seminar offers a space for people to gather and compare the expectations of teachers and educational researchers regarding Mathematics in order to promote the professionalization of teachers through their approach to the research findings (Ramírez, Suárez, Ortega y Ruiz, 2009). The use of information and communication technologies favors both synchronous and asynchronous interaction and virtual cooperation, as well as the creation of a repository of materials. The model also contemplates the creation of sites, groups of teachers within the same school who have a face-to-face interaction, which is parallel to the virtual interaction with other sites, and communities within the seminar.

The core of the organization of this seminar is the presentation. Each presentation revolves around a product of research (thesis, article, report, chapter of a book, or book) that is provided by the researcher —author— to the participants. These participants establish different interactions regarding this product of research. We particularly promote the concept of dialogue as the established interaction between teachers and researchers at three different points. The first one is established among the researcher and one or two teachers invited to talk directly to the researcher. The second one takes place when the dialogue is open to participation of different teachers who follow the broadcast of the presentation through a conference call or on the Internet. At this point, the interaction is simultaneous or almost simultaneous, the teachers who are present through the conference call ask questions orally, and those who follow it on the Internet, by sharing their thoughts on a discussion forum. The third one is the asynchronous interaction through a general discussion forum (Riieeme, 2015). This third point offers an interaction not only among teachers and researchers, but also among teachers. It is important to observe that the research product is the trigger of such three stages for the following discussion, in an effort of the teachers to understand it and of the researcher to be able to communicate it.

The seminar dynamic requires a pre-reading of the research product that, for the most part, is complicated for the teachers, particularly for those who experience the reading of a research product for the first time. From the dynamics of the SRM, the notion of 'site' serves the purpose of promoting the shared reading among teachers in-situ.

The session consists of the following products: the video conference, the product of investigation, the discussion forum, and the various materials developed around it —profiles, incentives, etc.— are kept in an open web site (Riieeme, 2019). This allows others to take these materials up for asynchronous discussions or other teacher upgrading programs. Until now, we have concluded 13 cycles. Moreover, cycle 14 is being developed, with an average of 7 presentations per cycle.

The Network of Investigation and Innovation in Teaching Statistics and Educational Mathematics (Red de Investigación e Innovación en Enseñanza de la Estadística y Matemática Educativa, Riieemee) oversees the academic selection and organization of the conferences that constitute the Seminar on Re-thinking Mathematics. Its founding members are a group of teachers from different institutions who experienced first-hand a confrontation with research findings followed by a working methodology similar to the one that would be established for the SRM later on. The authors of the research products were invited to establish a dialogue on the concerns raised by the shared reading of these. These two elements have provided the critical support to the seminar; therefore, we will give a more thorough look at them.

Dialogue among researchers and teachers

Brousseau (1989) describes the difficulty in the interaction among teachers and researchers due to the hard process of convergence among the practical requirements of teachers and the theoretical concern and searching of foundations of researchers. In his analysis, Brousseau implies that the activity of both is opposite in many senses, because it cares about different matters in spite of the fact that the concern of both (the proposal of dialogue among researchers and teachers: Seminar on Re-thinking Mathematics 837) is centered in the teaching of Mathematics. Sadovsky (2005a) presents this duality (proximity-distance) as a necessity for the existence of both activities, also Sarama & Clements (2019), Silver & Lunsford (2018) and Bauer & Fischer (2007) had finding in that way. Sadovsky discusses different aspects in which these two tasks converge and diverge, emphasizing the necessity of a mutual coexistence. According to Sadovsky, the theory should address the practice, but it cannot mirror it. The theory does not provide rules, regulations, or specifications in order to act, but it can deepen the understanding of events within the class, offer tools to think about the reality in the classroom, and make visible matters on teaching that are not available in the day-to-day activities. Moreover, there occur, on the meeting-ground for teachers, students, and knowledge, events that the theory cannot contemplate because unexpected events happen in the daily work which are beyond the control of any theoretical prediction. However, the theory can help us notice that what was always there was the result of human decisions, which do not owe their existence to a pre-established natural order.

We agree that investigation and teaching maintain a certain distance that is needed due to their different referential frames and to their own practical needs, but the mutual improvement of both tasks demands a healthy coexistence. However, the way in which this last one can be materialized cannot be neither simple nor automatic. Carr and Kemmis (quote by Rodríguez y Castañeda, 2001) emphasize the fact that we should include information oriented to make the cooperative discussion easier within the profession as one of our requests to research. They propose the professionalization of the instruction in which teachers should be able to demand more than just a research paper, but something that benefits directly their classes, or adds educational techniques, and researchers are as well open to teaching needs.

The dialogue among teachers and researchers is the starting point of our proposal. For us, teachers are scholars who need to think about their practice, find explanations to the facts and base their decisions on research findings (Sadovsky 2005b). Their contact with these outcomes together with the interaction with the researcher will encourage such needed thinking. Thus, we take back from Ochoviet and Oktac (2011) the idea that a researcher must be concerned about the understanding, interpretation and appropriation of his research findings by the teachers, as well as the new ideas that such interchange may bring about. In this way, researchers have the opportunity to look back at their work, and to define, based on the outcomes, the difficulties brought out from the teachers to focus on their new studies.

A shared research reading

The SRM identifies an implicit concept of that, in the long run, the reading materials of the sessions will facilitate the understanding to teachers who are not familiar with research papers. However, reading alone may become complicated and even overwhelming at first. While it is true that thinking is an internal process and that writing should pass first through the eyes, so that the individual perspective can be broaden, often, thoughts need actions in order to materialize. Thus, our thoughts on a reading become deeper once we communicate them. A dialogue between colleagues builds individual knowledge and adds to the advantage of creating links. From our point of view in the SRM, reading not only gives the pleasure of knowledge, but it also has a practical end: To rethink the mathematics that we teach and those that our students learn individually and as a group, virtually and in person.

The exchange of questions, comments, and concerns about the reference material of the sessions, made by the teachers who share the same institutional problems around their teaching practice favors the creation of academic communities within the same school or institution. In turn, the dialogue with the researcher is organized by more reasoned questions that are committed to the teaching activity, in order to use and understand the material in the writing phase.

Our goal is that the 'site' will serve as a promoter of shared readings among teachers in situ. The site consists of teachers gathering to share their concerns about a reading and helping each other understand it. We also hope that the meetings in the sites allow people to follow a simultaneous transmission of the dialogue through conference calls, if the school or academic unit equipment allows, or otherwise, through open Internet. That way, teachers who are not familiar with the use of technology will also benefit from the socialization of such difficulties. A later peer discussion will also favor the understanding not only of the research findings, but also of the dialogue itself.

The idea of site is properly institutionalized, and this favors the fact that it may be monitored and advised on by the seminar organizing team. It is also desirable that teachers come from the same institution, though they may be from different ones, but they must be able to attend physically as well. Nowadays, there are three national and two foreign sites, but their quantity has varied from one cycle to the other. For the eighth cycle there were six national sites, along with the ones from Instituto Politécnico Nacional, which is the official body hosting the SRM.

We have also observed that the individual participation grows when there are many teachers or colleagues who get together to share their reading or transmitting the session. Thus, for example, in session 56 of the eighth cycle, 44% of participations in discussion forums where from Colegio de Bachilleres de San Luis Potosi (Ramírez, Zenteno, García y Suárez, 2014). The incorporation of sites has also extended the SRM organizing network, and with it, many more speakers, such as teachers and researchers, have been invited. It has happened that, in some occasions, some sites cease to be so, but the link with some of the most active teachers remains, as they continue to be part of the organization. Teachers can take advantage of the information contained in the repository of the SRM to which all participants have access.

Nevertheless, this assistance mode for monitoring all stages of the sessions of the SRM has some problems. For example, one of them is that only between 20% and 30% of participations in forums, are based on the reference material presented, the rest is related to the first two stages of the session, and others relate to other comments on the same forums (Ramírez, Zenteno, García y Suárez, 2014). This indicates the difficulty of reading or understanding the research results by teachers. However, there is also evidence of some successes. In the following two examples of collaboration that have arisen from the SRM will be described. The first exemplifies the way of working in the SRM with the participation of the Colombia site in the SRM. The second talks about the transfer of the methodology to other areas that also require the incorporation of research into teaching.

SRM: an example thought the participation of the Colombia site.

From the beginning, the Seminar on Re-thinking Mathematics has been recognized by Latin-American teachers and researchers. Particularly in Colombia, this recognition has become clear in the different forms of participation of teachers and researchers in the SRM sessions. Through the different cycles of the SRM, participation has been consolidated as a concept that describes the collaboration between Colombia and the SRM.

Participation, in terms of Parra-Zapata and Villa-Ochoa (2016), may be understood as the "joint action of a group that shares the same goals and interests", it is also described as the commitment stated by subjects that makes them become 'part of'. For Parra-Zapata et al. (2018), this commitment and joint action goes beyond the understanding of participation in terms of visibility, i.e., 'stand out', and highlight the content of participation.

Colombian participation in the SRM is reflected through the roles that subjects have in the different sessions.

Cycle 7 (2012) marked the close link between some Colombian universities, among them, Universidad de Antioquia and Medellín, mainly, and the Riieeme. This collaboration has been evidenced in the different modalities of participation of Colombian professors and researchers in the SRM sessions throughout their cycles.Among which are:

Practicing teachers: a significant part of the Colombian participants in the SRM are practicing teachers who are interested in a specific topic, or in addressing or supplementing their commitment to training programs at a master's or Doctoral degree. For some of them, the SRM has become as a consolidated environment offering opportunities to interact with researchers, about relevant subjects in mathematics, within their school practices. Particularly, teachers have shown a strong interest in the creation of opportunities to transfer the research to the classroom. For instance, in session 83 held on April 2016, Olga, a Colombian teacher, asked in the Forum:

Olga: "[..] when I think about the assessment processes, I think that, when modeling and projecting overall situations, it is important to previously determine 'Categories' in which the items that will be considered for evaluating the overall performance of students are defined. I would like to know how you have managed to do so, and if you consider it relevant to use such Categories, and along with what kinds of important items."

In this respect, Isaura, one of the speaking persons in the Seminar, answered:

Isaura: "In the process of the Comprehensive Reform of the high school education in our country, it is suggested that many different assessment tools are used. We are currently working in Colegio de Bachilleres on the generic skills, and one of the references we have for this is: http://www.copeems.mx/guia-para-el-registro-evaluacion-y-seguimiento-de-las-competencias-genericas where you can find a description of some examples of how students can show their skills, some categories, and the way of monitoring them, as well as the progress achieved by students."

The Session guest also answered:

Researcher: "With regards to how students' activities are assessed during the modeling process, I can say that we have designed some worksheets for each activity, an opinion survey, a presentation of their project before the rest of the group, a record book for direct observations, a report on the group activity, and the survey. We also shoot most of the work in the classroom, in order to offer a critical analysis of the students' performance before this alternative to learn and to motivate them to study Mathematics."

Afterward, the researcher described one by one all working sessions with their students, the way in which data collected along with students in order to develop processes of mathematical model are obtained and processed. Adriana, another participant, also answered Olga's comment, and she pointed out that:

Adriana: "I would like to say that I have also used categories as a process of metacognition, conforming them along with the students after the activity to be assessed, and they have been very useful for the students to be aware of what they want, what they did correctly, and what they have to improve."

As Olga, other teachers have participated in the Seminar sessions searching out ways to give new sense to certain school practices of everyday life. Teachers also make frequent questions about the possibilities of taking action concerning certain peculiarities of Mathematics classes. For example, regarding the work within the classroom with certain mathematics concepts, subjects, processes, or procedures; as well as actions that teachers could develop towards factors such as: dropouts, high number of students, students with special educational needs, or lack of motivation to study, among others.

The example above is evidence that teacher Olga participated moved by the desire of putting into practice the developments of the guest researcher in her evaluations. This 'desire' made other participants contribute with examples, situations illustrating their own experiences and perspectives towards this need. This reflects how participation puts collaboration among teachers and researchers into practice in the everyday life as topics of common interest.

The commitment that these teachers made with the SRM reveals an asynchronous follow-up of the session because they are not able to follow it directly, due to their working schedules. It is also highlighted that, sometimes, teachers engage in additional activities of the SRM, and have received a certificate of a Diploma course taken.

Researchers. Some Colombian researchers have participated in the SRM as guests. For some of them, it offers them opportunities to disclose their work, but, above all, to reorganize the findings of their research from the perspective of the teachers and their practices in class. For example, in session 70 of August 2014, a Colombian researcher offered the findings of his research about Frechet and deaxiomatization of Mathematics. In this work, the researcher problematizes the existence and representation of mathematical objects. He also states that their presence may be in an 'abstract world' or in a 'world of contents', i.e., we sometimes work with abstract objects that get the meaning given to them through their structure. However, we sometimes must introduce objects with content —with a meaning external to its structure. The guest was questioned about the characteristics and training that teachers should have. This led the researcher to suggest a breaking point from the traditional formalistic pedagogy. And to promote a 'transformational' pedagogy, based in two different aspects: The first of which is the excessive formalism in the teaching of Mathematics at all levels that is sometimes justified by the accuracy, and the second one, the dual trait of such formalism, would be the rejection by the students. From all this, the researcher warns that the instruction based on an excessive formalism is at risk of deleting/concealing the heuristics of the constituent processes of formal Mathematics. Thus, focusing on formalism would disregard the heuristics of instruction, and for that, the history of Mathematics and the teaching practices would be relevant aspects of such heuristics.

Participation in this session highlights a collaboration based on interactions among teachers and researchers as a continuous call between history and epistemology of mathematics, and the training of teachers as well as didactics of mathematics.

Students (teachers in pre-service). The SRM sessions have been used in master's and doctoral degrees, and, especially, in some courses for the Pre-service Training Programs. As an example of the developed experiences, Villa-Ochoa, Suárez and Rendon-Mesa (in press) noted that students engaged in the searching of sessions related to the modeling of mathematics. The researchers reactivated some of the forums, analyzed the session videos, and used their face-to-face tutorials to discuss their own questions about some of the sessions, and they addressed these matters with the questions made in the forum. Collaboration in this case occurs from a participation in a type of *authentic experiences* that reflect an interaction with experts on a subject, and, above all, with other teachers in service who question their everyday practices in their classrooms.

Other roles among Colombian teachers and researchers in the SRM are through dialogue teachers and organizers of the event.

Model transfer to the "Seminar on Re-thinking" Network

The Seminar on Re-thinking Mathematics could be conceptualized as an innovative project, with potential for innovation within the classroom based on the educational investigation. And, above all, as a possibility of offering teachers and researchers a community environment to interact around their teaching or research practice. These characteristics build up a relevant model to be transferred to other areas of knowledge. Communication, Financial Culture (Navarro y Cruz, 2011), and Biochemistry (Luna, Suárez y Ortega, 2001) were the main areas to which it was proposed to transfer this educational innovation (Torres, 2011). All these seminars, such as SRM, aim to generate a forum for teachers´ reflection based on the findings of educational research in order to strengthen the professional teaching practice. However, each of them addresses different needs, academic interests, and educational research centered in particular teaching methods and, sometimes, in the emerging teaching methods along with their own difficulties and adaptations based on the methodology and philosophy of the SRM, using flexible spaces such as conference calls, forums and the Internet.

The "Seminar on Re-thinking" Network officially came into existence on June 2015 (IPN, 2015). This constitutes the recognition of the work made with the SRM since 2004, and the institutional effort for linking research and teaching in order to improve the instruction of specific didactics. This year, new seminars have been registered adding to the previous ones: Re-thinking Engineering (Seminario Repensar la Ingeniería, SRI), Physics (SRF), Chemistry (SRQ), Economics (SERECO). In the light of the concept of academic networks, the communities of the Seminar on Re-thinking have the opportunity of expressing themselves with hopes of transforming their teaching practice by taking into account their context, but by learning as a community the tendencies that are currently defined in high-school education, both at a national and at a Latin American level.

Conclusions

The analysis of the products generated through the SRM shows the need of such spaces for linking research and teaching, not only for mathematics, but for other areas of knowledge. Other factors that must be highlighted are the support from institutions, the interest of other communities and the transfer of this SRM model to other disciplines, along with communities responsible of other disciplines. The example given of other communities interested in this —the collaboration with Colombia regarding the way the SRM works— shows the success that may be achieved through these systematic collaborations. There are many opportunity areas in the generated products. There are many questions on research that it might bring about, and the answers to these questions may be obtained by innovative projects that can contribute to the professionalization of teachers. Efforts are being made (Ruiz y Suárez, 2015; Suárez y Mena-Morales 2014; Martínez, Suárez y Ruiz, 2017) in order to analyze these products to improve the SRM and to learn from the transfers that have been made.

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CONSTITUTION OF A COLLABORATIVE WORKING GROUP CONDUCTING LESSON STUDIES

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Assuming as promising for the development of professional teachers proposals of a reflective and collaborative nature that include and articulate different aspects of the knowledge, in this text we present and analyze the process of constitution and functioning of a collaborative group that carries out lesson studies. The data analyzed were collected by participant observation, with the elaboration of a logbook, audio and video recording of the meetings and a semi-structured interview with the participants. Relevant episodes were considered to understand the work dynamics developed in the group in two different moments, in 2018 and 2019, highlighting the challenges the team faced to implement a collaborative and reflective work perspective. The results show some of the different characteristics that influence the constitution and functioning of the collaborative group, of which the experience of the teacher educators has proved to be central.

The teachers' professional development is an essential aspect for the success of teaching; research on teacher education point to promising proposals of a reflective and collaborative nature (Boavida & Ponte, 2002; Jaworski, Chapman, Clark-Wilson, Cusi, Esteley, Goos & Robutti, 2017), which include and articulate different characteristics of the teachers' knowledge (Ball, Thames & Phelps, 2008; Carrillo, Climent, Contreras & Muñoz-Catalán, 2013; Shulman, 1986), related to the contexts in which such knowledges are mobilized (Silver, Clark, Ghousseini, Charalambous & Sealy, 2007). Thus, lesson study is presented as a process of professional development with these characteristics, and with repercussions in different countries (Murata, 2011; Perry & Lewis, 2009; Ponte, Baptista, Velez & Costa, 2012).

In this paper, we aim to present and analyze the process of constitution and operation of a collaborative group that conducts lesson study, formed by teachers of the Beginners in Elementary School[1] in a city in southern of Brazil. For the analysis, we consider relevant episodes to understand the dynamics of work developed in the group, confronting what was foreseen by the teachers educators and what was actually accomplished, evidencing the challenges the team faced to implement a collaborative and reflective work perspective.

Teachers' professional development

A widely held belief is that teachers' practices change as a product of curriculum changes, external guidance and assessments, and their professional development is often seen as something that just "happens" of course, from experience or as a result of training in specific methods or curricula. "Learning by observating", which reinforces the conservatism of practice, coupled with ineffective teacher training practices, inherited from conservative traditions, result in little professional capacity to learn and change (Ball & Cohen, 1999).

[1] In Brazil, the early years of elementary school goes from the 1st to the 5th grade (6 to 10 years of age).

A comprehensive approach to the professional development of teachers, proposed by Ball & Cohen (1999), requires changes in two aspects: (i) to acquire a fundamentally different content from the usual ones, enabling the acquisition of the knowledges, skills and dispositions that allow teachers to encourage this type of learning in their students; (ii) to immunize teachers against the conservative lessons they learn from practice. Thus, practice-based training proposals (Ponte, 2012; Opfer & Pedder, 2011; Silver et al., 2007; Webster-Wright, 2009) show to be promising for the professional development of teachers , offering opportunities for their professional learning (OAP - Ribeiro & Ponte, 2019) and having a positive effect on student learning.

Webster-Wright (2009), for example, highlights the need to think of proposals that take into account authentic experiences, aligned with the reality of the workplace and professional responsibilities, based on cases of professional practice, based on " artifacts "from his daily work. Training proposals should offer "opportunities for teachers to assess pedagogical problems and seek potential solutions through shared reflection and knowledge building processes" (Silver et al., 2007, p.262), as well as propose, discuss and consider solutions to pedagogical dilemmas and explore pedagogical possibilities. It is a question of focusing the training processes on critical activities of the profession - that is, on teaching and learning practices (Ball & Cohen, 1999), selecting materials that do not reinforce existing practices, beliefs or ideas. According to the authors, contact with practices different from the usual allows teachers to broaden and diversify their knowledge, creates opportunities from them to see new versions of teaching and learning and leads them to understand things differently.

Regarding knowledge of teachers, based on Shulman (1986), Ball, Thames and Phelps (2008) present the theoretical framework of *Mathematical Knowledge for Teaching* - MKT, considering six domains for mathematical knowledge for teaching. Later, Carrillo, Climent, Contreras & Muñoz-Catalán (2013) proposed the *Mathematics Teacher Specialized Knowledge* (MTSK), a theoretical model that reformulates the categories proposed by Ball, Thames and Phelps (2008), also including as central elements in the model both teachers beliefs about mathematics and mathematics teaching and learning. According to Zakarya & Ribeiro (2019), these aspects take a central position in the conceptualization of MTSK, as they influence and are influenced by all other aspects of the teacher's specialized knowledge .

Collaborative work in mathematics teachers training

Teachers should be offered opportunities to reconsider their practices and examine others, as well as learn more about the content they teach and the students they teach. (Ball & Cohen, 1999); various forms of collaboration have been used for this. The perspective of collaborative work contributes to the teachers' self-esteem, self-efficacy and motivation and to the diversification of teaching strategies, which affects students' learning (Schleicher, 2015). Professional learning opportunities from the perspective of collaborative work involve changes in teacher participation in the group, in knowledge, beliefs and/or teaching activities (Akkerman & Bakker, 2011; Goldsmith, Doerr & Lewis, 2014). Tensions emerging from the collaborative context, supported by discussions about these contradictions, enable the teachers to overcome them and engage differently in mathematics teaching and learning process, potentializing changes in classroom practices and constituting themselves as indicators of professional learning (Stouraitis, Potari & Skott, 2017).

There are several ways of collaboration that seek to promote interactions between the teachers involved, and to have them share their experiences (Boavida & Ponte, 2002). One of them are the lesson studies, that originated in Asia and has variations in several parts of the world, both in initial and continuing education (Murata, 2011). It is a cycle that aims to improve teaching, in which teachers work together by formulating objectives for students' learning; collaboratively planning a lesson from these objectives; implementing the lesson, with one member of the team in charge of the teaching activities and others gathering evidence on the students' learning; reflecting on and discussing the evidence collected during the lesson, using it to improve it, if possible replicating it and restarting the cycle (Perry & Lewis, 2009). The lesson studies have potential in terms of professional development closely related to the way they are conducted, evidencing opportunities for professional learning regarding selecting tasks proposed to students and the processes of reasoning, and favoring the communication in the classroom, especially the conduction of collective discussions with students (Ponte, Baptista, Velez & Costa, 2012).

About the context and the formation process

The work was developed in 2018 and 2019 in a group of teachers that seeks to work from the perspective of collaborative work. The group was formed for a continuing education project in mathematics, intended to teachers of the initial years of elementary education, in partnership with the Secretaria Municipal de Educação (SME) of a municipality in the southern region of Brazil, in which the two authors of this article and a another researcher acted as teacher educators. These teacher educators were proponents of the proposal for continuing education and SME acted on the condition of inviting teachers (in 2018 delegating to the direction of some schools chosen by her to appoint teachers, and in 2019 extending the invitation to whom wanted to participate).

The training processes of 2018 and 2019 presented different characteristics, both in their constitution and operation. In 2018, the SME decided the day of the week in which the face-to-face meetings would take place, because the SME was responsible for releasing the teachers during activity time[2] to participate in the continuing education. Regarding the periodicity, the SME also considered that the meetings should not be held every fortnight, as suggested by the teacher educators, because it would be more difficult for the schools to release the teachers. Thus, eight meetings were held between April and October 2018, one meeting per month. The SME chose the schools and sent invitations to the principals, who indicated the teachers who would participate. The group started with six teachers of the $4th$ grade (9 years of age), one postgraduation student and three teacher educators. Only one teacher remained until the last meeting.

In 2019, after circulation to all the teachers of 4 and 5 years in the city, interested contacted directly with teacher educators have negotiated directly with the directors of your school to release your time activity (without the intervention of the SME). The activities started in April, with 13 teachers, two postgraduation students and two educators. Considering the possibility of their being released from their schools, the meetings took place once a month. Until July 2019, after four meetings, the group had 10 teachers.

[2] Regulated by law, it takes place during teachers' working hours, when they carry out extraclass activities such as planning, organizing and assessing didactic activities, besides studying and engaging in continuing education.

The three teacher educators that conducted the 2018 process had little experience with working with a group in the intended collaborative perspective, where choices and decisions should be negotiated in the group. For the 2019 group, two of the three teacher educators remained (the authors of this article), but this time with some changes in the way they conducted the formative process, as a consequence of their learning from their experience in the previous year. In both years, the educators tried to encourage the teachers to work together, choosing a mathematical content into which they should deepen; performing studies that gathered mathematical and didactic discussions from the content chosen; identifying students' difficulties in learning content; preparing in detail a lesson to be implemented, observed and analyzed.

Methodological procedures

This study followed the principles of a qualitative research (Esteban, 2010) from a theoretical interpretive perspective (Crotty, 1998). The data analyzed were collected in 2018 and 2019 by participant observation with: (i) elaboration of logbooks (educators and postgraduate students), (ii) audio and video recording of the meetings, (iii) semi-structured interview conducted with the teachers who taught the lesson in their classes, with their respective transcriptions. For analysis, we tried to identify, in the collected data set, relevant episodes to understand the work dynamics developed in the group, highlighting the challenges that face the implementation of a collaborative and reflective work perspective. In the following session, we present evidence and interpretations that emerged from the data collected. The teachers' speech transcripts (in italics) point to evidence that supported our analysis. The names of the teachers are fictitious.

Data presentation and analysis

The training process of 2018 taught the teacher educators some lessons, which eventually made them change the way they conducted the group in 2019. In making comparisons between the two groups, we do not want to establish a value judgment by saying that one group was more or less productive than the other, even though the members of the 2019 group were different from those of the 2018 group. Only the two teachers educators remained the same. Hence, this text deals with the perception and the attitude of the teachers educators in face of the experiences. The agenda of the meetings proved to be common to both formative processes. Different contingencies led to the cancellation of a previously agreed encounter. The presence of the teachers was not unfailing, with messages warning the educators they would be absent a few hours before the meeting began.

One learning of teacher educators in 2018 that led to changes in 2019 refers to the perspective of collaboration adopted. In 2018, during the first four meetings the mathematical theme discussed was division, the theme chosen by the teachers. Academic texts, mathematical tasks brought by the teachers and curricular documents were discussed. However, at each meeting, the teacher teachers maintained an expectation that the teachers would engage in the proposal and make decisions moving towards the planning and further development and study of a class. However, they seemed to resist any new proposition, and teacher educators often failed to lead to what they intended. In the excerpt below, teacher Ana externalizes her feelings about a text that questioned the overvaluation given by teachers to the use of the algorithm when teaching the four operations:

Ana: *When I was reading I felt a little embarrassed, but then I went back and said, "I'm not ashamed, no". Why? What are we missing? Because I do not have mathematical* *training [...]. So, the ways [we see] there for teachers to teach mathematics as the theorists teach in their books, in their articles, there, they are hard work for us*.

Ana, a very experienced teacher, who played a leading role in the group, did not seem to make room for any proposal for change and this seemed to echo with other teachers. Teacher Paula, for example, agrees with Ana, when she shows to be worried about the exploration tasks being proposed, instead of the exercises they were used to:

Paula: *[...] so, thinking here, for example, as Ana said there, I could not visualize this. As long as our teachers cannot build it the way you [teacher educator] did, we cannot pass it on to the student. […] Because they will have many doubts. This confidence you had here, I will have to have it there*.

Although the continuing education aimed at working this mathematical confidence with the teachers, their resistance and the choice of teacher educators to always make collective decisions delayed lesson planning and development. A final meeting scheduled for the first half of 2018 needed to be postponed until August, and the group chose to change the mathematical theme to the one that would be worked in the second half, that is, fractions. This required new theoretical studies, new discussions and a new attempt at class planning, which took place in the penultimate meeting of 2018, which resulted in the class developed by teacher Tânia.

For the 2019 group, the teacher educators decided to change some forms of conducting the activities, directing more the group's actions and seeking to involve the teachers' classroom as soon as possible. Taking advantage of the class developed by teacher Tânia in 2018, at the very first meeting a lesson study was developed aiming that teachers would experience this work strategy. With this, the mathematical theme was defined for the next meetings: fraction. In the second meeting, based on the studies involving the different meanings of fractions from the tasks brought by the teachers themselves and on the lesson study, the development of a class was proposed and accepted by Maria, a teacher of the $5th$ grade with little classroom experience. The task Tânia used was re-elaborated from the reflection in the group, and developed by teacher Maria. In the following section, in an interview with teacher Maria after the class she developed with her students, we recognized the opportunities that the continuing education had given her and the demonstration of trust in the group to help in their difficulties.

Maria: *If I was prepared? First of all, I would not even have given the class if it were not for the course you're offering us. And so, what we learned from you gave me a lot of ground, so I could solve whatever came up. There may have been some other questions, but just like any other class, you might have questions that you don't know how to answer.*

In the third meeting of the group of 2019, was the study of the class of teacher Maria, different from the proposal of the previous year, we managed to work the class study of one of the members of the group itself, which was well accepted. In the following excerpts, we bring speeches from the teachers Maria and Sônia who show their approval for the class study and, especially, the appreciation of the collaborative work effectively developed by the 2019 group.

Maria: *The discussions are very important, because, besides the knowledge that you [educators] transmit, with the studies you have in the university, the exchange of experiences with the other teachers, how they work in their classes, it certainly helps us a lot, it helps us to see a way, to feel a little more confident of our practice or even to speak: no, I need to change, I need to improve it.*

Sônia: *When I saw you filming the classroom and people debating, the cooperation, not that "oh, my, how terrible, see the way she said it", no! You know, everyone there is aiming at growth, learning, experience. I thought it was fantastic.*

In the fourth and final meeting held by the 2019 group to date, discussions have taken place regarding new mathematical tasks and the planning of a new class on fractions. This time, another teacher set out to develop it July 2019. Taking into account that there would be few meetings of the group, as well as the difficulties perceived in 2018 for the group to achieve a planning and development of a class, the teacher educators took a more directive position, making decisions without allowing much room for the involved to refuse the course proposal even before they knew it. Despite the different characteristics of the members of the 2018 and 2019 groups, the new teacher educators' attitude seems to have favored that more actions be developed, and in a shorter time. Table 1 synthesize the comparison between the formations carried out in 2018 and 2019, highlighting those elements that characterize the constitution and the functioning of the group.

These elements were organized according to three aspects selected to to account for the constitution of the collaborative group, to include: the teacher educators, the teachers and the whole group. Regarding the teacher educators, we highlight aspects related to their experiences and approach used in the formative process. In the case of teachers, we were considered: participation scheme, identification (or not) with the approach and the conduction of the formative process, and their mathematical knowledge. Finally, in relation to the group as a whole, the following are relevant: the number of members, the periodicity of the meetings, the existence (or not) of a leadership profile among the participating teachers and the timing of the proposal's calendar with the environment. participants' work.

Table 1: Synthesis of the Constitution Process and the Process of Operation

Aspects	2018	2019
Teacher educators	Little experience of teacher educators with a more open work perspective.	Both teacher educators with the experience they lived in the group the previous year.
	Teacher educators' concern in maintaining a less directive approach, greater openness to define work conduct with the participation of teachers.	Tasks more focused and with predefined time. A little more directive approach.
Teachers	Participation based on indication of SME.	Voluntary participation.
	Lack of identification with the lesson study.	Greater openness to the lesson study.
	Resistance to proposals different from the usual ones.	Openness to different proposals from the usual ones.
	Difficulties with mathematical content (division).	Less difficulty with mathematical content (fraction).
Whole group	Group with few members.	Group with more members.
	"Leadership" profile of a more experienced teacher, based on conservative traditions, reinforcing beliefs about mathematics and its teaching.	There is no such leadership profile. Teachers more open to innovative proposals (higher number of less-experienced teachers).
	Monthly meetings and unforeseen events that led to the cancellation of meetings	
	Difficulty in "fitting" the class in the teacher's planning	

Discussion of data and final considerations

In 2018, the educators expected that the participating teachers engaged in the proposal of conducting a lesson study and made decisions, moving towards planning and later development and study of a classroom (Perry & Lewis, 2009). To that end, they sought to adopt a work dynamics that would allow them to explore the content of division in a different way, selecting materials that did not reinforce existing practices, beliefs or ideas (Carrillo, Climent, Contreras & Muñoz-Catalán, 2013), in an attempt to "immunize" the teachers from the conservative practices coming from everyday experience (Ball & Cohen, 1999). Thus, they selected as study materials the curriculum guidelines, proposals of differentiated tasks involving division and academic texts. However, although collaboration modes assume the creation of an environment that offers participants opportunities for their professional learning (Ribeiro & Ponte, 2019), our data regarding the formation of the group in that year reflect the strong resistance to change among the participants. The difficulty in scheduling the meetings, coupled with the unpredictability of the participation of the teachers, made it difficult to guarantee their engagement in the sharing of experiences and interactions. The leading role teacher Ana played evidenced the presence of practice conservatism (Ball & Cohen, 1999) and the influence of beliefs regarding mathematics and its teaching (Carrillo et al., 2013). These aspects, linked to the fact that the teachers who participated in the meetings that year were nominated by the direction of their schools, by a request from the SME from a sample of schools in the municipality, and not a voluntary participation, reflect in a context of limited engagement.

Unlike the results pointed out by Stouraitis, Potari & Skott (2017), tensions resulting from this context limited the engagement of teachers in differentiated processes of mathematics teaching and learning, making fragile the possibilities of changes in classroom practices and offering few indicators of effective professional learning (Akkerman & Bakker, 2011; Goldsmith, Doerr & Lewis, 2014).

Therefore, new elements were part of the constitution of the 2019. The now more experienced teacher educators, having gone deeper into the theoretical studies about collaborative work perspectives, have changed some of the ways they developed the work, more aligned with the local reality (Webster-Wright, 2009), seeking to select "artifacts" closer to the teachers' own practice (Silver et al., 2007), such as tasks they use in their planning, or practice records of their own classes. They also opted for the experience of working with lesson study to occur as early as possible. The fact that teachers are more open to innovative proposals (a greater number of teachers with little experience) has proved promising in the development of shared reflection and knowledge-building processes (Silver et al., 2007). In addition, the higher number of participants (compared to the previous year) has potentiated the performance of works in small groups to share experiences and promote interaction among its members (Boavida & Ponte, 2002), leading to an improvement in teachers' engagement, especially in moments of collective discussions in the format of plenary sessions (Ponte, Baptista, Velez & Costa, 2012).

This reconstitution of the group in 2019 made the educators to adopt a more directive approach, with the development of more focused tasks and with predefined time, without opening for many classroom story-telling that could lead teachers to refuse something that was being proposed by the educators. This approach, although "closer" to a type of training model to which teachers are used (compared to 2018), has proved more promising in terms of professional learning opportunities (Ribeiro & Ponte, 2019), as it could be seen in Maria's and Sônia's statements.

We reinforce our aim, in this study, to compare experiences lived by two teacher educators in the constitution and development of a collaborative group. Among the many aspects that influence the dynamics of collaborative work, it has become evident that the experience of the teacher educators in proposing and conducting work in this format is essential and demands further research.

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Theme C

Theme Chairs: Shelley Dole & Ronnie Karsenty

Roles, Identities and Interactions of Various Participants in Mathematics Teacher Collaboration

AN INVESTIGATION OF CHINESE MATHEMATICS TEACHERS' INFORMAL INTERACTIONS AND COLLABORATION IN MIDDLE SCHOOLS

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The aim of this paper is to present an investigation of Chinese mathematics teachers' daily interactions and collaborations in middle school. Some preliminary results will be reported on the different groups that teachers choose to involve in discussion, the content of teachers' daily interactions and a comparison among teachers from different cities. The study results suggest that Chinese teachers tend to seek interactions with colleagues in teaching research groups and lesson planning groups. The content of these interactions can also be ranked based on the frequency with which they occur. This ranking suggests some implications for teachers' professional learning in daily practice.

Introduction

The assumption that teachers learn through collaboration has underlain research into teacher collaboration and teacher learning, revealing that teacher learning may occur through organised collaborative programs of professional development (e.g. Bell, Wilson, Higgins and McCoach, 2010). Various forms of collaboration are used to support teacher learning and teaching practice within schools (e.g. lesson study). One line of research on teacher learning through collaboration indicates that teachers engage in significant learning through their daily practice, the importance of which has been stated in various studies (Shulman, 1987; Chan, Clarke, Clarke, Roche, Cao, and Peter-Koop, 2018; Chen and Yang, 2013). In another study, an investigation into the daily interactions of Chinese mathematics teachers contributed to an understanding of the culturalised contextual characteristics of teachers' informal interactions within Chinese schools (Chen and Yang, 2013; Zhang, Cao,Wang & Li, 2019). Influenced by Chinese collective culture, Chinese teachers have traditionally worked in groups. A variety of groups exist in the school organisation, including grade groups, subject matter groups, lesson planning groups, and apprenticeship and master teaching stations. Normally, groups have their own offices, allowing teachers to work together geographically and to more easily access interactions with different groups of people (Chen & Yang, 2013). In terms of teacher learning, the Chinese context has always been related to the influence of the lesson study group and other group activities within the school (Cao & Li, 2018). Limited educational resources have been made use of through teachers imitating the 'good teaching practices' of experienced teachers and learning from other teachers in the same groups, both of which play critical roles in guiding teachers to learn. However, aside from these formal group activities, which allowed for teachers to more easily engage in different forms of interaction with different groups, we were also interested in teachers' informal, daily interactions. We asked: Who do Chinese teachers informally interact with more frequently in daily practice? What are the focus and content of informal teacher interactions? How do they differ from the interactions of teachers in other cities in China?

Literature Review

Cross, Borgatti and Parker (2002) noted that some significant outcomes of teacher learning occur in daily conversations during which teachers exchange their ideas and opinions about teaching. These informal talks promote the development of teachers' understanding of knowledge, their creativity in teaching and any changes that may be occuring in the classroom. Also, with the wide popularity in Asia of the lesson study—a highly structured, practice-based approach to studying and teaching within the community of teachers, educators and researchers (Chen and Yang, 2013; Lewis and Teuchida, 1998)—collaboration among different groups of people in the community of mathematics education has traditionally been emphasized to promote the quality of teaching (Robutti et al., 2016). These assumptions, as well as the research, focus on teachers' daily interactions and involvement with colleagues.

Teachers' interactions and collaboration have been identified as a significant issue influencing professional collaboration and improvement (Hargreaves, 2001). To study their emotional experiences in connecting with colleagues, Hargreaves (2001) interviewed 53 Canadian teachers about their collaboration with colleagues. Their research found that teachers value the personal support and acceptance they receive from colleagues, which (the researchers argued) supported their positive and professional approach to teaching. Penuel, Krause and Frank (2009) studied the role that formal and informal teacher interactions play in helping teachers enact instructional changes in practical teaching. They argued that teachers' interactions draw not only on the social context of the school, but also the expertise and resources that can be exchanged through these interactions. Similarly, in Chen and Yang (2013), researchers provided an understanding of Chinese teachers' construction of a reform-based teaching strategy. They concluded that teachers within the school context have a shared interpretation system which may be revealed by their 'native discourse' (consisting of teachers' daily language use, concepts and interactions). The significance of teachers' informal interactions, as revealed by various studies, urges us to focus our attention on investigating the informal interactions of Chinese mathematics teachers' within schools.

Research Methods

Data Sources

The data reported in this paper came from the project *Alignment Between Idea and Practice of Chinese Mathematics Curriculum Reform* (GOA107015, 2010-2012), which selected teachers from three cities in China: BJ, in the north of China, CQ in the south of the country and SY in the northeast. Participants in this study taught at different upper-secondary schools in each city. Seven schools were chosen in each city according to a ranking of the students' grades provided by the local educational institution. Two of the schools were key schools, three of them were ranked in the middle and the remaining two were ordinary schools. We assume that choosing teachers from different schools could provide us with more rich information from both teachers' and schools' perspectives.

Data collection

Questionnaire surveys were sent to a hundred local teachers at different schools. Eighty-six questionnaires were collected back, of which seventy-two were valid for data analysis. Twenty-three of the twenty-nine questionnaires from teachers at BJ were valid, twenty-two of the twenty-six questionnaires from teachers at CQ were valid, and twenty-eight of the forty-five questionnaires from teachers at SY were valid. The questionnaire consisted of two main questions:

- Who do you normally seek out for interaction in terms of teaching-related issues? What is your relation to this person? (Teachers could choose one or more of the following five options: A. Friends; B. Leaders in the lesson planning group or teaching research group; C. Colleagues in the same subject group; D. The neighbour who is sitting next to you; E. The mentor in the apprenticeship.)
- What do you talk about with other teachers at the school? Participants were asked to choose the highest-frequency items from a list of thirteen options; they could also add another item as the fourteenth option (see Table 11).

In terms of the research design, semi-structured interviews with a group of the teachers were conducted after completion of the questionnaire survey. These interviews provided additional understanding of the teachers in terms of their interactions within schools—information that complemented our qualitative data. Detailed results from the semi-structured interviews (see Guo, 2012) are not reported in this paper.

Analysis and Results

Groups of Interaction

Data analysis. Teachers were presented with five groups from which to choose those with whom they shared daily interactions, including: *friends*, *leaders of the teaching research/lesson planning group*, *colleagues in the same subject group*, *the 'neighbour' who is sitting next to you in the office* and *the mentor of apprenticeship*. According to the survey, teachers showed their tendency towards different groups when seeking help with regard to teaching, as shown in the following table (Table 1).

Table 1: The number of teachers choosing each category in three cities

	Friends	Leaders	Colleagues	Neighbour	Mentor
BJ	8	10	20	4	0
CQ	11	11	22	13	9
SY	9	15	24	10	5

Table 2 shows the percentage of participant teachers from each city who chose one particular group/category. For instance, 34.80% means that 34.80% of teachers from BJ in this study choose to interact with *friends*.

Table 2: Percentage of teachers in each city choosing one category

	Friends	Leaders	Colleagues	Neighbour	Mentor
BJ	34.80%	43.50%	87.00%	17.40%	0.00%
CQ	50.00%	50.00%	100.00%	59.10%	40.90%
SY	32.10%	53.60%	85.70%	35.70%	17.90%

Table 3 shows the percentage of choices made in one category by teachers in each of the three cities. For instance, 19.00% means that 19.00% of choices made by the teachers in BJ were in the category of *friends* .

Table 3: The percentage of choices made in each category by teachers from each city

	Friends	Leader	Colleagues	Neighbour	Mentor
BJ	19.00%	23.80%	47.60%	9.50%	0.00%
CQ	16.70%	16.70%	33.30%	19.70%	13.60%
SY	14.30%	23.80%	38.10%	15.90%	7.90%

Comparison among different groups and cities. According to the above three tables, teachers tend to interact with those colleagues who teach the same subject. The percentages of interactions with *colleagues in the same subject group* were higher than the percentage of interactions with other groups. Teachers were less likely to informally interact with their *mentors of the apprenticeship*, especially teachers in BJ, none of whom chose to interact daily with the mentor of the apprenticeship. *Neighbours* and *friends* also were not addressed much by teachers in terms of daily interactions.

With regard to a comparison among the different cities, Table 3 shows that teachers in BJ appear to seek interactions more than teachers in the other two cities. The three cities maintain a relative alignment among their choice of *leaders* in either the *lesson planning group* or the *teaching research group*. The percentages were only slightly lower than the percentages for *colleagues*, but were still significant when teachers sought out interactions for teaching-related questions.

Content of interaction.

Thirteen questionnaire items, labelled a,b, c…m, (see Table 11) concerning the content of their interactions were provided to teachers, who to choose the topics most frequently mentioned in daily interaction. Collected information from the questionnaires is summarized in Table 4.

Table 4: Overall feedback from teachers regarding the content of interactions

Cities	BJ	CQ	SY
Total of chosen items	149	112	146
Total number of teachers	19	24	36
Average number of items chosen	7.84	4.67	4.06

Table 4 shows that the average number of items chosen by teachers in BJ was significantly higher than that of teachers in the other two cities. We assume that, for mathematics teachers, some items were more likely to be mentioned in daily interactions than others. In order to verify the frequency of the thirteen items occurring in daily interactions, two possible hypotheses were proposed and are analysed in the following section.

Hypothesis 1: Assuming that the questionnaire accurately represents teachers' daily interactions, then the different average numbers of items shows the differences among teachers' choices in the three cities. We calculated the percentage of every item among the total number of participant teachers. That is, the proportion of teachers from a particular city (BJ, CQ, SY) who chose that item is shown in Table 5. For example, 89% of teachers from city BJ chose item b.

Table 5: The proportion1 of teachers from each city (BJ, CQ, SY) who chose each item

City	a	b	c	d	e	f	g	h	i	j	k	l	m
BJ	63%	89%	47%	21%	63%	68%	68%	37%	68%	74%	37%	89%	58%
CQ	36%	59%	32%	18%	82%	41%	41%	32%	36%	41%	0%	50%	41%
SY	48%	63%	30%	33%	63%	44%	33%	33%	52%	37%	26%	41%	37%

The ranges vary from 21%~89%, 0%~82% and 26%~63% in the three cities BJ, CQ, and SY, respectively. The range in CQ was significantly wider than the ranges in the other two cities. In CQ, among the total of 24 teachers, item k: *share whether students have learned the content taught during the lesson* was not selected by any teachers, while most teachers selected item e: *analyzing students' learning examples*. The table shows that teachers' choices in CQ are similar to those in SY, but different from those in BJ. The three cities maintain a similar percentage for items c, d, e and h, but teachers from BJ tended to talk more frequently on the aspects of items b, f, j and l than teachers in the other two cities. Table 6 summarizes the rank of the frequency of the items in the three cities.

Table 6: The rank of items in three cities based on proportion1

Rank	1	2	3	4	5	6	7	8	9	10	11	12	13
BJ	d	h	k	c	m	a	e	f	g	i	j	b	l
CQ	k	d	c	h	a	i	f	g	j	m	l	b	e
SY	k	c	d	g	h	j	m	l	f	a	i	b	e

Table 6 shows that the four items k, c, d, h were the ones least frequently discussed in teachers' daily interactions. Out of the three cities, all teachers were more likely to select item b and item l. Item e was most frequently selected by teachers in CQ and SY, but was a relatively low frequency choice for teachers in BJ. Item i was mentioned more frequently by teachers in BJ and SY and less frequently by teachers in CQ. Items can be classified in terms of the frequency with which teachers selected them: high frequency items (b, e, i, l), middle frequency items (a, f, g, j, m) and low frequency items (c, d, h, k).

Hypothesis 2: Here we assume that the differences among teachers' selections were the result of the participant teachers' individual characteristics. That is, teachers may show different answering habits, where some tend to answer more questions than others. To contend with this situation, we calculated the percentage of the times that each item was chosen out of all items in each city, as shown in Table 7.

Table 7: The percentage2 of times each item was chosen from the total times being chosen out of all items in each city

	a	b	c	d	e	f	g	h	i	j	k	l	m
BJ	8%	11%	6%	3%	8%	9%	9%	5%	9%	9%	5%	11%	7%

CQ	7%	12%	6%	4%	16%	8%	8%	6%	7%	8%	0%	10%	8%
SY	9%	12%	5%	6%	12%	8%	6%	6%	10%	7%	5%	8%	7%
Average	8%	12%	6%	4%	12%	8%	8%	6%	8%	8%	3%	10%	7%

Table 7 shows that the ranges of the percentages in different cities varied from 3%~11%, 0%~16%, and 5%~12% in BJ, CQ and SY, respectively. The square deviation and standard deviation for each city have been calculated in Table 8, which shows that the percentages for each item in the three cities are relatively stable and reasonable.

Table 8: The square deviation (μ2) and standard deviation (μ) of the percentage2

	BJ	CQ	SY
Square deviation (μ2)	0.021%	0.024%	0.013%
Standard deviation (μ)	1.438%	1.550%	1.161%

Table 9 shows the distance from the ratio of every item to the average level for all three cities.

Table 9: Distance analysis between the percentage of each item selected to the average of that item selected among teachers in each city

	a	b	c	d	e	f	g	h	i	j	k	l	m	μ2	μ
BJ	0.02%	0.14%	0.12%	1.46%	3.87%	0.40%	1.08%	1.01%	0.24%	1.30%	1.53%	1.82%	0.04%	0.02%	1.44%
CQ	0.89%	0.05%	0.33%	0.57%	4.15%	0.29%	0.39%	0.55%	1.34%	0.06%	3.16%	0.23%	0.61%	0.02%	1.55%
SY	0.87%	0.09%	0.44%	2.02%	0.28%	0.11%	1.48%	0.46%	1.10%	1.24%	1.63%	2.05%	0.57%	0.01%	1.16%

The table shows that the distance to the average for each item is relatively small, meaning that the average level could be used to represent the characteristics of the data. We then used the average percentages to rank the different items. Table 10 shows the rank of frequency based on the average level calculated in hypothesis 2.

Table 10: A ranking of items based on the distance to an average percentage2

k	d	h	c	m	g	a	j	f	i	l	b	e
3.2%	4.1%	5.7%	5.9%	7.4%	7.4%	8.0%	8.1%	8.3%	8.5%	8.5%	11.6%	11.9%

According to the new ranking, we have four low frequency items (k, d, h, c), five middle frequency items (m, g, a, j, f) and four high frequency items (i, l, b, e).

To summarize: The two hypotheses and their related analysis methods finally result in a frequency ranking for the items mentioned by teachers. The frequency of teachers' interactions in terms of the content of their discussions and daily interactions can be ranked in the following order (Table 11):

Table 11: Frequency ranking of items in terms of the content of teachers' daily interactions

Item	Content
h	Discussions that occur during lesson group work/discussion concerning how to elicit students' opinions (solutions, strategies).
c	Sharing of the different ways in which students solve a problem.
d	Discussion as to why some students have not finished their learning tasks in preparation for future lessons.
k	Communicating whether students have implemented the teaching content after a lesson.
a	Discussion about the different ways of solving a problem.
m	Sharing opinions about students' learning progress.
f	Analysis of students' examples of learning so as to adjust teaching process.
g	Analysis of students' learning condition to ensure that students have mastered related knowledge.
j	Discussion on the use of teaching materials for teaching, practice and review.
b	Discussion on the choice of teaching content.
e	Analysis of students' learning so as to understand students' ways of solving a problem.
i	Discussion on the teaching process.
l	Sharing of teaching materials and activities designed for teaching.

Discussion and Conclusion

This study provides useful results for understanding Chinese mathematics teachers' daily interactions. Both differences and similarities were found in terms of the groups with whom the participants interact, as well as the content of these interactions in three different cities. First, the study addresses the significant roles that the *teaching research group* and the *lesson planning group* play in teachers' daily interactions. Since we are concerned with the tendency of teachers to seek help with teaching-related issues, the *teaching research group* and *lesson planning group* normally consist of teachers who are all teaching the same subject (mathematics) and who have a wide variety of practical experiences. Teachers' dependency on these groups shows the need for the trustworthiness of such groups. This specific Chinese school collective culture provides teachers with opportunities to seek help 'within their reach'. The school-based group activities and the correspondence informal interactions also provide implications for further developing teacher learning within schools. Further results from the semi-structured interviews are needed to understand how teachers seek learning and growth through interactions with colleagues in the same groups.

In terms of the content of the interactions, the depth of interactions surrounding classroom practice makes a difference in terms of supporting the improvement of teachers' classroom practices. Coburn and Russell (2008) found that deep interactions may include the mathematical intend of instructional tasks and the relative sophistication of student reasoning strategies. Interactions of less depth may involve the use of teaching materials or the mapping of curriculum to curriculum standards. This study suggests that Chinese mathematics teachers interact more when analysing student learning of content knowledge and when planning teaching process. Sharing and reflecting after a lesson in terms of student performances during the lesson is mentioned less frequently. Chinese mathematics teachers also rarely discuss teaching materials and curriculum standards, possibly because uniform textbooks and materials are used in China (Zhang et al., 2019) and may be taken for granted. More studies are needed to further study how informal interactions in terms of different content might help or hinder teacher learning in practice.

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SUPPORTING COLLABORATION BETWEEN VISUAL ART AND MATHEMATICS TEACHERS

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This research focuses on supporting an interdisciplinary collaboration between mathematics and visual art teachers with respect to developing an art-math integrated teaching practice. It is situated in two Greek art-based schools. Group meetings have been designed based on elements from Communities of Practice and Communities of Inquiry. This paper tries to shed some light into the challenges I have faced while committing to the role of broker between the art and mathematics teacher communities, facilitating their collaborative engagement in inquiry and reflection, and the actions I undertook to deal with them. In data analysis, I focus on challenges and actions undertaken and, in these actions, I try to identify theoretical constructs from CoP's Boundary Crossing. Three main challenges appeared: need for activation and motives; communication gap, transparency issues and other problems in discussion regulation; and challenge of succeeding in inquiry and reflection.

Many studies assert connections between mathematics and art (Cucker, 2013). Many others support the benefits of *arts integration* into other subject areas (Burnaford et al., 2007). Seeking creative paths to mathematics teaching, bridging the two could be of benefit: engaging in problem solving/modeling (Jacobs, 2000), supporting inquiry learning approaches through the art process (von Renesse & Ecke, 2016), developing mathematical thinking, linking classroom mathematics with personal experiences (Presmeg, 2009), etc. Connections between math and art instruction may reach a holistic-curriculum level, where art and math teachers are invited to collaborate to reform the two subjects (Bickley-Green, 1995). In mathematics education research, various studies use frameworks like Communities of Practice, to analyze how teachers work in groups (Robutti et al., 2016). Few studies focus on interdisciplinary mathematics collaboration (e.g. Nelson & Slavit, 2007, mathematics-science). In art education research, most studies concern professional development of teachers in artist-in-residence coaching programs, usually focusing on primary level (e.g. Burnaford et al., 2007). Few researches focus specifically on mathematics and art (mainly music/drama) (e.g. An & Tillman, 2014).

In this study, I am interested in visual art and math teacher collaboration. Constraints may exist in a collaboration between art and non-art teachers. For example, due to lack of communication tensions may arise (Jacobs, 2000), and due to strict timelines, both disciplines' teachers might need extra time for collaboration (Gullatt, 2008). Support of visual art and math teacher collaboration is needed. Late research focuses on facilitators' roles and practices (e.g. van Es et al., 2014). Few focuses on interdisciplinarity and some focus on inquiry ways of working (e.g. Nelson & Slavit, 2007) and challenges confronted (Cestari et al. 2005). More research is needed on how one can facilitate a collaboration between visual art and math teachers, as well as the challenges he may face and the ways he can overcome them. The research question is: What challenges does a facilitator face and what actions does he undertake to cope with them, in supporting visual art and mathematics teachers' engagement in a collaboration through which a visual art-math integrated practice is to be developed?

Theoretical perspectives

Arts integration has been characterized as integration of people (Burnaford et al., 2007). In that, I see a *collaboration* between people to be bound with the concept of *community*. Visual art teachers belong to the visual art (teaching) community and math teachers to the mathematics (teaching) community. I am interested in the boundaries of and between these two, envisioning the formation of visual art and mathematics teachers' community of practice through collaborative inquiry. I present briefly below constructs from Communities of Practice by Wenger (1998, 2006) and Communities of Inquiry by Jaworski (2006) that I have used as an analytical framework.

Communities of Practice (CoP) and Communities of Inquiry (CoI)

Wenger (1998) refers to *Communities of Practice*, i.e. groups formed by people that share practices reflecting their collective learning. For him, in these communities, practice and community are joint together. He describes three dimensions of the relation by which practice is the source of coherence of a community: mutual engagement, joint enterprise and shared repertoire. He talks about four components of learning: meaning (learning as experience), practice (learning as doing), community (learning as belonging) and identity (learning as becoming). He also focuses on brokering and boundary crossing between communities. Boundaries connect communities and offer learning opportunities. Learning here can be maximized through: *something to interact about, an intersection of interest*; *opening engagement with real differences as well as common ground*; commitment to *suspend judgement and competence of a community in its terms*; and *ways to translate between repertoires so that experience and competence interact*. Boundaries can be places where perspectives meet and new possibilities arise, but also be sources of potential difficulties. Three dimensions for boundary processes to create bridges that connect practices in deep ways are: *coordination* (enabling coordinated actions, accommodation and standardization of practices so that everyone can engage in them), *transparency* (access to meanings and understandings of practices involved) and *negotiability* (negotiation of perspectives and multiple voices). He speaks of brokers, introducing elements of one practice into another via creating connections, moving knowledge, exploring new territories. Boundary objects support connections between practices: artifacts (tools or documents that play crucial roles in connecting multiple practices), discourses (common language that allows communication and negotiation of meaning across boundaries) and processes (shared routines/procedures that allow multiple practices to coordinate their contributions). Use of these objects enable multiple practices to negotiate relationships and connect perspectives.

Wenger et al. (2002) report on seven principles for cultivating communities of practice. They talk about designing for evolution of a community (e.g. social and organizational structures such as a community coordinator or problem-solving meetings), opening a dialogue between inside and outside perspectives about the community, inviting different levels of participation (core, active or peripheral group), developing public and private spaces for members to interact with each other, focusing on the emerging value of the community and ways to harvest it, combining familiarity with excitement via events that will keep the members fully engaged in the community and creating a rhythm for the community (tempo of interactions, regular meetings, web site activity, heartbeat of the community).

Jaworski (2006) talks about *Communities of Inquiry*. She steps on Wells's (1999) perspective of dialogic inquiry - "a willingness to wonder, to ask questions, and to seek to make answers to them" (p. 122) - and proposes inquiry as a fundamental theoretical principle: inquiry as a *tool* and inquiry as a *way of being*. She focuses on inquiry in practice in: Addressing mathematical tasks in classrooms, developing approaches to mathematics teaching or finding ways of working with teachers to promote teaching development. She also uses the term "metacognitive awareness" and accords it to Wells "metaknowing through reflecting on what is being or has been contributed, and on the tools and practices involved..." (p. 124). Corresponding with Wenger's "modes of belonging", she says that for any individual, "belonging" to an inquiry community is both nurturing and challenging. She speaks of observation, action, questioning of actions and inquiry into actions as processes in which a novice practitioner is drawn into the community. In Cestari et al. (2005), they reflect on didactians' roles and tensions when working with teachers (ensuring full membership, inclusivity in dialogue and activity, keeping focus, not patronizing, sensitivity in judgement, trust-confidence or suggesting roles for participants, establishing an environment suitable to offer materials). A key issue for didacticians is how teachers will start to think in inquiry terms and to use inquiry in classroom.

Methodology

Context of the study and aims of the facilitator

The study is situated in two Greek art-based schools (grades 7–12). In the first part, an ethnographic approach was adopted (Allan 2017) (2 visits/week at each school, 8 hours/day, for 8 months, keeping field notes (observations of math and art classrooms, audio-visual records, informal discussions, self-reflections). Getting close to the art community, I acknowledged artifacts, tools and practices as boundary objects between art and mathematics community. I aim to support teachers' collaboration in inquiring boundaries in-between their discipline contexts and developing an art-math integrated practice. In the second part of the research, I intent to build on a developmental research (Jaworski, 2006), forming an integrated community and exploring its development. Interested in developing art-integrated approaches to mathematics teaching, I relate to CoI's second layer of inquiry in practice. In designing the meetings, I have drawn on inquiry-reflection and CoP's cultivating principles and boundary crossing, pursuing a sense of structure, rhythm and space for the upcoming community, as well as familiarity and excitement, perspectives and value. In the next sections, I will first describe the design of the meetings, as well as my aims and initially organized role in the collaboration context.

Design of the group meetings

For the last 3 months of my project, I initiated two art and mathematics teachers' groups [1 group at each school consisted of seven teachers (2 mathematics and 5 visual art teachers)]. Each group meets once every 2 weeks. The meetings are embedded in the daily school setting, conducted on a spare time that most of the teachers share (1 didactical hour, in the teachers' office or in the art lab). Until now, 6 meetings have been conducted by each group. The first meeting focused on an introduction to what the group meetings would be about. Boosting the general context of the research, a didactician from the university who relates to this research and whose expertise lies in mathematics education and teacher education research, came to offer some institutional form in the collaboration. In the next meetings, resources were chosen for the group to work on. Data collection includes field notes, audio recording and visual recording when needed (e.g. for gestures, visual descriptions). After each meeting, the transcription of the audio and visual record is undertaken. Then, I prepare a summary report with central points of the discussion and a summarized discussion flow, as well as the resources on which the group will work on in the next meeting. These are uploaded in the university's e-class platform (few days before each meeting). Teachers have been provided with password codes to have access in this platform. Future group meetings are scheduled. The last meetings will be dedicated to teachers' design and implementation of arts-integrated math tasks and reflections.

Brokering and facilitation in the organized collaboration context

My brokering and facilitative role in the collaboration is, also, designed based on CoPs' cultivating principles, boundary crossing and the notion of inquiry, as well as on the data I gathered via my observations and interactions with teachers and students in the 1st part of the study. I, also, draw on literature based on visual art and mathematics congruence. The sub-roles of: 1) provider of various resources; 2) forwarding, supporting, regulating or marginalizing discussions; and 3) promoting inquiry during the meetings on themes based on the resources and reflection on what has been discussed in respect to teaching and learning, are included.

I think of the resources as boundary objects that can transcend the boundaries in-between the two communities. In participants' stepping on these bridges, these boundary resources are meant to enable negotiation between the teachers and cultivation of math-art integrated practice. These resources include typical visual art resources (e.g. paintings), visual art students' actions (e.g. constructions), mathematics and art curriculum materials (common ground) and teaching issues (e.g. reflection on already implemented math-art tasks). I aim at these resources to serve as a basis to initiate inquiry and reflection of potential links between the two, focusing on 4 dimensions: opportunities for mathematics teachers to get informed about artistic creations and procedures (knowledge and skills, techniques etc.); group's inquiry and reflection on the artistic procedure in relation to mathematics; group's inquiry and brainstorming on how the above can be used in mathematics teaching and learning; and group's feedback. Each meetings' material is organized based on themes arisen in the previous meeting, in order to give the teachers a sense of "their" leading of the project through their discussions. I have considered two elements crucial; inquiry and reflection. I see both these as ways to engage teachers in negotiation towards developing integrated meaning and teaching practice. After inquiry and reflection on the given resources, each meeting ends with reflective questions asked by me and discussed among the group, such as: "What connections do you see here between mathematics and art?", "Would this help students of mathematics? How?", "How would you use this in mathematics classroom? What modifications would you make?", "What would a collaboration between the art and the mathematics teacher include when co-working on designing and conducting an integrated lesson? How one can help the other?". Sometimes, the questions may even concern art classroom and art students' learning accordingly. Gently regulating the discussions, I try for a full membership by the teachers and for their engagement in the negotiation of meanings rooted in inquiry and reflection. I have in mind for the discussion to stay focused and for time to be evenly distributed, aiming for mutuality in exploring possible trajectories towards building the potential integrated practice as a joint enterprise of the upcoming community.

Data analysis

For this paper I draw on the first 5 meetings of one of the schools' group. Adopting a Grounded Theory approach (Strauss & Corbin, 1998), data were coded and categorized based on the challenges I faced and the actions I undertook to cope with those in supporting visual art and mathematics teachers' mutual engagement in collaboration. In these actions, I have tried to identify theoretical constructs from Wenger (1998) concerning the four ways that learning can be maximized in boundaries and the three dimensions for boundary processes to create bridges that connect practices in deep, placed as codes in []. For each action accorded to a challenge I offer examples from data.

Results: Challenges and actions undertaken

Need for activation and motives

Difficulties have appeared concerning teachers' openness towards the research, willingness to participate and engage in the meetings and finding of meaning in doing so. They seem to usually be tired and under time pressure. Either mathematics or art teachers missing the whole or part of a meeting is not a rare thing (e.g. due to extra schoolwork). In addition, some teachers at first seemed to be quite skeptical about giving out their "ideas" and may be uncomfortable with inquiry and new territories. For example, a mathematics teacher (MT1) demanded: "Give me already designed tasks or even ideas to use them and if you do give me 10 of those then maybe I'll think about furthering one of them a step forward"**.** Furthermore, the following words of an art teacher (AT1) highlights a possibility of art teachers' feeling of not being needed or "wasting" their time: "I would like that what is brought to the group meetings would be something that concerns everyone and that everyone can earn from it. … Why should I waste my time on something that I already know? This is not mathematics; it is part of the art practice; it is an art technique…". Also, need for positive attitude is shown, for example, when after the 3rd meeting, two pairs of teachers (1 art and 1 mathematics teacher at each) decided to implement an art-math task. Due to unfamiliarity and time issues they did not inform me, although I have highlighted multiple times my interest in being present.

Actions. For supporting teachers' motives and positive attitudes towards participation, I try to encourage them when engaging in inquiry and sharing their ideas or suggestions [*suspending judgment*]. I even enlist humor to make them feel more comfortable and intimate and trying gently to build a positive atmosphere. For example: "oh that's a nice idea", "Oh cool! I did not have realized this before", or "How do you like this?". Moreover, in response to AT1' challenge above, trying to maintain my composure, I encouraged her to tell me more and justify this, saying that this will help me "arrange things better" [*suspending judgment*]. Furthermore, I choose resources, like examples from daily life or realistic contexts [*something to interact about*], to get them excited and enthusiastic about the potentiality of linking art and mathematics and engaging them in committing to the collaboration context and the development of art-based practice. Teachers find these "beautiful" and interesting, however these might not relate directly with the school curriculum which constitutes a new challenge, e.g. images with mathematical elements embedded in optical illusions [*something to interact about*]. Also, I may propose collaboration for out-of-the school projects, such as participation on a mathematical symposium or working on writing a paper for a mathematics education conference [*something to interact about*]. In response to MT1's demand, I chose to give them an art task implemented by another school (using a video presentation) that could facilitate inquiry on "what happened" and "better ways" to its design [*ways to translate between repertoires, negotiability*].

Communication gap, transparency issues and other problems in discussion regulation

Art teachers and mathematics teacher are unfamiliar with each other's contexts. There is a communication gap that rises transparency issues. Before reaching aspects of inquiry and reflection, issues of coordination between the upcoming community appear (e.g. issues of terminology that may hinder transparency and negotiability). I also experience a dilemma of whether to share personal points of views, when the discussion seems to need a boost or an explanation is required. Moreover, regulating the flow of the discussions is not always easy. As meetings go by, teachers are more and more talking to each other, ending up on forming subgroups that speak in parallel. Time limitation plays a role, too. This links with the fact that when many members participate there is sometimes a danger of the discussion not staying focused, as everyone carries on his/her personal experiences and concerns. It also, relates to teachers being late to come, not come at all or leave the meeting for a few minutes. Thus, the rhythm of the discussion is not always steady. Another need is that of private, quiet places for the meetings to take place in, as the school setting is usually crowded.

Actions. I focus on the material provided to have potential to enable mutual engagement and inquiry, trying so that everyone learns about each other's perception, thoughts, ideas, beliefs about what they see. For example, in the 5th meeting, still working on tessellations, I asked how they can remember a specific Escher's art painting in order to replicate it. The specific resource and the type of question gave space for inquiry into mathematical concepts embedded in the painting, as well as ways of replication, enabling all members' ideas and suggestion as well as focus to be revealed *[something to interact about, opening engagement with common ground and differences, ways to translate between repertoires, transparency, negotiability*]. I also ask for more explications and details on behalf of the teachers when I see that they do not understand something but do not feel like asking (first 3 meetings or so). For example, when an AT1 talked about negative and positive space in visual art, I had to ask her to explain a little further, as I could see in math teachers' faces that they did not understand what AT1 was saying but still remained silent [*coordination*]. In addition, I sum up what a teacher speaks about, to be sure the rest of us have understood and I link representations and pre-mentioned points of discussion [*coordination, transparency*]. I use simple terms, so as for the art teachers to be able to understand or not feel overwhelmed by "foreign" or "special" terminology, at least initially. For example, instead of "polygons" I use "shapes" [*coordination*]. When teachers talk all together, I try to decide a sequence, suggesting the person who is to speak each time, enabling *negotiability*. Also, if either a mathematics or art teacher misses a meeting or part of it, my role offers a repetition of the crucial aspects arisen and a call for their opinions or ideas on the critical questions and issues there [*negotiability*]. Also, I focus on finding *common ground* in art and mathematics teaching and learning issues, as well as *differences*, e.g. in the role of tools and problems the students face when using them.

Challenge of succeeding in inquiry and reflection

High levels of inquiry have not always been an easy task, due to two main sub-challenges. 1) Time scheduled for the meetings seems to not be enough, thus difficulty in keeping inquiry levels high emerge. Sometimes I face the dilemma of whether I should not worry about the time and stick to high quality of inquiry thus not having time for the reflection section and letting it aside to do it in the next meeting, thus getting late on the overall meetings' timeline, or respecting that timeline thus sacrificing high levels of inquiry. For example, in the 4th meeting, I brought material on tessellations. MT1 started talking about another task she designed with AT2 and implemented it in the classroom. I did not cut her off, as this is something important in respect to the development of their community. Due to not being strict in marginalization, that left only 15 minutes of break time to start the actual meeting. So, there was no time to engage in inquiry, for example, about why only 3 regular polygons tessellate a plane. 2) A need for teachers' boost for ideas and mathematics teachers' need for mathematics teacher development has appeared. This is true especially when few members participate. Teachers like to "see", they do not inherently care about inquiring or reflecting. When, in the last example, I asked if we should stop because their break time had come, they answered: "Em… maybe show us some more pictures. They are nice! Then we will leave", which does show an interest in the material, on which they would sacrifice their break time, but when we reached the reflection part, we stopped. They also show difficulty in making something "their own" or "of their own" instead of copying given ideas, and to design a "meaningful" instead of a "superficial" task. For example, MT1 in 3 out of 5 meetings would focus on the given questions or idea saying: "Oh I like that, I'll do it too", even if it was not an idea or task of "good quality", having difficulty to inquire and reflect in ways to develop it."

Actions. Trying to face the 1st sub-challenge, I twice gave them the reflection part as a "homework" so they can prepare it and send it to me to upload it on the e-class for everyone to see it and thus being more ready in the next meeting (earn some time), but it did not work out [*something to interact about*]. Trying to deal with the 2nd sub-challenge, I modify my intervention levels, e.g. if I am currently the only other member that relates to mathematics teaching, I have to play that role if the conversation starts to "die" [*ways to translate between repertoires*]. For example, when I tried to engage the group in inquiry about what mathematical concepts they could relate to a representation of a squared tessellated floor, there was only one mathematics teacher participating at that moment that would not come up with anything. Trying to gently facilitate the discussion, I drawn on a personal experience regarding the concept of area: "Em… you know… every time I see a squared floor, do you know what I think about? I remember my father asking: '-Do you know how much 1 square meter is? ... It is a floor tile that its side is 1 meter! -(Me:) Oooh really?? That big??' I really got excited!" [*something to interact about, opening engagement with common ground, ways to translate between repertoires*]. Moreover, I "play dumb" or do not answer them directly, to urge them to be more explicit about what they mean, give them time to inquire, to think for themselves, not forcing an answer by them, giving time for discussion between them to flourish [*coordination, transparency, negotiability*].

Discussion and future thoughts

In this paper I have tried to report on the challenges I face, and the actions I undertake to cope with them, in supporting visual art and math teachers' engagement in collaboration. Our findings reveal three main challenges in this collaborative context: *need for activation and motives*; *communication gap, transparency issues and other problems in discussion regulation*; and *challenge of succeeding in inquiry and reflection*. Community building is a challenge itself and, among other things, it requires a lot of time, something with which we struggle in all these challenges. Also, whereas most collaborations take place in workshops in university settings, this research chooses group meetings to take place in school setting. A stakeholder from "above" would make things easier (Gullat, 2008). Agreeing with Cestari et al. (2005), tensions seem to appear while avoiding a "top-down" model (p. 1357), regarding participation levels, teachers asking for ideas and given ideas or questions working as a model for MT1. The actions I undertake to cope with the main challenges mostly concern *coordination*, *transparency* and *negotiability* aspects, but also *something to interact about* and *opening engagement in common ground*, as well as *suspending judgment*, *opening engagement in differences* and *ways to translate between repertoires* (boundary crossing, Wenger, 1998).

Early signs of teachers' boost of motives and mutual engagement are starting to get through. The efforts of 2 pairs of teachers to integrate art and mathematics, though not directly related to school curriculum, is also a "win". A shift in my role also starts to arise along. At first, I was struggling with making the teachers talk, now they form spontaneous sub-groups talking in parallel. Though not efficient for the whole group's focus, it nevertheless gives a sense of community building and bonding. The importance of the research, also, emerge in MT1's words: "This helps me answer to my students when asking: "Why should I learn mathematics?". Future thoughts include issues related to: shifts in participants' roles; identities; resources; outcomes on teachers' development and effect on students' learning, sustainment and scalability. Reflecting on such issues may encourage better understanding on how math-art interdisciplinary collaborations may work in favor of math education.

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TOWARDS A MICRO ANALYSIS OF TEACHERS' INTERACTIONS IN A MATHEMATICS LESSON STUDY GROUP

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This contribution presents an ongoing study of dialogic interactions in a lesson study group about the teaching of problem-solving in mathematics. The paper focuses mainly on the construction of the grid of analysis, its roots in previous research and its potentiality to analyse the specific dialogic role of each facilitator and teacher during the phases of lesson study process.

Introduction

The theme of collaboration in teacher professional development is an increasingly important theme, particularly in mathematics education as shown in the ICME-13 Survey (Jaworski et al., 2017) and the future ICMI-25 study. Among the implied conditions for collaboration, effective communication and promotion of professional dialogue (Boavida & Ponte, 2002) are put forward in ICMI-25 study discussion document. The study of communication among teachers, more precisely on how teachers learn through interaction with others, questions particularly the nature of interactions between the facilitator and the teachers (van Es, Tunney, Goldsmith, & Seago, 2014). The goal of our research is to describe these dialogic interactions in order to better understand how teacher learning occurs in a lesson study process. By listening to the voice of every teacher and facilitator and by characterising their interaction, the ongoing micro analysis, conducted as a comprehensive case study, is a rather new direction of research. It would also make a significant contribution to the topic of ICMI-25 study, "Roles, identities and interactions of various participants in mathematics teacher collaboration", specifically to address the two questions:

- How are different roles and identities shaped and developed among various "actors" (teachers, leaders, mathematicians, researchers, etc.) within a collaborative group? How do lead teachers negotiate their dual roles and identities as both teachers and facilitators of peer-collaboration?
- How can different stakeholders impact teacher collaboration?

Our research is still in its initial stage and this paper concentrates mainly on its theoretical and methodological aspects. After presenting a brief background of lesson study, the context of our study and our research interests, the article focuses on the construction of our grid of analysis and, in link with the "Theoretical perspectives on studying mathematics teacher collaboration" theme of ICMI-25, its roots in previous research. The formulation of our research questions and the perspective for the research concludes this article.

Lesson Study

Jugyo Kenkyu, literally lesson study (LS), were born in Japan in the 1890s. They were popularised in the 2000s following international TIMSS comparisons (TIMSS & PIRLS International Study Center, n.d.; TIMSS Video, n.d.) and the comparison between mathematics education in Japan, Germany and the USA that Stigler and Hiebert (1999) presented in *The Teaching Gap*. Thanks to the efforts made to promote LS, and in particular to the work of Lewis, who contributed to formalising and popularising LS in the USA (C. Lewis, 2002, 2015; C. Lewis & Hurd, 2011), LS was initially introduced in the USA as a professional development approach to improve US mathematical classroom teaching and learning (Yoshida, 2012). As a mode of professional development, LS has developed all over the world leading to an establishment of the World Association of Lesson Studies (World Association of Lesson Studies, n.d.) and it has attracted the interest of many researchers in educational sciences, particularly in mathematics education (see, for e. g. among the most recent international edited books, Huang, Takahashi, & da Ponte, 2019; Quaresma et al., 2018).

LS starts from an area of difficulty in teaching and learning identified by a group of teachers. Teachers analyse the targeted learning, study the mathematical concept, consult the various teaching methods, study articles from professional journals and other resources. This study allows them to plan a lesson together. This lesson is implemented in the classroom of one of the group members. Other teachers observe the lesson in real-time and analyse its impact on students' learning. The group may decide to plan an improved version of the lesson to be conducted in another teacher's classroom and the loop begins again. The results of the work are disseminated, both in the form of a detailed lesson plan for future use by other teachers and also in the form of articles in professional journals.

LS groups are usually led by an experienced teacher or trainer, called a facilitator, who "keeps the conversation moving and fair. Involves all participants. Follows an agreed-upon agenda" (C. Lewis & Hurd, 2011, p. 124). While in Japan, LS are sometimes facilitated directly by the group members, they almost always involve one knowledgeable other who provide feedback in the discussion after the research lesson and sometimes another knowledgeable other who can draw attention to key elements during the planning phase (Watanabe & Wang-Iverson, 2005). In countries where LS are developed (particularly in Japan) the role of facilitators as facilitators participating in the group and that of occasional external experts is very well defined, these two roles are often assumed by the same person or are confused in places where LS are starting to take root (Clivaz & Takahashi, 2018).

Elements of Context and Research Interests

Our research is part of the work of the Lausanne Laboratory Lesson Study (3LS, n.d.) which studies the work of several LS groups in the French-speaking part of Switzerland. We are currently analysing the work of a LS group composed of 8 grade 3 and 4 teachers from the Lausanne region and two facilitators. The two facilitators consist of a mathematics educator (the first author of this contribution) and a teacher from the institution who participated as a member of a previous LS group in mathematics. From 2018 to 2019, this group has completed three LS cycles with the question: "how to teach grade 3-4 students how to solve mathematical problems". The fact that the teachers will be in a (professional) problem-solving situation about problem solving teaching is coherent with Ball and Cohen (1999) suggestion that professional development programs should situate teacher learning in the types of practice they wish to encourage. The data from these two cycles were analysed, using Transana (Woods, 2002-2017) to encode video recordings which are integrated with the transcripts. The first cycle mentioned in this article, includes eight meetings that lasted for about 90 minutes each and two research lessons. Interviews with the two facilitators were also analysed as a way of data triangulation. Since other researches of our team used the same type of data (e.g.

Batteau, 2017; Clivaz & Ni Shuilleabhain, 2019), the obtained results show that this type of data is thorough, systematic, reliable and authentic regarding the perspectives and practices of participants.

The research interests of our team [1] are two-fold. The first series of questions is about the mathematical knowledge related to problem solving that teachers use during this LS, the second series aims to describe the types of interactions related to the construction of this mathematical knowledge and, ultimately, to describe how the mathematical knowledge is constructed in a LS group. This paper will concentrate on the analysis of the interactions and only show the links with the mathematical knowledge and the problem-solving-teaching. In order to do so, we will present some of the previous studies and the analytical frameworks which is currently developed by our group.

Previous Research

It is with the objective of accurately describing how teachers' knowledge is constructed or evolves and, more generally, to better understand what happens between actors within an LS process, that we have been led to focus on discourse analysis in a sociocultural perspective, rooted in the work of Vygotsky (1962, 1978) for whom the acquisition and use of language transforms children's thinking. One of our first inspirations was driven from the work of Vermunt and his colleagues (Vermunt, Vrikki, van Halem, Warwick, & Mercer, 2019; Vrikki, Warwick, Vermunt, Mercer, & Van Halem, 2017; Warwick, Vrikki, Vermunt, Mercer, & van Halem, 2016) who categorise the dialogic processes in LS groups in order to find statistic correlations between certain dialogic features and teachers' meaning-oriented learning in LS. These categories being too broad for a comprehensive analysis we were led to study the work of a sister group within the Cambridge Educational Dialogue Research group (CEDiR, n.d.), the Scheme for Educational Dialogue Analysis (SEDA, Hennessy et al., 2016) group. Rooted in the work of Alexander (2008) about dialogic teaching, this group produced a comprehensive grid to analyse classroom dialogue in problem solving situations. The grid and the method of an "inductive-deductive cycle that allowed to distil out the essence of dialogic interactions and operationalise them in the form of a new systematic indicators for these productive forms of educational dialogue" (Hennessy et al., 2016, p. 17) seemed to be a good choice to serve as a basis for the construction of a grid of systematic indicators able to capture the forms of professional dialogue within a professional development process. Nevertheless, adaptations have to be made to SEDA scheme to take into account our research context as well as previous research on teacher learning in LS.

Comprehensive research on LS groups and the fact that they appear to have an impact on teachers' professional knowledge often focuses on the essential role of facilitators (e.g. Akiba, Murata, Howard, & Wilkinson, 2019; Bjuland & Helgevold, 2018; Borko, Koellner, & Jacobs, 2014; Carlson, Moore, Bowling, & Ortiz, 2007; Hart & Carriere, 2011; C. Lewis & Hurd, 2011; J. M. Lewis, 2016; Schipper, Goei, de Vries, & van Veen, 2017; Stepanek, Appel, Leong, Turner Mangan, & Mitchell, 2007) and possible knowledgeable other (e.g. Amador & Weiland, 2015; Akihiko Takahashi, 2014; A. Takahashi & McDougal, 2018; Watanabe & Wang-Iverson, 2005). While many studies mention the importance of these roles and give examples of facilitator interventions or mention statements by teachers saying how important this role is to them, qualitative studies describing precisely how this

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role allows teachers to build professional knowledge are rare to date. While surveys such as Akiba et al. (2019) show that "facilitators' focus on student thinking, the quality of materials, and duration of lesson study were significantly associated with teacher participation in an effective inquiry process, which in turn is associated with perceived positive changes in teacher knowledge, self-efficacy, and expectation (p. 352)", these studies do not address precisely what makes facilitator interventions facilitate the construction of professional knowledge and this group calls on further exploration "by analysing the detailed nature of interactions between facilitators and teachers in each stage of lesson study. (p. 362)".

For our part, in our previous research, we examined the evolution of the trainer's role in terms of knowledge sharing in a series of LS (Clerc-Georgy & Clivaz, 2016). Apart from that, we have also theorised the work of LS groups in relation with French *didactique des mathématiques* (Clivaz, 2015, 2018). We also showed which Mathematical Knowledge for Teaching (MKT, in the sense of Ball, Thames, & Phelps, 2008) teachers use during the LS process (Clivaz & Ni Shuilleabhain, 2019; Ni Shuilleabhain & Clivaz, 2017). Nevertheless, interactions within the group in particular between the facilitators and the teachers are yet to be explored.

Methodology for Analysing the Types of Interactions

In this section, we will describe our grid for analysing interactions as the result of a process that is both deductive and inductive. This grid currently focuses mainly on enunciative modalities but will be linked in the rest of our research with MKT and issues related to the topic of problem solving.

The Construction of the LS Interaction Analysis Grid

Composed of 33 codes grouped into 7 entries, the SEDA grid (Hennessy et al., 2016; SEDA, n.d.) had to be adapted from students' mathematics problem solving classroom situation to teachers' professional problem solving discussion. We started from the SEDA grid, using the same codes every time it was possible and adapting them when necessary. After a one-year coding work, and team discussion of the coding, we were able to set up, in an inductive way, our grid for analysing the interactions within a LS. This required a fairly radical adaptation of the original grid, as we had to take into account our particular context as well as the actors and their intentions.

The process of the modification of the SEDA scheme lies beyond the scope of this paper, but we will highlight here the two main modifications related to the type of exchange in a LS professional dialogue which differs from a students' dialogue in a classroom situation. The first characteristic is the Question-Response type of exchange among teachers that was often present in our data. This led us to reorganise the SEDA entries related to "Invite elaboration or reasoning", "Make reasoning explicit" and "Build on ideas" into "Arouse development or reasoning" and "Answer", both entries being specified into clarification-justification-hypothesising categories. The second modification is due to the observation that group participants often make reference to incidents or episodes of the LS cycle or to other teaching experience. This observation has already been illustrated in the analysis of the work of a different group (Ni Shuilleabhain & Clivaz, 2017) and we can relate it to the cumulative principle of Alexander (2018) "which underpins enquiry and knowledge growth in academic communities as well as classrooms, ensures that discussion is genuinely dialectical yet builds on what has gone before, advances understanding and is not merely circular" (p. 566).

Presentation of the Analysis Grid

The first unit of coding of our analysis is the conversational turn, which allows us to first code the identity of the speaker. Like the SEDA grid, our LS Dialogue Analysis (LSDA) grid is organised into 6 entries that group 37 codes that allow us to characterise interactions within the LS. Each code is characterised by indicators, assuring a good validity of the coding (interrater reliability tests will be conducted in Fall 2019). We have distinguished two levels of coding (see Table 1):

- Categories E, Q, R, P and G allow us to highlight a dynamic of talk. Each conversational turn is specifically coded. In some cases, a turn involves several types of utterances, in this case, it is cut out in order to have a more precise coding.
- Category C allow us to show what is being used as a reference in the conversational turn. This enables us to be aware of the connections that are made during the exchanges. In this case several turns are coded as a group.

The categories for LSDA are presented as follows in Table 1.

Table 1: Categories of codes for LS Dialogue Analysis

Category	Features
E – Express or invite new ideas	This category marks the entry of a new subject into the discussion, a new idea, an observation. Distinction between invitations to formulate new ideas and expression of a new idea is made.
Q – Arouse development or reasoning	This category is used with the next category R to code a series of exchanges around a subject. The Q-coded turn involves reference to a previous input. The three possible purposes of the Q-coded turn are, to better understand a factual statement or to understand the reasons for a previous statement or to consider other possibilities or hypotheses.
R – Answer	This category is used with the previous category Q in an exchange. The three possible purposes of the R-coded turns are, to provide clarification and explanation or to give a justification or to develop other possibilities or hypotheses.
P – Position or coordinate	This category is used to indicate a turn intended to mark one's stance or to coordinate ideas in relation to previous exchanges. It may involve synthesising ideas, evaluating different perspectives, challenging an idea or taking a position, approving, recognising a change in position.
G – Guide	This category is used to indicate a turn intended to guide the course of interaction by encouraging dialogue, by verbalising the rules of communication in order to promote discourse, by proposing an immediate action, by proposing an action in the future, by taking as an expert position, by providing feedback, by focusing.
C – Connect	This category is used to show what a series of exchanges refers to. It might refer to: • the content of a past episode of the discussion • the research lesson (past or future) • one's teaching experience

	• one's personal experience • a theory or to mathematical principles • the LS process (at a meta level)

Our Research Questions

The ongoing coding of our transcribed LS meetings allow us to have three levels of coding:

1) Identity of the speaker
2) Dynamic of talk (categories E, Q, R, P, G)
3) The topic to which the series of utterances are connected (category C)

These three levels and the relationships between them allow to operationalise our question about the interactions during LS. The description of the interactions in terms of dynamic of talk (2) will show the type of exchanges that can take place within a LS, answering to the questions: is the pattern in dialogue E then Q/R the most frequent? when do P and G take place?

The type of intervention (2) and the changes of reference (3) linked to the identity of the speaker (1) will allow us to determine the specific dialogic role of each facilitator and of each teacher during the phases of the LS process, answering to the questions: are the facilitators mainly in charge of making the discussion progress or is it a shared responsibility? are these roles evolving along the process?

Conclusion and Perspectives

This research is still in its initial phase and the analyses will be conducted, using the LSDA categories, as described above. Detailed analysis will be conducted for key moments of the three LS cycles about the teaching of problem solving conducted by this group for two years. A second series of analysis will be conducted about MKT in the direction of research which is in the similar vein with Clivaz and Ni Shuilleabhain (2019). The most important part will then be to linked these two parts in order to describe how the mathematical knowledge is constructed within a professional dialogue in a LS group and what the role of a facilitator is, in the construction of this knowledge. The fact that one of the two facilitators of the group is a teacher constitutes a way to not only listen to the voice of the teachers in their dialogue, but also to characterise their voice when they contribute to facilitating their own learning.

The facilitator's facet of this research is expected to give fruit in at least four aspects. The first is to contribute to the series of research of the Réseau Education et Formation group about "the role of the trainer in mathematics education programs". The second is to follow up of the research interest about MKT for primary teachers (Clivaz, 2014, 2017) with a LS contextualised exploration of Mathematical Knowledge for Teaching Teachers (Zopf, 2010) or of Mathematical Knowledge for Professional Development (Borko et al., 2014). The third and the fourth aspect are headed in the direction of the training of facilitators with a chapter in a collective book for LS facilitators (Murata & Lee, in preparation) and by the organisation of a summer school for training French-speaking LS (and other collaborative professional development) facilitators in Lausanne in July 2020. But our main hope is to participate in ICMI-25 study through the sharing of our preliminary results of this analysis and with the tools we developed, in order to analyse collaborative professional discussion of teachers working and learning in collaborative groups.

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FACILITATING TEACHER COLLABORATION IN SUBJECT-SPECIFIC PROFESSIONAL DEVELOPMENT: ATTENDING TO HOW AND WHAT QUESTIONS

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Teacher collaboration is not a fashion but a necessity in order to meet the challenges of teaching mathematics in a modern world. In teacher professional development (PD) which takes place in collaborative settings, facilitators take on a crucial role. They assume responsibility for organizing the collaboration, for teachers taking an inquiry stance, and for focussing on their students' learning trajectories. Qualifying facilitators and researching the processes behind teacher PD is therefore of interest. This paper presents practice examples for supporting teacher PD in subject-specific professional learning communities (PLCs) and draws theoretically corroborated conclusions of a general nature. The theory elements tackle how and what questions and attend to which facilitator moves seem expedient, which challenges occur, and what elements can be identified to foster successful teacher collaboration.

Introduction: From isolation to collaboration

In teacher education, the last 40 years are characterized by a shift from teachers working in isolation (Lortie & Clement, 1975) to collegial collaboration (Krainer, 2003), bringing a social dimension to teacher professionalization. Indeed, Talis (OECD, 2014) revealed positive effects of professional collaboration. That is, teachers who engage in extensive professional collaboration report on high self-efficacy and high job satisfaction. Although it is commonly acknowledged that teacher collaboration is an essential part of teachers' PD, it can be realized in different ways. Collaboration between teachers that is based on regular meetings in small groups to exchange experiences and mutually improve lessons has been established at more and more schools (Lachance & Confrey, 2003). Through sustainable cooperative teacher work in PLCs (Gräsel, Fussangel, & Parchmann, 2006), teachers share their experiences with other teachers in order to benefit from each other's knowledge. Such collaboration supports teachers in rethinking their professional performances in class so as to promote more meaningful (mathematics) learning for students (Lomos et al., 2011). In their meta-analysis, Lomos et al. (2011) reported medium effects of PLCs on student achievement. In addition, results by Hord (1997) showed that working within a PLC had a strong impact on teachers' educational practice, in particular on professionalization processes, self-efficacy, and planning and implementing high quality lessons.

The German Center of Mathematics Teacher Education (DZLM), which offers PD courses for mathematics teachers, pursues a subject-specific PLC approach. Teacher collaboration in PLCs is deemed essential for implementing issues addressed in subject- and content-specific PD courses in class. That is, teachers plan and discuss together how to translate specific aspects into their teaching. PLC members come with their personal "experience of self" and "learning trajectory" (Wenger, 1998, p. 150) and need to identify with the community in terms of their "joint enterprise" and "shared repertoire" (Wenger, 1998, p. 73). Facilitators orchestrate and sustain this passage (Schueler & Roesken-Winter, 2018), as indeed, they are "crucial to the success of the professional development program" in general (Borko, 2004, p. 10). In our contribution, we pursue the question of how a facilitator can support teachers' subject-specific learning. The literature agrees that facilitators' moves play an important role, particularly those referring to sustaining an inquiry stance (van Es et al., 2014; refined by González, Deal, & Skultety, 2016, and by Schueler & Roesken-Winter, 2018). To answer our overarching question of how to design PD for facilitating teacher collaboration we apply a topic-specific Design Research approach (Prediger, Gravemeijer & Confrey, 2015). In this paper we concentrate on the aspects of how a facilitator can support teachers in a PLC, working on teaching stochastics, both as individuals and as a team, what specific issues are to be addressed, and how this can be realized.

Facilitator PLC design with respect to learning trajectories for teaching stochastics

Stochastics is a teaching content that many teachers are unfamiliar with, even when they are otherwise experienced professionals. This constitutes both a chance and a challenge for facilitating such a PD course, as teachers are more likely to embrace teaching approaches for a new content, but there is content knowledge to be imparted, too. In order to conceptualize facilitating effective PD in a collaborative setting, the specificities of *what* is to be learned and of *how* this is to be orchestrated need to be defined (Prediger, 2019). Particularly, we follow Prediger's structure to arrange the questions and answers according to different theory elements which are categorized as descriptive, normative (including aims or reasons), explanatory, and predictive. There, the *what* questions cover "categories for distinguishing and relating aspects of the learning content", in our case of the learning content in reference to the PLC members, and the *how* questions (which are phrased as which questions) refer to "categories for design principles, process qualities, characteristics of design elements" (p. 8), to be addressed by the facilitators.

At this stage of our research, we attend to normative, descriptive, and supposed predictive theory elements; the explanatory and refined predictive ones are expected to be the result of further analysis. The collaborative learning arrangement is seen as a basis on which discourse and exchange can prosper, thus influencing the *how* and facilitating some aspects of the *what*.

Our overall research focusses on answering questions such as: Which theory elements can be identified in PLCs on stochastics, which explanatory elements occur in PLCs on stochastics, and which refinements of the predictive elements emerge in PLCs on stochastics? We present two examples that we deem to be representative of PD in stochastics and thus exemplify theory elements with the aim of compiling refined predictive heuristics that are universally valid for PD referring to new content or innovative approaches.

PLCs for stochastics at upper secondary level

In our first example, PLCs were formed with the aim of designing, conducting, and reflecting teaching sequences on stochastics at upper secondary level, following a five-day PD DZLM course that one teacher from the group had attended.

In regard to *what* was to be learned, the PD course addressed basics on student level, e.g. the preconditions and parameters of the binomial distribution were explained and illustrated with examples, thus creating a ground for discussing normative theory elements, e.g. non-rational approaches learners might have. Furthermore, regarding technical issues, there was input on e.g. how to use graphing calculators for simulations or how to visualize binomial distributions using software like GeoGebra, in order to encourage teachers to use these in class, thus addressing descriptive theory elements. Moreover, in reference to didactical and methodical considerations, issues on intended learning trajectories and divergent learning pathways were presented, e.g. on how to use simulations as a phenomenological approach to probability distributions. Hereby, a (vague first approach of a) concept of predictive heuristics emerged: That teachers can be encouraged to accept an innovative teaching concept by re-enacting the learning trajectories intended for students, while allowing them to voice their inhibitions and time for discussion and the exchange of ideas.

It is the PLC facilitator's role to take responsibility of *how* the PD is orchestrated. From the normative perspective, her moves ensure that all PLC members are actively participating, that mutual respect is expressed in regard to possibly naïve initial assumptions, and that the possible rationales are discussed. The descriptive elements include refreshing PLC members' memories of the greater curricular goal behind teaching a specific content, that beyond the next examination students are to learn about the possibilities and limitations of describing reality with the help of idealized models. This is particularly relevant in stochastics, where real data can be the basis for explorations (Oesterhaus & Biehler, 2013), and thus veritable problems can be posed and solved, which means authentic modeling can take place, including phases of simplifying, interpreting, and validating (Blum & Leiß, 2007). As such, the predictive heuristics emerged that PLC facilitators act as mediators between academic aspirations and school reality, displaying moves that point out stepping stones between these two extremes, and a more balanced approach can be obtained. For example, the facilitator can elaborate on how to (re-)introduce probability in grade 10 via presenting an interesting problem (see Figure 1), then systematizing observations by first hands-on and later automated digital simulations, and later explaining the phenomenon by calculations of binomial distributions and their characteristic values. The phenomenological approach yields the insight that improbable events (i.e. guessing nearly all answers correctly) happen, how often they happen in comparison to other events when the experiment is repeated very often, and how this differs, depending on the number of questions in the test. The realization that the *percentage* of correct answers varies more in a test with less questions (but the *number* of correct answers varies less) can be obtained in the experiment or simulation. Through these moves of "sustaining an inquiry stance" (van Es et al., 2014), a relevant basic idea becomes accessible (mean and spread in binomial distributions depending on the parameter n, later also the quantification of "large" in the law of large numbers), background knowledge is introduced ($1/\sqrt{n}$-law, sigma-rules), and there are options for internal differentiation for learners of varying ability or interests.

One test has ten questions with two answers each, one correct and one false. Another test has twenty questions with two answers each, one correct and one false. You pass each test if you have 60 % or more correct answers.
If you are merely guessing, which test is easier to pass: the test with ten, or the test with twenty questions – or are both equally difficult?

Figure 1: The 10/20-test problem, for (re-)introducing probability

The quintessence behind suggesting a teaching sequence like the one described above, however, lies not only in the normative and descriptive theory elements (the knowledge, tasks, and solution processes), but in the predictive heuristics, in combination with the enthusiasm to convey the belief that it is possible to indeed teach stochastics in a competence-oriented and demanding yet accessible way that makes allowances for learners of varying capability. This is an asset that a PLC consisting of teachers with no experience worth mentioning in teaching stochastics can only get as input from outside of their own group, e.g. from a PLC facilitator. In consistence with the results by Hord (1997), this addresses PD via its impact on affective aspects and planning competences, and it enables experiences of communal interpretation of the "joint enterprise" (Wenger, 1998, p. 73) that motivate the individual teacher to dare new teaching approaches.

Teacher tandems for stochastics at primary and lower secondary level

The course "Stochastics in primary school: Data, frequency and probability", designed as one of DZLM's several qualification courses focussing on mathematical content from a didactic perspective for teachers in the Berlin area, serves as our second example. In this course, participants worked together on solving stochastic problems and on integrating what they have learned into their teaching practice. In order to support sustainable cooperation, teachers are asked to form tandems, often consisting of one more qualified or experienced mathematics teacher and one with less or less specifically mathematical-stochastic qualification.

The teacher tandems are to become experts and to function as resources and mentors within their school and their school district, ideally initiating further PLCs. Therefore, working in and as a PLC is an essential element of the qualification process in the course. In this course, participants are trained in practicing cooperation when developing their teaching of stochastics. During course sessions, the PLC facilitator guides the group in their professional development: First, participants are to analyze their individual initial situation referring to their respective subject-specific focus (e.g. a general lack of experience on teaching probability and statistics). Then they identify and describe a teaching problem that is also relevant to the group (e.g. how to choose a field of activities that foster stochastic thinking in primary school students). The problem can be one from their own experiences, or a content that is scheduled to be taught at their school in the near future - or an example from the course can be put to the test and developed further. At this point, the PLC facilitator is required to provide input and ideas in order to focus and narrow down this huge field. In the above example, children's misconception regarding dice seemed a promising context, as most children have authentic experience referring to board games where casting dice is an important element to succeed in the game. Often, there are preconceptions, i.e. connected to dependencies between individual castings of a dice (*6 is more likely now because it hasn't shown for a long time*) or varying probability of different incidents (*6 is less likely than other numbers because 6 is especially lucky*). The facilitator actively generates cognitive tension to pursue the teacher learning goals. That is, she facilitates discussion with other participants, particularly with the tandem partner, to develop approaches for solving the problem, i.e. tasks, activities, and teaching sequences, in the above example particularly problems that challenge typical arguments and modes of thinking. As predictive heuristics we consider the facilitators' actions towards creating tension between students' learning and teachers' expectations. Focussing on experiments that are planned with the goal to test assumptions and to form a basis on which to make assertions concerning probability helps the PLCs to keep an inquiry stance.

Between course sessions, participants have the opportunity to put their newly developed approaches and ideas into practice, to sit in on each other's classes, and to reflect their pupils' and their own learning processes. The courses of action are tracked and documented so that they can be presented to the other participants at the next course session. When compiling and comparing the experiences made when addressing misconceptions concerning dice, the PLC members found that they had universally let their pupils express their expectations and give reasons for them and then conduct experiments, which led to the insight that all incidents (when throwing one dice) are equally probable, and some children connected this to the regular form of dice, a cube. Still, when they later had to decide if the statement "A 6 is harder to obtain than other numbers", many children opted for *correct*, where their teachers had expected an obvious *wrong*. This sparked a discussion with two main foci: (1) the wording of the statement ("6" as opposed to "a number unequal to 6" is indeed harder to obtain, as their respective probabilities are 1/6 and 5/6), and (2) the fact that conceptions concerning dice are mostly developed and experienced in emotionally charged situations (playing a game, winning or losing) and thus are rather stable. The PLC agreed it would be advisable to phrase the test statement as "A 6 is harder to obtain than a 2", and to accept that overcoming misconceptions is a longterm process that needs an ongoing teaching concept (Hawkins & Kapadia, 1984, Martignon & Wassner, 2005, Wollring, 1994). In this exemplary case, the PLC facilitator is challenged on more than one level. As predicitive heuristics emerge: supporting the reflection expediently, moderating the discussion regarding to content, intervening and correcting when necessary, and supplying background knowledge, e.g. from a psychological or pedagogical perspective.

Referring to the aim of developing teaching practice, during course sessions the focus is directed towards the cooperation processes in general and the cooperation with the tandem partner in particular. This also means that participants are forced to consider the work relations and organization at their school. Here, the PLC facilitator guides and supports the group in structuring the reflections and focusses on obstacles and conditions for success, therefore showing alternative ways of organization, and keeps an eye on normative theory elements. These are debated among the course participants, guided by the PLC facilitator. That way, due to the productive exchange of ideas enabled by the PLC's cooperation, impediments and prerequisites for successful professional development emerge, and paths to realization are revealed. Participants become aware of features for successful teacher cooperation, e.g. when listening to best practice examples. What is more, information on the goals, the central ideas, the operating principles, and the work processes of a PLC are addressed and experienced hands-on. A descriptive theory element becomes salient here: the facilitator moves the focus from the single phenomenon to a overarching structure.

The teacher tandems are then requested to phrase their next goal and the process steps to reach it, to test these in their day-to-day work, to reflect these, and to exchange views with other teachers. That way, there are repeated cycles of planning, testing, and reflecting throughout the course. The PLC facilitator takes responsibility to make the course participants aware of these cyclic processes. This way a consensus can be reached on how to sustain a valuable cooperative exchange among teachers, thus nearing the goal to create impact beyond the course itself, namely on the professional teams of mathematics teachers at the individual schools.

Conclusions and outlook

The analysis of two representative examples has led to answers to the *what* and *how* questions for subject-specific teacher PD, see Table 1. Whereas the *what* answers refer to stochastics in specific, the *how* answers, which describe the tasks and the role of the PLC facilitator in detail, are of general interest for the design of teacher PLCs.

Table 1: *What* and *how* questions for normative and descriptive theory elements for teacher PD on stochastics

	What questions for theory elements: structuring of the PD content	*How* questions for theory elements: design of the PD arrangement
normative theory elements	What are the learning goals for teachers? Teachers are aware of non-rational approaches their students might have towards stochastic problems and create learning opportunities where these can be amended. Teachers move on from exploring isolated events and learn to consider a distribution (all possible events) as a whole and in comparison to other similar distributions. Teachers realize that being able to describe, compare, and understand the characteristics of distributions is a challenging yet teachable competence.	Which process quality should be reached in order to achieve a later learning goal? The PLC facilitator encourages active participation of all PLC members, demonstrates respect for their initial assumptions, and guides the discussion of possible rationales – with the aim of including all participants in the learning process and enabling PD.
descriptive theory elements	What learning pathways do teachers usually take along the intended learning trajectory? Teachers might feel unsure about experiments or simulations because they cannot control the exact outcomes – discussing different possible outcomes and how to use them for students' learning can help. Teachers might recoil from the technical hassle involved in digital simulations – supporting them with software and ready-made learning arrangements and allowing time for questions and trying out can help.	Which situational effects can the design principles and design elements unfold in the teaching-learning pathways? And how does that relate to the intended effects? The PLC facilitator refers to the greater goal behind teaching stochastics: describing reality with the help of idealized models and critically reflecting this process; she generates cognitive tension and thus the wish to know and to understand; she moves the focus from the phenomenon to the structure to enable generalizable insights.

The consideration of these elements leads to more general predictive heuristics for teacher PD in stochastics, i.e. for answers to what should be addressed in order to facilitate successful PD, see Table 2. In particular, expedient facilitating moves emerge, mostly referring to sustaining an inquiry stance (cf. van Es et al., 2014), like guiding the discussion by *lifting up* (identifying an important idea raised by a PLC member) or *pressing* (prompting PLC members to explain or elaborate their ideas), or *countering* misgivings when elaborating on learning trajectories or illustrating how learners of mixed capability can be reached, thus conveying the belief that the content can be taught appropriately. These insights impact on the role of the facilitator: They are the advocates of the innovative teaching approach and in charge not only of moderating the discussion, but also of convincing PLC members of the practicability of the learning trajectories. Certain preconditions support these facilitation goals, when there is a common challenge, e.g. in the form of content to be taught, or technology to be mastered:

- repeated meetings of the PLC,
- a PLC facilitator who bridges the gap between academic requirements and school practice, and
- a store of tasks, examples, and activities that can be adapted according to individual needs.

Table 2: *What* and *how* questions for predictive heuristics for teacher PD on stochastics

predictive heuristics	What is the learning trajectory for the specific learning content? It is important a) to let teachers experience the intended learning trajectories for their students, b) to guide their exchange of ideas, c) to discuss students' various rationales, d) to allow teachers to voice their inhibitions, e) to allow time to discuss reactions to students' reasoning in detail.	Which design principles should be applied for which aim? The PLC facilitator's moves aim at a) creating tension between what students / PLC members believe to be true and what they observe happening, b) moving from assessing a concrete phenomenon to understanding a structure c) mediating between aspiration and reality by elaborating on learning trajectories, d) illustrating how learners of mixed capability can be reached, e) awareness of the cyclic nature of the process of PD at individual and at school level, f) conveying the belief that the content can be taught appropriately.

The repeated meetings of the PLCs are a necessary precondition for the facilitator and the participating teachers to develop a feeling of community and trust in reference to their common goal. The phases in between group meetings are of equal importance as they stress that it is essential to elaborate on ideas and teaching approaches that can be put into authentic practice. A facilitator taking practicability seriously is a feature that cannot be overestimated in teacher PD. Moreover, looking back on teachers' reports of their lessons during PLC sessions appreciates their professional practice and allows space and time for its reflection.

The PLC facilitator is a key feature in fostering teacher collaboration. Research has shown that elaborated forms of collaboration are relatively rare and seldom develop without outside management. However, a balance is needed between distance which enables a general view and the vision of reaching ambitious goals, and proximity which allows teachers to open up about their everyday problems so that solutions that matter in practice can be approached. Ideally, PLC facilitators should have a background both in teaching and in academic work.

Ultimately, the teaching materials presented and discussed in the PLC sessions should cover a range of interesting tasks, examples, and activities, that are both interesting and challenging for the learners, that enable reflection on different levels, and that represent the content adequately. In our above examples, addressing misconceptions regarding dice and the 10/20 test problem are interesting as learners can relate to these topics via their experiences with board games or tests. They are challenging because preconceptions may have to be revised when grappling with the problem. Both teaching approaches enable reflection on different levels: watching and documenting what happens, modelling and calculating, and comparing and evaluating the respective models.

Our explorations have yielded interesting insights into what supports successful facilitation of teacher PD. Our future research will be aimed at refining the predictive heuristics and at identifying explanatory elements, thus enabling a better understanding of the principles of these processes.

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CO-TEACHING MATHEMATICS IN AN INCLUSIVE CLASSROOM: CHALLENGES FOR COLLABORATION OF A PRIMARY AND A SPECIAL EDUCATION TEACHER

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The development of roles in a newly formed collaboration between a primary teacher and a special education teacher who co-teach mathematics in an inclusive classroom is subject of this paper. With different professional backgrounds but same personal attitudes and orientations towards collaboration as well as teaching children with and without special needs in one classroom, the conditions for a successful and harmonious collaboration seemed set. However, lesson observations and individual interviews showed the contrary. By comparative case descriptions along various categories we were able to reconstruct factors which influenced the uptake of flexible roles and hindered this specific collaboration, e.g. no set planning time, missing school administrative support, inability for openly exchanging mutual ideas and expectations. With our focus on mathematics lessons, a new factor which negatively affected the collaboration was identified: the missing mathematics education expertise in the team.

Following recommendations by the UNESCO, an increasing number of primary schools in Germany take on children with special needs, who were traditionally taught in special education schools, and include them in their classrooms. One organizational approach is to place a couple of children with special needs inside a traditional class and provide a teaching team consisting of a primary teacher and a special education teacher for this class during the majority of the lessons. These teachers have different professional backgrounds and traditionally are on their own in the classroom being responsible alone for their respective classes. In the newly formed inclusive classes, the teaching team is, therefore, faced with two challenges which are heavily intertwined: teaching an inclusive classroom and shaping collaboration. But how are collaborations and the different roles of the teachers shaped and developed? How do such collaborations look like on an organizational, personal, and lesson level? And which factors influence the collaboration of a primary and a special education teacher specifically for mathematics lessons? By investigating the first two years of collaboratively teaching mathematics lessons by one newly put together teaching team, we try to get closer to answering these questions.

Theoretical Background on Teacher Collaboration

Teacher collaboration – what is it and why is it important? Teacher collaboration is characterized by working in conjunction with others, by engaging in joint activity, and to share common goals (Jaworski et al., 2017, 263). It is intentional and communicative (Spieß, 2004, 199). Studies show positive effects of successful teacher collaboration on teacher-student-relationship (Seashore, Kruse, & Marks, 1996) and students' math achievement (Pil & Leana, 2009). If teachers talk about pedagogical topics with their team partner, they are more likely to use forms of inner differentiation in their instruction (Smit & Humpert, 2012, 1160). Successful schools (measured by the increased performance of their students) are characterized by a high degree of cooperation between staff members (Scheerens, 2000). Furthermore, a productive work relationship and feeling supported by colleagues seem to be a protective shield against work dissatisfaction and burnout (Johnson & Johnson, 2003). In summary, teacher collaboration can be considered as a significant motor for school quality and lesson quality.

Teacher collaboration can take different shapes. Gräsel, Fußangel, and Pröbstel (2006) differentiate three forms of collaboration: *exchange*, *collaboration with divided responsibilities*, and *co-construction*. *Exchange* is a very basic form of collaboration in which teachers inform each other about school related topics and share material. The division of labour and with it an efficiency enhancement is paramount to a *collaboration with divided responsibilities.* Necessary requirement of this collaboration is a shared goal. To achieve this goal, tasks are divided and assigned to each teacher based on his/her competencies. Collaboration evolves around communicating the goal, and dividing and bringing together the separately worked on tasks. Strictly speaking, this kind of collaboration does not require a joint activity. *Co-construction* is a form of collaboration in which the teachers have an intense exchange and relate their individual knowledge to one another. In doing so, they acquire new knowledge or develop shared solutions for tasks and problems. In contrast to the two previous forms of collaboration, both teachers work together for a long part of the process. Little (1990) regards co-construction ("joint work") as the highest developed and rarest form of collaboration. The special importance and benefit of co-construction for teachers lies in improving the quality of their work by stimulation and reflection, and in developing their competencies (Hargreaves, 1994).

For teaching inclusive classes, the collaboration of general and special education teachers is of significant importance (Werning & Arndt, 2013). A two-teacher-system like in our study, in which a permanent teaching-team teaches nearly all subjects in the inclusive classroom, creates advantageous prerequisites for co-construction. In practice, the two professional peers with different types of expertise can take up different roles. For co-teaching an inclusive classroom, Friend, Cook, Hurley-Chamberlain, & Shamberger (2010) defined six approaches. In the three approaches *one teach – one observe, one teach – one assist,* and *alternative teaching* a "main teacher" leads large-group instruction while the other observes, circles the class and offers individual assistance, or works with a small group of students. In *station teaching* and *parallel teaching*, both teachers have equal roles in separately teaching equal parts of the class. In *teaming*, both teachers lead large-group instruction together by both lecturing, representing opposing views in a debate, illustrating two ways to solve a problem, and so on. Friend et al. (2010) suggest that in everyday co-teaching "[t]he roles for the teachers are fluid, with each taking on any of the responsibilities suggested by the aforementioned approaches and sharing through appropriate negotiation the design and delivery of instruction […]" (p. 13). Contrary to this, Scruggs, Mastropieri, and McDuffie (2007) in their meta-analysis of studies on co-teaching in inclusive classrooms found the dominant co-teaching role to be *one teach – one assist*, with the special education teacher playing a subordinate role.

Research studies found that the collaboration between teachers depends on a couple of factors, e.g. the relation between each other, support systems like school environment, advanced training, and set time for joint planning (Lütje-Klose & Willenbring, 1999, 6). Missing time for collaboration outside lessons can impede collaboration and foster external differentiation instead of shared lessons for all children (Lütje-Klose et al., 2005, 85; Scruggs et al., 2007, 405). Furthermore, experience in former collaborations, the fit of individual personalities and personal attitudes as well as the didactical-methodical approaches of the teachers have an impact on the processes of collaboration (Scruggs et al., 2007, 405). Werning (2012, 58) highlights that an open exchange about mutual ideas, expectations and concerns as well as about the roles the teachers occupy during lessons influence the work as a team. Although co-teaching in inclusive settings seems challenging, teachers generally profit from each other regarding their professional development (Scruggs et al., 2007, 401).

As the reported findings mostly come from studies which investigated general teacher collaboration across different subjects, in this study, we follow and describe the evolving collaboration of a teaching team in an inclusive classroom and analyze the challenges the two teachers face with a focus on mathematics lessons. In particular, we aim to answer the following questions:

- How are the different roles and identities shaped and developed within the collaborating team of primary teacher and special education teacher?
- Which factors influence (enhance or hinder) the collaboration of these two teachers with different professional backgrounds, when teaching mathematics in an inclusive classroom?

Methods

Study design

The data reported in this article are part of a longitudinal study (2015–2019) comparing the development of mathematics learning and achievement as well as the social and emotional school experience of children in an inclusive vs. exclusive classroom during their four years of primary school education. Additional to assessing children's competencies and school experience on a yearly basis, we conducted three lesson observations during the second year and one individual interview with both teachers during the third year of the collaboration. Furthermore, the first author of this paper spent a couple of weeks each year at the school and conducted the children's assessments. During this time, she got unsystematic glimpses of mathematics lessons and informally talked to the teachers about their collaboration. These observations and conversations were documented in form of verbatim records from memory. The interviews, verbatim memory-records, and lesson observations are subject of this paper.

Data collection and analyses

Lesson observations were conducted to get first-hand impressions of the collaboration, in particular teachers' roles and tasks during math lessons, to develop guiding questions for the interviews, and to collect situations which the interviewer could refer to during the interviews. The first author attended three mathematics lessons as a participating observer; the observation was conducted overtly. Based on criteria for lesson observations by Inckemann and Dworschak (2018), observational protocols were kept. For this paper, the focus on the team (e.g., teaching approach, roles, responsibility for children with special needs) and the lesson structure are of special interest for this paper.

The interviews were conducted as guided, semi-structured interviews by the first author. The guideline included detailed questions (e.g., regarding education and profession of each teacher) as well as open invitations for narration (e.g., How would you describe your and your team partner's roles during mathematics lessons?). The open questions were used to give free space for teachers' subjective perceptions and experiences regarding the collaboration. The interview was structured by six topics: education & profession, organizational framework, mathematics lesson structure, collaboratively teaching, lesson preparation, and team-contentment. Topics and questions were adapted from work by Inckemann and Dworschak (2018) and Lütje-Klose and Willenbring (1999). The interviews took between 30 and 45 minutes and were audiotaped and transcribed.

The interviews and verbatim memory records were analyzed by means of "qualitative content analysis" (Mayring, 2008). Based on the relating literature, results of the lesson observations, and the interview guideline, seven main categories were developed deductively. The main categories were: *profession, organizational conditions, personal attitudes and orientation, mathematics lesson structure, collaboration during lessons, cooperation with other staff*, and *lesson structure of other subjects*. During coding of the teachers' answers, the main categories *personal attitudes and orientation, mathematics lesson structure*, and *collaboration during lessons* were inductively differentiated into subcategories. *Collaboration during lessons*, which is especially informative for our research interest, now comprises the subcategories *distribution of roles and tasks, lesson preparation, interpersonal level*, and *team-contentment*. For coding we used the qualitative data analysis software MAXQDA 12 (VERBI). As comprised summaries and for comparison reasons, we created tabular case overviews along the categories for each teacher.

Results

For the purpose of this paper, we selected and combined categories and present the findings in form of comparative case descriptions along the topics *teacher profiles, organizational conditions of the collaboration, collaboration during lessons* and *interpersonal level & team contentment*.

Teacher profiles: educational background and personal attitudes and orientation

Primary teacher. The primary teacher has completed a seven-semester primary education degree course at university with the first state examination and an 18-month teacher training with the second state examination. She has studied mathematics only as a didactical minor subject, not as one of her two main subjects. Therefore, she considers herself as "completely outside of the subject area math". She has been teaching for one year prior to the start of the cooperation. The primary teacher is the designated class teacher of the inclusive class and has no experience with collaborative teaching. For her, inclusion is creating a joint instruction for all students, in which the individual prerequisites of every student are considered and learning from and with each other is encouraged. She sees the work in a teacher tandem as a chance to individually support single students in small groups. She explicitly stresses the need to not only support children with special needs but to stimulate every child's individual competencies. Collaborative agreements with her special needs colleague and conversations about their work in the tandem are important for her. Mutual listening if "things" are addressed is so central for her that statements made during team meetings are written down in a protocol.

Special education teacher. The special teacher has completed a ten-semester special education degree course at university with the first state examination and a 24-month teacher training with the second state examination (which is a higher degree than the primary teaching degree, followed by higher payment). He has not been trained in mathematics but has recently completed a course in handling the heterogeneity when teaching mathematics (timeframe of four days). He considers himself not a "math person". He has been teaching as a special education teacher long before the start of this specific collaboration (approx. 20 years) and also has experience in collaborating with other primary teachers. Furthermore, he is moderator of the school's internal training "On the way to an inclusive school". The training is supposed to help teachers to acquire and develop competencies regarding the handling of heterogeneity. For teaching in inclusive settings, the special education teacher prefers team-teaching methods in which all students are taught collaboratively. He explicitly stresses that he feels more "comfortable" in a team in which "we say these are OUR children" and not "these are YOUR children, these are MY children", referring to his experience with both kinds of collaboration. He considers an inner differentiation with weekly lesson plans as appropriate for responding to students' individual prerequisites. In his opinion, both teachers should make the effort to plan instructions which enhance the learning of every student. Frequent conversations about their working goals in the tandem and collaborative agreements with his primary teacher colleague are important for him. Furthermore, he regards faith in each other, open handling of critique, and the inclusion of the individual professional competence as the basis for a successful work as a team.

Organizational conditions of the collaboration

The inclusive classroom under study, and with it the collaborating team, is the first at the school. The team was put together by the head of school; both teachers joined the team voluntarily. There are 26 students in the class, with five children in total having attested special needs. Two students have special needs in their emotional and behavioral development and three have learning difficulties. The team is co-teaching all subjects except arts (drawing) and physical education. There are five 45-minutes mathematics lessons each week. During the interview, two additional aspects regarding the organizational conditions are mentioned by the teachers which possibly influenced their collaboration. First, the primary teacher brought up that she wished for a structural framework and guidance by the school administrative which accompanied the collaboration: "It's important that there really are explicit structures given by the school, the system." In this specific case, it was the teachers' responsibility to work out the concept of collaboratively teaching in an inclusive setting, to organize and make time for joint lesson preparations and to mutually find together as a team. In particular the issue of planning time was repeatedly mentioned "If you work collaboratively, you'll definitely need three hours' time on any day for preparation. If you don't have the time, it just doesn't work" (special teacher). Second, both teachers report that the team was regularly split (the special education teacher estimates for at least one mathematics lesson each week), either by the need to substitute for an absent colleague and teach another classroom, or because of health issues by the special education teacher. In both cases, the primary teacher was left to teach the inclusive classroom on her own. She reports to feel overstrained with teaching the whole class alone on a regular basis, as there are so many children with special needs and, therefore, the children without special needs would get neglected.

Collaboration during lessons

The lesson observations as well as the interviews reveal that the mathematics lessons were executed in the way that the children with special needs were almost always taught in a separate room by the primary teacher. This approach manifested itself a couple of month after the collaboration started. The primary teacher and the children with special needs left the classroom right at the beginning of the lessons and primarily worked on the children's arithmetical competencies. Both teachers report that a joint but differentiated teaching was rarely executed. For mathematical contents like geometry, measurement, or modelling context problems, they sometimes found methods for all students to work together. This teaching approach of "separating right from the beginning" differs from what the teachers report about their German lessons (these were not observed by the research team). German lessons were conducted in a way that the whole class was starting the lesson together, led by the primary teacher, the special education teacher was assisting. After the primary teacher had introduced the lesson's topic, the special education teacher would work with the special needs children, the primary teacher aiding the children without special needs. At the end of the lesson, the whole class would meet again for reflecting what had been learnt. All lessons' preparations took place based on the division of labor with the primary teacher preparing German and science/history/social studies, and the special education teacher preparing mathematics. Both teachers also prepared the differentiated tasks for the children with special needs, which the collaborating teacher then taught. In summary, the interviews and lesson observations show that this team favors alternative teaching and collaboration with divided responsibilities, in which one teacher is responsible for the children with special needs and the other for the children without special needs. For which group the teacher is in charge and with it the teacher's role, however, changes dependent on the subject.

Interpersonal level & team-contentment

The category *interpersonal level* focuses on the fit of the individual personalities of the collaborating teachers. The results show that the collaboration is characterized by considerable tension on the interpersonal level; both teachers are "definitely not on the same wavelength" (primary teacher). The primary teacher is sure, that she and her colleague have different orientations towards inclusion and teaching an inclusive classroom collaboratively: "There is definitely a different orientation with us two." She is unhappy about the way, the children with special needs are taught separately and she indirectly criticizes her colleague about not preparing the math lessons well enough. Her impression is that her wishes and concepts for the collaboration and teaching structure are not taken seriously by her colleague. She feels "misunderstood". The special education teacher, on the other hand, is also unhappy about the collaboration during mathematics lessons and also assumes them having different attitudes towards teaching an inclusive classroom. He suspects that his age ("I could be my colleagues father.") and their different teaching experience made his colleague feel that he wanted to judge her ("That you don't hear something like 'It's not your responsibility to judge me'."). He does not feel appreciated in his special education expertise by his colleague ("And to make the primary teacher clear: 'Also use my competence and my expertise!'"). Repeatedly, he remarks on the limited time resource. Furthermore, he expresses discontent about the missing willingness of his colleague to actively participate in the collaboration. That each teaching related conversation is written down in a protocol, illustrates the dimension of the conflict. The described problems in the collaboration result in a personal discontentment of both teachers regarding their joint work as a team. In the interview, the primary teacher confesses: "I would like to completely go out of the class." It becomes clear that she does not believe in resolving the conflict and that she wants to end the collaboration.

Conclusion

The teachers in the collaboration under study have different professional backgrounds, with one being trained to teach whole classes, the other to focus on individual needs. Furthermore, they differ in their experience in team-teaching. Still, a lot of factors which have been found influential for successful collaborations (Lütje-Klose et al., 2005; Scruggs, et al., 2007) are given: Both teachers have similar requisites regarding their orientation towards inclusion, their didactical-methodical approaches, and their idea of working as a team. Both formulate the joint teaching of children with and without special needs in one classroom as their concept of collaboratively teaching in an inclusive setting. Both consider frequent exchange about the collaborative work as important. The two teachers also share their nearly non-existent mathematics education course work. Although the organizational framework of a permanent teaching-team and the concurrent personal orientations and attitudes towards teaching an inclusive class collaboratively seem excellent for *co-construction*, the daily routines are shaped by *collaboration with divided responsibilities* (Gräsel et al., 2006). Surprisingly and much in contrast to what both teachers report to wish for, the children with special needs are taught separately from the whole class in mathematics. The distribution of roles, with the special teacher being responsible for the preparation and teaching of mathematics for the whole class and the primary teacher separately teaching the children with special needs (*alternative teaching*), is rather unusual (Friend et al., 2010; Scruggs, et al., 2007). Moreover, the teachers' roles in mathematics differ from their roles in German lessons. Overall, the two professional peers work together as equal partners, with the special education teacher even being the expert in teaching mathematics. With this way of working, the team breaks through traditional roles regarding their professions (Scruggs, et al., 2007). The reasons for the development of these new roles seem threefold and mutually dependent. One recurring explanation is the amount of time which would be needed in a collaborative preparation of all lessons. In order to save time, the three main subjects were allocated to the team members with regard to their professional competence. With this distribution, a special difficulty becomes explicit. The primary teacher had been trained in German and science/history/social studies, and therefore took on the responsibility for preparing and executing the lessons for these subjects. None of the two felt competent in mathematics but as they wanted to divide the labor, and he recently took a special course in teaching mathematics, the special needs teacher had to take over the math instruction. The primary teacher "completely relied" on her colleague. Because of her lack of mathematical knowledge, she did not feel in the position to make alternative suggestions: "I could not stand my woman and say 'No, let's do it so and so'". The special education teacher on the other hand, felt unable to cope with the double requirement of preparing and teaching the whole class instruction and also prepare the tasks for his colleague who was lacking mathematical knowledge as well as special needs expertise but still was supposed to teach the children with special needs: "If this [the teaching of the special needs children] is taken over by the primary teacher, like it was in our collaboration, and she does not come from the area of mathematics […] then the special needs teacher has to prepare very much […]. And this was […] difficult." Both interviews clearly hint at the point that the lack of mathematical expertise in the team affected the teaching structure of the mathematics lessons negatively. Following, both teachers were unhappy about their daily teaching and their collaboration, which was reflected in an increasing tension on an interpersonal level. Unfortunately, the team was not yet situated in a supportive collaborative school culture. Next to professional support in teaching mathematics, an exchange with other collaborating teams might have been helpful. We find it rather tragic that the two teachers consider themselves to have different orientations towards teaching an inclusive classroom collaboratively, although the interviews show the opposite. The individual personalities possibly impeded an open exchange of ideas and orientations. Regular conversations about the collaboration which are coached by school administrates might have helped to clear the misunderstanding.

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METACOGNITIVE PROCESSES IN ONLINE COLLABORATIVE PROBLEM SOLVING FORUMS: MATHEMATICS TEACHERS' DUAL ROLES

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Collaborative mathematical problem solving, such as in Problem Solving Forums (PSF), was found to contribute to solvers' ability to handle challemging mathematical problems. Due to the important role of metacognition in directing learning processes, the current study goal is to examine the metacognitive process among teachers who collaboratively solve challenging problems in PSF in the role of students, and subsequently the metacognitive process of teachers, while experiencing PSF in the mentor's role. The paper focuses on a case study of one secondary school mathematics teacher, John, who was engaged in collaborative geometry problem solving in the context of a professional development program. In the program, John and his teaching fellows were exposed to a metacognitive support in and between the PSFs. In his experience as a student, John demonstrated a progress in metacognitive ability in the process of collaborative problem solving. As a mentor guiding his teaching fellows, John encouraged their collaborative learning by the use of metacognitve support. We present theoretical, methodological and practical contributions of the study.

Introduction

Studies carried out in recent years have indicated that teachers should experience learning methods for themselves as learners, prior to advancing similar processes among their students (Kramarski & Kohen, 2017; Kramarski & Revach, 2009). In particular, it was found that experience with learning methods that nurture teachers' metacognition assist the teachers while directing their own learning processes, specifically during mathematical problem solving (Flavell, 1976; Schoenfeld, 2013) as well as in directing their teaching processes (Kohen & Kramarski, 2012).

One strategy for learning, which encourages questioning, sharing ideas, and discussion of various issues is collaborative mathematics problem solving in small groups. Collaborative learning is based on discussions regarding solution methods, consulting with peers, and mutual assistance in understanding the material (Moss & Beatty, 2006). Technological developments have added possibilities for collaborative learning through discussion groups in social networks, which we refer to as Problem Solving Forums (PSF). Recent studies have shown the potential of collaborative problem solving through PSF (e.g., Koichu, 2015; 2018), and also demonstrated how strongly the efficient collaborative learning in small groups of students depends on the role of the teacher who facilitates and mediates the problem solving process (Zimmerman, 2008; Koichu & Keller, 2017; 2019).

The current study investigates a professional development program that enables teachers to experience the dual role– as solvers of challenging problems in PSF and as mentors of PSF for the other teacher participants. In continuation of work by Kohen and Kramarski (Kohen & Kramarski, 2018; Kramarski & Kohen, 2017), we focus on metacogtitive support during collaborative mathematical problem solving as provided and received by the teachers in their aforementioned two roles.

Theoretical framework

The current study is based on a combination of two theoretical frameworks: Collaborative Mathematical Problem Solving and Metacognition.

Solving a mathematical problem is a process which is based on previous relevant experience, where the path to the target is unknown, and passes through increasing understanding and reformulation of the possibilities for solving the problem at each stage of the solution (Kilpatrick, 2016; Lester & Cai, 2016; Pólya, 1945). Researchers particularly stress that solving a challenging problem must include a requirement for mathematical knowledge and skills and creation of a surprise challenge in the problem and its solution, so that in the eyes of the problem solver, it has no immediate solution. *Collaborative learning based on problem solving* refers to a learning method in which two or more students at different levels of ability and knowledge act together to attain a common goal (Schoenfeld, 1985). The students help each other and accept responsibility for both the group's as well as each participant's success. In recent years, several studies have been done that stressed interactions between students solving challenging mathematical problems in small groups, whereby interactions were classified by the type of question, such as questioning for assistance, or questioning for clarification (Carlson & Bloom, 2005; Clark, James, & Montelle, 2014). These studies relate particularly to interactions between learners (Group synergy) that illustrate collaborative thinking.

Metacognition is a central component of Self-Regulated Learning (SRL), referring to the ability of learners to activate a circular process of planning, monitoring, and reflection skills (Schraw et al., 2006; Schunk, 1996). In relation to teachers, scholars distinquish between the teacher ability to achieve their own SRL as learners or in the role of students, and the ability to promote their students' SRL while acting in their roles as teachers or mentors. The latter ability has been called SRT (stands for Self-Regulating Teaching) (Kramarski & Kohen, 2017; Peeters et al., 2014). Studies indicate that metacognitive skills do not develop spontaneously. These skills must be formed and developed through participation in environments that provide learners with opportunities to experience SRL actively (Zimmerman, 1998).

The study goal is to examine the metacognitive process among teachers who participate in solving challenging problems in PSF in the role of students, and subsequently in the metacognitive process of teachers, while experiencing PSF guidance of their teaching fellows in the mentor's role. Figure 1 presents the dual role of teachers, which will be investigated in this study in the context of collaborative problem solving. The present study focuses on interactions occurring in the PSF learning groups, whose purpose is collaborative solution of challenging geometric problems.

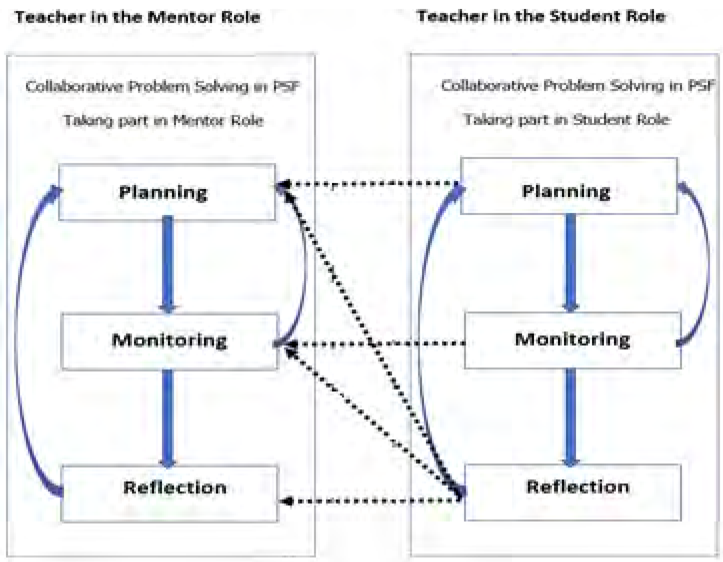

Figure 1: The dual role of teachers in a metacognitive process

Method

Setting

The study took place in the context of a professional development program for secondary school mathematics teachers. The two-year program aims at enhancing mathematical knowledge, mathematical thinking and at the enrichment of the repertuar of problem-solving methods of in-service mathematics teachers who teach or wish to teach the advanced-level of secondary school mathematics curriculum. The present study has been conducted in the context of two courses of the program, in which the emphasis was given to solving challenging geometric problems and to ways of including such problems in school practice.

The instructor for these courses (the second author) is a teacher with 30 years of teaching in secondary school in Israel, who moderates in-service training courses for math teachers, leads practicum courses in schools for pre-service teachers, and advises on development of digital learning environments. The study was approved by the Research with Humans Ethics Committee in Behavioral Sciences at the Technion.

Participants and procedure

Participants were 40 high school teachers from northern part of Israel, with professional experience ranging between 1-15 years. In this paper we focus on one teacher – John, who studied the course on basics of geometry. John has professional experience of 10 years of teaching advanced mathematics in secondary school. We chose to focus on John since he underwent an interesting process during the professional development program, as detailed in the findings section.

Stage A of the study focused on the process John went through as a problem solver in a group of teaching fellows. Namely, John and his fellows were required to make use of the online forums on WhatsApp for the purpose of collaborative problem solving of challenging geometrical problems in small groups (3-5 participants per group), mediated by the course instructor. Each teacher participated in six online meetings in the role of a student. On the forum, the teachers shared drawings, verbal messages, calculations and brief voice messages. A single discussion consisted of an average of 150 messages. Each discussion lasted between 1.5 – 2 hours and was designated for the solution of one problem. After each problem-solving session, a reflection session was conducted for all the teachers in the course framework. In this session, the teachers openly discussed their PSF experiences with the course's instructor. At this stage the instructor put forward metacognitive support for collaborative problem solving, which included explicit exposure to principles of a metacognitive approach to solving a challenging mathematical problem (planning, monitoring, reflection) and the method of metacognitive support while collaboratively solving challenging math problems in PSF. In addition, analytic sessions were held by the instructor between problem-solving sessions. These sessions focused on analysis of selected excerpts from one of the forums.

Stage B of the study focused on John as a mentor. John and each of his teaching fellows assumed a role of a PSF mentor, with support of the course instructor. Each teacher created a PSF for his/her teaching fellows, chose appropriate problems for mathematical discussion, and applied metacognitive support during the online forum activity.

Data Collection and Data Analysis

The protocols of three online forums were the primary data sourse. The analysis unit was each one of the messages appearing in the forum. For the purpose of analyzing the online forum protocols, each message was coded according to the different metacognitive components (Kohen & Kramarski, 2018): planning, monitoring and reflection. Data analysis was based on the mixed methods approach, including computation of frequency for each metacognitive component, as well as qualitative analysis of selected forum excerpts, which aimed at understanding the development of John’s dual role – as a student and then as a mentor, in a metacognitive perspective.

Findings

We will now present three forums in which John participated – in two of which he played the role of a student, and one more in which he participated as a mentor for his teaching fellows in the course. All forums took place concerning the subject "Isometric planar transformations".

Below is a table that summarizes the frequency of various responses in the three forums, according to their metacognitive, cognitive and other attributions. With regard to the metacognitive attribute, a statement was determined to belong to the *planning* criterion if that statement related to a general analysis of the task, choosing a problem solving strategy, defining a general goal and interim goals. The statement was attributed as belonging to the *monitoring* criterion if it included assessment of progress toward the solution, reflection on strategies that the group used to advance the solution in terms of correctness and efficiency to attain the goal. Finally, the statement was attributed to the *reflection* criterion if it included retrospection about the solution, that is, addressed such questions as: "Have the solution completely attained the goal?" "Were there other methods to solve the problem?" "Were the strategies used by the group appropriate for other problems?" "What have we learned while solving the problem?" When no relation to metacognition was revealed but the statement expressed the use of mathematical strategies, it was attributed to the *cognitive* component. Finally, a statement was attributed to the *other* criterion when it expressed general requests for assistance, e.g., "I can't keep up with you", "Can someone explain to me….?"

Table 1: Frequency of statements in three forums according to their attribution to: metacognition (P= planning; M= monitoring; R= reflection), cognition and other

		Forum 1 – John as a learner			Forum 2 – John as a learner			Forum 3 – John as a mentor	
		John	Teaching fellows	Instructor	John	Teaching fellows	Instructor	John	Teaching fellows
Metacognition	P	0	8	0	6	15	0	10	16
	M	5	33	20	14	27	3	2	7
	R	0	10	4	1	3	1	2	3
Cognition		22	13	0	11	24	0	12	19
Other		7	17	5	2	33	0	22	34
Total		34	81	29	34	102	4	48	79

In the first forum, the course instructor had the teachers solve collectively the following problem: *Given a square ABCD, point R is located on segment BC, and point M is on segment DC. AM bisects angle KAD. Prove that the length of segment AK is equal to the sum of lengths of segments BK and DM*. Forum 1 was the initial attempt of John to collaborate in a discussion forum. In this forum, after about 20 minutes he posted a full solution to the problem for his teaching fellows, and later in the discussion, each time other teachers raised questions, he would repeat his solution, without participating in the group discussion on particular issues and probelm solving steps of the forum activity. This process repeated itself 11 times. When posting the full problem solution, Johen mainly related cognitively to the problem: "Let's mark C'S'", "Let's use the theorem…", etc. By comparison, his teaching fellows made statements that can be attributed to metacognitive thinking in the problem solution process, as learners. For example, following is an excerpt that attests to the *planning* component in the metacognitive process:

T: Let's go over the question again.

S: Segment QM interests us.

R: I agree.

K: We need to place S' and C'.

S: Let's see where these points intersect, and in this way we can prove the segments are parallel.

Forum 2 was John's second attempt to collaborate in the forum as a learner. The instructor posed the following problem: *Given a triangule ABC, in which ACMN and BCKL squares are built on two of its segments (AC & BC) – assuming S is the middle of the segment AB, Q is the middle of the segment KM, R is the center of BCKL and T is the center of ACMN, prove that the polygon created SRQT is a square*. This time as well, John did not offer responses to the forum over a relatively significant period of time, about 15 minutes, during which he solved the problem on his own. Nevertheless, after solving, he didn't post the full solution, but rather tried to lead the group toward the solution using hints and directive metacognitive questions. He also demonstrated cognitive explanations in instances of misunderstanding, such as focused and detailed solutions. John was an active agent in all parts of the forum where collaborative learning occurred during the discussion, while attesting to metacognitive thinking that he had carried out while solving the problem. For example, he expressed *reflection* on the solution, when writing "*in order to prove that the polygon is a square, I thought about the definitions of the segments that need to be equal as well as the angles that must be 90°..* ". In this forum, it appears there is no significant change in the teaching fellows' responses that attest to metacognitive thinking, particularly in the monitoring aspect (see Table 1). Similarly, it appears that due to active participation by all the teaching fellows, the number of instructor's responses declined.

Forum 3 was the forum in which John took upon himself the role of a mentor, and raised the following problem: *Given a square ABCD in which AC equals and is perpendicular to BD, Segment AB equals 1, segment BC equals √2 segment CD equals √3. Diagnoals of the square are perpendicular to each other. Find the angle of angle ABC…* John chose an appropriate problem for the activity, which had no immediate solution, several stages of progress toward the solution had to be done, and after each stage, a path had to be chosen to continue from among several possibilities. After receiving an approval from the instructor for the chosen problem, John organized a group of three teaching fellows (both in terms of participants as well as in terms of scheduling the meeting), and led the entire learning process, while demsontraing planning, monitoring and reflection skills, in the SRT sense. In this forum John was very active. Such behavior is more intensive than what was expected from him as a mentor. However, it is possible to see that his attention was aimed primarily at technical support of the forum, and when he responded, it was a significant expression of his support of the metacognitive aspect, specifically referring to the planning component. For example: "*Which parallel traits could help with the present problem*?" or "*You've chosen the right theorem, but how do we proceed from here?*" His choice to focus on the planning component attests to his role as a mentor, who encourages the learners to think about the solution, but when they were in the midst of the process itself, he let them cope alone and supported only when necessary.

Between the forums in which John participated as a problem solver, and the forum in which he was in the mentor role, John participated in an analytic session focused on a particular forum excerpt. The analysis included attention to the instructor's behavior in leading the discussion. John wrote in the analysis: "*The instructor's intervened to explain how to generally approach the solution to the problem, but after the learners began to connect with each other and were focused on solving the given problem, the instructor almost did not interfere*."

Based on the excerpt analysis and personal participation in previous forums, John also mentioned the following: "… *As teachers, we need to encourage discussion about different strategies of solution, since they [the learners] don't have to accept a single solution that the instructor thought about. We need to provide learners with an opportunity to stimulate thinking about the issue… but if for a long time nothing interesting happens in the forum, it may sometimes require the instructor's intervention so there won't be boredom among the learners… In the forum I analyzed, the instructor's intervention was well placed, since a long time had elapsed and some of the participants had not paid attention to the central idea they were already discussing.*"

Discussion and contribution

John's progress is expressed by demonstrating metacognitive ability in the process of collaborative learning via problem solving as a student (Carlson & Bloom, 2005; Clark, James, & Montelle, 2014; Zimmerman, 2008), and afterwards when his role was that of the mentor (Kramarski & Kohen, 2017; Kramarski & Revach, 2009). This process was also expressed in the reflection John wrote at the conclusion of the course, attesting to self-awareness of the process of change he underwent: "*In the first forum I acted like a student who had to finish the assignment without letting others think, because the sharp students always give the answer, and that's that… In the second forum, I understood that most of my teaching fellows had no clue for how to solve the problem, so I acted differently and responded more through explanations and mediation… In the last forum, as the teacher, you need to act like a teacher in all manners. Therefore, I provided a direction for thinking about the solution, and encouraged them to solve the problem…*". In this process, John acknowledges the role of the instructor in supporting the collaborative problem solving process of the participating teachers as learners, and applied these conclusions in the forum he led as a mentor.

The current study demonstrates the use of an online environment for collaborative problem solving, a PSF. Addressing the difficulties mentioned in previous studies regarding the use of PSF (e.g. Koichu & Keller, 2017; 2019), the study suggests to metacognitively support the mutual collaborative interactions between teachers in the role of students (Kramarski & Kohen, 2017). Moreover, we explore the development of the teachers' dual roles in a metacognitive perspective – from their experience in PSF as students to establishing and guiding PSF as mentors to their teaching fellows. We specifically focus on the role of an instructor in supporting metacognitively teacher collaboration in the student's role and how it leads the development of the teachers in the mentor's role.

The contribution of the current study is twofold: Theoretically, the study connects two theoretical frameworks by exploring collaborative problem solving in online forums with a metacognitive view. The expected practical contribution is related to demonstrating a process of teachers who experience collaborative interactions as students, and then as mentors guiding their teaching fellows. This might be of significant importance to teacher educators in both teacher training and professional development programs.

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ANALYSIS OF MENTORING SITUATION BETWEEN MATHEMATICS TEACHER EDUCATORS DURING COLLABORATIVE COURSE FOR PROFESSIONAL DEVELOPMENT FOR IN-SERVICE MATHEMATICS TEACHERS

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The knowledge and development of the mathematics teacher educators has been investigated recently. We focus particularly on the nature of interactions between the members of co-mentoring group of mathematics teacher educators. Interactions between two mathematics teacher educators during preparation, implementation and reflection of the first session of longitudinal course for professional development for mathematics teachers was analysed. Framework for analysis of mentoring situations between mathematics teacher educators was adapted from a previous work of Blömecke et al. (2015). The proposed framework enables better understanding of observed interactions.

Introduction

Despite the huge amount of research regarding teacher knowledge (Ball, Thames, & Phelps, 2008) and teacher learning (Bakkenes, Vermunt, & Wubbels, 2010; Jaworski, 2006), only limited amount of work has been done in the field of preparation and development of mathematics teacher educators (MTE), particularly of the educators of in-service teachers (Margolin, 2011; Murray & Male, 2005; Zaslavsky & Leikin, 2004). Although almost no institutionalised form of education for MTEs exists, the MTEs are continually educated within their workplace, either by their colleagues and critical friends. Even reviewers can be considered as mentors (Jaworski, 2008). As early as in 1994, Jaworski and Watson (1994) characterised the three modes of mentoring: critical friends, inner mentoring and co-mentoring.

There are different pathways to become MTE and to affect their competences. Novotná, Margolinas, and Sarrazy (2013) describe some of them and hypothesise their impact on the specific knowledge of MTEs. Selmer, Bernstein, and Bolyard (2016) enhanced the widely used model of Shulman (1986) and depicted the different layers of knowledge of teacher educator. Zaslavsky and Leikin (2004) elaborated the three-layer model of MTE growth through their practice where MTEs' competencies are developed through reflection and generalisation of their solutions.

The difference as well as relationship between competence and competency is distinguished by Blömeke, Gustafsson, and Shavelson (2015): the competence represents complex ability divided into cognitive, conative, affective or motivational facets (resources, traits), marked as competencies. Competencies are not directly observable but are enabled for inferring from observable behaviour.

In this paper we will focus on professional learning of novice MTE. We will identify learning situations occurring during her first three sessions as educator of in-service teachers and following reflection on these sessions with the mathematics teacher educator's mentor (MTEM).

Methodology

Within the PRIMAS project (www.primas-project.eu) supported by the European Union FP7 the course aimed at the professional development (CPD) of upper-secondary mathematics teachers (MT) has been focused on the implementation of inquiry-based learning. The structure of the course followed the spiral model of teachers' professional development, consisting of the following three repeating steps: Implementation – Reflection – Analysis (Maass and Doorman, 2013). The course was based on the model of teacher growth proposed by Clarke and Hollingsworth (2002). During the course, the teachers went through the mentioned steps four times. They did specific changes in their practice and videotaped their lesson. There was a common reflection on the lesson. After the teacher reflected on it, the group came with specific conclusions that might influence their further practice. Majority of CPD sessions were co-taught by the two teachers, Daniela and Pavol.

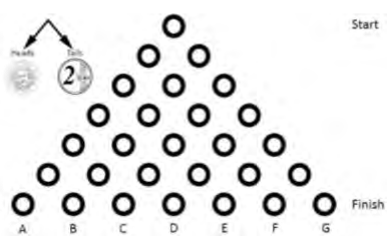
Daniela was a university teacher. She was quite experienced in identifying students' misconceptions and assessing the level of combinatorial thinking of tertiary students. She also had a very good understanding of horizontal content knowledge as well as from the perspective of pedagogical content knowledge, she had a good comprehension of the students of tertiary and upper-secondary level. She had limited experience with upper-secondary students or upper-secondary MTs before the CPD. Pavol had more than five years of experience in teaching mathematics at lower secondary level and tertiary level. He had two years of experience as an in-service teacher educator. Therefore, we can assume quite high level of pedagogical content knowledge (PCK), particularly knowledge about students and MTs as participants of the professional development. Within this cooperation they tried to establish the mode of work supporting their mutual development through co-teaching and shared reflection on the sessions of CPD leading to possible co-mentoring. The CPD sessions were videotaped and the following reflections were audiotaped or videotaped and transcribed. In this paper we focus on the first session within the described CPD and we analyse co-teaching and reflections of Daniela and Pavol.

To characterize learning situations of teacher educator, we will use framework introduced by Müller (2003) defining three main strands of learning: (i) learning-off-job means mainly theoretical preparation for CPD sessions. In our case preparation was based mainly on the study of literature about IBL followed by discourses among group of MTEs in the national group. Searching for suitable tasks is also included in this strand; (ii) learning-by-job is understood as the learning assisted by the colleagues giving MTEs feedback on their work. It occurs mostly as the reflection-in-action (Schön, 1987). Most of CPD sessions were reflected and analysed immediately or a day after the session; (iii) learning-on-job describes the self-education and the realization of MTE. It is based on both, reflection-in action and reflection-on-action.

Analysis

Preparation of the content of CPD (Learning-off-job)

Daniela had focused on combinatorics teaching for a long period of time and she had a quite rich content knowledge as well as a special content knowledge from combinatorics and from teaching combinatorics. Therefore, she picked the combinatorial problem as an example of good inquiry-task (Figure 1). She considered the problem effective in limited time and beneficial for teachers' day-to-day teaching. She had good experience with this problem from the CPD for lower-secondary MTs and therefore she felt confident with its implementation. Daniela explained to Pavol the different mathematical nuances of this problem, solutions on different levels of combinatorial thinking (listing elements of set of outcomes, rule of product, formulas), relation between combinations and permutations with repetition, Pascal's triangle and binomial distribution of probability, during preparation of the CPD session.



The playing-board on the Figure 1, can be used for foretelling the fated profession of the player. The starting point is on the top of the playing-board. Player will move according to the result of tossing a coin: if the head falls the player moves left-down, if the head falls, one moves right-down. After six rounds, the last row is reached on the position of player´s fated profession is revealed (e.g. position D means cook, position E teacher). Where should be your fated profession situated? Explain your answer.

Figure 1: The combinatorial problem (Páleníková et al., 2018)

Implementation of the CPD session (Learning-by-job)

Pavol's introduction of the theoretical background of IBL was followed by the above-mentioned hands-on activity led by Daniela. The MTs solved the task very quickly. Daniela was disturbed as she was not able to elaborate the mathematics that the solvers could learn through the task.

Daniela: So, the Pascal's triangle can be introduced also in this way… in slower way… Of course, you use your own verified approaches, but there is also this option.

Pavol: Mm… and so?

After Daniela's short attempt to address the pedagogical aspects of the problem she focused on the mathematical relationships.

Daniela: … and during the lessons aimed at combinatorics… wait, we have to get back a little. We calculated all these tasks… the solutions are… we get the probability that any results… **It is pointless to use other methods to solve the task, if you already found the approach with permutations with repetition [1].** We needed to reach the (some) line in the Pascal's triangle. So, we found one solution.

Pavol interrupted Daniela's continuing lecture with the aim to get the maximum out of the activity.

Pavol: (to Daniela) Maybe you should ask one more time, where the combinatorics is in there? Or…

(to MT group) When you were solving the task, so more or less, what was your first idea? (2 sec) Find the appropriate formula, wasn't it?

Pavol moderated the discussion about the task and led it from using formulas towards mathematical inquiry.

Pavol: **What if you don't know the formulas? [2]**

In the described situation, Daniela's lecturing was observed by Pavol. He perceived it as transmissive and decided to intervene as he agrees with Zaslavsky & Leikin (2004) that MTs should learn challenging mathematics in the way they are expected to teach, but Daniela was giving a lecture. However, Daniela was reluctant to his intervention, she explicitly expressed it in [1]. Pavol insisted as he wanted to establish the inquiry culture in the CPD since solving the first problem. Given that the MTs just skipped the essential combinatorial relationships and started the solution by using formulas, Pavol tried to decrease the level of combinatorial thinking needed for solving the problem, i.e. to use the sum and product rule instead of formulas. He was motivated as he saw the inquiry affordances of the problem. In his opinion, the connection to IBL was missing and he used questioning (Jančařík, Jančaříková, & Novotná, 2013), one of inquiry pedagogies introduced in the CPD, [2] to evoke a fruitful discussion. MTs shared their ideas about other possible solutions of the problem.

Daniela: Ah, there is one more interesting issue, all of you here … **I described everything what I wanted [3]** and you decided that permutations with repetition are the answer and … **when I started to draw [4]** the situation, we obtained the Pascal triangle. When we are talking about operations in combinatorics, is there any reasonable relation or **is it possible to use combinations instead of the permutations with repetition? [5]** … somehow … well? How? **Ok, I am not going to test you [6].**

Pavol: Well… actually, we can say that…. We can look at the triangle and… The triangle represents the result of some…

While Daniela summarised the MTs' inquiry she focused on her performance [3, 4] what is in the discrepancy with intended classroom culture. She stayed stacked on the solution using the rule of sum in the Pascal's triangle and MTs' solution using permutations. She tried to draw MTs' attention to the relation between permutations and combinations but as soon as she posed the question, she closed her speech by [6]. Her perception of the questioning as testing might be influenced by her previous experience with insufficient common content knowledge of participants of CPD for lower-secondary teachers (Melušová, Šunderlík, & Čeretková, 2014). Pavol did not agree, he wanted to address this issue and come back to this relation as he saw that this group has much better common content knowledge.

Reflection on the CPD session (Learning-on-job)

Pavol and Daniela showed the overall satisfaction with the CPD session during the immediate reflection. Pavol raised up the issue [5] not covered by discussion with MTs.

Pavol: But we could ask them what is the connection the permutations with repetition and… why there are combinations in Pascal's triangle, so silly question.

Daniela: I did ask them and **they pointed out this combinatorial number [7]**… or Matej said.. or… I can ask **"which place will I give to the character?" [8]**… I have to say that their common content knowledge is on level A+.

Pavol:	Yes, but what about pedagogical content knowledge? **Matej was talking too much [9]** and **I did not want one spokesperson to come out [10]**.
Daniela:	**But your managing of this situation was great, I liked it [11].**
Pavol:	Because other participants wanted to say their opinions, but Matej responded without raising a hand.
Daniela:	Well… you tried to ask them more and they followed you…
Pavol:	**The question is whether it was adequate [12]**.

Daniela perceived that MTs answered the question [5] implicitly [7]. One of the MTs (Matej) mumbled the bijection between category I and category II combinations in sense of (Lockwood, Wasserman, & McGuffey, 2018) in a form of a question suitable even for young pupils [8]. So, Daniela considered Pavol's question as correctly answered and did not see any point to continue elaboration of this relation. On the other side, Pavol missed the conversation about the mathematical aspect of the problem he regarded as important. In his opinion, not all the MTs did see the connection and understood Matej's answer [9, 10] and Daniela actually did not ask if they understood it. Although Pavol expressed the uncertainty with adequacy of his intervention [12], Daniela did not respond, and the discussion moved toward more administrative issues.

Conceptualisation of findings

In line with the original intention to co-mentoring, Daniela prepared the mathematical content of the CPD session and initiated Pavol into the combinatorial situation with all its facets. On the other hand, Pavol put himself in the main role as mathematics teacher educator mentor (MTEM) during the session and reflection. This may be result of Pavol's and Daniela's different background their different knowledge. Pavol had better knowledge in teachers' development, more experience with IBL, experience as a teacher and as a teacher educator. However, Daniela had strong background in problem-solving, combinatorics, combinatorics education and she can be considered a novice MTE. On the other hand, Pavol and Daniela shared fundamental knowledge as they completed the same teacher education programme. Knowledge sharing, as an essential part of planning the sessions and of reflection on action, led to construction of knowledge of participating MTE and MTEM. In contrast to cognition, the affect and motivation are not necessarily changed by sharing them, but they can influence each other.

In order to better understand this episode, we adapt the framework of Blömecke et al. (2015) for the context of collaboration of two professionals (Figure 2), namely Pavol (MTEM) and Daniela (MTE). The original frame of Blömecke et al. (2015) supposed that dispositions influence situation-specific skills where perception of the situation leads to its interpretation and subsequent decision making. The decision-making process has direct impact on observable behaviour of MTE and MTEM. As both, Pavol and Daniela considered reflection as a usual part of their profession, they both reflected in action during the CPD sessions. Thus, the behaviour of the co-educator was critically perceived and led to another PID process. As Pavol has gradually identified with his role of a mentor, beside his reflection in action, he had to prepare for consequent co-reflection on action with Daniela. In our adaptation of the framework, the reflection-in-action is marked as red (dotted) arrow and reflection-on-action is visualized as yellow (dashed) arrow. Interactions in affective level are visualized as violet (semi-dashed) line.

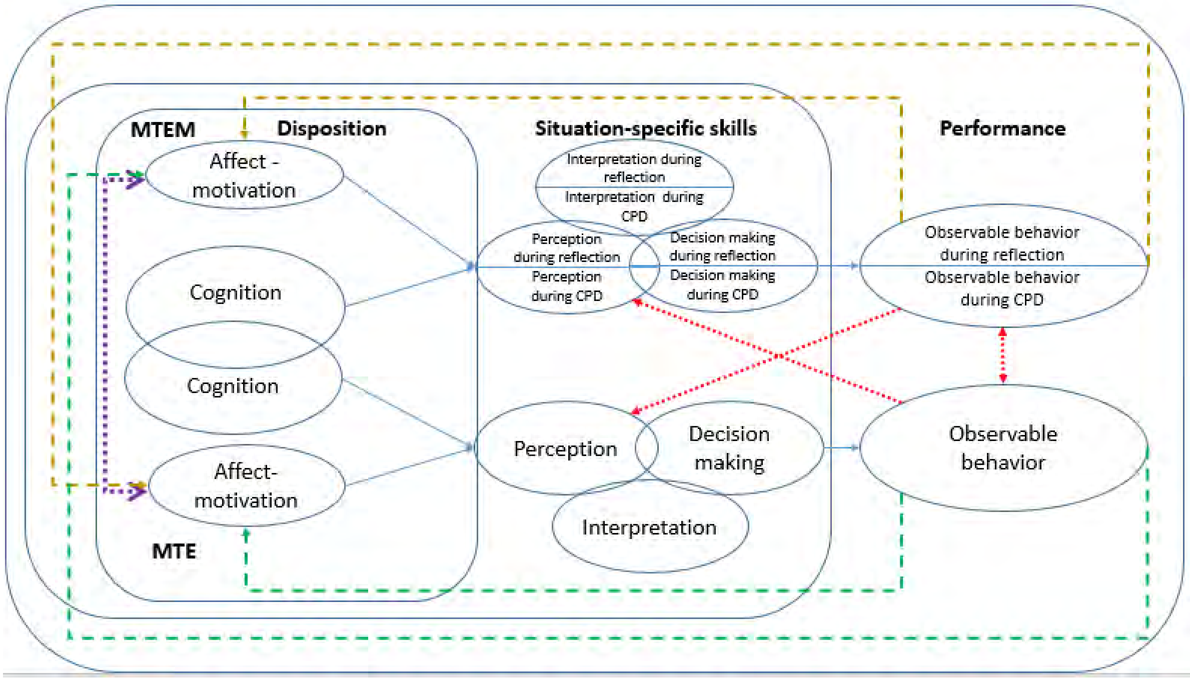


Figure 2: Framework for analysis of mentoring situations between mathematics teacher educators

Discussion and conclusions

In this paper we built on the previous work of Blömecke et al. (2015) and adapted her framework for the context of collaboration of two professionals, namely MTE and MTEM. In the first described situation from the CPD session, Daniela's transmissive teaching within the CPD aimed to IBL is demonstrated. Her observable behaviour initiated Pavol's process of perception – interpretation – decision making (PID) leading to his intervention [2] when he encouraged the discussion of MTs.

In the second situation the discrepancy between Daniela's and Pavol's perception of the same situation is accentuated. While Daniela perceived the problem as solved and decided to move on with the session agenda, Pavol felt the need to continue elaborating the problem. He might be motivated by his freshly obtained knowledge about the relation [5] as well as by his awareness of the quite strong common content knowledge in combinatorics of involved MTs. Thus, he decided to intervene again.

The reflection originated in the second situation which enlightened Pavol's further rationale for the intervention. He wanted to use the problem-solving process as a tool for establishing sharing and supportive culture in the CPD besides exhausting all the affordances of the problem. One-to-one interaction observed by the rest of the participants is not in line with Pavol's belief about classroom (in this case CPD) culture. As he considered the pedagogical content knowledge as equally or more important as common content knowledge, Daniela's intention to finish the problem was perceived as wasting the opportunity to implement IBL in the CPD session. In other words, teachers were not enabled to engage in alternative pedagogy from learners' perspective. This situation might be the result of a disagreement in the affective level between Daniela and Pavol.

The difference in their usual way of work is observable also by the way how they mentored their colleague. Daniela explained in advance and tried to cover all the structured affordances of the problem. Pavol tried to develop Daniela's pedagogical content knowledge based on the same spiral model (Maass and Doorman, 2013) as they intended for MTs.

In the analysis, we focused on the very first session of the longitudinal CPD. Daniela, as novice MTE, had room for possible improvement and her professional growth can be observed. Contrasting with Pavol's proficiency, her knowledge for teaching MTs has not been accommodated yet. In consonance with Novotna et al. (2013), different backgrounds of Daniela and Pavol resulted in different pieces of knowledge. All their specific knowledge showed to be important for good implementation of the CPD.

Proposed framework allowed us to identify learning situations crucial for development of both, Daniela and Pavol. Furthermore, the frame can be used for analysis of the collaboration between MTEs. Because of the collaborative mode of work in the CPD, the mentor's role was transferred between the involved MTEs. The proposed framework can simplify the recognition of the current mentor.

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FACILITATORS' AND SCHOOL LEADERS' ROLE IN ESTABLISHING AN INQUIRY-ORIENTED COMMUNITY OF MATHEMATICS TEACHERS

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This paper addresses the role school leaders and professional learning facilitators play in establishing communities of mathematics teachers characterized by authentic, collective inquiry into instructional practice and oriented toward ambitious and equitable learning aims for students. We report on an ongoing qualitative analysis instructional leaders' work designing and facilitating a professional learning system to develop an inquiry-oriented teacher community. We highlight the commitments and principles that guided the leaders' decisions and the challenges that arose.

Decades of research indicates that establishing teacher communities in subject-specific departments can benefit teachers and students (e.g., Horn, 2005; McLaughlin & Talbert, 2001). In inquiry-oriented communities,[1] teachers wrestle with dilemmas of teaching, they press on one another's views about teaching, subject matter, and students' capabilities, and they identify and work towards collective aims for their practice (Grossman, Wineburg, & Woolworth, 2001). In such communities, learning together about and from teaching is inherent to the work of teaching. For example, within the U.S., studies of the mathematics department at "Railside High School" provide powerful images of teachers engaging in authentic, collective inquiry into instructional practice oriented toward ambitious and equitable learning aims for students, accompanied by compelling evidence that students' learning and senses of themselves as mathematics learners improved (e.g., Boaler, 2002; Nasir, 2014).

As a field, while we know the value of teacher communities, there is minimal scholarship that explores how inquiry-oriented communities of mathematics teachers such as the one at Railside are established. In this paper, we report on an ongoing qualitative analysis of instructional leaders' support of the development of an inquiry-oriented community of mathematics teachers in a U.S. middle-school. In particular, we focus on the work of professional learning facilitators and the school leader, especially the principles that guided these instructional leaders' decisions in designing and facilitating a system of professional learning toward establishing such a community. Our focus on understanding the role of facilitators and school leaders is guided by existing literature that suggests that both matter for the development of teacher communities (Bryk et al., 2010; van Es et al., 2014).

Supporting Community Through Professional Learning

Developing inquiry-oriented teacher communities is incredibly difficult for a number of reasons. One reason, as Grossman et al. (2001) observed, is that, in general, the teaching profession lacks a collective vision. In addition, such communities challenge deeply established norms of privatization in U.S. schools (Little, 1990; Lortie, 1975). Historically, schools in the U.S. have not been organized to support teachers' learning. Until relatively recently, it was somewhat unusual for teachers to have time built into their workday to collaborate on teaching, and it remains unusual for teachers to have

[1] We use the term "inquiry-oriented community" to specify the kind of work happening in a subject-specific department, that being inquiry into instructional practice in relation to, e.g., students' mathematical thinking and participation.

routine opportunities to see one another teach, let alone teach together. However, creating a structure in which teachers are expected to collaborate, on its own, does not necessarily lead to strong community nor does it necessarily benefit teachers or students (Horn et al., 2016). As Grossman et al. (2001) argue, becoming a community often requires "transformation" of individuals' current practices and of the workplace setting (p. 948). Hence, as we will elaborate in what follows, the role of the school leader and facilitators of professional learning are critical.

Establishing a community is, in part, dependent on creating a coherent instructional system (Newmann et al., 2001). A coherent instructional system is one in which instructional decisions and policies are organized around an explicit set of goals for students' learning and a complimentary, shared vision of high-quality instruction (Cobb et al., 2018). Key elements of an instructional system include instructional materials and assessments, and professional learning opportunities.

Professional learning in the U.S., more broadly, tends to be piecemeal and fragmented (Borko, 2004). Within a school, it is increasingly common for U.S. mathematics teachers to be provided with time to collaborate with colleagues, often in grade level teams, as well as opportunities to work individually with a "coach" (Banilower et al., 2018). However, these various opportunities to collaborate are often not well connected or coordinated, such that what teachers work on in one structure is elaborated on in another (Cobb et al., 2018). Research indicates the value in designing professional learning opportunities (e.g., time for teachers to collaborate in a grade-level team, department-wide meetings, individual coaching) that are anchored to an explicit and specific set of student learning goals and instructional vision, and through which teachers can deepen their collective inquiry and instructional practice.

From a learning perspective, especially when teachers' aspirations for their practice requires substantial shifts in their current practice, it is especially important that teachers have opportunities to engage in cycles in which they *investigate* and *enact* key aspects of instructional practice (Grossman et al., 2009). A well-designed professional learning system ideally allows for teachers to analyze their current practice, through, for example, viewing video-records of their teaching and/or making sense of student work, perhaps in the context of time to collaborate during the school day. Teachers might then identify next steps for their practice, and then work with one another or an instructional coach to try out the key aspect of practice, followed by further analysis, and so forth.

Further, a well-designed professional learning system is *principled* in its design and facilitation (Kennedy, 2016). As an example, in the case of Railside, principles organizing the mathematics department's collaborative work included: all teachers and students are learners; working from strengths while making space for vulnerability; redefining "smart;" redefining what it means to do math in school; and the importance of relationships (Nasir, 2014). Nasir and colleagues argue that these principles were crucial for the work of the community of mathematics teachers at Railside: "The aspects of teaching practice and professional community at Railside were only powerful when connected to important and meaningful overarching principles" (p. 235). Because school leaders and facilitators of professional learning (e.g., instructional coaches) play a crucial role in the design and facilitation of professional learning events in schools, it is critical to understand the principles that organize their work toward establishing inquiry-oriented communities (Cobb et al., 2018).

Methods

This ongoing analysis was guided by the following research question: What emerged as central to instructional leaders' work in establishing an inquiry-oriented community of mathematics teachers?

Research Context

This analysis is part of a research-practice partnership which aims to understand how district math leaders, coaches, and teachers can use data about instructional practice to support system-wide instructional improvement. We focus on the development of an inquiry-oriented community in the Forest Middle School mathematics department during the year 2018 - 2019. Forest is an ethnically, racially, and linguistically diverse school in the Northwest U.S. The school had undergone significant changes in the two years prior to this analysis. The school had "detracked" all of its mathematics classes, meaning that students were no longer assigned to mathematics classes based on presumed ability. All seven of the mathematics teachers, along with more than three quarters of the teaching staff as a whole, were in their first two years of teaching at Forest (though some had been teaching for longer). The school leader and facilitators of professional learning were in their first year of a coordinated effort to design and facilitate a connected set of professional learning experiences, toward the aim of developing an inquiry-oriented community of teachers.

These experiences included a professional development model called Math Labs, providing opportunities for teachers to investigate and enact instructional practice together (Kazemi et al., 2018). Similar to Lesson Study, Math Labs involve opportunities for teachers and instructional leaders to enact instruction with each other present and attend carefully to the mathematical thinking and participation of students in response to instructional decisions. Different from Lesson Study, the lesson enactment is collective, and as such, teachers engage in enacting instruction with each other's students. During enactment, they pause to confer about instructional decisions they might make in response to students' thinking and participation. After enactment, they inquire into their instructional decisions in relation to students' thinking and participation, grounded in artifacts from the classroom visit, including student work, video of interactions, and/or teachers' and leaders' observations of students' thinking and participation. As we describe further below, professional learning experiences at Forest also included opportunities for grade-level teams to investigate practice together and opportunities for individual teachers to investigate and enact practice with an instructional coach.

The school leader (principal; hereafter, Jack) had been a principal for six years total and three years at Forest and, prior to becoming a principal, had worked as a secondary mathematics teacher. Jack expressed a strong commitment to equitable outcomes for students at Forest and to teacher and leader collaboration. He had participated in district-supported Math Labs in his previous principalship, facilitated by the third author and her colleagues. The facilitators of professional learning (school-based instructional coaches; hereafter, Patty and Ada) were responsible for planning and facilitating professional learning experiences at Forest, alongside Jack. Patty was in her first year as the mathematics instructional coach at the school after having taught middle grades mathematics at Forest for 13 years, during which she participated in Math Labs. Ada was in her eighth year as an instructional coach at Forest and her second year focused especially on English Language Arts. Several years prior to this study, Ada had collaborated with the third author and other instructional coaches to support teachers' implementation of rich tasks through their design and facilitation of

Math Labs. Prior to coaching, she taught in elementary and middle schools for a total of 19 years. Patty, Ada, and Jack saw themselves as "co-learners and co-facilitators and co-planners" in the work of supporting the community of mathematics teachers at Forest (Ada).

Data Sources

Our team participated in 12 collaborative planning sessions with Patty, Ada, and Jack throughout the year; the planning sessions were organized prior to and after key professional learning experiences. We shared ideas and resources and offered suggestions grounded in research on mathematics education, teacher learning, and systems for instructional improvement, and we audio-recorded and collected field notes and artifacts at each meeting. Additionally, at least one member of our team attended, audio-recorded, and collect field notes and artifacts at key professional learning events (13 total events, spanning a range of forms). A district mathematics leader participated in half of the planning and professional learning events.

The first author conducted semi-structured interviews with Patty, Ada, and Jack. Initial interviews in the fall included questions aimed at understanding their goals for students' and teachers' learning, and particular goals for each form of professional learning. She conducted a second semi-structured interview with each of them after the final set of professional learning events. These interviews included questions aimed at understanding how they saw the community developing in relation to their goals, and their rationale for specific planning decisions. She conducted one additional interview with Patty (the primary facilitator of professional learning events specific to grade levels) in order to understand her in-the-moment facilitation decisions.

Data Analysis

To date, our team has engaged in analysis of the instructional leaders' visions and goals in relation to developing an inquiry-oriented community. We especially focused on the leaders' design and facilitation of what we came to view as a system of professional learning, with a focus on detailing the theory of action underpinning the design of each structure for professional learning, as well as the intended connections, or threads, across the system. We asked a set of analytic questions of the observation data from the instructional leaders' planning for and enactment of professional learning and the interviews with facilitators including, *Why were particular planning and facilitation decisions made?* to identify principles guiding the instructional leaders' work.

Results

In what follows, we detail three categories that emerged in our analysis of how the instructional leaders worked to support the developing community of mathematics teachers.

Commitments Regarding the Community's Work

Establishing a shared vision of instruction. Based on interviews and planning conversations, it was evident that Patty, Ada, and Jack shared a vision of high-quality mathematics instruction in which students engage in rich conversations about their and others' mathematical ideas in order to develop meaningful mathematical understandings and come see themselves and others as mathematicians. They articulated an overarching aim that students "feel competent and safe as mathematicians, as people that can mess up and solve things … to collaborate, to feel safe enough to speak and share what they're thinking and value the diverse voices in the room" (Jack). The three leaders saw this vision as married to the school's broad aims to work toward justice and equity in relation to race, ethnicity, ability, and language status.

Jack described explicitly hiring teachers who shared broad commitments to justice and equity in relation to their students' learning. However, the leaders recognized that the teachers had differing ways of talking about mathematics instruction in relation to these commitments. As Ada articulated, "it's one thing to be committed and to say you're committed to [justice and equity] and it's another thing to see what that looks like and to experience what that looks like."

To work toward a shared vision of instruction, for example, the leaders organized professional learning around a set of shared goals for students' learning and teachers' instruction that were co-developed with the mathematics department. The leadership team first met in the fall to draft ideas grounded in research and documents such as the National Council of Teachers of Mathematics' (2014) *Principles to Actions*. They specified and refined these goals in an initial meeting with teachers. The resulting goals for students included listening to and making sense of the ideas and reasoning of others and seeing themselves has having a mathematical voice and agency in their classrooms. The resulting goals for teachers' instruction included facilitating dialogue among students that supports sense-making of a variety of strategies and approaches. These goals were referenced in each of the planning meetings.

Leaders further offered opportunities for teachers to deepen their instructional vision in relation to their shared commitments to justice and equity by engaging in strategic professional development together outside of the school day. For example, the department attended a conference together centered on "equity in the day to day work of math teaching, learning and leading." They found that a teacher who had been teaching in more didactic ways came back from the conference ready to "really think about [students'] agency and voice" in relation to their teaching practice (Ada).

Establishing a shared vision for how the community works together. Patty, Ada, and Jack also shared a vision for how the community would work together, characterized by authentic, collective inquiry into instructional practice. They aimed for teachers to see immense value and even necessity in collaboration with colleagues ("see that they need to collaborate, that it's better that way," Jack; "it's so powerful to be collaborating with other folks, and that's what really shifts practice," Ada). Relatedly, they aimed for teachers to see themselves and each other as having valuable resources for the community to build from in their work together, disrupting typical hierarchies of status among teachers and leaders in schools. They also noted the importance of teachers and leaders making their instructional practice public, positioning it as an object of inquiry for their own and others' analysis.

While willingness to collaborate was also a hiring criterion, the mathematics teachers oriented in different ways to collaboration, as they did with their vision of instruction. To work toward a shared vision for the community's work together, for example, the leadership team made careful facilitation decisions during enactments of instruction. They described intentionally framing collective instruction (e.g., before going into a classroom together in a Math Lab), emphasizing that their work would not be characterized by perfect, polished instructional practice ("...it's not going to be perfect the first time and we have to stick with it…," Patty). In our observations of professional learning involving enactments of instruction, the three leaders used language like "try it out," and "let's see what happens" to underscore the experimental nature of their collective instruction.

Structuring Collaboration: Designing and Facilitating A System of Professional Learning

Patty, Ada, and Jack designed a system of professional learning structures all organized around the vision of instruction and vision for the community's work together described above. Jack crucially built these structures into the school calendar. The structures involved a) collaboration within the mathematics department; b) collaboration among grade-level teams of mathematics teachers; and c) individual collaboration with Patty.

The mathematics department met three times in the 2018 - 2019 school year for Math Labs, which were co-facilitated by Patty, Ada, and Jack. Because Math Labs were typically full-day department-wide professional learning experiences, Jack arranged for full-day substitute teachers for each of the seven mathematics teachers for each of the three Math Lab days. Patty, Ada, and Jack "purposefully chose" classrooms to host the Math Labs to counter particular narratives such as "'Well, I can't do that because …' or, 'That's just a good group of kids'" (Patty). Math Labs were organized around particular instructional routines (e.g., Contemplate then Calculate; Kelemanik, Lucenta, & Creighton, 2016), which served as a predictable structure through which teachers could work on broader principles and practices of teaching (e.g., supporting students' mathematical identity and agency). These routines were selected by Patty, Ada, and Jack as conducive for working on the goals for students and teachers they co-developed with the mathematics department.

Between Math Labs, Patty visited teachers' classrooms, in which she and/or the teacher worked on particular aspects of instruction, implementing the same instructional routine used in the Math Lab. These visits were co-planned by Patty and the teacher to involve co-teaching or strategic observation of the teacher's instruction, aligned to goals set in the prior Math Lab and in response to the particular teacher's practice and narratives about their students' capabilities to support teachers to see that their developing vision of instruction "can happen … in the context of [their own] classroom" (Jack).

In addition, Jack organized the schedule such that teachers in the same grade level had a common planning period. Teachers met weekly in grade-level teams with the mathematics coach to explore mathematics content and to investigate instruction. Some of this grade-level planning time was used to interview students about their mathematical thinking and collectively debrief the interviews (adapted from the Math Reasoning Inventory; Burns et al., 2012). Other grade-level planning time involved joint analysis of students' work on common formative assessments and conversations about instructional decisions they would make on the basis of their analysis.

Principles Guiding Instructional Leaders' Design and Facilitation of the System of Professional Learning

We identified three principles that appeared to underpin the instructional leaders' planning and facilitation decisions for the system of professional learning as they worked to establish an inquiry-oriented community of mathematics teachers at Forest.

(1) Intersecting lines of inquiry. Each structure in Forest's system of professional learning was organized to engage teachers and leaders in joint inquiry in relation to students' mathematical thinking, participation, and identities; mathematics content; instructional practice; and narratives about students' capabilities. During each professional learning event, instructional leaders pressed teachers to reason across multiple of these categories (e.g., a student's participation in relation to how that student's mathematical ideas were positioned). This is commensurate with what Rosebery, Warren, and Tucker-Raymond (2016) found to be generative in supporting science teachers' learning: "investigating dilemmas in their everyday practice at the intersection of student sense-making, academic subject matter, and structural inequalities" (p. 2).

(2) Teachers' experience of coherence. There were common "threads" across the professional learning system at Forest, which appeared in each of the professional learning events we analyzed:

- A specific set of goals for students that the department collectively defined and specified (e.g., "What does mathematical agency look like?")
- A specific high-leverage aspect of instruction that the department was committed to improving (e.g., facilitating meaningful mathematical discussions)

These common foci connect directly to the shared vision and set of goals the instructional team worked to foster and support a coherent instructional system (Cobb et al., 2018), as discussed above. In an initial analysis of teachers' experiences of the system of professional learning, these common foci appear crucial to teachers' experience of coherence across various professional learning events.

(3) Collective investigation and enactment of instruction. Each of the professional learning structures designed made space for collective investigation or collective enactment of instruction, or both. Literature points to the importance of opportunities to both investigate and enact key aspects of instructional practice (e.g., Grossman et al., 2009). It appeared crucial in the work at Forest that these opportunities were *collective.* That is, teachers had opportunities to enact instruction *together* (with colleagues during Math Labs and with Patty during classroom visits) and engage in inquiry in relation to that collective enactment.

(4) Responsiveness to teachers. Key decisions were made in response to teachers, informed for example by teachers' current thinking, participation in the system of professional learning, instructional practice, and narratives about their students' capabilities. In an initial analysis of teachers' experiences of the system of professional learning, these responsive decisions appear crucial to teachers' developing vision of mathematics teaching and goals for their teaching practice.

Discussion and Conclusions

We have provided an image of how an inquiry-oriented community of mathematics teachers is established. In doing so we contribute to the literature on teacher communities by highlighting (1) the importance of facilitators' and school leaders' work and (2) the potential of building community through a principled system of professional learning organized around shared commitments.

Important questions are raised by our examination of this case. The development of the Forest community appeared fragile at least in part because it was at tension with normative U.S. expectations for teachers and leaders. We hypothesize that the sustainability of such a community requires decisions responsive to the department context (e.g., current district initiatives; needs of the students). How do leaders' design and facilitation decisions change in response to contextual changes?

A second issue concerns our finding that community at Forest appeared to be contingent at least in part on the work of a school leader who valued teachers' collaboration and created the necessary conditions to enact the principles named above. How do communities of mathematics teachers form in contexts in which the school leaders and facilitators do not start with shared vision for the work?

In this paper, we have focused on instructional leaders' planning and facilitation decisions in support of the community. In our future analyses, we plan to make sense of teachers' experiences of the system of professional learning and their goals in relation to the instructional leaders' goals for the community. Understanding the role and experiences of various community members is crucial in making sense of the development of an inquiry-oriented teacher community.

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THE BENEFITS AND CHALLENGES OF UNDERTAKING COLLABORATIVE WORK FOR THE PEDAGOGICAL RESIDENCY PROGRAM IN UFPA

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As is pointed out in the literature, collaboration between teachers and other members of the school community not only enhances the teacher's education, but also leads to better student learning. However, it is not always easy to work in partnership, because this requires constant evaluation and the ability to take account of different opinions. For this reason, this study involves an investigation of the benefits and challenges of collaborative work, carried out by the Faculty of Mathematics at the Federal University of Pará, within the scope of the Pedagogical Residency Program (CAPES, 06/2018), and under the supervision of a preceptor and three residents. Our methodology entailed conducting semi-structured interviews and the results of our analysis showed that the Pedagogical Residency was a means of providing new ideas for the teaching activities of the field schools. It also revealed that the different players involved had difficulty in understanding their roles, when dealing with a new project that was carried out in Brazil.

Introduction

It is not unexpected that, according to assessment schemes such as SAEB [National Basic Education Assessment], students in Brazil have difficulties in learning Mathematics[1]. At the same time, research indicates that teacher education still needs to be more advanced, whether with regard to mathematical content, pedagogical practices, or inclusion of people with disabilities, etc. (Giraldo, 2018).

Hence, teachers and researchers must seek alternative methods of overcoming the difficulties encountered in teaching mathematics in our schools. Among a number of suggestions, collaborative work has attracted a good deal of attention, as it combines improvements in students' learning, with a means of enhancing teacher training (Gideon, 2002, Ronfeldt, Farmer, McQueen & Grissom, 2015).

Within this context, the aim of this article is to discuss the benefits and challenges of the collaborative work carried out by the Pedagogical Residency Program (PRP) included in the Mathematics Degree course, at the Institute of Exact and Natural Sciences (ICEN), Federal University of Pará (UFPA), Belém-Pará-Brazil. For that, we report examples of some of the ativities that were developed and the improvement attained, as well as analyze the excerpts of the interviews with preceptors and residents, to evaluate the success of our attempts.

The Pedagogical Residence Program in the Bachelor's Degree course in Mathematics at UFPA

[1]Basic Education Assessment System (*in portuguese*): http://portal.inep.gov.br/educacao-basica/saeb

The collaboration project we are carrying out is a part of an institutional UFPA scheme that emerged from a declaration made by CAPES[2] (06/2018). This is called Pedagogical Residency and is the first national project of its kind. The objective of the project is to optimize the training of future Mathematics teachers and, at the same time, to cooperate with the field schools in establishing the new common ground that has been approved for Brazil. Our residential operations center is situated in 3 schools of the public network, in the city of Belém, Pará State (Amazonia) Brazil .

Our project group members are:

- 3 supervising teachers (counselors), who are university lecturers in Mathematics . One of them is a project fellow . One of the teachers does research in the area of applied mathematics, another in the area of pure mathematics and another in the area of mathematics education. All three run teacher - training courses at UFPA;
- 3 preceptors (facilitators), each of whom is a teacher in one of the schools in the project . All three are scholarship holders;
- 30 students (residents) doing the Mathematics Degree course at UFPA, 24 of whom are scholarship holders and six volunteers.

How does the collaborative process take place?

The Role of the Teaching Staff :

- The counselors form a link between the university and school administrators. They must discuss with the managers the role played by the preceptor and the residents within the school. It is imperative for the project to be carried out in a way that benefits the school community.
- They must have periodic meetings with with the preceptor of each school, so that they can provide the residents with clear and objective guidance, and also to allow the preceptor to report on the progress of the project. The counselors and preceptors should work together to assess the results of the different activities carried out by the residents, and recommend new tasks .
- They should incorporate periods of “socialization” for the residents of the three schools, to enable them to learn about the activities that have been undertaken. This should include addressing areas that may have given rise to problems, and discussing ways of tackling them. .
- They must encourage the residents to conduct diagnostic tests and learn how to interpret the data obtained from the activities carried out in school.
- They should offer guidance to residents about how to produce academic papers for publication.

The Role of the preceptors:

- Helping residents adaptation to the life of the school community Assisting the residents during the first phase of the project, which involves observing the school activities.
- Helping the students to conduct diagnostic tests.
- Offering guidance to the residents when planning and taking interventionary measures within the school.

[2] Coordination for the Improvement of Higher Education Personnel

- Evaluating the residents performance and the results they obtained. Informing counselors of any unusual situations involving trainees.

The Role of the Residents:

- Finding out how the school operates and integrating its activities.
- Establishing the kind of difficulties that the students of the school face when learning mathematics.
- Drawing up an interventionary plan to help the students overcome their difficulties.
- Carrying out strategic tasks and assessing the results.
- Writing about their experiences during the project.

Conducting collaborative work according to the literature

Although there is no formula or single way to carry out collaborative work, there seems to be no doubt that peer collaboration leads to improvements in a) teaching institutions, b) the training of the professionals involved and c) the goals they share (Gideon, 2002, Boavida & Ponte, 2002, Crames & Stivers, 2007, Ronfeldt et al., 2015). This is particularly the case of teachers who usually talk about everyday subjects, but are not in the habit of exchanging experiences about the classroom, which means that isolated work is common, sometimes under the pretext of respecting their right to autonomy (Gideon, 2002).

As Crames and Stivers (2007) argue, collaboration in the context of schools has long ceased to be based on a simple "ingredient", such as an insignificant factor that depends on subjective desires, which is understood as an essential feature in teaching. According to Ronfeldt et al. (2015), the more there is collaboration in the school, the greater the amount of student learning achieved.

From our tradition of working in a solitary way, it can be expected that we may feel insecure when working with someone else. As Crames and Stivers (2007, p.7) point out, "Fortunately, the skills needed for successful collaboration can be learned" that is, learning how to collaborate can be acquired through dialogue, negotiation and adjustment.

> By bringing together a wide range of people who can interact, enter into a dialog and reflect together, synergies are created that allow for an increased capacity for reflection and an increase in the opportunities for mutual learning. This allows teachers to go much further and create better conditions for their students to successfully face any uncertainties and obstacles that might arise (Boavida & Ponte, 2002, p. 45).

It is thus the sharing of learning, exchange of experiences, communication, negotiation and dialogue, which are the collaborative activities that are carried out within the scope of Pedagogical Residency. These can enable us to investigate the benefits of collaborative work in what concerns the training of the teacher (preceptor), future teachers (residents) and students learning.

Activities developed and improvement in student performance

Here we would like to present two examples of activities developed in collaboration within the Pedagogical Residency, in two different schools, pointing preliminary results regarding the students' performance and the relevance of the tasks performed.

In our first example, in school A, activities were proposed to retake the mathematical contents of Basic Education (addition, subtraction, multiplication, division, potentiation, rooting, numerical expressions, fractions, rule of three and fundamental geometric concepts). As is well known, it is not uncommon for high school students to have difficulties with basic mathematical concepts, which compromises their performance in the following school years. This activity was performed by the residents, under the guidance of the supervising school teachers.

In this sense, a diagnostic evaluation was initially performed through a test in order to investigate the main difficulties faced by these students. During the months of April and May of 2019, the 120 students of the first year of high school of this school received classes with diversified methodologies, involving lectures and dialogues, the use of games and mathematical objects, working the mathematical contents mentioned. There were a total of eight lessons, each 45 minutes long, taught at four different opportunities.

Then, a new test was applied, pointing the progress regarding students' performance, as follows: in the first test, 19.16% of the students obtained a minimum score (zero). In the following test, the minimum grade was 2.0. Regarding the maximum grade, there was an increase: from 8.75 to 9.5. Considering the simple arithmetic mean obtained in both applications, the values are 3.06 (first application) and 5.95 (second application), an increase of more than 94%.

As it turns out, the work of the residents brought a significant advance regarding student performance, which should not be limited to the performance on the applied test.

In our second example, at school B, activities were conducted with a mobile app to improve students' skills in solving equations in a 9th grade elementary class with 26 students. Again, the task was performed by residents under the supervision of the preceptor and the mentoring teachers. This development was due to the difficulties presented by the learners in the mentioned content, as well as the observation that they used their smartphones in school, but not in school activities.

Initially a diagnostic evaluation with five questions was performed, pointing out the great difficulty of the students, despite the fact that they had already done several activities considered conventional, of exercise resolution. Then, through the MIT App Inventor 2 platform, an application for solving the 2nd degree equations was created, which was called the 2nd degree calculator, to assist students in their studies. This platform addresses the programming language through logic by assembling blocks that fit together, resulting in the commands that will determine the actions of the application.

The application was made available to students, who used it during the following fourteen classes, taught by their teacher. After this development, a new test with five questions was applied to the students in order to assess their progress, with an average growth of 58.8% in grades. Thus, this activity served as a positive experience, emphasizing how important it is to intervene in order to use technological means such as software and applications, and to define a good way to use the smartphone inside the classroom.

The results achieved in the 2 mencioned examples show the relevance of collaborative work in the Pedagogical Residency Program, in view of the exchange of experiences of the various actors. As Redmon, Brown and Sheelhy emphasize:

> Learning about the teaching of mathematics occurs most productively when the professional audience is diverse and includes both local community members of teachers and others, such as

> university academics, whose taken for granted perspectives suggest novel ways of 'seeing' and interpreting the local practices. (Redmond, Brown, & Sheehy, 2011, p. 655).

As we will see below, in the statements of the interviewed residents and preceptor, both enrich their training with shared experiences: residents bring suggestions for new teaching methods and tools, while the preceptor collaborates with their teaching experience, culminating in improved school performance, regarding the learning of mathematics by their students.

Methodology and research tools

The research was qualitative (Gil, 2008) and included semi-structured interviews, which were conducted as follows: i) with one of the preceptors, called Jocimar - who was enthusiastic about making a contribution to the research project and appeared to be very much at ease when expressing his opinions - and ii) with three residents, called Renan, Silvano and Yuri, who assisted this preceptor. The initial draft for the interviews contained the same questions, and was designed to reflect the attitudes to the Pedagogical Residency Program from different standpoints.

The significance of the Pedagogical Residency Program from the standpoint of the preceptor and residents

Here are some excerpts from the interviews that we believe are of value in achieving our objective, together with a brief analysis conducted by the authors of the importance of collaborative work.

In the first question, on the significance of the Pedagogical Residency, the opinions of the residents are given in the following excerpts:

Interviewer:	In your opinion, what is the importance of the Pedagogical Residency Program?
Yuri:	The PRP is a different experience from the others we have inside the University, because it gives us an experience of the school. In the other programs, even though there were practical subjects (like workshops), they did not prepare us as well as the PR. When we had to go to the classroom and do activities...we obtained useful experience there
Renan:	PRP broadened the scope of the Internship, by giving us useful knowledge and experience.
Silvano:	In PRP we were able to be active players, not just sit and watch. We trained to teach classes, having feedback from the students and the school teachers.

About this question, the residents highlight the importance of PRP as a way of experiencing teaching practice in a supervised and collaborative way, unlike having theoretical discussions in the classroom. It also upgraded de supervised internship, having the opportunity to actively participate. They improved their teacher-training through an exchange of experiences with school teachers.

In the second question, about the collaborative process between the different actors, we have included some excerpts from the responses of the preceptor and one of the residents:

Interviewer:	How do the different "actors" of this project (the preceptor, resident and guiding teacher) influence this collaboration?
Jocimar:	The resident has this exchange of experiences with the school, as well as introducing the ideas of the university and we end up with an awareness of our reality [...]. This helps in improving the school and also his own view of education that sometimes … as I always tell them, what we see in university in theoretical terms is one thing, it is

different when we come to practice; we have to adjust our ideas [...]. This exchange is really useful for the resident because he is getting ready for challenging situations.

Renan: At first, we had problems to understand the difference between the Internship and the PRP. Maybe we should have more meetings with the counselors, because sometimes the meetings with the preceptor included issues that nobody knew about, and we were in a state of uncertainty.

As the preceptor stated, the exchange of experiences was the main influence exerted by the various actors who took part. The emphasis was on the collaboration between the residents and the school teachers, and how this enhanced the views of each collaborator with regard to education and the teaching of Mathematics. As Crames and Sivers (2007, p.10) explain, "Collaborative relationships can be a rich source of professional and personal growth, well worth the investment of time and effort that may be necessary to nurture them" by putting into practice the principles and methods that are taught in pedagogical theories.

However, the residents reported a problem regarding the lack of information, which they say could have been overcome by having more meetings with all the participants. Because of the innovative character of the PRP, some information requested by the residents, in fact, caused uncertainty among the whole group. This obstacle was also pointed out by the preceptor Jocimar and will be discussed here.

As Boavida and Ponte (2002) point out, the importance of dialogue and negotiation is becoming more prominent in collaborative activities. When entering into a dialogue that involves a clash of ideas, it is no longer a mere instrument for reaching a consensus and becomes a factor in increasing one´s understanding of a particular theme or even opening up a new understanding. At the same time, negotiating meanings, objectives, priorities, or working methods, etc. is of crucial importance , given the expectations of each participant. For this reason, we asked the interviewed, the following questions:

Interviewer: What is the role of dialogue and negotiation for decision-making in collaborative work? How has this been happening in the Pedagogical Residency project ?

Jocimar: We try to see what is most feasible for the school, and then adapt it to the needs of the residents [...], so we always talk to the students and try to match their needs with ours.

Renan: The preceptor was open to dialogue and negotiation; We were able to choose between activities that better adjusted to our schedule and timetable. At the beginning, we collaborated with several mathematics teachers, but after some time each of us chose one teacher to work with. Regarding the school activities, we discussed some ideas with the preceptor, (i.e. each person expressing his/her own ideas), until we decided which projects would be implemented. Then each resident chose the project he/she would take part in.

The preceptor and resident emphasized that the requirement that each part involved should be listened when taking decisions about the project. However, this meant aiming at an overlapping goal, which was to meet the needs of the school, while improving the effective learning of Mathematics. Gideon (2002, p. 34) states that different points of view should be heard to ensure a successful collaboration: "successful collaboration requires all teachers to be heard and that administrators must be willing to accept differing viewpoints". This means that the participants must be open-minded when making decisions together.

When assessing how collaborative work influences classroom teacher practice, we asked them to comment on the following question:

Interviewer: Does this collaborative work mean that the learning you acquire influences your pedagogical practice? Please give a reply.

Jocimar: Yes. In what sense? When I'm advicing the residents [...] I try to understand the approach thy are adopting [by delivering some theoretical content or solving a problem] and I explain to them whether the approach they're adopting is proper […]. When we are working, we cannot ask a question like "if I teach this way, is it valid?", Sometimes the resident teaches in an easier way, and this leads to an exchange of learning. Sometimes we realize of some mistakes we make, when we see someone else making the same mistake.

Jocimar says that collaboration with residents not only enhances their methods, but also makes him recognize his shortcomings. When he is in the classroom as the lecturer, he is not always able to see the weaknesses of his own methodology, which he noted as an observer, when correcting the residents' activities. This assertion can be better understood by what Olson (1977, p. 25, apud Boavida & Ponte, 2002, p. 50) says:

> Each one will come with their own goals, purposes,needs, understandings and through the sharing process, each one will depart having learned from the other. Each one will learn more about himself, more about the other, and more about the topic at hand.

Thus it is clear that Jocimar understands more about himself when collaborating with others and, in reflecting on his own practice, he is able to adjust it to suit the interests of his students' learning. Hence, it is not surprising to find it pointed out in the literature that collaborative work, by enhancing teacher training, also improves student learning (Gideon, 2002, Crames & Stivers, 2007, Ronfeldt et al, 2015).

While several good points emerge as a result of collaborative work, this does not mean that there are no further difficulties and challenges to overcome. At the beginning, collaborative work requires constant evaluation in order to make the adjustments that are needed. For this reason, we asked the interviewed:

Interviewer: Which obstacles did you have to overcome during this project?

Jocimar: The main obstacle that we had was finding out the meaning of Pedagogical Residency. Anything else? At the beginning, there was not enough information about how to decide what was right and what was wrong, which meant that each person had his own interpretation [...]. At school [...], we sometimes covered holes [...], because the school interpreted the Residency as a pinhole.

When embarking on a collaborative endeavor, it is natural that there will be uncertainty, doubts and different understandings about the objectives, the way of proceeding and the role of each participant involved. For this reason, Boavida and Ponte (2002) emphasize that collaboration is characterized by unpredictability, since not every detail can be decided in advance. Hence the different directions and roles must be adjusted, in light of the importance of the negotiations, in this case, specifically, with regard to the expectations of the school and the expectations of the residents and preceptor.

Conclusion

As can be seen, collaborative work within the Pedagogical Residency Program has increased the learning of the preceptors and residents, with regard to their activities and experiences. The preceptor/tutor has had experience of teaching Mathematics classes for many years, while the students can collaborate by making suggestions for innovative activities, resulting from their learning in the degree course. This collaboration suggests that the learning of field school students can also benefit from enhanced learning.

At the same time, the analysis conducted shows that the innovation brought about by the RPP causes doubts, uncertainty and the need for adjustments and negotiation to enable the good work to progress. Since it is an innovative project, it is expected that these difficulties will appear, and hence everyone will be required to listen to each other, be flexible and learn how to deal with the unpredictable.

Finally, in addition to conducting research on the students' learning in the field school, it is necessary to investigate its benefits to the teaching practice of the advisors, as well as, more broadly, the school communities of the three field schools.

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ROLE OF FACILITATORS IN SUPPORTING TEACHER COLLABORATION DURING PD COURSES ON LANGUAGE-RESPONSIVE MATHEMATICS TEACHING

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Facilitators play a crucial role in initiating and maintaining teacher collaboration in formal and informal professional development (PD) settings. Although facilitators recently attracted a growing research interest, more empirical studies on their role and professionalization processes in teacher collaborative settings are needed. The presented case study investigates how (and based on which rationale) two facilitators support teacher collaboration within their PD course on language-responsive mathematics teaching, and how they relate the discussion to the objects of learning. The qualitative analysis focuses on the enacted facilitation moves after eliciting presentations of group work. It investigates to which principle of language-responsive teaching they refer and the effects for teachers' collective learning. The first results show the high relevance of facilitators' content-related framing of teachers' contributions in order to strengthen and connect fruitful ideas, also as basis of their further work at school.

For establishing new teaching approaches (e. g. language responsive mathematics teaching) PD courses are widespread. As teachers' collaborative work is a crucial way to bring educational innovation into the everyday practice of teaching, it is highly relevant that the facilitators support teachers working and learning in collaboration also in these formal PD settings (Jaworski et al., 2017). Even if teacher collaboration heavily depends on the self-regulation of their participating teachers, the facilitators can play a substantial role in supporting it. However, little is known so far about the role of facilitators in supporting teacher collaboration during PD courses and about key elements of a successful support with respect to the PD content as objects of teachers' collective learning. In order to reduce this research gap, this contribution presents a case study of two facilitators which investigates their role by supporting teacher collaboration during their PD course on language-responsive mathematics teaching. For the analysis an existing content-independent framework for identifying the enacted facilitation moves (van Es et al., 2014) is used and the facilitation moves are specified with respect to the addressed principles of language-responsive mathematics teaching (Prediger, 2019). Thereby, the case study contributes to responding the following rather broader research question*: Which role can facilitators play in supporting teacher collaboration during PD courses*?

Theoretical and empirical background

Faciltators' role in supporting teacher collaboration during PD courses

There is a wide consensus that facilitators play a crucial role in PD courses (e. g. in supporting teachers' learning and development in collaborative work) and are to a large extent responsible for their quality. Due to their importance, their role has gained an increasing attention of mathematics education researcher within the last decades and has begun to be taken closer into account (e. g. Borko et al., 2011; Rösken-Winter et al., 2015).

The related research thereby often focusses on the design of facilitators' preparation programs (e.g. Kuzle & Biehler, 2015; Rösken-Winter et al., 2015; Karsenty, 2016; Borko et al., 2014) and their required skills and knowledge (Borko et al., 2014; Elliott et al., 2009; Lesseig et al., 2017). Recently, the question of how facilitators support teacher learning has been raised and primarily answered by the description of *facilitation moves* (van Es et al., 2014; Tekkumru-Kisa & Stein, 2017; Coles, 2019).

In the framework of van Es, Tunney, Goldsmith and Seago (2014) the identified facilitation moves have been categorized independently from the PD content and subsumed under four categories of central facilitation practices, whereby one practice refers directly to the *support of group collaboration*. This category of supporting group collaboration includes three different facilitation moves: *Standing back*, *distributing participation* and *validating participant ideas* (see Table 1 for definitions (adapted from van Es et al., 2014) and examples from our case study).

Table 1: Facilitations moves for supporting group collaboration

Facilitation move	Definition	Example
Standing back	Allow the group members time to discuss an issue and pursue ideas together	Not interjecting when the group is exploring an idea
Distributing participation	Invite participants to share different ideas based on who is (and is not) participating as a way to include all members in the conversation	"Do you have any contributions to that [*presented group work product*]?"
Validating participant ideas	Confirm and support participant contributions	"What you made there is great, because you think already about many things."

Nevertheless, this young field of research on the role of facilitators has not yet illuminated the ways facilitators support teacher collaboration, especially with respect to specific PD contents (Prediger, Rösken-Winter & Leuders, 2019). The relation to a certain PD content (like language-responsive mathematics teaching in this paper) makes it possible to shift the focus from the question of "How do facilitators support teacher learning?" (van Es. et al., 2014) to "By what do facilitators support teacher learning?". This could be for example helpful for explaining effects of the PD for teachers' collective learning.

Promoting teachers' expertise in language-responsive mathematics teaching

As language proficiency turned out to be very relevant for achieving mathematical conceptual understanding (Secada, 1992; Haag, 2013; Prediger et al., 2018), the need for language-responsive mathematics teaching is widely accepted. In order to be able to realize it in a way that supports mathematics and language learning, the teachers must be aware (inter alia) of the following important principles of language-responsive mathematics teaching (slightly adapted from Prediger, 2019):

- **Pushing rather than reducing language** (P-A): Teachers should not be concentrated on reducing language demands in tasks or word problems, because developing language proficiency requires pushing language production with language demands in the zone of their proximal development (neither over- nor underchallenged).

- **Focus on discourse level, not only on word level** (P-B): Teachers should not only do vocabulary work (in the sense of clarifyiing unknown words or providing word lists), instead they also should foster discourse practices (like explaining meanings or arguing).
- **Conceptual understanding before procedures** (P-C): Fostering conceptual understanding is crucial for language learners' mathematics learning. For realizing this principle in language-responsive mathematics teaching it is important to differentiate between conceptual and procedural knowledge, the related discourse practices of reporting procedures (e. g. how you expand fractions) and explaining meanings (e.g. what expanding fractions means) and between meaning-related (e. g. finer structured) and formal vocabulary (e. g. to expand) .
- **Integrative rather than additive language learning** (P-D): Teachers should perceive language learning not as a additum, but understand language-responsive mathematics teaching as connection of mathematical content goals and language learning. Therefore it is crucial to be aware of links between content goals, discourse practices and lexical means.

A case study shows that at the beginning of an PD course, teachers rarely take the last principle of integrative rather than additive language learning sufficiently into account. For example the teachers often don't link their vocabulary support to the mathematical goal of supporting the consolidation of conceptual understanding (Prediger, 2019). As the aim of the PD course focused here is to strengthen teacher communities to teach mathematics language-responsively and to share lesson planning with colleagues, it is interesting to investigate whether and how the facilitators refer to the four principles of language-responsive mathematics teaching by supporting teacher collaboration with regard to simulations of the practical collaborative work at school. Hence, the research question can be specified to the following: *By which moves do facilitators support teachers' collaboration and which principles of language-responsive mathematics teaching do they address or not address?*

Research context and methodological framework

The research is embedded in a bigger project on facilitators, in which researchers from Israel (Ronnie Karsenty, Abraham Arcavi and Gil Schwartz from the Weizmann Institute of Science) and Germany (Susanne Prediger and Birte Pöhler from the TU Dortmund University) investagate the practices of facilitators in two different PD programs. The case study presented here stems from the German PD program focusing on the PD content of language-responsive mathematics teaching (Prediger, 2019).

The PD program aims at engaging teachers in professional learning communities around this (for Germany innovative) PD content. Schools often book the PD courses for (some of their) mathematics teacher teams with the aim that they are afterwards able to teach mathematics language-responsively and share their lesson plannings with other teacher colleagues. Therefore, facilitators engage them in collaborative activities (such as video analysis, analyzing student's products or task design), The PD courses usually take four hours and are conducted by two facilitators.

Methods of data collection and analysis

The video data corpus of the case study currently consists of fourteen videotaped PD sessions conducted by n = 5 facilitators in changing pairs and video-based post-PD reflection sessions between the researchers and the facilitators (in total about 70 hours of video material). For the presented case below, episodes of the first PD course of Fred and John are selected, in which they support teacher collaboration. The focused facilitators are both new in conducting PD courses on language-responsive mathematics teaching and have both much experience as teachers at school and as facilitators for other mathematics PD contents. Additionally Fred has experience as researcher and teacher educator in university.

The qualitative data analysis followed deductive category-led analytic procedures (Mayring, 2015) and consists of four steps: (1) Identifying and transcribing relevant sequences of supporting teacher collaboration (in this concrete case of initiating and accompanying an group work and managing the presentation of the emerging products); (2) Classifying the enacted facilitations moves especially on the practice of supporting group collaboration according to the framework of van Es et al. (2014) (see Table 1 above for definitions of the facilitation moves and examples from our data); (3) Interpreting and classifying the addressed principles on language-responsive mathematics teaching as P-A to P-D (see above the list of important orientations); (4) Taking teachers' feedback on the PD session into account.

Empirical insights into a case study of supporting teacher collaboration during group work presentations

The first case presented here stems from the first PD session of Fred and John on the introduction of language-responsive mathematics teaching. Relatively at the end of this session Fred introduce a teacher activity with three steps: Based on the topics and pedagogical tools they worked out before in the PD session, the teachers have (for their own mathematics classrooms in the next weeks) to (1) choose tasks, e. g. from textbooks, (2) modify the task to include conceptual understanding, and (3) provide written vocabulary scaffolds so that students can cope with the task. Activity (3) is most central to the PD content. During his introduction, Fred describes how an adequate language support could look like:

> 1-Fred: Not only support […] in the sense of the how [language which helps to describe procedures]: Not only separate words of the technical vocabulary, but also to think meaning-relatedly [language which helps to express the meaning of mathematical concepts]. […] I don't know how you work together [at school]. If you work in parallel [on the same topic in the same school years] in some ways. Otherwise it could be nice, if you do it now. The idea of this activity is really that you produce something what you can try next week.

In his initiation of teachers' collaboration, Fred addresses the principles of *conceptual understanding before procedures* (P-C from the list in the first section) and *focus on discourse level, not only on word level* (P-B) as he emphasizes the importance of taking into account also meaning-related lexical means and of not only using words as language support. He explicitly encourages teacher collaboration during the group work ("I don't know how you work together… Otherwise it could be nice if you do it now."). By inviting the teachers to share their ideas for modifying a language-responsive task, he applies the facilitation move of *distributing participations* (van Es et al., 2014).

After Fred's introduction, the teachers work in groups composed by topic interests or teaching school years for 20 minutes. During this group work, we identified Fred's facilitation moves of *standing back* (as Fred and John gives the teachers time to discuss an issue of practical relevance (for them) in groups) and *validating participant ideas* (as they support the work of several groups situationally during their discussions).

During the subsequent presentations of the group work products John and Fred invite the other participating teachers to comment on the presentations and they try to connect ideas. In the following analysis, we investigate which facilitation moves they use und to which principles of language-responsive mathematics teaching they refer.

The first group presents their tasks on volume of different three-dimensional shapes that ask student to explain for which of the pictured three-dimensional shapes the formula $V = base\ area \cdot height$ applies. The teacher group scaffolds students mathematical thinking and articulation by (a) suggesting first to identify differences and similarities between the pictured three-dimensional shapes (some have identical polygon ends and some not), (b) graphical hints (3D shapes with colored base areas) and (c) some vocabulary scaffolds (like same end shape). After their presentation, John invites the other teachers to comment: “Do you have any contributions to that?” (Turn 8). This could be categorized as facilitation move of *distributing participation*. When teachers do not react, John makes the following comment:

9-John: […] I really like that very much, what you have said [*about*] the explanation for why you can multiply the base area with the height in a prism. As support you can also take a cuboid which is filled with cubes, for example.

John strengthens one idea of the presenting group with regard to fostering the conceptual understanding of measuring volumes and adds another way of for visualizing it for students, this facilitation move can be categorized as *validating participants' ideas*. He thereby addresses the principle of fostering *conceptual understanding before* teaching *procedures* (P-C) and focusses on reaching the content goal of building conceptual knowledge, while neglecting related discourse practices or lexical means.

The mathematical content of the second teacher group is probability. The group selects and reformulates a task asking whether a situation (A thief has six keys which appear to be the same. Only one key fits. He tries one after the other and remembers the tried ones.) can be simulated better by throwing a die six times or by putting balls in a ballot box and taking them out one after the other. The group’s language support consists of graphical representations (a die and a ballot box) and meaning-related (“trying no key more than once”) and formal vocabulary (“with and without replacement”).

As neither the teachers nor John comment the presentation of the second group, Fred take the opportunity to make the following contribution:

12-Fred: […] In my opinion that what you made there is great, because you think already about many things. We discussed that when I joined your group work: Putting keys away […] or I use a key two times or not two times. That for you it is a term for with and without replacement. And this is a meaning-related language, which you use again and again.

By confirming the diverse group’s ideas (“what you made there is great, because you think already about many things”) and framing one idea (using terms as “putting keys away” as language support) by the related theoretical background (categorizing the terms as meaning-related vocabulary), Fred applies the facilitation move of *validating participants' ideas*. Thereby he addresses the relevance of *conceptual understanding* (P-C) and focusses on linking meaning-related and formal vocabulary.

The third group works on a statistical task on understanding and calculating the mean (“The average temperature during the past three days was 17° C. What could be the temperatures on the three days? Give two examples and compare them with your partner.“)

The group adds two subtasks, (b) asking for describing and comparing the solution strategies and (c) explaining how to identify all solutions (c). As language support, the group provides sentence starters (like "First I have…" or "I search for the middle by…") and words (like "adding" or "balancing"). John's first move in response to the group presentation can be identified as *distributing participation,* by asking for the rationale behind the (new) task structure:

14-John: Can you even tell me why you integrate this progression [*increasing difficulty*]? […]

After two teachers respond John's question (with a focus on organizational aspects) he commends their task modification (move of *confirming participants' ideas*) and values its suitability for differentiated instruction with subtasks of increasing difficulty.

Only at the end of the discussion, John focusses on the discourse practices and tries to figure out why the group integrates the discourse practice of describing instead of explaining into the task.

25-John: I joined shortly your group work. We thought about the usefulness of deleting "explaining" and replacing then by "describing".

26-T6: We thought that "describing" is a bit easier than "explaining".

27-John: But don't we want to have a correspondingly high level for leading the learners to more language skills?

28-T5: Yes, but we broke down the step for reaching the highest [*explaining*] a bit and made smaller steps so that they have the chance to understand the mathematical content.

29-John: That actually makes sense here, for having then a higher level in subtask c).

Besides the facilitation move of *confirming participants' ideas* (in Turn 29), the analysis reveals that John's contribution (in Turn 27) addresses the principle of *pushing instead of reducing language* (P-A) by making transparent why the teachers integrated the discourse practice of describing before striving for explaining in the second subtask.

The analysis of the enacted facilitations moves in the reported episode reveals that the facilitators often confirm (Turns 9, 12, 29) and support (Turns 9, 12, 22) participants' contributions (facilitation move of *validating participants' ideas*) during the group work presentations. The facilitation move of *distributing participations* mainly occurs while introducing the group work (Turn 1), but only occasionally while discussion the group presentations (Turns 8, 14, 25, 27).

At the end of the PD session, the participating teachers are invited to give a feedback. Besides positive comments on atmospheric and organizational aspects, several teachers express that the PD was a confirmation of the practices they already enact and that it brought some aspects back to their attention. No teacher explicitly refers to the newly offered principle of language-responsive mathematics teaching.

For explaining why the teachers perceive the PD session in particular as a confirmation of their previous practices, it is useful to analyze the principles of language-responsive mathematics teaching which the facilitators addressed within their comments on the group work presentations. This analysis shows that Fred and John indeed address several principles (P-A, P-B and P-C) more or less explicitly in their first PD session. But thereby especially John seems to support teacher collaboration with regard to reach the content goals (e. g. fostering conceptual understanding; Turn 9 and 22) rather than fostering them to find connections between content goals, discourse practices and lexical means. This observation is on the one hand also reflected by the fact that he (and also Fred) doesn't address the fourth principle of language-responsive mathematics teaching explicitly (P-D), which emphasize the relevance of *integrative rather than additive language learning*. On the other hand it resonates with a teacher's comment to her neighbor after John provides an alternative for fostering the conceptual understanding of volumes of three-dimensional shapes (Turn 9): "That is then more a mathematical didactial support. But here it was a matter of lexical means."

Conclusion and Outlook

What can we learn about the role of facilitators in supporting teacher collaboration in and after PD sessions from this case study with its limitations in sample size, scope and topic?

As the facilitation move of *validating participants ideas* occur frequently, it seems that confirming and supporting participants ideas is common and probably useful. Nevertheless, the teachers' feedback after the PD session revealed that the teacher perceived the PD particularly as a confirmation of the practices they already enact. In order to analyze the source of this limited perception, it was necessary to analyze the facilitation with respect to the foregrounded PD contents.

Therefore the analysis of topic-specifically formulated principles (in this paper the principles of language-responsive mathematics teaching of Prediger, 2019) seems to be a promising analytical tool for capturing facilitators' practices, even if the adressed principles often occur in combination and aren't always explicit. The case shows that the facilitators addresses different principles (P-A, P-B and P-C). Whereas a tendency to focus on the principle of *conceptual understanding before procedures* (P-C) could be identified, the facilitators did not address the very important principle of *integrative rather additive language learning* (P-D). Given earlier empirical results that teachers rarely take this relevant principle at the beginning of a PD course sufficiently into account (Prediger, 2019), it should be necessary that the facilitators make explicit how conceptual understanding is linked to language learning and other orientations.

The case presented in this paper can be contrast with a second case study of the same facilitator (Fred), in which the facilitators and the participating teachers (in their feedback) address the missing principle (P-D). In this PD, teachers' feedback also addressed language-related issues as a crucial learning content. For this contrasting case, the chance that the teachers' professional learning communities continue working on language is substantially higher.

Zooming out from the case studies to a more general level, the paper could modestly contribute to the successive and empirical grounded development of a research framework for understanding facilitation practices. For understanding restrictions of teachers learning uptake, the case study shows that identifying facilitation moves can be insufficient to capture the role of facilitators of focusing the attention to a specific PD content while supporting teacher collaboration. Besides identifying addressed content-related principles, which was focused here, also a interpretation of underlying goals, knowledge aspects and identity aspects (e. g. with regard to their dual role) could be fruitful (e. g. Prediger & Pöhler, 2019).

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DEVELOPING COLLABORATIVE RELATIONSHIPS IN LESSON STUDY GROUPS

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This paper aims to discuss how collaborative relationships may develop in lesson study groups. We pay attention to the critical incidents that may arise in the development of a lesson study. This qualitative work is based on two lesson studies, one with primary and the other with middle school teachers. Data were collected through a researchers' journal, audio records of working sessions, video records of research lessons and post lesson reflections. Individual interviews were also made to participant teachers after the research lesson. In the two cases, the collaborative relationships developed as common goals and ways of productive joint work emerged. Several critical incidents had a strong influence, positive and negative, in the development of these relationships. As lesson study is a bound activity, these relationships are ephemeral unless the participants establish new common goals and new common working processes.

Collaboration is a way of work often recommended to educational professionals to deal with the problems that they face in their practice and in school life and to improve the quality of teaching and learning (Boavida & Ponte, 2002; Hargreaves, 1994). However, collaboration is often regarded in a simplistic way, with no attention to its associated complexities and uncertainties. Recognizing the need of collaboration among teachers to deal with complex educational problems, we must understand how it may develop and what difficulties it may face, so that its potential is fulfilled.

Lesson study is a professional development process with collaboration as a main feature (Lewis, 2016; Takahashi & McDougal, 2016; Wood & Cajkler, 2018). However, the way collaboration takes place in a lesson study seldom is analyzed in detail. How do the participant teachers establish such collaborative relationships? What is the depth that such relationships may attain in the frame of a process of limited duration as a lesson study? What difficulties and problems arise in developing collaborative relationships in a lesson study? With these questions as starting points, the aim of this paper is to contribute to knowledge about how are constituted collaborative relationships in a lesson study and what problems may emerge in their development.

Issues in collaborative processes

The educational literature offers several quite distinct definitions of collaboration. For example, Vangrieken et al. (2015) define collaboration "as joint interaction in the group in all activities that are needed to perform a shared task" (p. 23). A more precise definition, put forward in the frame of collaboration among organizations, is provided by Wood and Gray (1991), stating that collaboration "occurs when a group of autonomous stakeholders of a problem domain engage in an interactive process, using shared rules, norms and structures, to act or decide on issues related to that domain" (p. 146). Several aspects are usually regarded as important in collaborative processes, including (i) the active involvement and effort of participants, mobilizing specific knowledge and skills, (ii) the setting up of a structure of work in which the different participants have different and balanced roles concurring to the common aim, and (iii) the establishment of caring, open and trustful relationships

(Achinstein, 2002; Boavida & Ponte, 2002; Vangrieken et al., 2015). In this paper, we consider that a collaborative process unfolds from the identification of a set of aims assumed as common for the participants and also from adopting working processes agreed upon by all group members. In addition, both the common aims as the working processes must be freely assumed by the participants and not imposed by an external entity. As Dooner, Mandzuk and Clifton (2008) indicate, means and aims are critical aspects of collaboration and tend to evolve as this process unfolds. In the view of these authors, from diverse aims, common means emerge, leading to common aims, and finally to diverse means.

Vangrieken et al. (2015) also suggest that collaboration is a dynamic and multifaceted process and that different types of collaboration may develop with varying depth. In their perspective, collaboration is a general term admitting a variety of possibilities. To overcome the lack of precision of the notion of collaboration, Cohen and Bailey (1997) define a team as “a collection of individuals who are interdependent in their tasks, who share responsibility for outcomes, who see themselves and who are seen by others as an intact social entity embedded in one or more larger social systems (for example, business unit or the corporation), and who manage their relationships across organizational boundaries” (p. 241). This leads Vangrieken et al. (2015) to consider a continuum of forms of collaboration which span from “a mere aggregate of individuals to high levels of collaboration in teams” (p. 26) and to designate “the degree to which a collection of individuals possesses the quality of being a team” (p. 26) by “team entitativity”.

Achinstein (2002), assuming a micro political perspective, considers that conflict constitutes an inherent part of collaborative groups. In her view, the forms and outcomes of organizational learning are related to the way how these groups deal with their differences and conflicts: “Communities that can productively engage in conflict, rather than those with low levels of conflict or those that suppress their differences, have a greater potential for continual growth and renewal” (p. 448). She considers that a strong challenge for professional groups is to find a balance between handling controversies that emerge from the participants’ different opinions and beliefs and maintaining strong interpersonal ties and connectedness proper of a caring community. In this way, collaboration does not exclude conflict. On the contrary, intensive collaboration depends on the existence of some conflict, that is managed so that it remains constructive and generator of new ideas for the members of the group.

Research Methodology

This qualitative work is based on two lesson studies. The first case that we analyze is based on a lesson study with primary school teachers teaching grade 3. Initially, the group had more teachers but from some point on it was carried out with just three teachers. In this case, the collaborative group is constituted by the three teachers and a teacher educator (the author of this paper). One of these teachers (Irina) (all names are pseudonyms) had a strong professional knowledge in mathematics and mathematics teaching and self-confidence as teacher of this subject, whereas the other two teachers (Antónia and Manuela) had a much weaker professional knowledge and were quite insecure regarding mathematics teaching. The second case refers to a lesson study with five middle school teachers, teaching grades 5 and 6, and the collaborative group is made by these teachers and the same teacher educator that led the whole process. From the five teachers, three (Maria, Isabel, and Francisca) were tenured in the school and two (Luísa and Teresa) had annual contracts. In both cases a second teacher educator participated in several sessions, moving in and out of the group. The role of the teacher educators was to set the agenda for the sessions, propose activities for the group to undertake, and make suggestions regarding the division of labor.

Data were collected through a research journal, audio record of working sessions and video record of the research lesson and post-lesson discussions. In addition, individual interviews were made to the participant teachers right after the research lessons. Data from sessions (S*x*) and Interviews (I) was analyzed seeking critical incidents (Estrela & Estrela, 1994) as situations or events that stand out by their features that make them critical, distinct and relevant for the understanding of a given phenomenon or process. The analysis considers what were the aims and working processes during the lesson studies, the critical incidents that influenced these processes in a positive or negative way, and the collaborative relationships that developed during the process.

A Lesson Study with Primary Teachers

Aims and Working Processes

This lesson study developed framed by two distinct but interrelated goals. In a first phase of the work, the goal of the group was to plan and teach a research lesson on addition of rational numbers. In a second phase, the goal was to prepare and carry out new lessons about other topics related to rational numbers. Underlying these goals was the teachers' interest in deepening their mathematical and didactical preparation and in getting to know how better teach these topics taking into account a new curriculum that was being introduced. The teachers might also be interested in corresponding to the invitation from the school principal to participate in this professional development process as well as in doing a certified teacher education process. The goals were proposed in both phases by the teacher educator and discussed by all group, that reshaped and detailed some aspects.

The first phase of the work included a study about the teaching of rational numbers, a survey of curriculum materials, and an analysis of students' difficulties. In this work the proposals were mostly made by the teacher educator. There was a strong involvement of Irina, the teacher that had a good relationship with mathematics teaching and there was very little involvement of Antónia and Manuela, the two teachers who were not confident regarding it. These two teachers seemed quite lost in the working processes used in planning the research lesson. The second phase took place after the post-lesson reflection, during a follow-up in joint work, with work towards a new common goal, shared decisions, and an effective division of labor. In this second phase, all teachers new the working processes and what was expected to do, given their experience in the first phase. That was apparent in the way the teachers got involved in selecting and adapting tasks for the new lesson.

However, Antónia and Manuela begun their planning quite insecure. They opened the textbook and began scanning it. They seemed uncomfortable with the perspective of selecting tasks, perhaps thinking that would not be regarded as appropriate. Taking into account the struggle of the teachers, the teacher educator suggested that, instead of just selecting a task, they could adapt it. Manuela liked this idea: "I think so, increasing the difficulty, isn't it? Because that one is very basic. But I think yes, mixing up tenths, hundredths and thousandths, with different denominators" (S10). At that point, Antónia and Irina also began giving further suggestions to elaborate the task (Figure 1):

Antónia: … Or A, B and C, in order to have two equivalent and one different . . . For example, to have two equivalent fractions. They understand that . . . [those] are equivalent fractions, albeit having different denominators.

Manuela:	Why we do not give a hypothesis here… That is, why do not give equivalent fractions?
Marisa:	Ah, one of these being as a fraction. Yes, instead of all being decimals . . .
Manuela:	For example, here, there are four as decimals, isn't? And one in words. Why we do not take out one that is as decimal and put it as fraction?
Irina:	Exactly.
Marisa:	May be.
Irina:	And then, during the discussion, we can ask them to write this also as a fraction. (S10)

1. In the grid, you should paint 0.4 green; 40/100 blue and four hundredths of yellow.

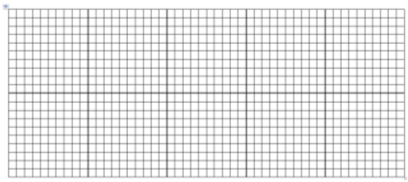

Figure 1: Task prepared by the teachers in session 10.

The two teachers that had little participation in the first phase, now begun having a strong participation, making proposals for the group and assuming the responsibility for carrying out an important part of the tasks to undertake. The teachers become more confident in their knowledge and that seemed to help them to assume a joint responsibility for the results of the second phase in which the role of the teacher educator in leading the sessions become less prominent.

Critical Incidents along the Lesson Study

A first critical incident experienced by the group was taking the decision regarding who would teach the research lesson. Initially, none of the teachers volunteered. Irina thought that it should be one of her colleagues, who had much more to learn. The other two teachers, aware of their limitations, though that it should be Irina to teach the lesson. This issue created some discomfort in the group:

> I think that the tension begun vanishing when I said "I do not mind being me". Then, it calmed down a bit. Did it really calm down? Not. It alleviated. I knew it would come to me. (Irina, S12)

When it became necessary to make a decision, the fears of Antónia and Manuela weighted more than the incentive of Irina to the learning of her colleagues. Unsatisfied, she assumed to teach the lesson that she prepared with great care and detail.

A second critical incident was the research lesson. Antónia and Manuela, the teachers that had less participation so far, felt that the lesson taught by her colleague had an interesting dynamic and seemed to have appreciated the deep reflection that was made on the lesson:

Manuela:	What happened in our group may not apply to other groups. I think that for me it was an added-value to observe Irina's lesson, I have no doubt about that!
Marisa:	Why?
Manuela:	Because I think that the security and the way she looks at mathematics… I was listening to her… I think I have always to learn with Irina.
Irina:	We always have much to learn with each other.
Manuela:	. . . I learned much more than if I was teaching the lesson, perhaps. (S12)

The third critical incident occurred just after the research lesson. We made individual interviews, in which we sought to know how the teachers regarded their experience so far in the lesson study. For Antónia and Manuela it was a moment in which they could voice their difficulties and insecurities.

Both teachers indicated that they felt tiring the detailed work of analysis of tasks and students' solutions and Manuela referred that often she did not understand what was being discussed, given the limitations of her mathematics knowledge. These interviews assumed the role of exorcising ghosts, creating a close relation with the teacher educator.

Collaborative Relationships

In this lesson study, the collaborative relationships in the first phase developed only between two participants (Irina and the teacher educator), with the two other participants assuming a peripheral role. At the middle of the process, two critical incidents (the research lesson and the interviews) led the four participants to develop closer relationships and the group moved to a kind of work rather collaborative. Within the group, the decision power begun by being centered in the teacher educator, who made most of the working proposals, being shared by one of the teachers who assumed a strong involvement. In the second phase, the teacher educator maintained an important role in formulating the most general working proposals but the decisions about their enactment become distributed by all group members. In the first phase of the lesson study the teachers did not know yet what was expected from them at each moment and that influenced their involvement. In the second phase, the teachers were more confident about their knowledge and knew the working processes. That led them to be able to adopt common goals and working processes. This enabled them to work in a more autonomous way regarding the teacher educator, sharing also the responsibility for the results of the work.

A Lesson Study with Middle School Teachers

Aims and Working Processes

This lesson study had also two phases, each one with its own goal. The first phase aimed to prepare and carry out a research lesson about comparing rational numbers; the second phase aimed to prepare and undertake new lessons about other topics related to rational numbers. These goals developed in a similar way to the primary group. The teachers showed very similar interests regarding the primary teachers and showed also interest in making their lessons more dynamic, not only in this but also in other topics, in order to improve students' learning and to develop their own understanding of the new curriculum.

The structure of this lesson study is very similar to the primary group, following the usual model of defining the topic to address, study curriculum materials, prepare, enact, and reflect about a research lesson. There was also a follow-up. In the first phase, the working proposals were mostly made by the teacher educator, with a strong involvement of the teachers, albeit in different degrees, with many cases of joint work. In the second phase, the follow-up went on also in joint work, with decisions assumed in a shared way, with division of labor to reach a common goal.

Critical Incidents along the Lesson Study

Despite their similar structure, the two lesson studies had quite different dynamics. In this second case, at the very beginning, there was a movement of questioning of why doing the lesson study. Its realization had been agreed in a meeting with the school principal and several teachers of different grade levels, among whom Maria, in the role of coordinator of the middle school teachers. In this meeting, Maria put no problems, but in the first lesson study session she raised many questions, showing high mistrust regarding this kind of work. She questioned why to center all the attention in just one topic when there are so many problematic topics for pupils ("so much time to tackle just a topic!" S1), questioned if the presence of a high number of observers would not disturb the pupils and, with particular emphasis, indicated that she felt very uncomfortable by having a lesson observed, as she "did not want to be evaluated" (S1). The other teachers seemed to share these concerns. This reaction created a dilemma to the teacher educator team, between trying to persuade the teachers and dropping the lesson study. We presented several arguments, indicating benefits that may arise from the deep study of a single topic, explaining that experience shows that pupils tend to not become disturbed in the research lesson and referring that the focus of this lesson would be in the students' learning and not in the work of the teacher. Finally, the teachers decided to accept to experiment, to see how it would go. This unforeseen beginning was the first critical incident in the lesson study.

A remarkable turnover took place in the beginning of the following session, providing a second critical incident. From a position of great reservation, the teachers moved to a position of active participation. Several mathematical tasks that were proposed made them feel challenged, enjoying the experience. The teachers also got involved in other activities such as the analysis of students' solutions, the elaboration of a diagnostic of students' knowledge, the preparation and undertaking of the research lesson, and the follow-up activities. Such involvement is noticeable, for example, as the teachers planed the whole class discussion with little support of the teacher educator, taking into account ideas already discussed, and calling the pupils to present and justify their strategies:

Maria: So the idea was that Luísa would put the answers at the board…

Marisa: Some.

Luísa: Two or three.

Maria: And then to put them for class discussion . . .

Luísa: We could analyze one and say: "what is going on here? Who agree?" Isn't it? . . .

Maria: Who thinks this is correct? . . .

Tânia: To give the word to who is at the board, because he/she then ends up arguing: "I liked this because I thought that…"

Luísa: Normally when they go the board they explain and then I ask: "and then, you agree? Ah! No? Why not?" So, the one that is at the board explains and then if there is someone who does not agree… (S5).

A third critical incident arose, also in this group, with the decision of who would be the teacher for the research lesson. The structure of the group already indicated that the choice should be among one of the grade 5 teachers (Maria, Francisca and Luísa). Maria and Francisca refused to assume this role and the decision fall on Luísa, the teacher from this group with less professional status, since she had only an annual contract. Once the decision was made, without much protests from Luísa, the good climate returned to the group. This is a clear example how micro political power balance played an important role on how a decision was made.

A fourth critical incident, this time positive, occurred with the research class that, albeit it did not go as planned regarding the tasks proposed and regarding students' learning, even so it yielded good moments of students' work and originated an interesting and participated post lesson discussion. In the final reflection, the teachers underlined that they enjoyed the experience of observing the research lesson:

> It was fruitful for all of us, because we verified that when we are in the classroom things do not go so well as we foresee. And other things, the kids surprise us with the solutions that they present . . . I thought it was a pretty lesson, different from usual… (Francisca, S12).

Collaborative relationships

In this lesson study, after its problematic beginning, the collaborative relationships among all members of the group developed in a very positive way. The mood of the sessions was enjoyable, the teachers showed willingness to carry out the tasks that were defined and the activities undertaken enabled the movement of the group towards its goal. The single exception to this pattern concerned the decision of the teacher for the research lesson. The refusal of the teachers in being observed has several explanations. It results, first of all, on the strong tradition of individualism and privacy in the work of the teacher that exists in Portugal as in many other countries. In addition, this reservation is also a consequence of previous attempts from the Ministry of Education to establish a system for teacher evaluation in which the observation of lessons was an essential feature. This reservation concerning the observation of lessons is a very strong cultural element in our country that requires special attention in the adaptation of lesson study to our educational context.

In the final reflection made in the last session of this lesson study we asked the teachers what was the most salient feature of the work that was carried out. For our surprise, the teachers highlighted the collaborative work that they indicated to have expanded to outside the lesson study work:

> Marisa: What was the most salient feature of the lesson study?
>
> Maria: It was collaboration! It improved the relation among the five of us because we were three tenured teachers in the school for many years and we did no longer pay attention to each other, because we know the working methods of each other . . . And I find that this lesson study yielded this contact and also with the new colleagues . . . And yesterday when we made that lesson planning . . . We were two old women with a young one, and I like this contact. I am frank.
>
> Francisca: For me it was very positive… Because… We worked together, we shared information… And the work among teachers is absolutely important. Many times we feel alone, isn't it? . . . I find this the main issue. We opened ourselves to each other. We did not had fear of… Because sometimes there is people that have fear of showing their weak points, and I find sharing very productive, that is, working in group . . . (S12)

In fact, the collaborative work had never been object of explicit discussion in the sessions. However, the teachers showed to be aware of it and valued it strongly. The collaborative work carried out in the lesson study also influenced the daily relations of the five teachers that had good personal relations but had little professional interactions. Before, there was very little communication among the tenured and the contracted teachers who were new at the school and the lesson study brought another dynamic to the group, providing moments of joint work that did not exist.

Conclusion

Collaboration is often seen as a process that develops in a natural and non-problematic way. In lesson study, collaboration tends to be regarded as something inherent and not as something that has to be constructed by the participants by their joint activity, negotiating roles and relationships. As any other social process, lesson study unfolds in movements forward and backwards with critical incidents that sometimes favor the development of collaborative relationships and other times oppose them. The cases presented in this paper show that, in our context, lesson study framed by the preparation and teaching of lessons about a given topic, may sustain the development of these relationships, but these are not given at the beginning – they need to be constructed by the participants in their joint activity.

The two lesson studies that we presented had several critical incidents that generated conflicts (Achistein, 2002). These critical incidents originated in the unfamiliarity of the teachers who faced a teacher education model very different from those that they know and are used to participate and, most especially, on their reservations in being observed by the whole group in a research lesson. These conflicts and difficulties were dealt with diplomacy and patience and the stimulating nature of the working proposals of the teacher educator during the sessions was the main factor to overcome them. It must be noted, in addition, that there were critical incidents with a positive role, namely the research lesson (in both groups), the individual interviews (with the primary group) and the active involvement in doing challenging mathematical tasks and analyzing students' solutions (with the middle school group). The goals proposed in the lesson studies, in each phase, provided the necessary direction to the activities and the working processes that emerged, following a pattern of progressive development quite different from that indicated by Dooner et al. (2008). In both lesson study groups these processes were particularly efficient in the last phase of the process, during the follow-up, in which all the teachers had opportunity to prepare tasks directly for their classes and share their experiences. We may say that in this phase they worked as a team (Vangrieken et al., 2015).

In these two lesson studies, the teacher educators were an integral part of the collaborative group, albeit with a very specific role. As the lesson study experts, they led the whole process, but did so in permanent negotiation with the other participants. Collaborative relations developed, supported by assuming common goals with enough strength and by establishing productive joint ways of work associated to the preparation and teaching of a research lesson. As our data indicates, especially with the middle school group, this may provide a strong collaborative experience. But these relationships are ephemeral, unless new common goals emerge as well as of new productive joint ways of work.

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MATHEMATICS TEACHERS AND TEACHER EDUCATORS LEARNING IN A COLLABORATIVE SETTING

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The promotion of fruitful collaboration between teachers and teacher educators is an important issue in mathematics teacher education. This research project investigated how teachers and teacher educators, working collaboratively, may create opportunities to reflect on their knowledge and share their practical classroom experiences. The aim of this paper is to discuss findings from a teacher education process regarding the concept of function. Data were collected using video and audio recordings, collection of documents and observation of lessons carried out by teachers in schools. The results indicate that the collaborative learning environment favored the emergence of learning opportunities that enabled teachers and teacher educators to (re)think and (re)organize their mathematical and didactical knowledge related to school algebra.

In collaborative processes, several participants pursue common objectives working in a coordinated way. These processes enable them to face difficult professional challenges and provide powerful learning environments (Robutti et al, 2016). However, the development of relationships of trust, which is essential in collaboration, is a slow and negotiated process (Boavida & Ponte, 2002). The promotion of collaboration between teachers and teacher educators in mathematics education constitutes an important issue for research and production of knowledge (Robutti et al, 2016).

The concern with the mishaps in algebra teaching and learning identified during many years of studies and investigations (Ribeiro, 2007), motivated the author of this paper to idealize, organize and develop a research project[1] seeking to show alternative pathways to (re)think algebra teaching, in order to support teachers in developing a practice that promotes students' learning. Results from other studies also pointed out to problems regarding students' failure in learning algebra (Matos & Ponte, 2009; Stephens & Ribeiro, 2012), and still others documented the difficulties encountered by teachers in their teaching (Doerr, 2004; Ribeiro & Cury, 2015; Wasserman, 2015).

Although there is considerable scholarly production on mathematics teacher education (Fiorentini, Passos & Lima, 2016; Stahnke, Schueler & Roesken-Winter, 2016), Ponte and Chapman (2008) identified a strong demand for research that prioritize teachers' practice. Expanding the problem of "teaching and learning of algebra", Ribeiro's research project considered issues related to mathematics teacher education in algebra in relation to teachers' practice. As part of the teacher education process, the project highlighted the experience of teachers in spaces for discussion and collective work (Cristovão & Fiorentini, 2018), providing opportunities for collaboration among all participants in different stages of research and training (Boavida & Ponte, 2002; Jaworski, 2006),

[1] Research project "Mathematical Knowledge for Teaching Algebra: an approach based on conceptual profiles", a 4-years longitudinal project (2013-2017), funded by Capes (Brazil).

and creating opportunities to the participants to reflect on their knowledge and to share their practical classroom experiences, mediated by tasks that support their professional learning (Ball & Cohen, 1999; Smith, 2001). Drawing on the work of this project, this paper addresses the following questions: *How do we create a collaborative work environment that promotes learning for those involved in algebra teaching? What learning about algebra teaching was promoted to different actors who worked in collaboration?*

Professional knowledge for teach mathematics

Ball, Thames and Phelps (2008), in their seminal work, present a set of six domains that characterize what they call *Mathematical Knowledge for Teaching:* (1) common content knowledge (CCK); (2) specialized content knowledge (SCK); (3) knowledge of content and students (KCS); (4) knowledge of content and teaching (KCT); (5) horizon content knowledge (HCK); and (6) knowledge of content and curriculum (KCC). Among these domains, specialized content knowledge is a key conceptual notion of the present study, since this kind of knowledge is specific for the craft of teaching mathematics in order to promote students' learning (Loucks-Horsley, 1997).

Arguing that teachers' knowledge is action-oriented, Ponte (1999) presents a perspective of professional teacher knowledge strongly anchored in practice. In his perspective, a key element of professional knowledge is didactical knowledge which unfolds in four domains: knowledge of teaching content, knowledge of the curriculum, knowledge of the student, and knowledge of the teaching process. For the author, this knowledge "relates very closely to various aspects of the teacher's personal and informal knowledge of everyday life as the knowledge of the context (school, community, society) and the knowledge about him/herself" (Ponte, 1999, p. 3). The analysis in this study draws on this perspective, in particular, in knowledge of the student and knowledge of the teaching processes.

Teacher professional learning to teach mathematics

Teacher professional learning has been studied, discussed and investigated for several decades (Opfer & Pedder, 2011). The perspective assumed in this paper is that such learning is strongly anchored in classroom practice (Ball & Cohen, 1999; Smith, 2001). This also applies to preservice teacher education, which, as Webster-Wright (2009) point out, is only the first stage of the teachers' learning process, which later continues over many years and in the contexts of professional practice.

Thus, we assume the importance of organizing and developing professional learning opportunities based on the teachers' practice (Ribeiro & Ponte, 2019), as an important context for teachers to learn throughout their careers. In our understanding, professional learning opportunities may emerge during teacher education processes, for example, considering professional learning tasks (PLT), understood as "tasks that involve teachers in the work of teaching, [which] can be developed in order to find a specifics goals for teacher's learning and take into account the previous knowledge and the experience that the teachers bring of their activity" (Ball & Cohen, 1999, p. 27).

Methodology and context of the study

Regarding methodology, a qualitative-interpretative approach was adopted, in the frame of a *design-based research (DBR)* (Cobb, Confrey, diSessa, Lehrer, & Shaube, 2003), in particular a type of DBR that contributes to teachers – working together and in collaboration with teacher educators – to develop learning that will enable them to carry out innovative teaching practices in their classrooms (Cobb, Jackson, & Dunlap, 2016). The "practical" component was a key element of the professional learning tasks (PLT) used in the teacher education processes, which contained records of practice (Ball, Ben-Peretz, & Cohen, 2014), such as curricular materials, videos or narratives of episodes of lessons, and written samples of students' work. Beyond the PLT, the collaborative learning environment included the discursive interactions between all participants and the role and actions of the teacher educators.

Data were collected throughout the teacher education process using video and audio recordings, collection of documents (protocols produced by students and teachers, lesson plans), observation of professional learning opportunities and observation of lessons carried out by teachers in basic schools. The process of analysis took place inductively and through different techniques, since the data were collected by multiple sources. We coded the collected data and elaborated *vignettes* (Borko, Jacobs, Eiteljor, & Pittman, 2008), composed by episodes from audio and video records of lessons taught in basic schools.

DBR has a cyclical and iterative nature. However, given the space limitations in this paper, it is not possible to explore the different cycles that made up the longitudinal study. For the purpose of illustrating results that allow us to answer our questions, we present an episode that exemplifies a collaborative teacher education process that addresses the teaching of functions in the basic school (Ribeiro, Aguiar, & Pazuch, 2018).

The team of participants underwent several changes in its composition, since it was a 4-year project. There were usually about twenty pre-service and in-service teachers (PT) and five teacher educators (TE). The PT were from different levels of teaching and from different schools. Regarding the TE, the team was led by researchers and other university instructors, sometimes by mathematics education graduate students, other times by university instructors in partnership with teachers. It is important to highlight that several "change of roles" occurred during the project, in special, when PT became "facilitators", working with TE in preparing and developing some teacher education sessions (e.g., when some PT supported the TE in choosing video episodes to be discussed and analyzed together with all participants). This dynamic enabled change of roles among the participants, acting sometimes as learners, other times as educators, and encouraged an environment "that the various players work together, not in a hierarchical relationship, but on an equal basis so that there is mutual help and reach objectives that benefit all" (Boavida & Ponte, 2002, p. 3).

The common goals of PT and TE in the collaborative process were to understand the different forms of students' mathematical reasoning when they engaged in tasks involving algebraic concepts. For the TE, another goal was to understand how the PTs learned to interpret students' reasoning. As said above, a particular kind of PT and TE work together was when they selected and watched video from lessons and, when all of them were working together, they had the opportunity to exchange their knowledge about students and algebraic concepts regarding, for example, solving equations or manipulating functions using different mathematical representations.

The teacher educators were preparing for the formative sessions throughout the week preceding each meeting. In particular, regarding the episodes that will be discussed in this article, the TE watched the video of the whole lesson to identify episodes that would allow reflections on the role of the teacher in conducting the class previously prepared by the group. In addition, the TE organized professional learning tasks (PLT) that contributed to the discussions and reflections about, for example, the teacher's actions in presenting the task to the students or when faced with a teaching situation not foreseen in the planning. As important to watched the video of lesson and selected interesting episodes, the TE had to choose adequate records of practice, like protocols of students activities or transcripts of students' dialogues, to complete the PLT and to complement the video episodes. Finally, the TE had to prepare themselves to orchestrate collective discussions during the meetings, and they organized this, mainly, by anticipating mathematical and didactical knowledge of participating teachers and making a forethought of the questions and difficulties that the TAP could generate when developed in small groups and plenary sessions.

Teacher learning in a collaborative learning environment

The teacher education process was always developed in a collaborative working environment. The participating teachers were initially invited to share their knowledge and their experiences about the contents to be addressed in the teacher education meetings, in order to break with the hierarchical form of knowledge construction that normally dominates the spaces of teacher education. The participants chose to use "videos of lessons" as resource during the teacher education process, seeing this as an important tool to promote teacher professional development (Coles, 2013; Louis & Steven, 2018).

Data analyzed and discussed in this paper were collected from the lesson plans on the concept of function by the participating teachers (PT). Among the lesson plans developed, the PT chose a lesson to be carried out with secondary school students (14-15 years) by the teacher Carlos[2]. The lesson was videotaped and, later, watched and analyzed by the teacher educators, in order to select episodes to be used in the teacher education process. The moment of analysis and collective reflection involving the participating teachers occurred in the university, days after the development of the lesson at a school.

The episodes selected by the teacher educators took into account the importance of promoting PT activation in terms of immersion, resonance, authenticity, and motivation compared with viewing the practice of teachers unknown to them (Borko et al., 2008). The episodes were used in conjunction with a professional learning task designed to promote professional learning opportunities about the concept of function and its teaching.

The planned lesson was taught in a Brazilian public school, with 32 students divided into groups of four, in a conventional classroom, using computer and multimedia equipment. The lesson plan had a mathematical task to be solved using *Geogebra*. The task had potential to promote an inquiry-based approach for the students. The episodes prepared for the session focused on the actions of Carlos and the two episodes analyzed in this paper were drawn from two different moments of the lesson. The selected episodes could serve as scaffolding "to provoke the development of descriptive and critical reflection" (Gaudin & Chalies, 2015, p. 51).

[2] Carlos, who developed the lesson at basic school, was 21 years old and he was studying the last semester of a mathematics teaching degree. He had no experience in the classroom, but acted as a tutor at a private elementary school and was monitor in the algebra course at the college where he studied.

The first episode shows his actions during the presentation of the task and of the software, and his discussion with the students over the different mathematical representations of the concept of function (numerical/table, algebraic, graphs):

Carlos: Today we are going to work with *Geogebra* (...) Has anyone ever heard of it?

Student 1: Yes.

Carlos: Already? So you already know how to move? Already familiar?

Students: No, no [laughing].

Carlos: So, come on. Who has already moved? Already moved, but do not remember? It has interfaces here (...) each has a name, a different type of function [Teacher shows the functions of software to students] (...) I'm going to show you now, first of all, how we're going to put some things in spreadsheets, so we can turn that into a graphic, okay? You know what is graph, right? It is a Cartesian representation. (...) You know what a Cartesian plane is, right? Remember that?

Students: Yes [laughs].

Carlos: So, go here in "Spreadsheet" [shows the software function] and this will appear [Window 'Spreadsheet'], a lot of boxes here and it will not end, ok [refers to the cells that make up the Spreadsheet]. (...) The operations in the Spreadsheet will look like this [uses the blackboard to present the operations to be used in the *Geogebra* Spreadsheet] (...) For example, if I do this here (…) [constructs a table with values 2, 3, 4, and 5 distributed in one column and writes $\mathrm{f(x) = x + 5}$]. What does this mean here? [Referring to the $\mathrm{f(x) = x + 5}$]. Does anyone know how to respond?

Student 2: *f*(2) It's going to be two plus five... *f*(3) It's going to be three plus five... (...)

The second episode shows the actions of Carlos when he experienced a critical moment in relation to what had been previously prepared for the lesson. This unexpected and unforeseen moment led him to have an insight to solve the impasse:

Carlos: Folks, in the application [*Geogebra*], the configuration is not to use decimal place. So, for people not to waste time in configuring, I put here in *Excel*. You've heard of *Excel*, right?

Students: Yes.

Carlos: I put that spreadsheet here in *Excel*, it's pretty much the same thing, even a little easier to do.

Student: And the colors? [The spreadsheets built for each telephone plan initially had different colors]

Carlos: So, I tried putting here in *Excel* the approximate colors with those of *Geogebra*. So, oh, again, what's your name [retakes the previous conversation]?

Student: Victor.

Carlos: He [Victor] told us that it was the fixed amount plus the minute value multiplied by the number of minutes [continues carrying out the operations to determine the graphical representations referring to the three phone plans].

Coming back to the teacher education process, the participating teachers, organized in a plenary session, watched and analyzed the two episodes aiming to identify the mathematical and didactical knowledge mobilized by Carlos during the lesson. The teacher educators prepared a script to guide the analysis conducted by teachers. Next, there is an excerpt of the discussion generated between the participating teachers (PT) (among them, Carlos) and a teacher educator (TE):

PT1: It would be interesting to do this activity in the laboratory.

TE: Yes, but since the school did not provide this, Carlos used the Data Show and made a dialogue with the students using *Geogebra*.

PT2: Ah, *Geogebra*, it's beautiful and wonderful, but I cannot work with *Geogebra*.

TE: Anything else? Do you agree or disagree with teacher Carlos's examples?

PT3: (...) This is our problem, we have to let the student do, let he/she take over [the software]. Move there, participate in that, feel part of that lesson, not only in the "dialogue part" of lesson, but not make it happen.

PT2: The idea [of the lesson plan] was for the student to do it on his computer, so he could understand... use the graph and the table and create the algorithm together as well (...).

PT1: [asking Carlos] And during class did they [the students] just watch? Or did they take notes?

Carlos: They did the activity on the sheet that was delivered to them. Then we checked their results on *Geogebra* (…).

From the discussion of the lesson episodes, the participants were offered a panoramic view of the entire lesson. The use of video was fundamental in this process, since it allowed them to experience classroom practice, yet in an indirect way. In analyzing the episodes of this lesson, the teachers, together with Carlos, pointed out aspects of the class and promoted reflections regarding the dynamics of the class, the teacher's actions, the students' participation and the way the concept was discussed.

In general, considering the analysis of the class episodes through the use of an organized script for this, as well as the environment of trust and exchange of ideas and reflections, we infer that participating teachers perceived themselves (re)organizing their knowledge of: (1) the use of different representations (algebraic, geometric, tabular) for the study of affine function; (2) the use of *Excel* spreadsheet as a deviation from the lesson plan, since an unforeseen configuration of the *Geogebra* software was presented; (3) the low participation rate of students in the moments of dialogue with the teacher; (4) how a proposed lesson plan that envisaged the use of a computer lab could not be enacted due to the school structure.

Conclusion

In order to answer the two questions of this paper, data was collected in a teacher education process structured as a collaborative learning environment, aiming to favor the emergence of learning opportunities to enable the participants – teachers and teacher educators – to (re)think and (re)organize their mathematical and didactical knowledge related to school algebra.

With regard to the question *How do we create a collaborative work environment that promotes learning for those involved in algebra teaching?*, we conclude that the use of professional learning tasks (PLT) (Ball & Cohen, 1999), structured to favor discussions between teachers and between teachers and teacher educators, favored the elaboration and development of lesson plans on the topic of functions and the subsequent reflection on the classes carried out. The PLT used in the teacher education process culminated in the analysis of video episodes of lessons, which contributed to the teachers to "get in" the classroom and (re)think their own learning practice (Coles, 2013; Taylan, 2017). In addition to the mediation by PLT, the role of teacher educators, as organizers of the collaborative learning environment (e.g., organizing records of practice to include in the PLT), as when change of roles that took place during the teacher education process (e.g. working together to select video episodes from lessons), seems to have increased shared knowledge in all participants (Ball, Ben-Peretz, & Cohen, 2014).

Regarding the question *What learning was promoted to different actors who worked in collaboration about knowledge for algebra teaching?*, we note that the experience in a collaborative learning environment gave to the participating teachers opportunities to mobilize mathematical and didactical knowledge when preparing, developing and reflecting on a lesson involving the concept of function. In particular, they mobilized and used specialized content knowledge (Ball, Thames & Phelps, 2008) about different representations (algebra, tabular, geometric) of the concept of function. They also mobilized different didactical knowledge, including teaching strategies, such as carrying out a lesson based on the use of software and, when faced with a situation not foreseen in the lesson plan, they had autonomy to search for another didactic resource to overcome the obstacle (Ponte, 1999). With regard to teacher educators, the possibility of sharing a collaborative learning environment with presence and participation of teachers from basic school, led them to perceive the dynamics of a “real” classroom, through the episodes and respective discussions that took place throughout the teacher education process.

This process has some advice to be considered. One of them is the time necessary to develop a teacher education process in a collaborative learning environment, since several meetings had to be used to establish trust and mutual respect among participants, as well as the involvement and willingness to expose themselves to colleagues (Boavida & Ponte, 2002).

The collaborative work in a teacher education process based on practice, such as what was discussed in this paper, had an important role in the learning experiences of the participating teachers about the mathematical and didactical knowledge for the teaching of function in basic school. Also very important was the use of video to provide an approximation of real teaching situations to a teacher education process.

One of the features identified in the teachers learning, resulting from collaborative work, refers to the movement of sharing questions about the best resources for teaching and how to consider students’ difficulties in carrying out mathematical tasks. Regarding teacher educators, another feature we identified was they had the opportunity, sharing viewing videos with teachers, to understand the dynamics of practice within the school classroom and knowing the learning of algebraic concepts of younger students.

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FACILITATING A COLLABORATIVE PROFESSIONAL DEVELOPMENT FOR THE FIRST TIME

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Collaborative professional development initiatives require skillful facilitation. The purpose of this paper is to characterize challenges in leading a collaborative video-based professional development for the first time, and highlight the importance of the program's support team in addressing them. A case-study approach was used to illustrate one novice facilitator's professionalization process over one year. It was found that although the facilitator did not perceive herself as a leader at the beginning of the year, towards its end she fostered collaborative video-based discussions, in parallel with the consolidation of her facilitator identity. This study provides insight into how novice facilitators' practices and identities evolve, as well as what is required to support this process.

Introduction and research questions

Teachers' collaboration, as defined by an ICME-13 working group dedicated to this subject, can take many different forms: "collaboration involves mathematics teachers engaging in joint activity, common purpose, critical dialogue and inquiry, mutual support; addressing issues that challenge teachers professionally and reflecting on their role in school and in society" (Jaworsky et al., 2017, p. 263). In this study, I focus on a unique video-based professional development (PD) program named VIDEO-LM (details below), where teachers discuss issues of their practice together, under the guidance of a facilitator who directs them towards *collective reflection*. Collective reflection is a discussion where teachers not only share their thoughts, but also listen carefully, comment to each other, re-consider their views and develop a shared language and understanding related to their joint work. Teachers' collaboration is manifested in this context as an involvement in the collective reflection, as well as in an ongoing process: an effective collective reflection may leave its trace on the group after the PD course has ended, by generating changes in teachers' practices and creating an ongoing dialogue using the acquired shared language. This ambitious goal requires facilitation that deliberately directs discussions toward reflection. Since practice-oriented discussions are becoming more prevalent in PDs, there is a focus in the literature on the skills needed to orchestrate them (van Es et al., 2014, Tekkumuru-Kisa & Stein, 2017). Nevertheless, leading discussions is still a major challenge for facilitators, especially for novices (Borko et al., 2014). Novice facilitators who are experienced teachers may also face the challenge of navigating the middle ground between being a colleague and a leader (Knapp, 2017). The studies that have investigated novice facilitators' challenges in fostering teachers' collaboration recommend changes in facilitators' preparation and support, but to date there is little published data on the *implementation* of this kind of support, and of how it can address tensions in facilitators' multiple, sometimes conflicting, identities.

This study aims to contribute to this growing area of research by exploring the following research questions: What are the challenges faced by a novice facilitator when leading a collaborative professional development course? How do they shape and how are they shaped by multiple identities? How can these challenges be addressed?

In this paper I report on one case from a long-term study that investigates the professionalization processes of 8 novice facilitators in a specific PD program. The selected case demonstrates what hinders a first-time facilitator in fostering peer-collaboration in the PD, and how the program's support system assisted her in achieving her own goals, as well as the program's.

Context

VIDEO-LM (Viewing, Investigating and Discussing Environments of Learning Mathematics) is a PD program developed at the Weizmann Institute of Science in Israel. The program aims to foster teachers' reflective skills and mathematical knowledge for teaching, and develop a shared language for discussing practice through peer-analysis of lessons taught by unfamiliar teachers. The VIDEO-LM team developed a six-lens framework, SLF, to guide teachers' observations of the lessons (Karsenty & Arcavi, 2017) that comprises: (1) mathematical and meta-mathematical ideas in the lesson, (2) teacher goals as they can be inferred from the lesson, (3) the tasks used, (4) interactions in the lesson, (5) teacher dilemmas and decision-making and (6) teacher beliefs. To achieve the VIDEO-LM goals, facilitators should choose which videos, lenses and activities to use in each session, according to the particular context; there is no rigid curriculum they are required to follow. However, the project team has determined that each VIDEO-LM session should include the following four components: a videotaped lesson; mathematical content; use of SLF (all or some of the lenses); engaging activities for teachers. This quartet is intended for the cultivation of reflective discussions inspired by the observed lesson, in a non-judgmental atmosphere that allows of sharing insights and dilemmas (enabled by the use of SLF). In previous studies we found that such candid peer-conversations, where teachers exchange views, discuss and convince each other, are a catalyst for reflective processes and changes in practice (Karsenty & Arcavi, 2017; Schwarts & Karsenty, 2019).

The first steps of the project consisted of a few PD courses every year, mostly led by project team members. However, the project was gradually scaled-up, which required new skilled facilitators. To meet this need, the VIDEO-LM team developed a program for training and ongoing support for novice facilitators (who themselves experienced the PD as participants). The support system includes bimonthly facilitators meetings led by the project's team, and one-on-one sessions with a personal mentor, who is an expert VIDEO-LM facilitator. The mentors and mentees meet occasionally, plan PD sessions and discuss them afterward. The mentors themselves have monthly meetings of the support team, where they share problems and discuss solutions.

Methods

This paper uses a qualitative case-study approach (Stake, 2000) to investigate in-depth a novice facilitator' challenges in fostering collaboration. The facilitator, Rose, is an experienced mathematics teacher who participated in a VIDEO-LM PD as a teacher (in 2013-14), enrolled in the VIDEO-LM facilitators training course (in 2015-16), and facilitated her first VIDEO-LM PD course in her own school, with 12 participating teachers (in 2016-17). Rose had not previously led any PD.

The following data is used in this case study: (1) pre- and post-facilitation questionnaires; (2) reflective journals that the facilitator wrote before and after each PD session, following guiding questions regarding goals, decisions and more; (3) videos from the 2nd and the last (the 8th) PD sessions led by the facilitator (the 2nd session was chosen, and not the 1st to ensure that the facilitator would feel comfortable); (4) videos of stimulated recall interviews (SRIs) that were held with the facilitator a few days after each of the filmed PD sessions. The researcher and the facilitator jointly watched the PD sessions' videos, and the facilitator was asked to stop the video whenever she noticed a decision she has made, and reflect on it; (5) protocols from the program's support team meetings.

All video data was transcribed. To track challenges in the novice facilitator's practice, I searched the first SRI for decisions that Rose was dissatisfied with, and the support team's protocols for challenges that were raised by her mentor. Then, I searched for evidence in the whole corpus of data for references to these challenges, ideas for resolving them, as well as the implementation of these resolutions and their effect on teachers' collective reflection. A content analysis was performed, representative excerpts were chosen and arranged in a narrative structure to highlight the evolution of the facilitator's practices and identities. During this analysis I used Gee's definition of identity: "Being recognized as a certain 'kind of person', in a given context" (Gee, 2000, p. 99) to find when Rose identified herself as a teacher, a colleague or a facilitator and how it guided her actions.

Findings

In this section I first present the two prominent identified challenges and highlight how they relate to the facilitator's practices and identity. Then, I describe how these challenges were addressed by the support team, and how the facilitator's practices and identity, as well as teachers' reflections, evolved. The notation "S2-5" is used for turn 5 in the 2nd session transcript, and "I1-3" for turn 3 in the transcript of SRI-1 (the 1st interview). The participating teachers are noted as T1, T2, etc.

First challenge: planning the session in accordance with the PD agenda

I begin with a challenge that was recognized by the support team (details follow), regarding the way Rose planned the 2nd PD session. While she structured the session with engaging activities, they were not connected to the VIDEO-LM goals, and did not include all four essential components of a VIDEO-LM session which were stressed in the facilitation course. For example, the session comprised *engagement of the teachers* in pedagogical and *mathematical discussions*, and the teachers watched a *videotaped lesson*, but the discussions were not about the chosen video, and were hardly guided by SLF. Actually, the videotaped lesson was observed only at the end of the 4-hour session, leaving 4 minutes to discuss it. The PD essence is watching unfamiliar teachers' lessons in order to "step into their shoes" and interpret their actions from their point of view, in order to introspect one's own practice in the form of *collective reflection*, but this kind of discussion could not occur when the video is the last activity on the schedule. To summarize, the facilitator faced the challenge *to plan the session in accordance with the PD agenda.* This challenge was identified by the researcher and the support team with the help of this study's data, but the facilitator herself did not refer to it at all in SRI-1 or in the journals, which strengthen the assumption that she was not aware of the disparity between the PD goals and her planned activities, and perhaps (as will later be illustrated) did not understand how the four components of a VIDEO-LM session should be integrated to achieve collective reflection.

Second challenge: managing a discussion where the teachers can reflect and collaborate

The second challenge I focus on is one that Rose herself noticed and reflected upon. The following representative exchange from the 2nd PD session exemplifies the challenge:

S2-267 Rose: What is the added value of solving a problem in several ways?

S2-268 T1: I want to answer, rather, not mathematically, but... from life, one of the things that I as a teacher try to teach the students, is that in many situations in life there are several possibilities...

[Turns 269-295: T1 elaborates on this example, Rose and the teachers comment on T1's contribution]

S2-296 Rose: Excellent. Actually, I found something else. I think, it also lets students [...] appreciate elegant solutions [...] And I think that when a student sees a number of solutions, he is learning to appreciate an elegant solution.

While the discussion started with an open-ended question that calls for reflection, T1's response was followed by the *facilitator's* answer. In SRI-1, she stopped the video after this episode, and said:

I1-33 Rose: We discussed it in the facilitators' course, should we intervene, should we share our opinion? I've come to the conclusion that when I ask questions, I first want to hear *them*. Then, if I have something that I think can add to the discussion, I think, like, if I was now sitting with the teachers around the table, I would also tell this story, I wouldn't have avoided it […] I feel that the right thing to do is not to refrain from expressing myself because I'm 'only a facilitator' and I'm 'only listening'. On the other hand, I shouldn't take over the discussion.

This quote reveals Rose's conflicting views regarding her roles and identities. As a teacher and a colleague, she feels she is entitled to share her opinions ("I would also tell this story"), but since she is the facilitator, she "first wants to hear *them"* and does not want to take over the discussion. She identifies facilitation as a practice where one "only listens", which hints that as a facilitator, she knows what she is not supposed to do (speak a lot), but there is no reference to active roles she should take (besides listening). This view begins to evolve as the interview continues and Rose observes more discussions where she is the dominant speaker:

I2-37 Rose: Perhaps I should have brought it up for discussion and not just said my opinion.

I2-73 Rose: Maybe after I gave them the definition of meta-mathematical ideas, I should have asked them, what do you think?

I2-125 Rose: Maybe I talked too much. […] we were told we are supposed to listen more, and talk less.

These excerpts show a possible change in Rose's view of facilitation: she may have realized that her goal to express herself comes at the expense of giving the teachers opportunities to comment and share. Still, she rarely mentions active facilitation roles. The identified challenge here is *managing a discussion where the teachers can reflect and collaborate*. This challenge stems from different sources: first, as was shown above, Rose's multiple identities (teacher/facilitator) generate multiple goals (speaking/listening) which seem to contradict each other. Second, it can be inferred that Rose does not perceive herself as someone whose *role* is to foster collective reflection, as was also demonstrated in the first challenge. Third, Rose has no experience in leading discussions which are focused on *mathematics teaching* rather than *mathematics*, and this shift in "subject-matter" might have limited her space of possible reactions to teachers' comments, as she expressed in SRI-1:

I2-59 Rose: There is no right or wrong answer here. In class it is much clearer. When a student raises some brilliant question, or a brilliant answer, it is clear that I will say "great!". Of course I will. But here it's like... Okay, that's what you brought…

Unlike when facilitating, as a teacher Rose has coherent practices to draw on due to her experience, but also due to the subject matter epistemology that allows her to distinguish between "right or wrong".

Addressing the challenges

After the aforementioned PD session and SRI, the researcher informed the program's support team about Rose's challenges in understanding her role as a VIDEO-LM facilitator. The team decided that these challenges would be addressed in a one-on-one session with Rose and her mentor Ada, where they would watch the PD video together and stop the video whenever they noticed something they wanted to raise. This method aligns with the program agenda: rather than evaluation, the session's goal was to allow Rose to reflect on her practice with an experienced practitioner. Ada reported on this session in the following support team's meeting:

> When I watched the video [beforehand, alone] there were a lot of issues to discuss, it is obvious that she is a novice, therefore, I had to choose my battles [...] So I decided that what is most important to me is placing the videotaped lesson in the center and trying to bring forth the lenses. We watched the PD session together [...] At one point, Rose asked: 'I'm acting as a teacher, right?' And I chose not to answer yet [...] We also discussed the question of whether she should be perceived as an authority that always provides answers? She said that she really wants to express her opinion [...] I suggested that she can express her opinion in the manner of 'I heard that someone in a different PD said so and so... What do you think about that?'. She liked it. She realized that she has to change the way she builds her sessions [...] She really wanted to talk about auxiliary lines in geometry in the next session, but she thought about showing a lesson in calculus, so we talked about how she could choose a videotaped lesson to fit such a discussion, a lesson more suitable for the emergence of auxiliary lines. It seems to me that she got a clue of how we use the videotaped lesson […] as a springboard for discussions.

Both of the presented challenges were raised and addressed during this mentor-mentee session. Ada provided Rose with resources to overcome the challenges, but did not enforce them. According to the journal Rose wrote after the 3rd PD session, it appears that she internalized some of these resources:

> **What was the most important decision you made during this session? What guided your decision?** Talk less. One teacher asked my opinion about a certain issue and I began to answer her, and then I stopped and told the group that it was important to hear their opinion, not necessarily mine. The teacher was pleased with this move.
>
> **What is the most important thing you take with you from this session, towards preparing the next session?** The most important thing, as I understood from my mentor Ada, is focusing on the videotaped lesson, which is what I will do in the next session. Beyond that, I will try to make the discussions longer and engage the quiet teachers as well. I intend to give a few minutes to think about the questions for discussion, rather than expecting immediate answers.

These excerpts point to the influence of the support on Rose's facilitation, that was expressed in her awareness and practices regarding the sessions' structure and the centrality of discussions. While facilitating, she noticed her conflicting goals of sharing her opinion versus eliciting teachers' ideas, and this time, chose to give more space to teachers' voices. She also mentions active roles she should take, such as engaging more teachers. These answers indicate the consolidation of her facilitator identity, as she is no longer "only listens", but has the agency to promote collective reflection.

The last PD session of the year

While the 3rd session's journal suggests that Rose wanted to change her practice, and that some changes were beginning to take place, further investigation was needed to check if these changes were observable and sustainable, and how she intertwined her professional identities as a result.

I begin with a short background of the session: the 8th (and last) PD session, which occurred about 5 months after the 3rd session, opened with a few minutes' viewing of a videotaped geometry lesson, where the filmed teacher first reviewed a theorem that was taught in the previous lesson: *triangles with the same base on a given line and a third vertex on a parallel line have equal areas*. Then, the filmed teacher introduced a related challenging task and asked the students to solve it. Rose screened this part, and asked the PD participants to solve the task themselves. One obstacle in solving this task is that the use of the aforementioned theorem is not immediate. The parallel lines in the solution are placed vertically, and not horizontally as is "usual" when this theorem is presented. After some teachers solved the task (and some did not), Rose launched a group discussion regarding this experience, using the lens of *tasks and their enactment*:

S8-247 Rose: Was this task difficult? What made it difficult? [...] And if you think this task was interesting, why do you think it was interesting? Let's think about it for 2 minutes.

S8-248 T2: First, it was difficult for me because I tried […] I kept looking for things I *already* know.

The beginning of this session shows some changes in Rose's practice: she structured the session around the videotaped lesson, engaged the teachers with *related* activities, and gave them time to think – as she decided to do a few months earlier. This opening move led to a 24-minute discussion where most of the teachers were involved and shared their experiences in solving the problem. Many of them mentioned, like T2 above, that they were "fixed" on using theorems that are usually associated with similar tasks. In response, Rose rotated the task figure by 90 degrees, and asked:

S8-423 Rose: If I had presented the task in such a way, would it have been easier for you to solve?

This question opened a 10-minute discussion on didactics of geometry: Should we draw shapes such as triangles in a variety of forms, or focus on their prototypical orientations and properties? Teacher T3 pointed at the gap between what she sees as desirable, versus what she actually does in her class:

S8-685/687 T3: I'm good with theories, but… [during the lesson] I draw all kinds of triangles on the board, and I say, 'now we'll prove the theorem for this triangle, then we'll prove it for that one, and then...' And what do I start with? With the acute triangle [the prototype].

S8-690 T2: From the easy one to the difficult…

S8-691 Rose: We, ourselves, have to let go of it.

S8-692 T3: Why should it be easy and difficult? Why can't an obtuse triangle be easy as well?

S8-693 T2: Because it's harder for *us* to see.

S8-696 T3: Exactly, that's why... because we cause fixation…

S8-734/740 T4: Should we teach them habits, but without fixating them? […] The question is whether fixation doesn't help them to think and solve things. Because if it was abstract then they would not even know... The fixation is not always a bad thing.

This representative discussion, where teachers share issues of teaching mathematics and reflect on their classroom practices, aligns with the PD agenda and shows evidence of change in Rose's actions. She seldom shared her opinion during this session, but kept asking key questions that steered the conversation into reflective places (Turns 247, 423), as she described in SRI-2:

I2-146 Rose: I led this discussion. I repeated things [...] teachers said…

I2-148 Rose: I listened, it was important for me to ask, one more time, 'Why, in your opinion, was the problem difficult?' And even when I got three answers, I asked again, 'Why do you think ...?' Maybe someone else will explain. […] I think I led it [the discussion] in a way that the teachers felt very comfortable [...] and participated, and shared [...] I don't feel at all that I was dragged along, but really... I actually led it, I led it.

Rose mentions numerous facilitation actions she initiated: repeating, re-voicing, listening, probing, engaging more teachers. She frames these actions as *leading*, a verb that was not used at all in the early session's data. At this stage, her facilitator identity is more crystallized: she sees herself as a leader whose role is to involve teachers in meaningful peer-discussions. This change in identity can also be found when comparing Rose's answers in the pre- and post-questionnaires to the question: *In your opinion, is there a difference between a good teacher and a good facilitator?*

> **Pre-questionnaire:** A teacher is supposed to transfer knowledge from scratch and instill it in the students' heads. The facilitator is mainly supposed to listen to the teachers' comments. A teacher is usually at the center and she is the source of knowledge, and a facilitator should not be at the center or the source of knowledge.
>
> **Post-questionnaire:** Usually, the teacher is the source of knowledge in the lesson. However, the facilitator does not hold the knowledge exclusively, rather, the teachers and the facilitator share it. A good facilitator is supposed to raise important and interesting topics for discussion, and from this point she is supposed to conduct the discussion properly: ask questions, ask for clarifications, link ideas, etc.

While in the pre-questionnaire Rose interpreted her new practice as, again, "mainly listening", after one year she perceives it as an active role that aims to promote collective reflection. She now sees knowledge generation during the PD as a mutual responsibility of all participants, and there is also a nuanced change in how she views her role as a *teacher* in that matter. The next quote, from SRI-2, suggests that towards the end of the year she had more capacity to intertwine these two identities:

> I2-123 Rose: It was the first time I felt so comfortable that I did things I wasn't exactly planning […] it was spontaneous, like when I feel very comfortable with my class […]. For example, when we talked about prototype figures, I asked them: if I gave you the figure like this, would it be easier for you? [Turn S8-423]. That's an idea that popped at that moment, I didn't think of it before, but it's something that if I was in class, of course I would ask.

Rose's ability to use her teaching resources while facilitating suggests that she no longer sees her teaching and facilitating practices as mutually exclusive, and benefits from their combination: using her experience as a teacher increases her flexibility and comfortableness during the PD session.

Summary

The aim of this case study was to describe a novice facilitator's challenges in fostering teachers' collaboration, how they were connected to her multiple identities, and how they were addressed by the PD program. Two prominent challenges were demonstrated: planning the session in accordance with the PD agenda, and managing a discussion where the teachers can reflect and collaborate. The program's offered support was presented, followed by a portrayal of the last PD session and a later SRI that revealed changes in her practice and identity, and changes in teachers' participation. The results suggest that novice facilitators who do not understand their leadership role in the context of the PD program they provide, and have trouble negotiating their identities as teachers, colleagues and facilitators (Knapp, 2017), can undergo a professionalization process and become proficient facilitators who lead reflective, collaborative discussions. With regard to the claim that good teachers do not necessarily have the ability to be good facilitators of other teachers' learning (Even, 2005), the results shed light on *how* this ability can be developed and learned: the mentoring sessions, as well as Rose's participation in this research and in bimonthly facilitators' meetings (which were not analyzed for this paper), provided opportunities to *reflect on her practice* with her mentor, her peers and on her own. The use of reflection as a tool for *facilitators*' professional growth aligns with Coles'

(2019) claim for coherence in programs' agenda both at the PD level and at the facilitator level. Indeed, the mentoring session included also *modeling* of fostering reflections. These reflections, that probably intensified Rose's professionalization process, allowed the consolidation of her facilitation identity and led to noticeable changes in her actions. She did not only structure the sessions according to the program agenda and extend her repertoire, but was also able to draw on her teaching resources and be more adaptive towards the end of the year. This suggests that conflicts between identities should not be resolved with one identity taking over the other, but rather, holding both identities can enrich practice and keep the facilitator practice-oriented while working with her colleagues.

In conclusion, the program's support team is a key factor in assisting facilitators to overcome challenges. The mentors themselves have multiple identities: they are part of a project established in academia, but they are also experienced teachers and emerging facilitators. Thus, they can create bridges between the research-based development with its intended goals, and the implementation in local contexts by teachers. I would recommend such a support system, and argue that it should make use of tools such as journals and SRIs to detect challenges, and should offer solutions that strengthen facilitators' agency and align with the program's spirit. Paraphrasing Simone de Beauvoir's words, "one is not born, but rather becomes, a facilitator", I claim that PD programs can and should guide novice facilitators during their "becoming" process.

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ROLES OF FACILITATORS AND TEACHERS IN MODELS OF TEACHER PROFESSIONAL LEARNING

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Three models of teacher professional learning are analysed using an analytical framework for studying the roles of facilitators and teachers in their collaboration. The PRIMAS model, the Lesson Design Study model and the Extended Interconnected Model of Professional Growth are analysed with respect to the agency involved and the role of teachers and facilitators, and two issues to be tackled in our future work are identified.

In this paper we contribute to the discussion in Theme C: Roles, identities and interactions of various participants in mathematics teacher collaboration. In Theme C, one of the important issues concerns the role of the facilitator and the nature of interactions between the facilitator and the teachers. There are three rationales for us to contribute to Theme C. Firstly, being a teacher (in Norway/the Western world) has changed from a highly individual approach where each teacher worked totally on his/her own behind closed doors to a more collaboative approach where teachers are encouraged to work together. Influenced by the East Asian lesson study models, researchers and teacher educators are seeking to collaborate with practicing teachers in ways that demand more openness and collaborative approaches (e.g., Vangrieken, Dochy, Raes, & Kyndt, 2015). Secondly, some fundamental questions of mathematics education have long been discussed at least as long back as antiquity, yet it has still remained difficult to arrive at a generally shared conception of these questions. For instance, what is mathematics, how does one learn mathematics, what does it mean to learn mathematics and how is it best learnt? (e.g., Wittmman, 1995). Thirdly, problems identified and disappointing results in national and international tests have been increasingly addressed by policy and research. However, new reforms in mathematics education are 1) not immediatlely embraced by teachers, and, partly as a consequence of 1), 2) not implemented in schools. This gap between research and practice has been identified and addressed by several scholars (e.g., teachers' resistance to change in Valoyes-Chávez, 2019), and several models have been developed to try bridging the gap (e.g., see models below).

In this paper we focus on the following questions: 1) What is the role of facilitator/teacher in supporting teacher collaboration? 2) What are the characteristics of a good facilitator of teacher collaboration? In the following section, we firstly give a brief description of the teacher professional learning models that we focus to study and discuss in this paper. Next, we briefly explain the analytical framework by Boylan, Coldwell, Maxwell and Jordan (2018) and the use of this framework in the analysis of the selected studies. Finally, we discuss the initial findings of our analysis of the selected studies and propose further issues to be tackled in our future work.

The models and their purposes

From an initial literature review of studies of mathematics teachers' collaboration and learning across professional communities, we selected three models that have been conducted in different cultural contexts for a more in-depth analysis. The three models, designed for supporting and studying inservice teachers' collaboration and learning, are: (1) The PRIMAS model (Maaß & Doorman, 2013); (2) The Lesson Design Study (LDS) model (Ding, Jones, & Sikko, 2019); (3) The Extended Model for Teacher Professional Growth (EIMPG) (Coenders & Terlouw, 2015).

Maaß and Doorman (2013) present a complex PRIMAS model about how to implement and scale up Inquiry-based learning (IBL) in day-to-day teaching in their international project PRIMAS that involved 14 universities across 12 European countries. The term IBL refers to a teaching culture and to classroom practices in which students inquire and pose questions, explore and evaluate. Their study addresses the necessity of designing a model for dissemination and implementation that both addresses core principles of IBL and has the flexibility for implementing and scaling up professional development in various national contexts.

Ding et al. (2019) examine the interconnectedness and difference between action research, as depicted by members of the American Educational Research Association's Action Research Special Interst Group (Rowell, Polush, Riel, & Bruewer, 2015), and a lesson design study conducted within a Teaching Research Group (TRG) in Shanghai, China. One of the contributions of the LDS is its focus on the teachers' implementation of the reformed textbooks and theoretical ideas (e.g., teaching with variation) through designing and acting on the targeted-theory-based teaching (Ding et al., 2017; Ding & Jones, 2018).

Coenders and Terlouw (2015) use Clarke and Hollingsworth's (2002) interconnected model of professional growth as a starting point to analyse teachers' changes in pedagogical content knowledge and beliefs during the implementation of a new content-based chemistry curriculum in The Netherlands. Two groups of teachers were involved in the research project: one group was involved in both the development and the enactement of new curriculum materials, whereas the other group only enacted the material without being involved in the curriculum development. The analysis necessitated an extension of the interconnected model of professional growth to include an additional domain, namely the Developed material domain.

An analytical framework for studying the roles of facilitator/teacher in professional learning

The analytical framework by Boylan et al. (2018) is based on a critical analysis of a number of models of theorising the nature and process of teachers' professional learning. Their framework focuses on categories of model components, scope, explicit and implicit theories of learning, location of agency and philosophical paradigms. Given the questions of this paper, we chiefly concentrate on the category of location of agency, together with the interrogatory question given in Boylan et al. (2018) for a further discussion about this category. *How is agency conceived within the programme—is it focused on individual teacher agency, or does it include broader conceptions?* In Boylan et al. (2018), the issue of agency is of the theoretical account of the agents that instigate or produce change processes in teachers' professional learning. Noticeablly, Boylan et al. (2018) pointed out that the models they studied did not include an explicit discussion of agency. That is, it is important to know that the particular views of agency in their selected models are implied by Boylan et al. (2018). For instance, Boylan et al.'s (2018, p. 130) view of agency in Clarke and Hollingsworth's (2002) interconnected model is teacher agency as central to process; students as actants. Agency in Opfer & Pedder (2011) is viewed as an emergent property of the system, arising out of relationships rather than a property or quality of individual elements of the system. Boylan et al. (2018) suggest that apart from teachers, there are other agents, for instance the role of learners, professional development (PD) facilitators, and others not directly involved in PD programs such as school leaders. For an initial analysis of the roles of different agents in the selected models, we mainly refer to two theoretical terms according to two approaches to theorising agency and their relationship to professional learning (for details of the references of the two theoretical approaches see Boylan et al., 2018):

- Teacher agency: individual teachers' roles, as 'active learners shaping their professional growth through reflective participation' (e.g., Clarke & Hollingsworth, 2002, p. 948). Sociological theories of agency are adopted, which focus on agency as individual action within social contexts.
- 'Actant': the role of materialities such as texts, tools, technologies, bodies, actions and objects (Fenwick et al., 2012). Sociomaterial and socialcultural theories are utilised, in which agency is treated as a product of sociomaterial relationships (e.g., Fenwick et al., 2012).

In the following sections, we briefly analyse the compoents and scopes of the three selected models for helping readers to gain general information of the features of the studies. Mostly, we focus on studying in depth the conceptions of agency implied in the selected models according to the theoretical account of the agents above. In particular, we are concerned with the following questions in the analysis: *Are facilitator/s and teacher/s posited in the centre of the secleted models*? *If not, what is emphasized*? *What are the characteristics of a good facilitator?* Finally, we propose further issues for studying the characteristics of a good facilitator in our future work of teacher collaboration.

Analyzing the roles of facilitators/teachers in the three selected models

Given the questions of this paper, in this section we illustrate our initial analysis of the roles of teachers or other agents emphasized in the selected models, with a focus on the theoretical models of teachers' professional learning and location of agency in the models.

Model 1. The PRIMAS model

Components and relationships. Maaß and Doorman (2013) highlight the interative character of four research cycles for scaling up the IBL-PD model across 12 European countries: Cycle 1 is of checking the consistency and practicality by members of the design team (all countries); Cycle 2 also includes checking the relevance by a group of international experts; Cycle 3 adds the factor of the practical, context-specific adaption of the overall model to the national requirements and a walkthrough was carried out with a group of target users, institutionalized within a so-called National Consultancy Panel (NCP) in each country (comprising, for example, heads of schools, teacher educators, school authorities); and Cycle 4 focuses on the evaluation of how the implementation of the national adaptations of the common international model were implemented in the partner countries. It is worth noting that a main design feature of the PRIMAS model is that it does not strive for ''context-free'' claims; rather, it sees context as central to its conceptual terrain (Kelly, 2006).

Scope. Each of the 12 countries had the goal of educating a minimum of 20 multipliers who in turn were to educate at least 100 teachers within the years 2012 and 2013. The multipliers' education was scheduled to take place in 2011 so that they would be ready to start their work as multipliers at the beginning of 2012.

Location of agency. Maaß and Doorman (2013) present three aspects of PRIMAS model and the complexity of its theoretical basis. Firstly, the term "teacher professional development" (TPD) relates to changes in the teachers' professional knowledge and competence. A spiral model including three main phases of analysis, implementation and reflection are repeated over the long-term. Teachers are expected to gradually develop their teaching practice towards IBL through the spiral model. Moreover, the intimate relationship between teacher agency and students as actants is emphasized in the PRIMAS model implementation and dissemination. This emphasis is clearly showed in Maaß and Doorman's (2013) reference to Putnam and Borko's (2000) recommendation, that teacher educators should treat teachers the way they expect teachers to treat students ("teach what you preach").

Secondly, the PRIMAS project developed the strategy of educating facilitators (called 'multipliers' in the project) through university-based TPD courses for scaling up the IBL with multipliers. That is, the characteristic of a good facilitator is the fact of being able to learn and collaborate with researchers and teacher educators in the PRIMAS project designed and implemented university-based TPD courses; and then, of being able to play as the project's multipliers who disseminated their knowledge to teachers. Multiplier education in PRIMAS followed the Müller model (2003) (for details of the reference see the article), which consists of three strands: learning-off-job (multipliers experienced the modules designed for the TPD courses themselves through university seminars), learning-by-job (multipliers worked in pairs, had project team members observe their professional development courses) and learning-on-job (giving multipliers literature to work with and by encouraging them to reflect on their competences in running professional development courses and their needs in further education). However, the overall effectiveness of the Müller model during the project period is not yet clear. For instance, which aspects of the model worked in which context and how did it work to support changes of multipliers and then to teachers.

Finally, the PRIMAS project adds one more layer of theoretical account to teacher professional learning - Dalton et al.'s (2007) socio-ecological system levels of supporting individual teachers in a complex system: micro-systems (e.g., family, colleagues); meso-level (e.g., organizations such as schools and localities such as the neighborhood and town in which an individual lives); and macro-system level (e.g., cultural, political, economic and structural factors). Maaß and Doorman (2013) explain that all these systems influence what teachers do in their classrooms and should thus be taken into account (using the term in Boylan et al., 2018, the actants) when aiming at a widespread implementation of innovative pedagogies.

Model 2. The LDS model

Components and relationships. The LDS (Ding et al., 2019) aimed at concurrently developing teachers' professional knowledge, beliefs, and identity through three teaching cycles of the LDS model. Each of the teaching cycles included multiple dimensions of targeted learning through lesson design, lesson implementation, TRG discussion, and reflection. The LDS model particularly attends to the teachers' targeted professional learning through the multiple layers of action and reflection through the three cycles (for details of the three cycles see below). It should be noted that the cycles in the LDS model are overlapping to indicate the accumulation of 'wisdom of action' through teacher action and reflection that the LDS aims to study closely in order to foster it within TPD.

Scope. The LDS was conducted in an international school (Grades 1–9) in the west suburb of Shanghai from 2013 to 2015. Seven Keli (exemplary lesson) topics were selected from Shanghai elementary mathematics textbooks. The choices of these Keli activities were based on a need at the time to foster teachers in the school to make a shift from traditional skill-based lecture pedagogies to a more student participation-oriented pedagogy so as to be able to teach well using the reformed textbooks. The LDS community included the participant groups of a number of researchers (regional, national, and international), two expert teachers (external teachers who are specialists in their teaching and provides in-service teacher education in their school district), and seven Keli mathematics teachers from the mathematics TRG in the elementary section, together with the head of the primary division of the school.

Location of agency. The overall approach utilised in the LDS project (Ding et al., 2019) is 'Action Education' (AE) (Xingdong Jiaoyu in Chinese) model (Gu and Wang 2003), that emphasises two fundamental ideas underpinning in-service TPD in China: (1) simultaneously to emphasise two dimensions of teacher learning in TPD, namely peer coaching among teachers in TRG (the horizontal dimension, or width, of teacher learning) and an expert's mentoring that provides theoretical and professional guidance (the vertical dimension, or depth, of teacher learning); (2) simultaneously to use Keli (a Chinese term for exemplary lesson development) in the TRG and address the whole process of teachers' reflection on professional learning through the AE model.

The first cycle of the LDS model focuses on the possible gap between the LDS teachers' own belief and assumptions of mathematics and its teaching and learning, on the one hand, and the targeted theoretical ideas and its implementation to be introduced to teachers in the re-designed and re-implemented lesson in the second cycle on the other hand. Teacher agency is central to the first cycle.

In the second cycle of the LDS, the researcher and the expert teachers played a significant mentoring role in supporting the teachers' deep learning of, and reflection on, the targeted theoretical ideas and implementation (the vertical dimension, or depth, of learning in Gu and Wang (2003)). That is, in this second cycle, the teachers were expected to learn, and to use, the targeted theory and teaching methods to re-design the lesson tasks. That is, in the transition from cycle 1 to cycle 2, teachers' 'wisdom of action' is emphasised through their reflective teaching experiment of the targeted theory. Here, the researcher and the expert teachers play an important role as actants to support teachers to reflect in learning the targeted theories and to make changes in teaching action.

The third cycle focuses on the teachers' effort in making changes to their teaching practice, and their reflection on their action changes from cycle 2 to cycle 3. Here, the expert teachers played a significant role in mentoring the teachers' 'wisdom of action' – in guiding them to improve their classroom action with a targeted theoretical idea and to learn methods of reflection on their action changes and develop their teacher identity over the period of LDS project.

Model 3. The extended model of teacher professional growth

Components and relationships. Coenders and Terlouw (2015) point to the crucial role of teachers in implementing curriculum changes in practice. For the new chemistry curriculum in the Netherlands, two aspects of the way of teaching were considered new and innovative, that of the context-based approach and the focus on cooperative learning. A context-based approach means taking a context that is known and appealing for students as a starting point for learning. To accomplish a change towards such an approach, teachers need to change their practices and beliefs with respect to materials, pedagogies, assessment, as well as purposes and goals of science teaching and learning. According to Coenders and Terlouw (2015) these changes in knowledge and beliefs about science and science teaching can be identified by changes in five components of pedagogical content knowledge (PCK) (Shulman, 1987), viz. orientations toward science teaching, knowledge and beliefs about the science curriculum, knowledge and beliefs about instructional strategies, knowledge and beliefs about students' understanding of specific topics, and knowledge and beliefs about assessment.

Scope. Two different groups of teachers were focused in the research. The first group consisted of 3 teachers and a mentor.This group was supposed to develop new learning material and also enact this material in their own teaching. The three teachers and the supervisor developed new student learning material during a time frame of 7 months. During the 7 months, the group met for nine sessions of 3-4 hours. Between the sessions, email was used for exchanging ideas and inform each other. After a teaching module was completed, the three teachers enacted the material in their own classes. Based on their own experiences with the material and group discussions amongst them, the material was revised before it was released for use by other teachers. This process took one year in all. The other group consisted of 5 teachers. This group was not involved in the development of the material but were supposed to enact the new material in their classes. They were invited to a half-day workshop where the teacher-developers explained the rationale behind and the use of the material from their own experiences.

Location of agency. The macro level comes into play concerning the introduction of a new (national) chemistry curriculum for upper high school in The Netherlands. The outcome of the professional development was new learning material for students, where the new elements concerned the content in the form of context-based teaching, and it concerned the teaching method in the form of collaborative learning. Both aspects where supposedly new to the participating teachers. The meso level is involved in the form of teachers in the context of a professional development programme. For the first group of 3 teachers this involved collaboration between themselves but also being supervised by a mentor, with work on the new material taking a year to complete in the sense that it would be ready to be used by others. The other group of 5 teachers who were only supposed to enact the new material without having been part of the planning where only admitted a half day workshop to familiarize with the material and the rationale behind it before they were supposed to enact it in their own clases. The micro level is appearant in the moment to moment learning experiences, with both groups of teachers, e.g, in their enacting of the material in their own classes with their students. The results concerning the group of teachers' professional growth were found to be clearly different. For instance, it was found that the second group of 5 teachers did not enact the material as intended. In terms of teachers' professional growth, the first group of 3 teachers experienced changes in PCK both during the development phase and during the enactment phase of the programme. The second group of 5 teachers did not change their knowledge and beliefs according to the new intended approach but rather changed the material to align with their previous knowledge and beliefs. Noticeably, however, the role of the mentor (facilitator) with the first group of 3 teacher-developers is not elaborated in the article.

Discussion and Conclusion

In this paper, we focus on studying the roles of the facilitators/teachers and the nature of interactions between the facilitators and the teachers emphasized in the three selected models. A summary of the main findings of our initial analysis of the selected models is shown in Table 1.

Table 1: A view of agency in the professional learning process

Model	Roles of facilitator/teacher and others in the model of learning	Characteristics of a good facilitator
PRIMAS	Educating multipliers is central to develop teachers' knowledge and competence for IBL-based teaching. Context is central in conception of the IBL-PD models. The intimate relationship between teacher agency and students as actants is addressed in the spiral model.	Multipliers: Being able to learn and collaborate with researchers and teacher educators in the PRIMAS project designed and implemented university-based TPD courses; and then, being able to play as the project's multipliers who "multiplied" their knowledge to teachers.
LDS	Teacher agency as central to process, with an emphasis on teachers' reflection and action on the targeted theories; the teacher-led-and-student-centred learning approach is valued in the Chinese classroom settings.	Expert teachers: Being able to lead teachers into deep professional learning (e.g., providing theoretical and professional guidance); to use Keli to support teachers to develop reflective teaching experiment of the targeted theory; to cultivate teachers' 'wisdom of action' and teacher identity.
EIMPG	It is central for teachers to take part in the development of new material, thus gaining ownership to it, for new curricula to be successfully implemented	Developing new material over an extended period of time in a network, including trying out new approaches with subsequent reflections and revision

Based on our findings in Table 1, we propose two further issues to be tackled in our future work:

- Developing a more holistic way of thinking of the relationship between theoretical framework of teacher professional learning and teachers as living beings in their professional life course.
- Developing simultaneously both theoretical and practical principles for designing TPD courses of preparing and supporting facilitators.

Firstly, our findings confirm Boylan et al.'s (2018) observation that the models selected omit an explicit discussion of theorization of agency in their models. For instance, Maaß and Doorman (2013) presented three layers of theoretical ideas and models of teachers' learning and collaboration (Putnam & Borko, 2000; Müller model, 2003; and Dalton et al., 2007), which shows a complicated mixture of both sociological theories of agency and that of sociomaterial and sociocultural theories discussed in Boylan et al. (2018). Ding et al. (2019) highlighted the idea of 'wisdom of action' (Gu & Wang, 2003) which is rooted in the ancient Chinese classic learning theory of Zhi-Xing-He-Yi, yet has not yet been identified as an alternative theory of agency in the western literature. It is interesting to note that Boylan et al. (2018) highlight the more or less anthropomorphic concept of agency. For us, it is essential to address teachers as living beings who not only work but also live through the period of any professional development. Thus, it is important that teachers should not only be treated as learners the same way as they treat their students in their professional learning process, but also be regarded as vivid living beings and be involved in empathetic interactions with the professional knowledge and experiences. Moreover, it should not oversimplify the complexity and dynamic social pattern of teachers' professional learning of knowledge and action (Ding et al., 2019). This leads to the second issue that we pursue in the future.

The commonality addressed in the three selected models we studied above should be noted: the concrete TPD modules given in PRIMAS, Keli designed and implemented in LDS, the involvement of teachers in curriculum materials development in EIMPG. That is, the question of how to make the tacit and abstract nature of professional knowledge (e.g., theoretical principles of IBL, teaching with variation) practical to teachers should be central in designing TPD courses for preparing facilitators. In so doing, the design principles can contribute to a more scientific approach to TPD with the nature of being context-free, and provide solutions to overcome the challenges of teachers' resistance and subjectivity in reforms across the world.

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NEGOTIATING DUAL ROLES IN TEACHER COLLABORATION THROUGH LESSON STUDY: LEAD TEACHERS' PERSPECTIVE

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Lead teachers play a vital role in supporting a successful teacher collaboration. Yet, relatively little is understood about the way in which leading teachers negotiate their roles and tensions or challenges they faced to support teacher collaboration. This paper seeks to examine the dual role of lead teachers as a numeracy coach or curriculum coordinator and a member of a planning team and explore the extent to which the diversity of the different leaders' experience and knowledge contribute to teacher collaboration in the context of Lesson Study. The analysis is based on field notes of planning meetings and three interviews carried out before and after the first and the second Lesson Study research cycles.

Research on mathematics teachers' collaboration and its role in advancing teacher professional learning has been well documented (Darling-Hammond & Richardson, 2009; Timperley, Wilson, Barrar, & Fung, 2007). Collaboration in itself does not necessarily lead to teachers' learning and change. Opfer and Pedder (2011) underlined the need to have a holistic view of various dynamics involved in teacher professional learning and argue that much more sophisticated conceptualisations are required in order to take account of the "various dynamics at work in social behavior and [how] these interact and combine in different ways" (p. 378).

Robinson and Timperley (2007) reported tensions faced by leaders in establishing collective responsibility in schools where the norms of teacher autonomy and the privacy of classroom practice are strong. Vangrieken, Dochy, Raes, and Kyndt (2015) also stated a similar point

> the long-standing culture of teacher isolation and individualism, together with teachers' preference to preserve their individual autonomy, may hinder deep-level collaboration to occur [while] … Critical reflection on and discussion of teaching practice seems to be rare which hampers the possibilities for teachers' collaborative learning. (Vangrieken et al., 2015, p. 35)

In the context of teacher collaboration, lead teachers and the leadership team at a school play a vital role in supporting the process of teacher collaboration and providing opportunities for teachers to 'break' the boundary of privacy of classroom practice (Robutti et al., 2016). Yet to date, relatively limited empirical evidence is available regarding the perspective and insights of lead teachers to understand how they negotiate their roles and overcome tensions and challenges in participating and facilitating teacher collaboration.

In this paper we report on the role of mathematics teacher leaders when implementing Lesson Study in a network of three primary schools in Victoria, Australian. In particular the paper will address the following question: How do lead teachers negotiate their dual roles as both teachers and facilitators of peer-collaboration in Lesson Study? (Theme C, ICMI Study 25).

Teacher Collaboration through Lesson Study

Lesson Study is highly valued as a platform for teacher-led professional learning which centre on teachers collaborating with their colleagues to plan, observe, and reflect on classroom teaching and learning as a community (Lewis, Perry & Hurd, 2009; Takahashi & McDougal, 2017). Features of Lesson Study such as collaborative and in-depth planning of teaching materials, ongoing inquiry of students' mathematical thinking, and critical reflection of teacher practice facilitated by an expert, all play an important role in facilitating teacher professional learning and growth.

The findings from a survey conducted by Robutti et al. (2016) revealed that teacher collaboration "takes many forms and involves different groups of people with differing roles" (p. 680). They underscored the value of diversity among group members. However, Akiba, Murata, Howard, and Wilkinson (2019) pointed out the challenges of negotiating the tension arising from having a learning community of teachers from diverse backgrounds, beliefs, and values.

Lesson Study does not rely on a teacher leader to facilitate the process. It is driven by teachers who play a central role in researching classroom practice and exploring ways to improve student learning (Takahashi & Yoshida, 2004). Leadership is provided through the involvement of the 'outside expert' in the research lesson observation and the subsequent post-lesson discussion. The 'outside expert' is the final speaker in the post lesson discussion providing a summary of the discussion as well as their critique of the task and its relationship to the learning goal, and the enactment of the task to achieve the learning goals.

Role of Lead Teacher and Leadership team in Teacher Collaboration

Studies of lead teachers of mathematics have focused on their knowledge and role in providing professional learning for teachers at their school (Borko, Jacobs, Koellner, & Swackhammer, 2015) or for schools in a district, mentoring and coaching individual teachers (Jackson et al., 2015) or facilitating and leading collaborative professional learning or planning teams (Grootenboer, 2018). In each of these roles productive leadership connects the discussion and work with teachers to their context(s) that is, their students, parents and community (Jackson et al., 2015; Lingard, Hayes, Mills, & Christie, 2003). Lead teachers actively listen and encourage intellectual debate and discussion about the mathematics, the goals for learning and teaching, students' thinking and teacher actions (Martinovic & Elkord, 2018; Lingard et al., 2003; Borko et al., 2015). They promote critical reflection and facilitate research of their own practice (Grootenboer, 2018; Borko et al., 2015). To support collaborative planning and reflection, Masters (2010) explained that teacher leaders jointly analyze samples of student work, co-plan, co-teach, review their teaching and celebrate the professional learning. These actions of collaborative work are typical of collaborative practices in Lesson Study. Effective leaders of mathematics develop a culture of high expectation (Martinovic & Elkord, 2018).

In their project to upscale mathematics leaders' support for teachers' learning, Jackson et al. (2015) articulated specific goals for leaders' learning which included designing support for teachers' learning and reflection and using talk moves to differentially press teachers' thinking and reflection. They found that their system leaders needed more support on questions to use in order to press teachers' understanding of the mathematics content, analysis of classroom video clips of students' thinking, the teacher's actions, and the pedagogical approach of ambitious instruction. In their design of professional learning for leaders to conduct collaborative professional learning, Borko et al. (2015) modelled the design of learning goals for teachers, anticipating students and teachers solutions for the mathematics tasks, and planning questions and prompts to use with teachers when working on the problem task and analysing video excerpts of enactment of the task in a lesson at their school. They found that the collaborative learning of mathematics leaders enabled them to deepen their mathematics content knowledge, develop their knowledge of content with respect to students and curriculum and to better anticipate teachers' thinking and be prepared with questions that press teachers to justify their thinking, and support teachers to connect their ideas.

Lewis (2016) reported on learning of two novice mathematics teacher leaders over 18 months as they implemented Lesson Study in their schools with the mathematics teachers. Neither leader had participated in a Lesson Study prior to implementing Lesson Study in their school. They found that the lead teachers lacked strategies to engage teachers and that they spent too much time on doing the mathematics and not enough time on planning the lesson. They expressed concerns about managing the process, especially the amount of time needed. They found themselves acting as a participator rather than a leader in the planning meetings and valued the observation of the research lesson and post-lesson discussion for their learning about what teachers noticed and learned through the process. Lewis (2016) also reported tensions experienced by teachers who acted as facilitators when they tried to step back and when to be directive and revealed that the facilitators were able to manage this tension more productively over time.

The Project

The implementing structured problem-solving mathematics lessons through Lesson Study project was carried out from 2012 until 2014. The project involved 3 schools in a network of schools from a metropolitan region of Melbourne, Australia. Teachers and the lead teachers worked collaboratively in two cross-school Lesson Study planning teams self-named as the Bobbies and the Matomes. Each team included five people comprising a Year 3, Year 4, and Year 3-4 classroom teacher, two leading teachers with a diversity of teaching experience and expertise and supported by two researchers. Each Lesson Study cycle involved each team of teachers planning a research lesson on the same topic, one member of each team teaching the co-planned research lesson in front of observers, and both teams participating in the research lesson observation and the sub-sequent post-lesson discussions.

Following an initial whole day workshop about Lesson Study led by the researchers, four planning meetings were conducted by each team for each Lesson Study Cycle. At the first meeting in the first cycle, one of the authors conducted a lesson using the task for the first cycle to model the lesson structure and teacher actions, and to enable the teachers to solve the problem. For the remainder of the first meeting and the other three meetings in the first cycle the planning teams collaborated to plan the research lesson. Each team developed their own Research Lesson plan. Similarly, in the second cycle the teachers solved the problem themselves in the first planning meeting. One member of each planning team taught the research lessons in each cyle in front of observers including key staff at each school, and other professionals. Both planning teams and observers participated in the subsequent post-lesson discussions.

Data and Methods

The project adopted a Design-Based research approach (Design-Based Research Collective, 2003) as teachers and the research team worked closely together to implement innovative forms of learning through Lesson Study. However, this paper presents a case study (Yin, 2009) of the lead teachers.

Drawing on field notes and semi-structured interview data from the four lead teachers (see Table 1), we will examine how they negotiated their roles and overcame tensions and challenges they faced as leaders in teacher collaboration through Lesson Study. Field notes were collected for each planning meeting for both Bobbies and Matomes. Three audio-recorded interviews were also carried out with each of the participating classroom and leading teachers, as well as the network numeracy coach, who also took part in all of the activities. All the interview data were transcribed and the field notes and interview data were subjected to an iterative, thematic content analysis (Keller, 2013).

Lead teacher participants

Four lead teachers of mathematics participated in both Lesson Study cycles (see Table 1). Paula, the mathematics coach for the network of 22 primary and secondary schools, played a vital role as a 'broker' by inviting the numeracy coaches and the leadership team from the schools in the region to be involved in the project. She provided regular professional development for the numeracy coaches and mathematics leaders in the network. Megan had been the numeracy coach at School C for 3 years, prior to which she had been a classroom teacher. She observed teachers' lessons, conducted coaching conversations with teachers and taught demonstration lessons. Paula and Megan were members of the Bobbies Lesson Study team. Narah, the curriculum co-ordinator at School B, had previously held the position of numeracy coach at School B. George was the numeracy leader and coach at School A. Narah and George were members of the Matome Lesson Study team.

Table 1: Lead teachers participants

Name (pseudonyms)	Role	School	Years of experience	Planning team
Paula	Regional coach	-	23	Bobbies
Megan	Numeracy coach[1]	C	7	Bobbies
Narah	Curriculum Leader	B	6	Matome
George	Numeracy leader	A	7	Matome

Collaborative practice and planning varied across the three schools. At Megan's school, teachers at each year level collaborated to plan the sequence of topics. At Narah's school, teachers used formative assessment data to plan the sequence lessons for each topic but there was less evidence of collaborative planning. The professional learning teams at School A collaboratively planned their lessons giving attention to differentiated learning. George facilitated this planning, providing professional reading and resources to support task selection and setting of learning goals.

Findings and Discussion

The role played by the leader teachers in their Lesson Study teams are described before discussing the challenges of their dual role as teacher and leader.

Dual roles of the lead teachers

The mathematics teacher leaders did not assume the role of leader or facilitator in their Lesson Study planning team meetings. Rather they contributed ideas, posed questions and challenged the other team members to reconsider their suggestions. In Matome's planning meetings, George was most provocative when the team was discussing the learning goals, drawing their attention to

[1] Numeracy coaches are experienced teachers who provide curriculum leadership in mathematics teaching by working with individual teachers and teams of teachers to improve teaching and student learning in mathematics.

distinguishing between overarching learning goals and goals specific to the lesson. Narah searched the curriculum documents during these discussion to provide options for the teachers to consider. She also focused on the language and checking mathematics knowledge: “Is algebra a concept or a strategy?” George and Narah were also active when the team was discussing the structure of the lesson, especially the whole class discussion of solutions. George wanted to ensure that “After the sharing – pointing back the board with all the solutions – the penny might drop for the additive thinkers that multiplication works.” Narah focused on the questions, probes and prompts that teachers might use when interacting with students and to orchestrate the whole class discussion such as “What do you notice?” and “Can you see a problem here (i.e., with counting all)? Can you show me another way?” As a consequence of raising the importance of providing enabling and challenging prompts, the Matome team listed prompts and questions for each of the anticipated student solutions in the Research Lesson Plan. These questions and prompts were used by the teacher enacting the Research Lesson. Narah saw her leadership role in the school involved “knowing how to resource the school, to support teachers in doing that and knowing how to guide them, to want to go and research that sort of thing, before teaching.” (Int2). This is consistent with earlier findings reported by Borko et al. (2015) and Masters (2010) about the importance of setting clear goals, expectations, and planning for prompts.

Initially the Bobbies team in the first cycle were concerned with the wording and context for the task (The Matchstick Problem, Healy & Hoyles, 1999). Paula in her role of as the regional coach would ensure teachers in the planning team reflected on their experience by asking “Why are we doing this?”. She challenged whether or not the use of concrete materials would support students to test and justify their solutions. The teachers in both Lesson Study teams were keen to trial the lesson study with a different problem solving task and Paula provided a task that teachers from both teams used so that they could trial a longer period of time for discussion student solutions at the end of the lesson. Megan was conscious of supporting the teacher who would teach the Research Lesson in front of a large audience. So she arranged to co-teach the Research Lesson to a different class at School C in order to test their draft of the Research Lesson Plan. Megan also visited School A to support the Matome’s teacher to trial the Research Lesson with another class in Cycle 2. Megan and the teachers reflected on their trial at the final planning meeting of their respective Lesson Study teams. She highlighted the benefit of the trial process in the third planning meeting in Cycle 1:

> Trevor and myself and Lynn were all in the same room doing the lesson at the same time so it was good to sort of bounce off each other and say yep that makes sense for that students’ work to be spoken about first, second, third, fourth and so on (Megan, PM3, C1)

This experience was instrumental for the professional growth of the teachers (Widjaja, Vale, Groves & Doig, 2017) and the lead teacher played a vital role as catalyst to ‘break’ the boundary of privacy of classroom practice (Robutti et al., 2016). Lewis (2016) also reported the lead teachers managed their dual roles more productively over time by putting themselves as members of the team and giving teachers more agency to figure things out on their own.

Navigating tension and expectations

All lead teachers experienced tensions at various stages of Lesson Study process. At the very beginning, the lead teachers expected the research team would lead and provided explicit direction during the planning of the research lessons. Both planning teams struggled to articulate the goals for the lesson in the first two planning meetings. Hannah expressed her frustration and said “there was no guidance and it was a bit frustrating but sometimes that’s the best way to learn”. The breakthrough came after both planning teams decided to trial a similar problem to help them in formulating the goal of the lesson and anticipating student solutions.

Working together in cross-school planning teams with different levels of experience was challenging at the beginning. Both George and Megan mentioned the challenges associated with working with teachers with diverse experiences whom they had no professional interaction before.

> Working in a group of people that I don’t know as well, like Keith and Henry, I’ve never had anything to do with them before, so, to sort of bounce ideas off each other and not know where each other comes from and those kinds of things, I think was a bit challenging at first, but then, at the end, a positive because it takes in different aspects of different people’s knowledge. (Megan, Int2).

This tension was resolved in the next cycle when they develop mutual engagement and the level of trust. The challenges also came from making sure their leadership did not imply they dominated the conversation but allowed everyone in the planning team shared equal responsibility and contribution.

> It's a challenge to make sure that levels of experience and different roles don't allow you to perhaps dominate your thoughts and opinions and things like that because I want everyone in the group to make sure that they had their equal contribution. (George, Int 2)

Paula, the regional coach, did not see that having cross-school planning teams as a challenge and she said “you get the benefit too of yeah more than one mind, you know more than – and especially across schools and resources and across yeah background”. But as a very experienced coach who worked to provide professional learning for all numeracy coaches in the region, she expressed her dissatisfaction from being asked to step back “I was sort of being very much directed not to share, but I found that really difficult when they came to brick walls” (Int3). A similar tension was reported in Lewis (2016) where the lead teachers experienced tensions about when to step back or to be directive and jump in to support the teacher participants.

Hannah raised a concern about the vast amount of work for the school-based coaches to mentor teachers so that teachers in the school can can use a consistent practice. She was also concerned with teachers’ mathematical content knowledge and identified this as her major challenge “The hardest thing for me with teachers is getting them to understand that [the maths in the task] and then modelling to them how to find that out if I’m not sitting next to them.” (Int2). Jackson et al. (2015) identified the need for mathematics leaders to acknoweledge the different professional learning needs of teachers. When conducting their design-based research they found that leaders needed to differentiate the prompts used with teachers when planning structured problem-solving lessons to account for their knowedge, routine practices and different classroom context.

Support to overcome challenges

The lead teachers identified some common things but also different ways in overcoming challenges involved in teacher collaboration. Paula identified the benefit of having a school-based coach involved in the planning team “to support them within the school, through the leadership and was making sure that you just didn’t get swept under the carpet” (Int3). She highlighted the benefit of giving the planning team time to look closely at the students’ work collected during the research lessons and consolidate their plan as a team prior to the post-lesson discussion. According to Paula, this opportunity “takes it away from the individual teacher and all of a sudden it's a team planning”.

She also valued the contribution of the knowledgeable other and the critical friends who observed the research lessons. According to her, they "have done a really good job of modelling, how to be critically evaluating a lesson without being negative or nasty" (Int3).

Megan attributed the trial lessons with Lyn and Trevor as instrumental to overcome the challenges faced in earlier stages of planning and to build her confidence. She valued the role of Lyn and Trevor, the teachers from her school, as "a model teacher for the rest of the school" because "they're the ones that have sort of made it realistic in their classroom which is believable for the other teachers to know that it's realistic for them, as well" (Int3). Megan underscored the significance of planning as a group, rather than distributing planning as the normal planning practice in her school.

> So, getting conversations happening about one particular lesson, rather than one person doing all the thinking behind it … I think is really important. (Megan, Int2)

Narah highlighted the importance of having time out of the classrooms for her and members of the planning team considering "there's so many different priorities in schools these days and we don't have the set curriculum like in Japan" (Int2). George shared Narrah's view about the importance of time release and he identified ways to incorporate the in-depth planning involved in Lesson Study into the school structure. He commented that "at a team level we get 2 hours a week where we plan together and so if we structured those in a way that would suit this type of a process, I think we could easily achieve those goals" (Int3). While this comment indicated a direction to resolve the tension to find time for in-depth planning and ongoing inquiry of students' thinking and learning in Lesson Study (Akiba et al. 2019; Fujii, 2016; Takahashi & McDougal, 2017), the resolution to the issue depends on the support of the leadership team to provide the time allowance in the school structure. This challenge was noted by Fujii (2016) who said that the extensive time spent in planning in Lesson Study is often "under-appreciated by non-Japanese adopters of Lesson study, possibly because the effort involved is invisible to outsiders" (p. 411).

The vital role of having "the trust between the teams for a start" and "a very strong professional dialogue and a lot of respect amongst the group" was underscored by Narrah (Int2). These comments illustrate the importance of leaders holding high expectations of their teacher colleagues (Martinovic & Elkord, 2018). Paula noticed the teachers' growth and shared ownership of the project.

> The people who went in to share at the start were totally different people who came out at the other end. You know they had shared and they were really really proud of their schools, their achievements, the project (Int3).

Conclusion

Lesson Study positions teachers as the researcher of their own practice and hence they are seen as leading the agenda of their professional learning. However, where the participants include a diverse group of teachers, lead teachers, and teacher educators, the expectation for teachers to lead the agenda might not be understood well particularly at the beginning of the process (Akiba et al., 2019). Teacher participants including lead teachers expected the research team (teacher educators) to provide leadership and guidance and they experienced tension from stepping back (Lewis, 2016; Robinson & Timperley, 2007). While all the lead teachers expected more guidance from the research team, after the first cycle, the dynamic evolved overtime and they started to see the purpose of the research team to provide rooms for the participants to develop their agency and ownership of the research lessons. This is consistent with findings reported from Robutti et al. (2016). Engaging in collaborative planning with teachers from other schools with diverse background and experience was found to be challenging at the beginning. While a similar finding has been reported before (e.g., Akiba et al., 2019), the lead teachers in this study came to value having a diverse group of teachers to enrich the conversation among the planning teams.

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THE VALUE OF CHINESE LESSON STUDY IN DEVELOPING TEACHERS: AN ACTIVITY THEORY PERSPECITVE

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Research shows Chinese lesson study is powerful in developing teachers' expertise, however, the mechanism behind such effectiveness is still under-researched. This study contributes to this gap through exploring how a case of Chinese lesson study contributed to teachers' learning of using theories to plan lessons and reflect on their teaching. Taking activity theory as the theoretical lens, this study identified the contradictions between activity systems of research and teaching, and how contradictions were dealt with through the lesson study activities. This study can shed light on how teachers learn in lesson study and necessary conditions that support their learning.

Introduction

Research shows Chinese lesson study is effective in developing teachers' expertise (Huang, Gong, & Han, 2016; Huang & Shimizu, 2016). However, much is unknown about the mechanism behind teachers' learning. This study aims to contribute to this gap through a case of Chinese lesson study which aims to help teachers use theories (learning trajectory (Simon, 1995) and the Van Hiele model of geometric thinking (Van Hiele, 1986)) to design one lesson and reflect on their teaching.

Culture-historical activity theory (CHAT) is adopted to understand the complex and dynamic process of lesson study. One core tenet of CHAT is regarding contradiction as the inner force of change and source for learning (Engeström, 1999). From this perspective, this study focuses on contradictions between two distinct activity systems: research system and teaching system, as researchers concerned about theories and teachers concerned about practices. It is discussed how the dealing of contradictions promote teachers' learning in lesson study.

The question this study attempts to answer is: how does the Chinese lesson study contribute to teachers' development in expertise?

Chinese Lesson Study and Activity Theory

Chinese Lesson Study and Mathematical Curriculum Reform

Chinese lesson study is job-embedded, systematic and multi-tiered (Huang, Fang, & Chen, 2017). It plays important role in developing teachers' profession through different forms, for instance, public lessons, and exemplary lesson development (Wei, 2019). Research shows Chinese lesson study has contributed to Chinese students' outstanding performance in international assessment, such as the Program for International Students Assessment (PISA) and the Trends in International Math and Science Study (TIMSS), through improving teacher quality (Han & Huang, 2019). There are also evidences that Chinese lesson study is effective in link theories and practices to promote the implementation of the current curriculum reform (Huang et al., 2019). However, the mechanism of this process, that is to say, how teachers learn in lesson study is not clear. This study aims to make contributions to this gap.

Culture-historical Activity Theory

CHAT is a cross-disciplinary theory to understand people's objects and pursue of objects in a collaborative setting (Engeström, 2001). According to CHAT, people's objects are achieved through tools, which are also mediated by the cultural and historical roots of the system, such as, community, rules and division of labor (Engeström, 2001). CHAT has three generations (Engeström, 2001; Leonitiev, 1974; Vygotsky,1978). The third generation proposed by Engeström calls for a collective and dialectical view of human's learning and extends the analysis of single activity system to the interactions between different systems, to address the complexity of human activity (Engeström, 2001).

Literatures on using CHAT to understand teacher's learning in professional development program is not rare. Potari (2013) explored the relationship of theory and practice in mathematics teacher professional development from an activity theory perspective and suggested "AT can offer us a tool to relate individual and social perspectives in research focusing on mathematics teacher development" (p. 517). Karen (2019) used CHAT to describe three primary teachers' professional learning and practices in cycles of collaborative action research and argued CHAT "has considerable potential for understanding learning and how educational contexts promote learning (p. 364). Potari, Psycharis, Sakonidis, andZachariades (2018) used activity theory perspective to identify contradictions between activity system of mathematics teaching, research in mathematics education and educational policy interacting in the process of developing a reform-oriented mathematics curriculum and found it useful.

As teachers work and learn collaboratively in lesson study, CHAT is a suitable theoretical lens to understand its process. This study adopted the third generation of AT in the form of two interacting activity systems represented by an expanded meditational triangle (Figure 1). In this study, the two activity systems are research system consisting of university researchers and educators and teaching system consisting of school teachers.

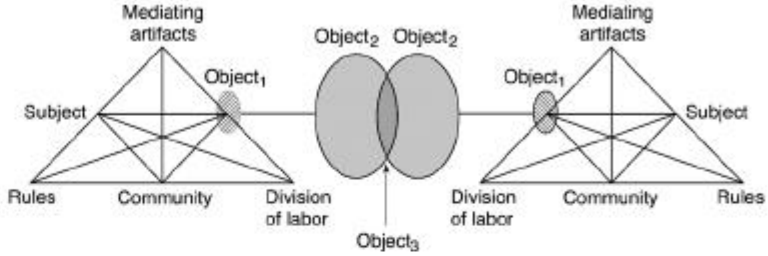

Figure 1: Two interacting activity systems (Engeström, 2001)

Method

Setting and Participants

The lesson study took place in a private school in southwestern China with more than 450 teachers and 4400 students. The lesson study group is made up with three university researchers in mathematics education, one teacher educator, four graduate students and 7 teachers from one teaching research group. Among the three university researchers, one has rich experience in working with teachers through lesson study; another researcher has around 20 years of experience in teaching in middle schools before joining the university; the third researcher just finished her PhD study and mainly in charge of data collection in the process; the teacher educator is a retired university researcher and now working in the teacher development center. The teacher educator is very familiar with teachers in this study. The students involved in the study are in grade 7 and at an average level in terms of mathematics achievement.

The Process of Conducting the Lesson Study

The orientation of the lesson study is to develop lessons that are driven by theories and can promote students' thinking. The topic selected is reflection symmetry. Students had learnt this topic in primary school; however, the requirement in middle school is higher. This topic opens a space to discuss different ways of teaching in different levels of learning. Two theories were introduced to guide the lesson plan and classroom teaching-learning trajectory and the Van Hiele model of geometric thinking.

The process of lesson study consisted of two phases of planning, instruction and reflection (see Figure 2). In phase one, lesson study group members collaboratively developed the research lessons (planning discussion) and one teacher taught the lesson in her class (research lesson 1). In phase two, the group worked to revise the lesson plan based on evidence of students' learning, students' pre-test and post-test (post-lesson debriefing 1), then the same teacher taught the revised lessons to a different group of students (research lesson 2). After the second research lesson, the lesson study group had another group discussion (post-lesson debriefing 2). The students of the two classes were both at an average level in mathematics achievement. Pre-test and post-test were analyzed before and after each research lesson. The lesson study lasted for the whole month of May 2019. The process is as below.

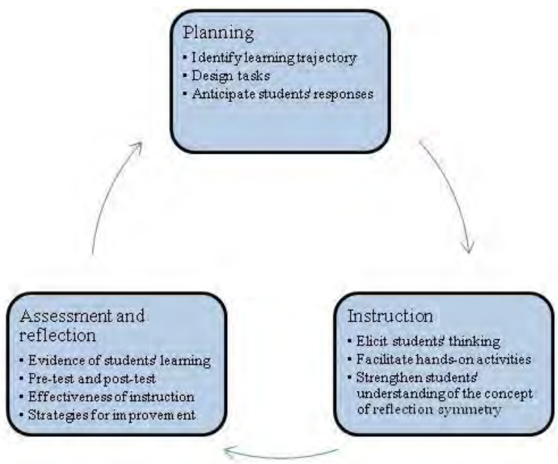

Figure 2: Lesson study cycle

Data Collection

The data collected in this study included:

- All the lesson plan created during the lesson study (four versions)
- Audio-record of planning discussion, post-lesson debriefing 1, post-lesson debriefing 2
- Videos of two research lessons
- Students' pre-test and post-test
- Interviews with the enacting teachers before and after each research lesson
- Interview with two other school teachers on learning from participating in the lesson study

Data Analysis

The audio-recorded interviews, videotaped lessons and debriefs were transcribed verbatim in Chinese. Data analysis was conducted on the Chinese documents with relevant transcripts translated into English.

The expanded meditational triangle presented in Figure 1 was used as framework to analyze the data. Two activity systems were distinguished from the lesson study: teaching system and research system. In each system, the components of activity theory, namely, subjects, objects, tools (artifacts), rules, community and division of labor were analyzed.

To be more specific, in the teaching system, the subjects are the school teachers. In the research system, the subjects are the three university researchers, educators and graduate students. The objects are the goals of the subject that motivating their participating. In this case, it is to develop an exemplary lesson on the topic of reflection symmetry. Tools or artifacts can be physical, cognitive or symbolic, that are used by the subject to achieve the objects. For example, tools teachers use to design the lesson include textbooks, teaching materials, and their teaching experiences. Community is the individuals or groups that the subject involved while engaged in the activity. Rules refer to the norms in the subject's community. Division of labor involves horizontal division of tasks and vertical division of power and status. Different kinds of data source, such as individual interviews, group discussions and videos of lessons were analyzed for triangulation.

After analyzing the six components of activity system of teaching and research respectively, comparison was made between them to identify contradictions. It needs to be noted that activity system is dynamic rather than unchangeable. Any change in one of the six components will caused changes in the system. Moreover, the two activity systems were connected and interacted, rather than separating from each other. The interaction between them explains how the contradictions raised at the beginning of the lesson study were dealt with, and how this process contributes to teacher's learning.

Results

This part reports the contradictions between the researchers who concern about the theoretical issues and the teachers who concern about practical issues in the lesson study. Furthermore, it is discussed how these contradictions were dealt with through activities in the lesson study process and how it promotes teachers' learning.

To begin with, main information of the two activity systems was summarized as Table 1.

Table 1: The analysis of activity systems in lesson study through activity theory

Activity system	Teaching system	Research system
Subject	Seven school teachers	Three university researchers, one teacher educator, four graduate students
Object	Develop an exemplary lesson	Develop a theory-driven exemplary lesson; guide teachers to use theories to plan their lessons
Tools	Textbooks, teaching materials (for example, previous lesson plans), teaching experiences	Research papers, textbooks, theories, teaching experiences
community	School community (colleagues, students)	University community (faculty, graduates)
rules	School culture, teaching research group norms, students' situation	Theories-driven, promote students' thinking
Division of labor	Design the lesson together; one teacher delivers the lesson	Researchers mainly provided theoretical notions and gave instructions on how to design lessons based on theories
Contradictions	Teachers found it difficult to understand and use the theories to design the lesson	
Outcome	Teachers realize the value of theories in guiding lesson plan and can use theories to reflect their teaching	

Contradiction between Teaching System and Research System

During the lesson study, contradiction between the two activity systems could be easily identified, especially in the planning discussion before the first research lesson.

Before the lesson planning discussion, the research group discussed the learning trajectory embedded in the topic through primary to high school. Furthermore, they discussed how the Van Hiele model of geometric thinking can be used to understand the topic and plan the lesson. They also found some tasks that had the potential to elicit students' thinking. For the teacher group, the enacting teacher provided an initial lesson plan for discussion, mainly based on her previous experiences of teaching the topic.

As the two groups came together to discuss and plan the lesson, contradictions between the two activity systems occurred. The researchers want to focus on how to use learning trajectory and Van Hiele model to design the lesson, while teachers hardly knew and showed little interest in these theories. They were satisfied with their existing way of teaching the topic and did not see a need to fundamentally change the lesson plan. During the discussion, we can see that the researchers tried to guide the direction of discussion to the theoretical level, but teachers did not have much to say. At the later stage of discussion, researchers made some compromise on the theories and started to discuss how to improve the tasks in the lesson plan provided by the teacher. They offered suggestions on how to revise the tasks and provided some new tasks for the teacher's consideration. The planning discussion lasted for two hours and a half. After the discussion, the teacher educator recommended two books to the teachers to learn the theories. One is about Van Hiele model of geometric thinking and the other is about theories on teaching concepts. However, the enacting teacher (Teacher W) found it very challenging to read the books as expressed in her interview below:

> "I tried to use the theories to guide my lesson plan. But they are too theoretical. For the first two hours of reading the books, I can't help falling asleep. It is very difficult to understand. After two

> hours, it started to get better. Normally, we seldom learn about theories. Our training is mainly about teaching skills" (first interview with teacher W)

The teacher educator is not satisfied with Teacher W's revised lesson plan. She shared: "I do not think the lesson plan is very good. The teacher did not plan the lesson based on the theories we introduce to them. She still does not understand the theories".

From Passive to Active Participation: Dealing with Contradictions

We can see that at the beginning of the lesson study, contradiction existed between researchers and teachers. On one hand, researchers intended to introduce theories to teachers; on the other hand, teachers preferred more practical suggestions. In the lesson study process, how the contradictions were dealt with? From the activity theory perspective, they may be dealt with through Subjects, Tools, Rule and Division of labor.

Subjects. The teacher educator, who actively work across the two systems as boundary broker, is necessary to ensure the communicating between researchers and teachers. Without her, the lesson study cannot be so effective.

Tools. Tools are very important in lesson study. At the beginning, teachers and researchers used different tools (teachers had little access to theories and research papers while researchers did not know the situation of students well), thus their interaction was not effective. However, in the first and second post-lesson debriefing, shared tools between researchers and teachers were created, such as lesson observation protocol, the lesson, and students' pre- and post-test. Also, teachers became more familiar with theories through reading books. Through creating shared tools, the communication between teachers and researchers are more harmonious and effective and teachers became more engaged.

Rules and division of labor. At the beginning, the norms in the lesson study is that researchers are knowledgeable others and teachers should do what they require. So teachers were passive participants. At later stage, this rule was broken. During the second debriefs, teachers and researchers were purposefully grouped together (researchers are arranged into different teacher groups) to fully express their thoughts on the lesson and provided suggestions for improvement. Teachers said this kind of discussion offered a platform for them to really participate in the lesson study activities, which promote their learning.

Influence of Lesson Study on Teachers

This study shows the case of lesson study on reflection symmetry has brought about several significant changes to participating teachers.

First, shift from "focusing on teacher's teaching" to "focus on students' learning". At the beginning, when teachers planned the lesson together, they mainly focused on how to teach instead of how students learn. For example, teachers thought students should have no difficult to understand the concept of reflection symmetry as they already learnt this topic in primary school. So in the first lesson, teacher W only used two minutes to introduce the concept. However, in the post-lesson test, it shows 80% of students cannot write down the key elements of the concept, that are, "one line", "fold" and "match exactly". Most of the students wrote down "the same size" or "have the same area". When researchers reported students' results, teachers felt quite surprised.

They realized they do not fully understand their students. One teacher said: “the most important thing I learn in the lesson study is that we need to understand our students better. We need to focus more on students’ thinking”.

In addition, in post-lesson debriefings, they used to focus on the teacher’s teaching skills, such as the teacher’s design of the lesson, the teacher’s explanation of concepts, or hand writing. However, in this lesson study, researchers required them to write down their observation of students. When they made comments, they should begin with “I observed that students…, so I think…, then I suggest…”. Interviews shows teachers prefer this way of post-lesson discussion.

Second, use theories to plan lesson and reflect on teaching. Another change this study observed is that teachers started to use theories to guide their lesson plan and reflect on their teaching. In the second lesson plan, teacher W started to use terms of theories. For example, in the objectives, she wrote: “help students reach level 2 of van Hiele Model of Geometric: Thinking-informal deduction.” To meet this objective, she designed several tasks that require students to explain why figures, for example, parallelogram is not reflection symmetry. To meet the higher level of learning trajectory (transformation) in this topic, she added some higher-order thinking tasks, for example, “there are four squares forming an L shape, can you move one square to make it reflection symmetry?” she shared that : “The exercises are more open and require more thinking. I will ask them how you find the answers to elicit students’ thinking”.

After the second research lesson, teacher W started to use theories to reflect on her teaching.

> “The theory of teaching concept introduced by the teacher educator defines different levels of students’ learning. First is for students to experience the concept in hand-on activities, second is to summarize some characteristics of the concept through their activity experiences. The last level is to use mathematical language to accurately describe the concept. I think in the second stage I should give more time for students to summarize the key features of reflection symmetry, so they can understand the nature of the concept better.” (Interview with teacher W after the second research lesson)

At the end of the lesson study, Teacher W summarized her learning as below:

> “I think in this lesson study I have learnt a lot of theoretical knowledge. When I first hear these theories from the university researchers, I feel so strange. But after I teach the topic again and again, I realize what happens in real class is just like what theories indicate. The theories are indeed come from our classroom, and they are very useful. For me, this is a very important progress. I think in my future teaching, I need to read more books on these theories”. (Interview with teacher W at the end of lesson study)

Conclusions

Through the case of lesson study on the topic of Symmetric figures, this study sheds light on the contradictions that may exist between the activity systems of research and teaching, as one focuses on theories and the other focuses on practices. However, contradictions became the opportunities and sources for teachers’ learning. This study observed two significant changes in participating teachers—putting more focus on students’ thinking and drawing on theories to guide lesson plan and improve teaching. To ensure the effectiveness of lesson study, it is suggested that there should have active boundary brokers in the lesson study group, shared tools should be created to promote interaction between teachers and researchers, and a more equal rules and division of labor should be provided. Theoretically, this study contributes to our understanding of how lesson study could help teachers to develop theory-informed mathematics instruction and build the connections between theory and practice. Practically, this study identifies some conditions that are necessary to ensure the effectiveness of lesson study. However, as this paper focuses on the value of Chinese lesson study on teachers' development, researchers' learning in the process is not discussed, which will be considered in further studies.

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Theme D

Theme Chairs: Ornella Robutti & Luc Trouche

Tools and Resources Used/Designed for Teacher Collaboration and Resulting from Teacher Collaboration

STRUCTURED ONLINE TEACHERS' COLLABORATION FOR FOSTERING PROFESSIONAL DEVELOPMENT

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This work illustrates the design and the outcomes of online activities for supporting teachers' collaboration within a professional development course for in-service secondary mathematics teachers. For this aim specific tools from the general purposes e-learning platform Moodle have been exploited to support virtual structured collaboration. Then the outcomes of the collaboration are analyzed and discussed, as well as of the answers to a post-questionnaire submitted to the trainees.

Introduction and theoretical background

In this paper, we report the design and the outcomes of online activities for supporting teachers' collaboration within a professional development course for in-service secondary mathematics teachers. In order to frame the use of an e-learning platform for professional development course of in-service teachers in mathematics, we extend the Focus\Role Model introduced by Albano, Coppola & Pacelli (2013), taking into account the Meta-Didactical Transposition (Arzarello et al, 2014) and the Instrumental Genesis (Rabardel, 1995).

The model, shown in Table 1, starts from the assumption that the use of a platform in a course for mathematics teachers' professional development course can affect a course attendee from various points of view. On the one hand, the course aims to draw trainees' attention to specific issues in mathematics education, which are pivotal to improve students' learning. In order to make effective such improvement, they need, as teachers, to redesign their classroom practices. On the other hand, the blended setting of the course requires the trainees to use an e-learning platform for carrying out some assigned tasks, allowing them also to explore the didactical potential of some specific artifacts (tools) of the platform.

Focus\Role	Trainee	Teacher
Mathematics Education	to acquire new competence in mathematics education	meta-didactical transposition
Educational Technology	instrumental genesis	meta-instrumental transposition

Table 1: the Focus/Role Model for online professional development

Let us give a more detailed look into each of the above aspects.

Mathematics Education/Trainee (ME/Tr): the interaction with mathematics education researchers foreseen in the course allows the participants to develop new awareness of the recent research results and new didactical competences in terms of effective teaching practices (Arzarello et al., 2014).

Educational Technology/Trainee (ET/Tr): according to Radarbel's *instrumental approach* (Rabardel, 1995), the complexity of the inclusion of an e-learning platform in the course context should be taken into account. The trainees experience for themselves as users an instrumental genesis linked to a proposed instrumented activity, making use of a specific tool (artifact) of the e-learning platform in order to perform a given task.

Mathematics Education/Teacher (ME/Te): according to Arzarello et al. (2014), the dialectic between researchers and teachers lead to the emergence of new teacher praxelogies. The emergence of a didactical transposition in line with recent educational trends comes out in the classrooms of the participants engaged in the course.

Education Technology/Teacher (ET/Te): the trainees become aware of the processes taken place in the experienced instrumented activity and they appropriate the underpinning models. This leads them to activate, in their classrooms, a meta-instrumental transposition, implementing similar instrumented activities in their classroom.

The paper aims to investigate the following research questions:

> RQ1: To what extent do digital platforms constitute opportunities for teacher collaboration? How can they be exploited to support teacher collaboration?
>
> RQ2: How are teachers engaged in the design of resources in collaboration? What are the outcomes of these collaborations?

To this aim, we present the design of structured virtual collaboration, implemented in the general purposes e-learning platform Moodle. Some of its specific tools have been suitably customized to foster professional development according to the mathematics education topics addressed by the course. Then the outcomes of the collaboration are analyzed and discussed, as well as of the answers to a post- questionnaire submitted to the trainees.

Didactical design

The course setting

The professional development course, focused on argumentative competence, aims to offer teachers an overview of most recently results in Mathematics Education Research and examples of how to implement them into teaching/learning contexts.

The course took place along six months (January/June) and involved about 60 secondary school teachers. It consisted in four face-to-face lectures (about 4 hours per lecture), in a laboratory setting, blended with 12 hours of online activities to accomplish through the e-learning platform Moodle. At the end, a final meeting for summing up the outcomes of the whole course in terms of professional development for the engaged participants.

The four lectures were held by invited researchers in mathematics education, particularly expert in argumentation, and concern the following issues: rules and explanations in mathematics; promoting educational assessment processes in mathematics with new technologies; argumentation and proof as an educational problem; argumentation between form and meaning. All the in-depth materials (e.g. scientific papers) suggested by the experts has been recorded in a folder of the platform, shared with the trainees.

The timeline of the course activities can be described as the flow shown in Figure 1, consisting in six phases: course introduction (Phase 0); design of hypothetical learning trajectories (Phase 1); lectures and interaction with invited researchers (Phase 2); re-design of hypothetical learning trajectories

(Phase 3); peer-review (Phase 4); re-design of hypothetical learning trajectories (Phase 5); final discussion (Phase 6).

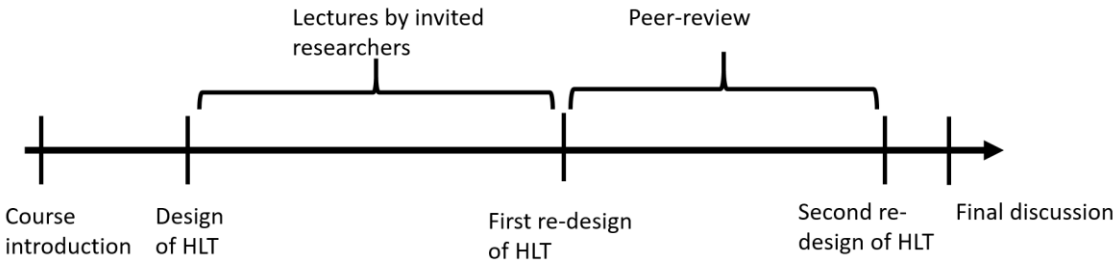

Figure 1: Course's flow

The online activities have been consisted in individual and collaborative work, whose detailed description will be given in the following. In order to perform the online activities, two main tool of Moodle have been exploited:

- *Assignment*: it allows the trainees to submit their work as response to a task given by the course staff. The work should be submitted within a deadline, and then the course staff can proceed to assess and give feedback to the trainees (possibly, requiring new submissions).
- *Workshop*: it allows the collection and the peer-to-peer revision/assessment of trainees' work. In the first step, the trainees submit their own work and in the second they receive a certain number of submissions from other users which they are required to revise/assess according to criteria given by the course staff. Both the steps are time-restricted.

The tool *Assignment* has been used to implement Phase 1 and Phase 5, whilst the tool *Workshop* has been used to implement Phase 3 and Phase 4.

The design of collaboration

The trainees have been engaged in online activities, consisting of two main components: design of learning trajectories and peers' review. Although each trainee is expected to work individually, they are engaged in being a resource one for another. The activities are performed and mediated by the tools Assignment and Workshop of the Moodle platform. According to Rabardel & Bourmau (2003), three mediations occurs in the instrumented activities, shown in Figure 2:

- the mediation to the object: it comes from the trainee activity oriented toward the content of the activity (i.e. HLT);
- the interpersonal mediation: it comes from the orientation of the trainees to colleagues;
- the reflexive mediation: it happens when the trainee is asked to submit the new design after interactions with experts as well as after the peer-review results;

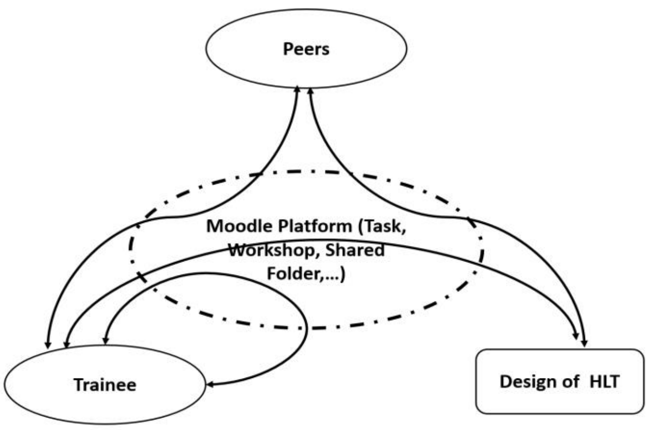

Figure 2: instrument mediated activities

Let us see the two components in detail.

Design of learning trajectories

- Phase 1 - Description of a hypothetical learning trajectories (HLT) that you usually carry out in class on a given mathematical content

At the beginning of the professional development course, before of the face-to-face lectures by the invited researchers, the trainees are required to submit a document (doc/pdf format) describing a hypothetical learning trajectory (Simon, 2014), concerning a certain chosen mathematical topic. The document should specify the learning goal, the detailed description of the tasks and the expected students' learning process, according to their usual classroom practice. Guidelines are provided to help the trainees in editing the document, expected to be submitted by using the Assignment tool.

[....Describe an educational path that you usually follow in class on a certain mathematical content. The focus should be on methodology, rather than on content and prerequisites.

Expose the phases of your course and, for each of them, specify them:

1. *which tools you use and how (e.g. textbook);*
2. *what methodology you adopt and how it is implemented (e.g. cooperatives learning). ....]*

- Phase 3: Re-designing the hypothetical learning trajectory at the end of the formative course

The trainees are asked for re-designing the hypothetical learning trajectory submitted in phase 1, according to their own knowledge changes due to the interactions with the course's materials and experts. Such knowledge changes can impact on one or more of the three components of the learning trajectory, that are a learning goal, a set of learning tasks, and a hypothesized learning process. Then a new document is produced by the trainee. Collection and re-distribution of the new documents is performed by the Workshop tool of the platform Moodle.

[...At the end of the four formative sessions, review the previously chosen activity in the light of the stimuli you have received. Write the activity in a word/pdf file, avoiding to include your name in the text and in any attachments, in order to optimize the subsequent peer review process…]

- Phase 5: Re-designing the hypothetical learning trajectory after revision from peers

After having received the feedback from their peers on the design submitted in phase 3, the trainees are expected to re-design once more the hypothetical learning trajectory, according to the changes occurred in their own knowledge due to peers' interaction (if any).

Peers' review

After the phase 3, the trainees are required to review the learning trajectory devised by a colleague (Phase 4). Such review has been steered by the researchers, providing the trainees with four criteria, each of them referring to one of the experts' seminars.

Phases 4 consisted in a peer review process implemented using the Workshop tool of the platform Moodle (Albano et al., 2017, Dello Iacono et al., 2019), which allowed to collect and redistribute all the documents produced in the phase 3, reviewed by colleagues, based on specific criteria, provided by the experts in the training meetings, listed in the following:

1) First reviewing criterion concerning "mathematical facts":
   Are there any specific activities within the didactic path you are reviewing for the students' understanding of the various "mathematical facts" dealt with (definitions, theorems)? If so, do you think these activities are adequate?
2) Second reviewing criterion concerning formative assessment:

Are there any moments of formative assessment within the didactic path you are reviewing? If so, are some of Black and William's strategies explicit? Does he/she use technology, and how? How much is the argument present as a tool for formative assessment?

3) Third reviewing criterion concerning open problems:

Are there any tasks proposed to the students that intend to describe the solving procedure and to justify the correctness? Are there any open problems in which the formulation and justification of conjectures is required? If not, do you think it is possible to introduce open-ended problem-solving activities?

4) Fourth reviewing criterion concerning linguistic issues:

Are there any tasks within the didactic path you are reviewing that require you to think about the validity of a statement through its interpretation in a mathematical context? Is it considered and evaluated that linguistic competence could influence the results of tasks that require the elaboration of argumentative texts?

Outcomes and discussion

In the experimentation 60 secondary school teachers were involved, of which 57 completed all the activities. The experimentation results have been analyzed both from a quantitative and from a qualitative point of view. From a quantitative point of view, to measure the impact of the course lessons (i.e. interaction with experts) and the impact of collaboration (i.e. peer review by means the online platform), we have identified a scale of levels, which takes into account the transition from Phase 1 to Phase 3 and from Phase 3 to Phase 5, as defined:

- *Level 0*: no changes or suggestions for changes not properly integrated into the hypothetical learning trajectory;
- *Level 1:* re-elaboration with modification/integration of design details (e.g. explanation of methodologies, tools, assessment tests) without evidence of elements deriving from interactions with experts or from peer review feedbacks;
- *Level 2*: re-elaboration that is explicitly affected by the interaction with the experts or from peer review feedbacks (e.g. focus on argumentative competence, evidence of use of experts' suggestions).

The levels do not refer to the quality of the learning trajectory in a specific phase, but rather indicate whether or not there has been a change from one phase to the next. We are interested in observing changes in the learning trajectory designed by teachers. In the following, we show an excerpt in which a teacher modifies her learning trajectory, due to the peer review feedback, reaching Level 2, moving from Phase 3 to Phase 5.

Determine the equations of parabolas with - $F(2,3)$ $d: x=-1$ - $V(0,0)$ $F\left(0,-\frac{1}{4}\right)$ - $V(-3,-2)$ $d: y=-\frac{9}{4}$ - $a: x=1$ for points $(0,1)$, $(-1,4)$ - for points $(1,0)$, $(0,2)$, $(3,2)$ For each item it is required to argue the various steps, explaining the reasoning.	What happens in the parabolas if a is greater than zero? - What happens in parabolas if a is less than zero? - Which other values can be assumed by a? What happens in that case? - What is the geometric meaning of the slider associated with parameter c? - What happens if c is 0? Justify your answers.

Table 2: Excerpts from a learning trajectory in two different phases

Indeed, in Phase 3, the teacher designed a summative assessment test, consisting of operational questions (Table 2 column 1), even if she asked for some kind of justification. In the Phase 4, she receives from the reviewer, the following feedback: “*In the summative assessment test it is asked: ‘for each item it is required to argue the various steps, explaining the reasoning’. Actually, these items are exercises and not open problems, and no formulation and justification of conjectures is required”.* This feedback has brought the teacher to change the foreseen summative assessment test in Phase 5. As shown Table 2 column 2, she introduced questions requiring the students to explore and to conjecture, justifying their statements.

The graph in Figure 3 shows the number of teachers of each level in the transitions Phase 1 → Phase 3 and Phase 3 → Phase 5.

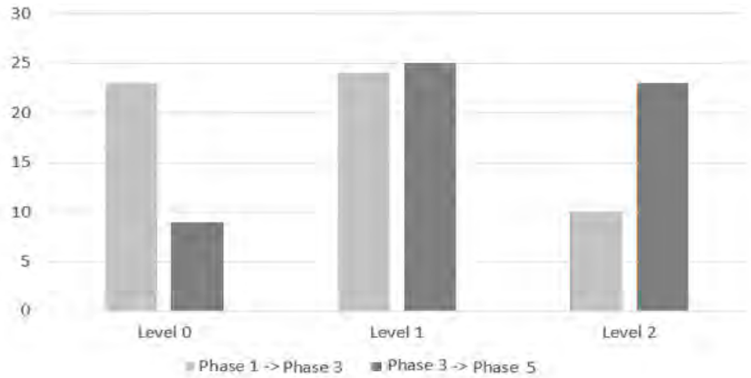

Figure 3: Number of teachers per level and phase passage

The number of Level 0 teachers decreases as they move from Phase 3 to Phase 5, while the number of Level 2 teachers increases significantly. Moving from Phase 3 to Phase 5 is a consequence of the review process and, therefore, it is strongly influenced by the collaboration between teachers. Figure 3 shows that this collaboration played an essential role in the ownership of the course content. It seems, however, that those who had slightly modified the design as a result of interactions with experts (Phase 1 → Phase 3) have not been too affected by the suggestions of the reviewer.

However, the graph in Figure 3 provides only global information on the course progress, but does not show the number of teachers who have moved from one Level to another in the phase transition. This information is presented in the Table 3 below:

Phase 1→3 \ Phase 3→5	L0	L1	L2
L0	4	14	5
L1	4	10	10
L2	1	3	6

Table 3: Number of teachers per Level and phase transition

Table 3 shows that the majority of teachers who were at Level 0 moving from Phase 1 to Phase 3 (i.e. about 60% of Level 0), reached Level 1 moving from Phase 3 to Phase 5. Therefore, the collaboration allowed them at least to modify the learning trajectory, even if without too much evidence of the interactions with the experts. About 22% of the teachers have moved to Level 2 and, in this case, the collaboration has also allowed the teachers to appropriate the specificities dealt with in the course. Only 4 out of 23 teachers remained at Level 0. For these students, the course, as a whole, did not have any obvious influence. Table 3 also shows that for only 4 teachers at Level 1, the collaboration did not have any effect in the sense that, after the review, there was no substantial change in the learning trajectory. About 42% of the teachers who were at Level 1 also reached the same level in moving from Phase 3 to Phase 5. Therefore, for these teachers, the collaboration influenced in equal measure the interaction with the experts. Moreover, about 42 % of the teachers who were at Level 1 reached Level 2. For them, collaboration had a greater influence than interaction with experts. Finally, we note that 6 teachers who were already at Level 2 and who, therefore, had strongly changed their learning trajectory, reach Level 2 even when moving from Phase 2 to Phase 3. For these teachers there have been significant changes both as a result of interaction with experts and of collaboration. Therefore, their final delivery was completely different from the initial one.

Therefore, it seems that the collaboration has strongly influenced the re-designing of the activities by the teachers. Collaboration would not have been possible without the platform for: the reduced time of the face-to-face phase; the difficulty of realizing in face-to-face an organization like the one described above (i.e. anonymous redistribution of HLT documents and returned feedback). Taking into account the impact that the platform had on the performance of the activity, we analyze the data from a qualitative point of view, using the Focus/Role model described in Table 1.

It seems that online activity and resources impact on improving the required learning trajectory, with attention to mathematics and, in particular, to argumentative competence (Mathematics Education-Trainee in Table 1). For example, a trainer writes in the new learning trajectory (Phase 3):

> In the problem-solving phase [...], all comments made during the peer discussion are asked to be made in writing. During the next phase of mathematical discussion each of them will express their reflections orally and argue the conjectures that have emerged. The meta-cognitive questionnaire has also been expanded so as to induce students to argue each phase of the followed course, and to give rise in them to the need to acquire and appropriate the specific terms of mathematics.

Moreover, it seems that the use of e-learning tools has influenced the change in the educational practices of the teacher in perspective, with reference to mathematics (Mathematics Education-Teacher in Table 1). For example, a teacher, in Phase 5 of re-designing, writes:

> Maybe it would be more appropriate, [...] to ask students to approach verifications consistent with this methodology, such as open problems or modelling situations, through which it is possible to evaluate not only their calculation skills but also the competences acquired during the educational path. It would also be appropriate to try to accustom students to a conversion between one semiotic register and another, asking them not to always start from symbolic writing and then move on to a graphic representation, but also on the contrary: interpreting different graphic representations.

Surely the teachers, during the activities, familiarized themselves with some tools offered by the platform (Education Technology-Trainee in Table 1): Forum (used for communication between trainees and organizers); Folders and Files (used to organize the material made available to the trainees by the organizers and experts); Task (used for deliveries in Phase 1 and Phase 3); Choice (used to arrange specific meetings); Workshop (used for delivery in Phase 3 and for the review phase in Phase 4). The main focus of the teachers was on the Workshop tool and on the peer review phase as a methodology to be used in class with their students and to investigate educational paths and methods (Education Technology -Teacher in Table 1). In this regard, a teacher, in answering the question of the questionnaire "What advantages for your teaching profession can you identify in the peer review activity", writes:

> I find the peer review activity useful as a moment of comparison and mutual growth and for this reason it is a useful tool for my students

Another teacher writes:

> In my opinion, peer review is an effective tool for comparing colleagues. Each comment, for those who receive it, can be a useful starting point for reflection to improve their work. It can also be useful even if carried out among students.

The results obtained show that the designed online collaboration among secondary school teachers, within the professional development course, has positively influenced the re-planning of their teaching activities. In particular, the collaborative peer review activity, provided through a specific tool of the e-learning platform, has contributed significantly to this redesign.

Thus, concerning RQ1, the outcomes show how a suitable design exploiting the Moodle Workshop tool can promote collaboration among teachers. More in depth, the Workshop feature of specifying reviewing criteria allows to scaffold the collaboration in order to be successful with respect to the aim of the collaboration foreseen by the designers of the professional development course.

Concerning RQ2, the accuracy of comments made by the teachers during the reviewing phases show their engagement in collaboration. Moreover, the re-designing based on such comments highlight the effectiveness of collaboration in terms of quality of the new resources.

Finally, we note that the proposed methodology seems to impact on daily practice in the classroom, as it has been seen by teachers as a possible educational strategy to be used also with their students.

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SOCIAL MEDIA FACILITATED COLLABORATION: AN ANALYSIS OF IN-THE-MOMENT SUPPORT IN A MATHEMATICS EDUCATION FACEBOOK GROUP

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As participation in social media has become a reflexive practice, teachers—like the rest of society—have turned to online spaces to find support. The purpose of this study is to investigate how a public Facebook group mediates contextually relevant teacher collaboration. Using a qualitative case study research design, a Facebook group tailored to mathematics education is analyzed based on 1) how group members share knowledge, 2) how different discourse structures support collaboration, and 3) how the online platform affords teacher collaboration. Findings suggest that teachers are collaborating within this Facebook group around in-the-moment problems by leveraging the mechanisms of social media to create generative discourse structures. Future research should address the importance and influence of social media interactions within the larger scope of all professional learning opportunities available to teachers.

Teaching is complex. Teachers must attend to the needs of multiple learners simultaneously, integrate newly adopted content standards, and deploy technology integrations often without proper training. Teachers need, and desire, opportunities to learn as professionals to properly react to the complex nature of teaching. Unfortunately, when teachers reflect on their professional development, few report satisfaction with what their schools offer (Boston Consulting Group, 2014). To supplement these inadequate learning opportunities, teachers look outside of their school for support.

Social media is one place where teachers look to find professional learning opportunities. Within these spaces, teachers find communities where they participate in critical reflection of their practice, learn about new methodologies, access experts outside their personal network, and develop their teaching identity through discussion (Macia & Garcia 2016). Through these virtual interactions, teachers are able to ask and respond to questions, read previous posts, and participate asynchronously in a learning environment. However, there is little research on online collaborative interactions and potential learning outcomes. This research study examines the interactions that occur within social media to better understand how teachers use these spaces for collaborative learning.

Conceptual Framing

Many professional development opportunities offered at school sites are built upon a growing consensus around how professional development can best support teacher learning. Experts agree on core principles of teacher professional development: being centered in practice, engaging teachers in active learning and inquiry, and building a community of learners (Ball & Cohen, 1999; Desimone 2009; Borko, Jacobs & Koellner, 2010, Darling-Hammond, Hyler, & Gardner, 2017). These guiding principles allow for the creation of communities of practice (Lave & Wenger, 1991), which provide teachers a place to learn and reflect together. These professional development opportunities allow teachers to access new knowledge that is focused on the needs of their students. But what happens when teachers do not have access to an active, collaborative community of practice that supports their individual needs? Teachers must go outside of what is formally offered to find a learning community.

While the motivation for going online for professional development has been documented (Macia & Garcia, 2016), little is known about whether these environments are supportive of the best practices of teacher collaborative learning. Researchers of professional development have documented the importance of collaboration grounded in teacher practice, but how do these interactions manifest when teachers are collaborating in online spaces? Building from recent frameworks of professional development and specifically the importance of collaboration, this paper will analyze interactions within one Facebook group of mathematics educators to better understand discourse structures and how these collaborative interactions support teacher professional development.

Methods

This research project is approached using a qualitative revelatory case study design (Yin, 1984). A revelatory case study is used to illuminate the interaction among a group of individuals that have rarely been studied, or have not been examined through the current theoretical lens. The case study is intended to illuminate detailed examples of teacher collaboration within a public Facebook group.

Research Context and Participants

The group used in this study, is a public mathematics education Facebook group. At the time of data collection, there were 1,738 active members of the group. Members are considered active if they have interacted in the group at least once. An interaction is considered one of the following activities: posting, commenting, or reacting. The active members have an average of 11 interactions. The top 100 members of the group average 80 interactions, the creator of the group was withheld from this calculation due to her increased participation as a moderator.

Data Collection

All posts, comments, interactions, and member activity were collected for a 39-month period. There were 1,476 original posts to the group, which contained 5950 comments.

With the intention of looking for instances of teacher collaboration initiated by in-the-moment help seeking, the data set was initially partitioned into questions (N=357) and non-questions (N=1,119). Questions posed by members were indicators of a member's desire to initiate collaboration. Due to the size of data set and research interest, purposive sampling of posts was used and posts requesting in-the-moment support were selected for analysis in an attempt to create a revelatory case for online teacher collaboration.

Findings

Through an iterative process of coding, three response types (answers to requests) and four discourse structures (ways members interacted) were identified. The three response types identified: 1) *Show* – offer idea without providing detail, 2) *Share* – offer idea with details such as description of idea or how it was implemented in their classroom, and 3) *Discuss* – no new idea offered instead commenter discusses ideas already offered. Throughout the interactions four discourse structures emerged between group members: 1) *Desired* – commenters provides desired support, 2) *Reframe* – commenters offers different idea than requested, 3) *Challenge* – commenters challenges requested support or ideas that were presented by previous commenters, and 4) *Generate* – commenters and/or original poster work together to build a new understanding of desired support. These codes will be used below to illustrate four cases of online mathematics teacher collaboration.

Case #1 – Providing Desired Help – Collecting Online Math Games

The first type of discourse structure observed contained a majority of response patterns that directly met the requested support. The *desired* discourse structure code was applied to posts when replies rarely deviated from requested support. While all three response types occurred during these interactions, most replies were coded as *show*. These types of posts were transactional in nature, rarely developing into a collaborative discourse pattern.

The below network (see Figure 1) represents interactions when a member of the group requested resources for online math games. Using social network analysis, and its representation in graphical form, the interactions between members were quantified. Each member is represented by a node and the total number of interactions a member participated in is represented by the node size. The direction of the arrows indicates who is receiving the information, if a response is provided to the original post, it is assumed that the original poster is the recipient of the information. The different patterned arrows represent the different response types. The "Follow" code emerged for this group, which did not often occur in the large data set. The code was applied to responses that said "following" or ".", which is a mechanic often used by group members to bookmark a specific post.

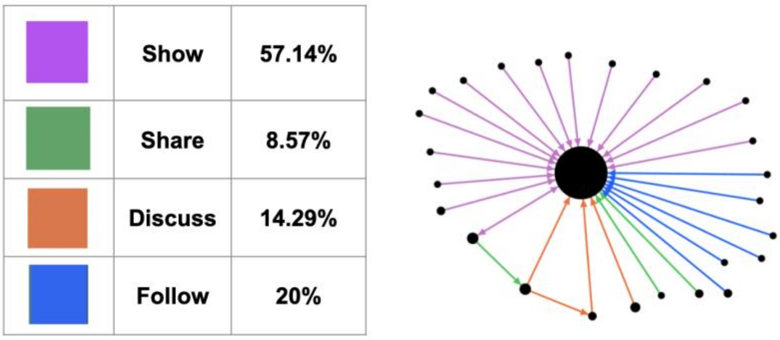

Figure 1: Case #1 Network

Affordances of the Technology. This discussion lasted for 44 days, with the original poster checking in after 41 days indicating that he used the resources and showed his appreciation to the group. The first day had nine interactions which was the highest daily number of interactions. Towards the end of the thread there was a seven-day stretch of inactivity. The use of an online group allows for participants to collaborate at convenient times in their schedules, not limiting collaboration to specific times. The asynchronous aspect also allowed for a lengthy collaboration time, permitting individuals to join the conversation late or return multiple times to provide more information.

Members of the Facebook group used common social media techniques to follow the activity within the group without actively participating in the conversation. Six members of the group replied to a post with the word "following" or the punctuation mark "." indicating to the group their interest in the information and desire to return later to reference. These six individuals made public their intention to continue following the interaction, which provides legitimacy to the conversation.

Collaborating within a large public group allows for access to a vast reservoir of knowledge. The initial requester was able to access a community of 26 members interested in offering ideas, clarifying questions, or simply following the conversation.

Integrating these conversations into online platforms allows members to benefit from instant access to resources. Within the replies, 11 responders provided links to the content they recommended. This allowed for the requester to easily access content, evaluate it quickly, and if needed ask any clarifying questions in a timely manner.

Case #2 – Reframing Help – Deficit Minded Request

The second type of discourse structure emerged when replies offered alternate solutions not originally intended by requesting group member. The *reframing* discourse structure code was applied to posts when interactions between group members diverged from original request based on philosophical differences about mathematics education.

The following interaction provides an example of offering alternative information. The offering of alternative information allows the responder to reframe the help provided to the requester. The interaction below (see Figure 2) was initiated by a member of the community asking for a periodic assessment tool to use for her students that have "very low" number sense scores. Quickly, two members join the discussion to reframe how the teacher supports building number sense. Both responders recommend not using an assessment. Instead, they recommended use activities that have students practice these skills. While this interaction only includes three individuals, it highlights how the community quickly responses to a deficit-minded comment and reframes the knowledge that is shared.

	Show	66.67%
	Discuss	33.33%

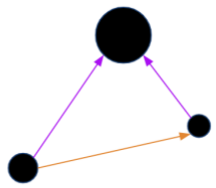

Figure 2: Case #2 Network

Affordances of the Technology. The online platform provides a place to interact with individuals who are different in many ways. Case #2 is an example of a collaborative opportunity that occurred between individuals with philosophical differences. While individuals often choose to interact with people who are philosophically aligned with them (Grannovetter, 1973), online spaces, which networks numerous individuals together, provides an opportunity for diversity in collaborative opportunities. When asking a question in the Facebook group, requesters cannot limit who can see or respond to the question. Requesters have provided a topic for all to discuss and as noted in the above interaction, while explicitly indicating what they needed help on, requesters cannot control the flow of the conversation. It is evident that within this public Facebook group, members have access to a variety of views both similar and different to their own.

Case #3 – Challenging Help – Deconstructing Common Core Standard

The third type of discourse structure occurred when group members challenged and debated the requester's, or other responder's, ideas. The *challenge* discourse code is different than the *reframing* code, because the challenger highlights why they believe the proposed idea or concept is wrong, and a debate ensues. Often times in these interactions, the challenger, along with highlighting what is wrong, will also provide a countering idea—similar to the *reframing* discourse move—to help redirect the conversation.

The following network (see Figure 3) represents a post centered around the concept of simplifying expressions and the introduction of integers within the Common Core standards. The original requester is looking for help with aligning his old teaching practice with the new requirements of Common Core. The requester provides how he has taught the concept in the past, the teaching idea that he would like to enact moving forward, and also a case that might cause his new idea not to work. The thread starts with many responders agreeing with him and sharing his frustration. When the challenger joins the conversation, he highlights that many respondents are providing shortcuts to teaching simplifying expressions without developing conceptual understanding. His challenge is countered by two other members who believe that their way of teaching helps students "read" the mathematics. The debate lasts for 12 comments with all three participants providing evidence for their method of teaching.

	Show	22.73%
	Share	11.36%
	Discuss	65.91%

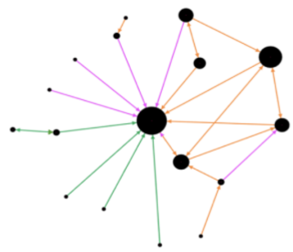

Figure 3: Case #3 Network

Affordances of the Technology. As the debate unfolded within the responses, it was clear that one individual was taking the attack role while another was defending their idea. The defending member eventually removes herself from the conversation. While she does not return to any part of the thread with her own comments, she does follow the rest of the debate between the original attacker and another community member who joined the conversation to defend her. This is shown through her usage of the "like" button. She continues to like all of the comments made by her defender throughout the rest of the debate. The technology provides her a way of removing herself from being at the center of the interaction, without having to leave the interaction completely. She is able to decrease the attacks on herself while still providing support within the debate.

Case #4 – Generative Help – Co-creating Understanding of a Resource

The final type of discourse structure observed within this public Facebook group occurred when members worked together to build a common understanding of an idea. The *generative* discourse code was applied when members, through their interactions, generated a more nuanced understanding of a requested or offered idea. The collective understanding of the idea was heightened by the collaboration of the group members. Generative discourse often happened when a member asked a clarifying question of a previous responder, or when they provide additional information to develop a better understanding of a previous proposed idea.

The network below (see Figure 4) represents an interaction between five individuals as they discuss enrichment activities for elementary mathematics. This network has a self-loop, which indicates an individual responded to their own post. One respondent offers an activity by brand name including that they use it with students to make mathematics visual. Following this response, another member joined the conversation and agreed with the resource and mentioned that they enjoy using the app on the iPad. The first responder did not know an app existed and reflected that it would be a great activity to use on an iPad. This is an example of how the community collaborated to build a more complete understanding of a resource.

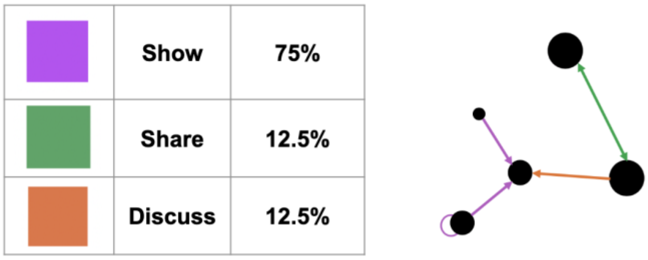

Figure 4: Case #4 Network

Affordances of the Technology. The use of the public Facebook group provided a way for individuals to collaborate from long distances. In this collaborative interaction, one individual was on the east coast of the United States while the other was on the west coast of the United States. The collaborative aspect of this interaction happened over an eight-minute time period. The use of an online technology allowed for individuals with similar interests to collaborate over large distances quickly and build collective knowledge.

Discussion

This case study of a public Facebook group focused on mathematics education highlights interactions among group members that provide opportunities for teacher collaboration. Drawing from literature on professional development (Ball & Cohen, 1999; Desimone 2009; Borko, Jacobs & Koellner, 2010, Darling-Hammond, Hyler, & Gardner, 2017) and specifically the best practices of mathematics teacher professional learning by Ball and Cohen (1999)—being centered in practice, engaging teachers in active learning and inquiry, and building a community of learners—I will argue that this Facebook group is a space where teacher professional develop is possible.

Being centered in teacher practice is a key feature of productive professional development. Ball and Cohen (1999) argue that while it is critical for professional learning opportunities to be centered in teacher practice, it does not have to occur within the walls of a classroom. The examples from practice in the cases presented above provide members a chance to examine critical part of teacher practice outside of traditional structures. While the members of the Facebook group do not directly work together within a school setting to investigate teacher practice, they are able to participate in professional development through discussions of artifacts from members' practice which generate collaborative learning opportunities.

The second key feature of productive professional development emphasizes that professional learning opportunities must be investigations of practice. Teachers must be given an opportunity to dig into issues that surface in their classrooms. Artifacts, imagined by Ball and Cohen, are strategically documented teaching practices such as "written cases of teaching, multimedia cases or the raw materials of such cases, observations of teaching, teachers' journals, and examples of student work that are embedded in evidence about teaching" (p. 15). Within teacher learning opportunities, artifacts do not have to come from participants' practice; but rather, artifacts can be from other's practice used as representative examples. Within the online Facebook group, members often bring artifacts from their practice to investigate, as detailed in the cases above. These artifacts provide all members of the group access to real classroom situations that center the discussions in genuine issues that arise within mathematics classrooms.

The final key feature of professional development reflects that the interactions among members of the group should support a learning community. Questioning, investigating, analyzing, and criticizing are types of discourse that are reflective of a community of teacher learners (Ball & Cohen, 1999). These discourse structures create a learning environment that is centered on discussion, which allows for reflection and co-construction of knowledge. The discourse moves highlighted in the above section are detailed examples of how social media facilitates interactions among the group members which cultivate a collaborative learning environment. Ball and Cohen reflect that "by enabling encounters with very different practices, such work would broaden and diversify teachers' knowledge and create opportunities to see new versions of teaching and learning, and to understand things differently" (p. 18). Social media is a space where teachers bring and discuss activities from their practice with members from diverse backgrounds to build a unique community of practice.

Conclusion

The cases above highlight how an existing social media platform can facilitate mathematics teacher collaboration. Using a purposive sampling method to highlight relevant interactions, discourse structures between members were analyzed based on response types, and technology affordances. Finally, using Ball and Cohen's (1999) description of professional learning, the professional learning opportunities within the group were analyzed. From the analysis, it is clear that the interactions within the Facebook group have the potential to promote professional collaboration. The members of the Facebook group studied are able to increase their access to knowledge for mathematics teaching by requesting help through questions posed to the group. These questions are responded to through multiple response types within different discourse structures, all creating an environment where analysis, questioning, challenging, and debating increase the potential for teacher learning.

Based on teacher responses to The Boston Consulting Group's Survey on Professional Development (2014), finding alternative professional learning opportunities is critical for teachers to adequately develop. With only 25% of teachers satisfied with the current professional development offered by their schools, online collaboration platforms, like the Facebook group studied, can become places where teachers network together virtually to build learning communities. If teachers feel supported in online learning communities, how can the characteristics of these virtual collaboration influence what is currently being offered to teachers as learning opportunities at their school sites? A future study should address how to build structures within formal teacher learning environments that capitalize on how teachers are collaborating in these informal online settings.

Within the public Facebook group, few teachers are reporting on their enactment of new learning. At the conclusion of a meta-analysis of informal online teacher learning, Macia and Garcia (2016) highlight that little evidence of enactment exists within online learning spaces. While it is not often clear that teachers are enacting what they are learning from this public group, teachers within the space are providing evidence of the value of online collaboration. The following is an example of the value members are placing on the group.

> Original Poster (OP): How many of you are [a] "lone wolf" in your math department?? I have one other math teacher (out of 10) that's on the same page as me on teaching philosophy. I start PD next week and feel like I need to prepare for an uphill climb of skepticism and critique!
>
> Responder 4 (R4): You have everyone here for support! Fight the good fight
>
> R5: My thoughts exactly!
>
> OP: Yes, so thankful I found this group!

While many of the interactions that members experience within the Facebook group are valued by returning members of the community, a future study should address the impact of these collaborative opportunities on teachers' practice.

Finally, while this Facebook group is valuable to a certain type of mathematics educator, it is not appropriate for all. As the technological barriers prohibiting some teachers from going online are decreasing with the advancement of online accessibility, the intellectual, or philosophical, barriers will continue to limit access. Some teachers will find the content offered or discourse structures used within the interactions to be unapproachable and limiting to their personal growth. This concern draws out another topic for future research. How can collaboration within social media be structured so more individuals feel welcomed and find value? This question would bring together the literature on facilitation of learning opportunities with current findings of teacher online learning.

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THE EFFECT OF A COLLABORATIVE PROFESSIONAL DEVELOPMENT ON QUESTIONING SKILLS OF TWO NOVICE MATHEMATICS TEACHERS

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The purpose of this study was to investigate changes in question types of two novice mathematics teachers who participated in professional development (PD) focused on implementing cognitively-demanding tasks (CDTs) through 5 practices (5P) (Stein & Smith, 2011). The results showed that before PD teachers usually asked low-level questions (LQ) and hardly ever asked high-level questions (HQ). However, implementing CDTs through 5P provided significant changes in the classroom environment of teachers. It occurred various types of HQs that hadn't been experienced frequently in their classrooms before PD.

Introduction

Cognitively challenging questions can help students be active thinkers and broaden their understanding of mathematical concepts. It has ensued in many studies that asking higher-order questions promote mathematical skills such as justification and generalization, make students engage in solving mathematical tasks in motivating ways, support classroom learning environment and provide opportunities for a deeper understanding of mathematical concepts. However, the studies indicated that teachers relied mostly on the LQs that include recalling facts, rules, and procedures in their classrooms. For example, in Boaler and Brodie's work (2004), teachers' questions served mostly for gathering information and leading students through certain procedures. In another research that focused on teacher questioning (Sullivan & Clarke, 1991), only 5% of the questions "required the pupils to think independently or to give more than one answer" (p. 9). Studies also indicated that novice teachers had similar questioning styles. For example, Moyer and Mileseviscz (2002) acknowledged the fact that novice teachers have ill-developed questioning skills. To improve questioning skills, teachers need to include CDTs that have a high potential to reveal students' mathematical thinking. Boston (2013) found that teachers working with CDTs were able to improve their questioning and discussion routines.

The cognitive demand of a task implemented in a classroom is considerably important for generating HQs. CDTs focus on the meaning of mathematical concepts and ideas. These kinds of tasks require high cognitive effort, using different strategies, explore and understand the nature of mathematical concepts, processes or relationships. The questions in such an environment focus on revealing different strategies, lead to a discussion to point mathematical relationships and meanings, generate ideas, extend thinking to different situations and focus on key elements of a situation. Whereas, low-demanding tasks require students to memorize facts, rules, and procedures or use algorithmic procedures and there is no connection between concepts or meaning that underlies the procedures being used. These tasks usually require no explanation and limited cognitive effort (Stein & Smith, 1998). Therefore, these questions focus on not bringing out strategies, explanations or ideas of students, but quickly revealing one correct answer of a task.

The purpose of this study was to investigate changes in question types of two novice mathematics teachers who participated in a PD focused on implementing CDTs. Following research questions were addressed:

a) What kind of question types occurred in classrooms of two novice teachers before PD?

b) How did using CDTs along with PD affects two novice teachers' question types?

Methodology

This qualitative study aimed to find out how teachers' questioning dispositions evolved with participating in a PD which focused on implementing CDTs. For this purpose, the lessons of teachers observed before and during a PD. The aim of the observations before PD was to determine in which level the tasks implemented in terms of cognitive demand and what type of questions used in this period. Subsequently, along with the PD period, it was aimed to determine the effects of PD in questioning styles of teachers. In other words, to find an answer to whether there were any changes in question types of students as a result of participating PD. The main purpose of this PD was not to improve teachers' question styles, but it was assumed that a PD based on the implementation of CDTs through 5P would also improve teachers' questioning styles. Figure 1 illustrates the context of this study.

The participants Aisha and Ece were both in the first year of their profession when the study was conducted in the 2015-2016 academic year. Both of the teachers were working in two different public schools in a rural part of Turkey. Data were collected through video and audio recordings of 5th and 6th classes before and during the PD. Respectively, ten and twenty-two hours of observations were made before and during a PD for each teacher. All of the questioning patterns in these recordings were completely transcribed.

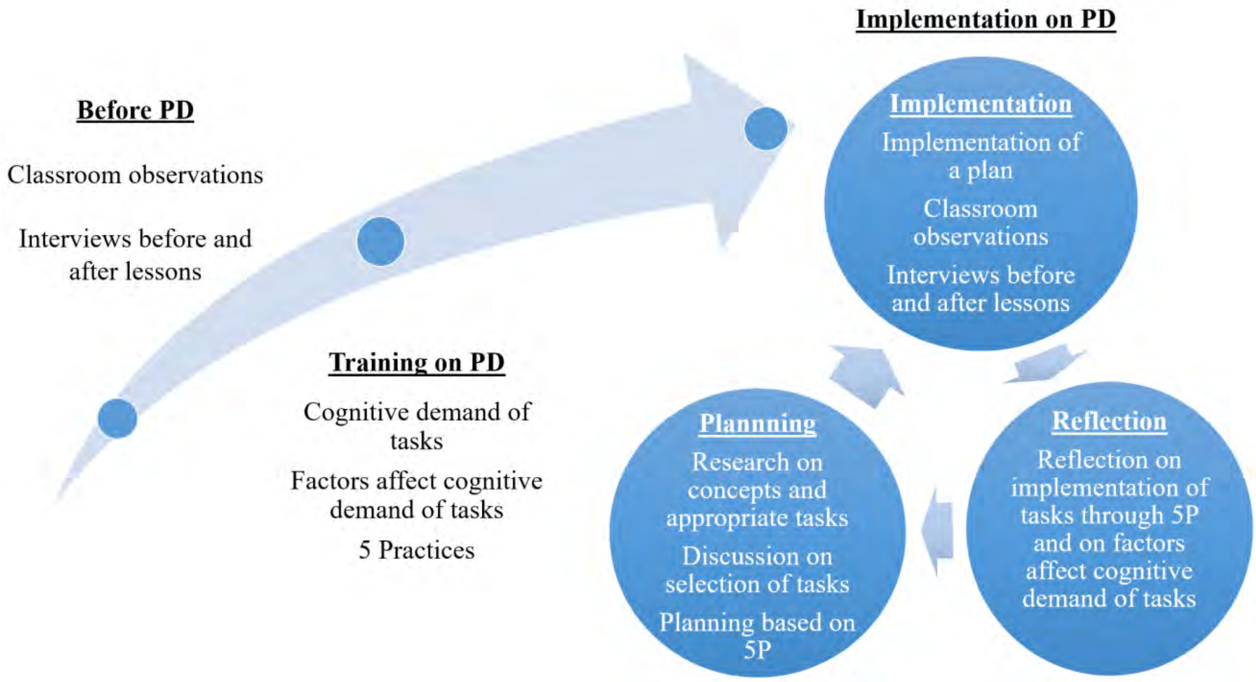


Figure 1: The context of the study

Collaborative Professional Development

As illustrated in Figure 1, the PD was twofold. Firstly, the teachers participated in seven sessions of training at the beginning of the 2015-2016 academic year. The training focused on cognitive demand of mathematical tasks, the factors associated with the decline or maintenance of CDTs, modifying mathematical tasks to increase cognitive demand and using 5P *(anticipating, monitoring, selecting, sequencing, connecting)* which provide teachers a systematic approach to maintain the cognitive demand of the tasks at a high level. (Smith & Stein, 2011). Secondly, teachers continued in a PD which includes collaborative meetings, implementation and reflecting cycles throughout the remainder of the year. They came together seven times once or twice a month. In collaborative meetings, teachers researched important mathematical ideas of subjects, considered the task to implement in their classrooms and made student thinking-centered planning based on 5P. After the implementation of tasks, they watched important sections of their classroom recordings, examined student artifacts and reflected on them.

Data Analysis

The data collected before PD were analyzed concerning the cognitive demand of the tasks and question types. The cognitive demand of the tasks was classified either low or high-level according to Stein and Smith's (1998) task analysis guide and the question types were categorized according to Boaler and Brodie's (2004) question types illustrated in Table 1. The first two types on Table 1 identified as a low-level type of questions, others were high level. The analysis of data collected during the PD focused on the types of questions involved in high-level tasks.

Table 1: Question types (Boaler & Brodie, 2004)

Type	Question type	Description
1	Gathering information, leading students through a procedure	Requires immediate answer. Rehearses known facts/procedures. Enables students to state facts/procedures.
2	Inserting technology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them.
3	Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations.
4	Probing, getting students to explain their thinking	Asks student to articulate, elaborate, or clarify ideas.
5	Generating discussion	Solicits contributions from other members of the class.
6	Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life.
7	Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used.
8	Orienting and focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem-solving.
9	Establishing context	Talks about issues outside of mathematics in order to enable links to be made with mathematics.

Findings

Task implementation before PD

Conducted observations and interviews showed that both of the teachers have similar task implementation routines. Their lessons were usually based on explaining the subject rapidly and solving several exercises to reinforce it. They both had some routines which cause to decline of cognitive demand of the tasks, such as not giving enough time while working on problems, focusing only one correct answer, having classroom management problems, routinizing the problematic aspect of a task, making no connections between concepts, ideas, representations, and real-life situations. During this period, Aisha implemented 59 tasks and performed an average of 5,9 tasks per lesson. She selected 27 tasks at a high level but only 12 out of 59 (%20) tasks were implemented without a decline in the cognitive demand level. Similarly, Ece implemented 50 tasks and performed an average of 5 tasks per lesson. She selected and implemented only 11 out of 50 (%22) tasks at a high level. Once question types used by teachers before PD were examined as you will see below in Figures 2 and 3, it was seen that low-level question types used more frequently. Because of this reason, this PD aimed to provide teachers to plan and implement CDTs by following 5P and to seek whether there was an alteration in question types used by teachers.

Question Types before and during PD

PD which focused on implementing CDTs through 5P provided significant changes to question types of teachers. Figure 2 and Figure 3 illustrate the question types used by teachers before and during PD. Totally 50 out of 84 (%60) and 126 out of 193 (%65) questions classified as Type 1 before PD. However along with PD only 11 out of 111 (%10) and 35 out of 170 (%20) questions classified as Type 1.

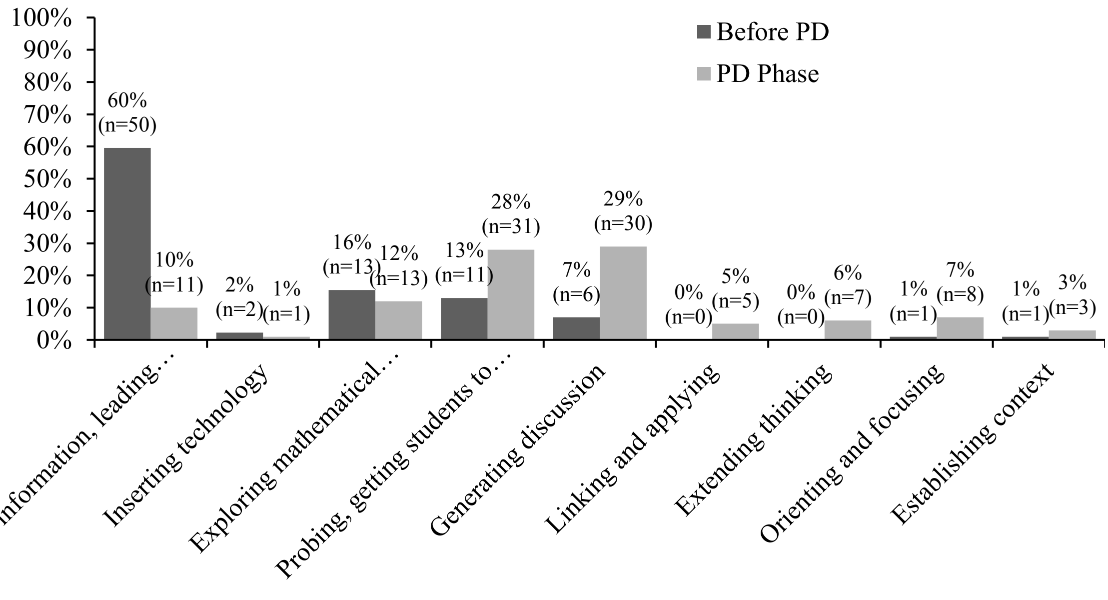


Figure 2: Question types used by Aisha before and during PD

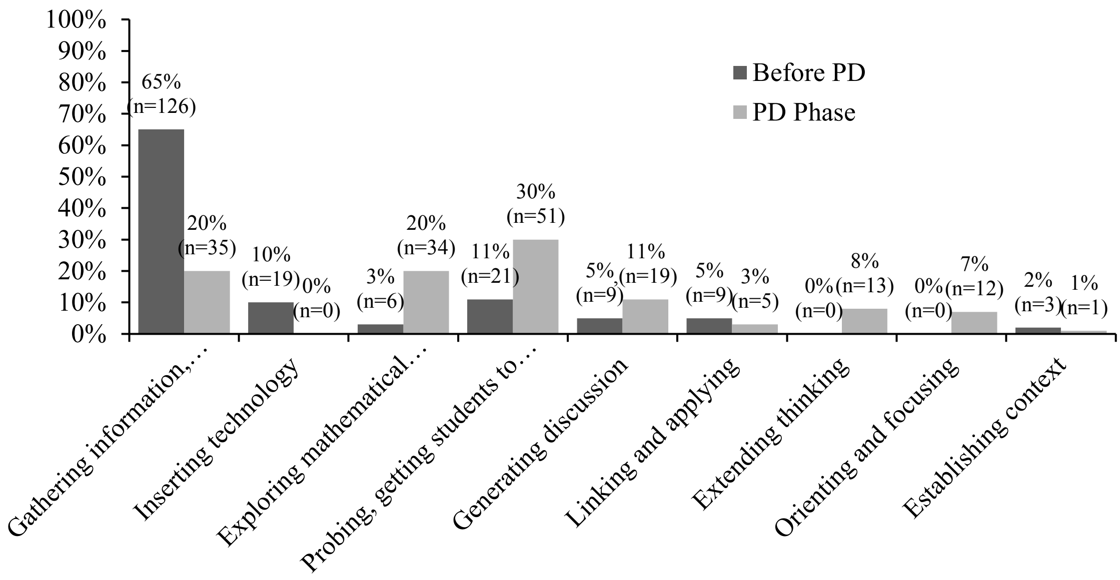


Figure 3: Question types used by Ece before and during PD

Aisha more frequently used Type 4 and Type 5 of questions as compared with before PD. Also, Type 6, Type 7 and Type 8 of questions occurred in classrooms which were hardly ever used. Similarly, Ece more frequently used Type 3, Type 4 and Type 5 of questions as compared with before the PD. Also, Type 7 and Type 8 of questions occurred in classrooms that had never used before.

In the following transcript, all of the questions classified as Type 1. The lesson implemented by Ece before the PD and it was aimed to teach length measurement conversion. She planned to teach length measurement conversions by drawing stairs. Every stair corresponds to a metric length measure and it is multiplied or divided by 10 when going a step up or down.

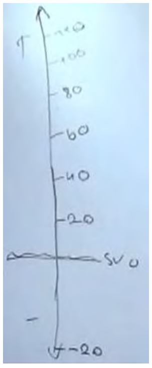
Table 2: Ece's sample lesson before PD

Transcript	Type
T. Now, the topic is measurement conversions. For example, I will ask you how many meters equal to 200 kilometers. How many centimeters is 1 meter? How many stairs are there between meter and kilometer? One, two [counting with students], you are going one step down and one step down again. So, **how many steps will you go down?**	1
C [Class]. Two.	
T. Two steps down. Well, how many centimeters are there in one meter, in other words, how many centimeters equal to one meter [showing the stairs by her hands]. I know that there are two stairs [she usually answers her own questions] but I don't understand either it will increase or decrease as going down. Can anybody explain? Ali? I will covert one meter to the centimeter. But, **will it increase or decrease?**	1
A**.** [Hesitantly] It will increase.	
T. **Well, how many centimeters are there in one meter?**	1
C. 100.	
T. **What happened then?**	1
C. Increased.	
T. Is it increased? **Then, will it always increase as going down?**	1
C. Yes.	
T. Well, I divided to 10 here, I divided to 100 here. I divided it to 1000 here. So, **what is the difference between them?**	1
C. 10	

T. Well, **which operation will I perform to increase as going down?**	1
C. Multiplication	
T. I would multiply it by 10. Well, **how many steps am I going down as converting meter to centimeter?**	1
C. Two	
T. Then, I will add two zeros. In other words, multiplying by 100 equals adding two zeros, so I will add two zeros. Ok? For example, how many decimeters is one kilometer? It will increase as going down, so we will multiply. **How many stairs are there, Ela?**	1
B. One minute… One, two, three…	

The second transcript from Aisha's lesson on the PD phase in which all of the questions were classified as high-level. The purpose of the lesson was to provide students to comprehend integers in a context of temperature and place them on a number line.

Table 3: Aisha's sample lesson during PD

Transcript	Type
[The teacher firstly monitored solution strategies and selected two different students to explain their strategy.]	
T. Ok. **Where are positive and negative numbers on the number line?**	3
..	
T. It increases one at a time here but it increases ten at a times there. **Do you think all of the intervals are equal? Is it consistent?** [Intervals are not equal in student's solution]	8
…	
B. The temperature increases while going right, and decreases while going left.	
T. **Is the same thing valid here as well?**	4
B. Yes.	
T. **Is it valid for whole numbers on the number line?**	4
B. Yes.	
….	
[Isa sketched the following vertical number line]	
	
T. Isa, explain your idea. **How did you sketch? What did you notice while sketching the number line?**	4
I. I noticed that the intervals between numbers were equal.	
T. Ok. You sketched the number line, and **which number did you place first?**	4
I. I placed zero degrees first.	4
T. **What does zero mean?**	
I. Sea level.	5
T. Ok. Do you have a question for Isa? Ahmet has one.	
A. Intervals continue equally above zero but why -20 is at the bottom? [the interval between 0 to -20 is not equal to the interval between 0 to +20]	
….	
[Fatih sketched a horizontal number line and they discussed on between two number lines]	3
T. **Do you notice anything while placing numbers to the number line, for instance, while going to the right or left side?**	5
B. Positive integers are on the right side and negative integers are on the left side. But on Isa's number line positive integers are on the upper side and negative integers are on the bottom side.	3
T. **Do you notice a change in numbers? Do they increase or decrease?**	5
B. It increases while going right side and decreases while going left side.	

Discussion And Conclusion

First of all, there was a decrease for both teachers on using Type 1 questions which are known as low-level question types. Implementing CDTs based on 5P provided teachers to use high-level questions frequently. Teachers learned to ask questions that reveal students' ideas rather than make students rehearse known facts or procedures. In Boston and Smith's (2011) and Boston's (2013) task-centric studies which examined teachers' selection and implementation of CDTs before and after a PD, teachers improved their ability to implement CDTs; specifically, their questioning and discussion techniques. Similarly, Polly (2015) stated that "the extent to which students communicated their mathematical thinking was directly aligned to the levels of mathematical tasks that were posed". PD participants in this study asked more "why" questions within the period.

It occurred various types of questions that hadn't been experienced too much in their classrooms before PD. Orienting and focusing type questions mostly occurred while teachers were monitoring students' work and asking them assessing and advancing questions. Thus, teachers made students focus on key elements or aspects of the situation to enable problem-solving. Before PD, Ece didn't have monitoring practices at all. Aisha sometimes performed monitoring to look for whether the correct answer was found or not, rather than getting students to explain their ideas. Extending thinking was yet another type of question that never used both of teachers before PD. A limited number of discussions initiated by teachers were usually aimed to make students explain their ideas. Teachers weren't used to extending the situation under discussion to other situations which aim to make a generalization. The structure of CDTs in a way led to the discussion of the underlying concepts, provided to conceptualize student understanding and meaningful learning. In the current study, there was a significant change in questioning patterns before and during PD while some other studies (e.g. Polly and Hannafin, 2011; Boaler and Staples, 2008) hadn't observed such advances much more. Boaler and Staples (2008) aimed to find out question types of teachers at the project and traditional classrooms. While sixty-two percent of their questions were procedural which is too high compared to the current study, project teachers used more varied questions like as current study than the teachers of traditional classes.

In conclusion, implementing CDTs within a systematic approach using 5P led to students to maintain the cognitive demand of tasks at a high level. This systematic approach led to teachers to use various types of questions as well. On the other hand, using high-level questions provide teachers to maintain the cognitive demand of tasks at a high level as Boaler and Staples (2008) stated.

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A MATHEMATICS TEACHER'S LEARNING IN DESIGN-BASED RESEARCH: THE BROKERING SUPPORTS THROUGH DIFFERENT COLLABORATIVE GROUPS

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This paper reports a mathematics teacher (MT), Jean, her learning trajectory in design-based research with the supports of collaborative groups at different stages of encountering various pedagogical problems. The dynamic working and learning processes among collaborative groups at different stages were analyzed with the meta-didactical transition model (Arzarello, Cusi, Garuti, Malara, Martignone, Robutti, & Sabena, 2014). The individual and shared praxeologies of three forms of collaborative groups and Jean's were the evidences to analyze the evolutions of her professional learning. The components of designs (tasks), techniques, and discourses (incl. technology and theories) in each praxeology were presented to elaborate Jean's learning trajectory including changes and fixations. Also, the members of the collaborative groups as brokers played important roles supporting Jean's changes on designs. The results contributed that though the external components apparently drove Jean's changes, the didactical discourses among different periods showed her growth evolved gradually in design-based research.

The approach to student-centered and competence-based teaching is the latest educational reform and emergent issues to deal with in teachers professional development in Taiwan. Recently, one mature transformative cascade professional development (PD) model (Lin, Yang, & Wang, 2016) composed of mathematics-teacher-educator-researchers (MTERs), mathematics-teacher-educators (MTEs) and mathematics teachers (MTs), to cultivate MTs to make use of mathematics-grounding activities (MGAs), designed by MTs, for enhancing students' learning motivation and conceptual understanding in mathematics via conducting the Fun Math Camp, has been worked since 2014 in the Just Do Math (JDM) project (Lin & Chang, 2019). In addition, beyond the MGAs for enhancing students' learning engagement (Lin, Wang, & Yang, 2018) and understanding in out-of-school-lesson context, the activities of MGA-in-class have been developed and worked in school classes with a revised transformative cascade PD model since 2018.

The JDM project provides various PD programs to cultivate MTs to design and to use mathematical tasks and activities, i.e. MGA and MGA-in-class, and gradually to become MTEs who facilitate other MTs in working MGAs and MGA-in-class. In addition, such PD programs promote MTs' professional learning through collaboration among groups. The case Jean presented in this study is the one who started to attend the design workshop designing one MGA, further attended another design workshop and transformed the designed MGA into MGA-in-class, and was selected to shoot her instruction with MGA-in-class and edited in 15 minutes as the material for facilitating other MTs learning how to teach with the specific MGA-in-class activity in class, i.e. co-planning the mathematical lesson with MGA-in-class design.

In this study, we aim to report how Jean's learning with the supports of three different types of collaborative groups function in such PD model, through exploring Jean's changes in a series of design-based workshop in the PD programs. The changes among the versions of Jean's designs from the first to the last one provided evident transformations of learning during her four-year (from 2015–2019) PD among different collaborative groups. The supports given by three different types of collaborative groups will be presented based on Jean's pedagogical problems found in her design-based research, and will be analyzed with the meta-didactical transposition model (Arzarello, Cusi, Garuti, Malara, Martignone, Robutti, & Sabena, 2014).

Theoretical Perspectives

The factors influencing the evolution of teachers' professional learning in PD programs are complex (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010) and therefore challenge researchers in evaluating and analyzing teachers' actual learning. Considering the sustaining interactions between members in the PD, a practical framework to discuss the influence of collaboration on teachers' working and learning is important.

The Meta-Didactical Transposition Model

The meta-didactical transposition model (see Figure 1) for analyzing teachers' learning trajectories through collaboration are considered as an effective framework for that it is focused on both researchers' and teachers' roles with the description of the evolution of their praxeologies over time (Arzarello et al., 2014). A praxeology is composed of four interrelated components: task, technique, technology and theory (Robutti, 2018). The given task and the technique applied to solve the task are the so called *praxis*, i.e. the practical praxeology, while the technology and the theory are the so called *logos*, i.e. the theoretical praxeology validating the use of technique.

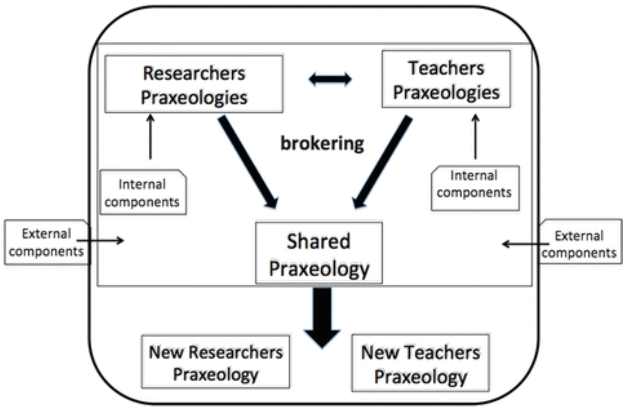


Figure1: The meta-didactical transposition model (Arzarello et al., 2014)

Moreover, similar to Clarke and Hollingsworth's (2002) internal and external domains, the meta-didactical transposition can investigate the dynamics between the internal and external components among various praxeologies (Prodromou, Robutti, & Panero, 2018). The views on these internal and external components of praxeologies can differ from researchers and teachers within the PD program. Indeed, these internal and external components are not easy to measure and analyze quantitatively. Nevertheless, the important factors could be summarized from them to analyze how and why the praxeologies do evolve among specific people/community but not the others.

Brokering Processes for Professional Growth

Wenger (1998) defined that brokering involves processes of translation, coordination, and alignment between perspectives, and it requires the ability to link practices by facilitating transactions between these processes. The brokering processes within the meta-didactical transposition model require the specific roles of broker to bring and accelerate the changes. That is, the work of brokering can be viewed an act with specific intention or purpose, and in the brokering process the broker seeks to work in collaborative and creative ways with people, communities, networks, organizations, ideas, knowledge, and resources to develop something new or change something (Jackson, 2003). The roles of brokers can be the people or organizations move the goal objects such as knowledge (see knowledge broker; Meyer, 2010) around and create connections between researchers and the various audiences, e.g. PD participants in the PD program.

Method

Participants

The case MT Jean reported in this study has 17-year experiences in junior high school as a mathematics teacher, and she is one member of the Local Counseling Teams (LCTs) of Taiwanese official teacher counseling organization (Lin, Hsu, & Chen, 2017). She started to join the series of PD programs provided by the JDM project since 2016, and continually participating the PD programs in the project. Before attending the series of PD programs, she started in 2015 to design the projection activity for grounding the similarity property and the content was continually revised among different collaborative groups in the following series of PD provided by the JDM project. Further, she was a teacher with the style of traditional instruction, i.e. more teacher-centered and less student-centered instruction, for the "efficiency" of teaching. She has received the annual awards of excellent teacher for several times from the local department of education in north Taiwan.

There are three forms of collaborative groups through Jean's learning: among MTs of whole group (WG), among MTs of small group (SG), and among MTERs of professional group (PG). Regarding the MTs of the collaborative groups that Jean worked with, most of them appeared in PD programs belonged to different LCTs from different places in Taiwan, especially from two cities of the north Taiwan: NC_1 and NC_2, and some were from the Central Counseling Team (CCT; Lin et al., 2017). Since the collaborative learning of this PD model processed with the intertwined *whole group* discussion and *small groups* working/discussion (among 2 to 6 MTs), the MTs' contributions to Jean were categorized based on the supports given from whole group or small group. Furthermore, the collaborative group as a professional group (PG) in the PD program is mostly composed of MTERs and they have abundant experiences of theories and practices of mathematics education. In the study, we particularly focused on $MTER_A$ who continually provided his brokering supports to Jean from beginning to the end during the period of her participation in the series of PD program from 2016 to 2019. To present the concise influences of collaborative groups (i.e. brokers) on Jean in this study, we mainly focused on supports from three collaborative groups: the discussion/comments originated from WG, the working/discussion on the active interactions of SG, and the comments/suggestions directly given from the PG.

Professional Development Contexts

As aforementioned, this study was underlying the framework of the transformative cascade PD model (Lin et al., 2016) and its revised model of JDM project. The contexts were selected from the PD programs that the subject Jean had participated as a *learner*, *instructor*, *designer*, and *facilitator* gradually. According to her professional learning with the collaborative groups, the contexts can be divided into different but sequential stages (see Figure 2).

Since the series of her design-based research were sequentially evolved among stages, and the praxeologies evolved between hers and the collaborative group's at different periods, we, therefore, report the complete history of her professional learning on designing projection activity for grounding the similarity property in geometry.

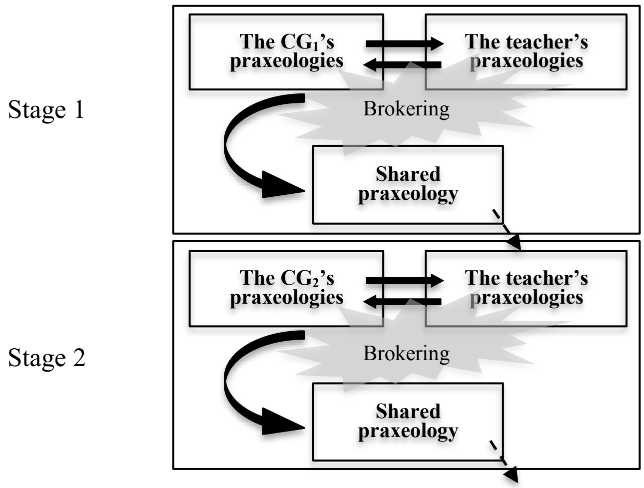


Figure 2: Jean's sequential meta-didactical transpositions

Data Collection and Analysis

In order to explore thoroughly Jean's working and learning through collaborations among the PD programs, the data were collected from five different sources. First one was the collection of various versions of Jean's designs as the lesson plans for analyzing their changes. Second one was the videotaped PD programs including the comments and suggestions given by the collaborative groups as the discourses within the communities. Third one was the videotaped and recorded interview with Jean about what and how she revised her lessons plans among the different stages. The recorded interview was transcribed. Fourth one was Jean's written notes on her designing/revision intentions and the backgrounds of the corresponding designs. Fifth one was the field notes of the researchers at different PD programs. The complete five sets of data were collected for data analysis and for triangulation of data. However, the later three sources of data were mainly applied in the preliminary analysis in this study. Furthermore, the data were analyzed with an inductive approach to group and look for their relationships.

Results

After analyzing the collected data on Jean's revised versions of design through collaboration during different periods of PD, we presented the findings regarding the evolution of her learning, and the brokering supports provided by different collaborative groups, i.e. WG, SG, and PG.

The Gradual Changes of Jean's Design-based Research

The embryo idea of her original design came from re-presenting the sentences given in the textbook: "*If using the light source to light up a figure, then its shadow could be found on the wall. The relationship between them is shrinking and enlarging*" of the topic of introducing the similarity property. According to her teaching experiences, she found that students were passive in learning the topic of similarity property and performed badly. At the moment, she thought it might intrigue students' motivation if giving such activity to them. She then applied the *leather-silhouette show* and a *Chinese fable* to design activities for motivating students learning this topic. In order to revised the design, she communicated with some MTs from different junior high schools. These colleagues encouraged her to join the design workshop (PD program of JDM project) to learn how to complete this design with other MTs and MTERs (interview, 2018). Overall, Jean's learning trajectory through collaboration were analyzed into four stages based on the significant pedagogical problems (Cooney, 1994) found in her professional learning (see Table 1). In addition, Jean's design-based research included (1) the process of designing the specific mathematical topic of similarity property as the lesson plans for MGA and MGA-in-class, (2) the process of making video for the facilitation of PD workshop, and (3) the process of being a facilitator in the PD workshop.

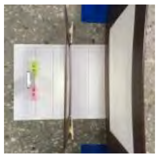
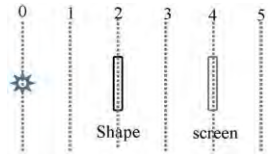
Table 1: Significant pedagogical problems found in Jean's professional learning

Stage	Pedagogical Problem
1	How to increase students' interest Focus of an interesting and metaphorical situation (concerns of students' interests and motivation, and the devices)
2	How to realize perspective projection Focus of designing proper manipulatives for perspective projection (concerns of devices)
3	How to make sense of properties of parallel lines with manipulatives Focus of constructing projection model in solid (concerns of devices)
4	How to design learning activities for active thinking Focus of adjusting conjecturing-testing tasks

After ascertaining Jean's pedagogical problems, we categorized the brokering supports provided by three forms of collaborative groups/brokers, i.e. MTs of WG, MTs of SG, and MTERs of PG) to Jean's professional learning in each pedagogical problem/stages (see Table 2) in her series of design-based research on designing foundation lessons for the topic of similarity property. Jean's design was set the activities from *stretch*, i.e. stretching on mobile phone and with powerpoint, to *dilation*, i.e. projection with a light source, object, and the screen, as the foundation of similarity property. The revised versions of her design were mainly led to improve the *dilation* activities involving the three phases, i.e. the last three pedagogical problems, gradually.

Table 2: The supports provided by three forms of collaborative groups in 2015–2019

Stage	Brokers	Supports (Praxeology)
1 (Metaphorical situation)	a) MT_A of LCT from NC_1 at the MGA design workshop (**MT_A of WG_1**)	Suggestion of the light source (mobile phone) (*)
	b) MT_B of LCT from NC_1 at the MGA design workshop (**MT_B of WG_1**)	Focus of changing different (fictional) shapes, and the (fictional) enlarging light
	c) $MTER_C$ as a module reviewer (**$MTER_C$ of PG_1**)	Comments on the metaphorical situation and the geometric representations
	d) MT_C of CCT at the preparation meeting for MGA instructor workshop (**MT_C of PG_1**)	Comments on replacing the title concerning projection activity
2	a) MT_D of LCT from other city at the MGA instructor cultivation workshop (**MT_D of WG_2**)	Suggestion of the light source (**; with experiment)

(Proper similarity projection)	b) MT_E of LCT from other city at the MGA instructor cultivation workshop (**MT_E of WG_2**)	Comments on the geometric shapes
	c) Jean herself among the group at the MGA instructor cultivation workshop	Jean's reflection on the design during the workshop w.r.t. proper similar figures  
	d) $MTER_B$ at the pre-shooting of MGA-in-class video (**$MTER_B$ of PG_2**)	Suggestions of adding one stretching task with mobile phone
	e) $MTER_A$ at the facilitators' one-day PD workshop (**$MTER_A$ of PG_2**)	Intervention in the focus of the revised design during co-planning the lesson & its theory of similarity transformation
3 (Projection model in solid)	a) $MTER_A$ at the MGA-in-class design workshop (**$MTER_A$ of PG_3**)	A mission of constructing a solid model of projection & its theory of similarity transformation
	b) MT_F of LCT from NC_2 at the MGA-in-class design workshop (**MT_F of SG_3**)	Collaboration of constructing a solid model 
	c) $MTER_A$ at the facilitators' one-day PD workshop (**$MTER_A$ of PG_3**)	Intervention in the focus of the revised design during co-planning the lesson
4 (Conjecturing-testing tasks)	a) MT_F of LCT from NC_2 at the MGA-in-class design workshop (**MT_F of SG_4**)	Comments of students' learning problem with proportional reasoning of similarity property 
	b) $MTER_A$ at the MGA-in-class design workshop and MGA-in-class video shooting (**$MTER_A$ of PG_4**)	Mission of proportional reasoning with conjecturing and testing approach (from intuitive double side length to triple side length) and its theories of learning from measuring (intuition) to quantity (formal), and diagnostic conjecturing
	c) $MTER_A$ at the facilitators' one-day PD workshop (**$MTER_A$ of PG_4**)	Intervention in the focus of the revised design during co-planning the lesson

The Whole Group's (WG) Practical Supports to Revising Jean's Instructional Devices

The supports provided by the WG were mainly focused on general pedagogical and technical issues of helping Jean to revise the devices of her projection design. These WG supports mainly happened in the first two pedagogical problems (metaphorical situation and proper similarity projection) she met. According to Table 2, it could be found that WG supports were mainly given by the LCTs from different cities (e.g. MT_A of WG_1, MT_B of WG_1, MT_D of WG_2, and MT_E of WG_2). In the preliminary PD period, it is not easy for Jean to accept others' comments, for example, she expressed that she did not accept MT_A's suggestion because of her professional proud and this miss made her revision lagged (interview, 2018). Until the next time, she was suggested the same technical support by another teacher (see MT_D of WG_2) with experiment, she realized that this suggestion is really workable.

The Small Group's (SG) Practical Supports to Criticizing and Refining Jean's Design

The SG provided Jean critical and key supports in revising her design. For example, the MT_F who was from LCT of NC_2 as Jean, collaborated closely with Jean to make the manipulation with a solid projection model realizable. With her supports, Jean solved the mission posed by $MTER_A$ quickly and correctly. They constructed a solid projection model with two geometric planes and three bamboo sticks (see the corresponding picture of the support given by MT_F of SG_3). The manipulative devices make the dilation (similarity transformation) being manipulated by students. Moreover, during the collaborative working, MT_F mentioned an error commonly seen among students in learning the similarity property (see MT_F of SG_4). She pointed out that students are easy to make mistake in deciding the ratio of *light-object* to *object(light)-shadow* based on the sides' ratio between the object and the shadow, e.g. when the side length of the shadow is triple of the object, students considered that the distance ratio of *light -object* to *object-shadow* is 1:3 (it is 1:4 instead; interview, 2018). This pedagogical comment triggered the fourth pedagogical problem for Jean to solve.

The Professional Group's (PG) Theoretical Supports to Completing Jean's Design

The PG provided key missions and questions underlying theories for Jean to better her design. For example, the $MTER_B$ suggested to add one stretching task with mobile phone before the stretching task with powerpoint (see $MTER_B$ of PG_2). It is because the stretch with mobile phone keeps the figures similar, while it may not the same with powerpoint. Such opportunity could create a conflict between them both for students, and spotlight the importance of the following projection activity.

In addition, the $MTER_A$ posed four critical questions at different stages for Jean to solve. The four critical questions/comments are: (1) What kind of projection can represent the similarity transformation (between the object and the shadow)? (2) Why do all similar figures follow the property of the proportional segments of parallel lines with (externally) intersectional transversals? (3) Learning approach from observing/manipulating the situations (specific) to quantifying them (general), and (4) How to involve the diagnostic conjecturing into students' learning? (interview, 2019). These critical questions and comments completed and bettered Jean's design gradually to differentiate the stretch and dilation (i.e. the need of projection activity), and to realize the manipulation with similarity transformation and then quantifying it.

Discussion

Reflecting on Jean's learning in this PD model of design-based research, it is found that the supports of different forms of collaborative groups acted various functions on Jean's professional learning (see the contributions of WG, SG, and PG). Such variety of collaborative groups could help the participant MTs produce better design in design-based PD. The peer MTs in WG and SG as brokers provided collaborative and creative practical-praxeologies, while the MTERs of PG as brokers provided constructive and critical ideas and interventions based on their theoretical praxeologies. Through collaboration among different collaborative groups, it helped Jean's professional growth from being too proud to accept others' advices (i.e. to ignore other's good ideas) to becoming one who could try to understand the connotation of the advices and gradually adjust herself in her learning. Moreover, through the brokering supports, the design-based research motivated Jean to explore students' learning problems and to solve them from different perspectives that she had never tried before.

Regarding the future studies, this study provides two implications. First is that the design-based research provides abundant theoretical and practical opportunities for MTs to learn and adjust their instructional approaches between teacher-centered and student-centered instruction. Second, the collaborative groups play an important role of design brokers providing both theoretical (esp. the MTERs) and practical (esp. the peer MTs) supports for the MT's evolution during professional learning. In this study, we only reported one MT's learning trajectory with the peer MTs and the MTER's brokering supports. However, such collaboration could be extended to discuss the whole community's evolution, such as the change of MTERs or the change of other MTs based on the reactions they received among collaborative groups in PD programs.

Last, the PD model of design-based research through collaboration not only provides teachers opportunities to practice, i.e. the praxis opportunity (practical praxeology), but also helps them to know the underlying knowledge, i.e. the logos opportunity (theoretical praxeology) of their designs through discourses. They could receive different levels (from direct to indirect) of comments from the professional communities in such PD model.

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THE COLLABORATIVE WORK ON SCENARIO DESIGN AS A TOOL TO FOSTER TEACHERS' PROFESSIONAL DEVELOPMENT

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In this paper we introduce the idea of scenario design as a method to support teacher professional development (PD). Specifically, we refer to the Meta-Didactical transposition framework to analyse data from a PD programme implemented in Italy, with the aim of highlighting how the collaborative work on scenario design could foster the evolution of teachers' meta-didactical praxeologies. To strenghten this analysis, we present teachers' reflections about the effects that their collaborative work on the scenario design had on their teaching practices and, in general, on their PD.

In this paper we introduce the idea of the *scenario design* as a way of extending the typical activity of task design. We define the scenario design as a process of envisioning possible implementations of mathematical classroom activities in which the interventions of the students and the teachers are made explicit. In particular, this process consists, not only in designing the tasks for students and the teaching methodology, but also in hypotesising possible students' answers to the tasks and hypothetical excerpts of classroom discussion, containing teachers' interventions. The product of this process is an ordered set of scenes - herein called scenario - representing the foreseen development of the classroom activity (i.e. teachers' interventions aimed at supporting students' learning processes, highlighting and discussing their difficulties, activating students' reflections…).

The scenario design has been implemented with a small group of in-service secondary school teachers, who volunteered to take part in a eight months PD Programme focused on teaching methodologies for mathematics education. Herein, our aim is to investigate how the teachers' collaborative work on scenario design triggered and fostered their professional development, while they worked together and also worked with mathematics teacher educators within University Professional Development (PD) Programmes. To develop this analysis, we will refer to the Meta-Didactical Transposition framework (Arzarello et al. 2014).

Theoretical Framework: the Meta-Didactical Transposition

Meta-Didactical Transposition (MDT) is a theoretical framework used to describe mathematics teacher professional development as a process that comprises a number of variables and their possible changes over time, in a dynamic way. It was first introduced in Italy, and then more widely by the international community (Arzarello et al. 2014, Aldon et al., 2013, Prodromou et al., 2018). It is based on Chevallard's Anthropological Theory of Didactics (Chevallard, 1991) - which is contextualised in the teaching of mathematics at school - and considers the relationships and reciprocal influences of teachers and researchers involved in professional development, within their target institutions. The term meta-didactical refers to the fact that important issues related to the didactical transposition of knowledge are faced at a meta-level. During an educational programme, teachers usually encounter a new didactical paradigm, and, after careful reflection on it, they may reconsider and discuss their practices, and they may also change them. This change is described in terms of evolution of teachers' praxeologies, made of four main components (according to Chevallard, 1991): on the pragmatic side, the task and technique and, on the theoretical side, the technology (term used by Chevallard to mean justification) and theory. If a didactical praxeology refers to teacher's activity into the class, a meta-didactical praxeology refers to teacher's activity in PD programmes. In our case, the task for teachers, given by the researcher (acting as a teacher educator), is to design a scenario. This task has to be solved taking into account specific criteria (the techniques), and justifying the tecniques referring to theories from mathematics education (the technology and theory components).

The possible change in teaching practices is described in terms of changing of the status of the components of teachers' praxeologies. These components, in fact, can pass from external - located outside the teacher's personal world (Clark & Hollingsworth, 2002) - to internal – constitutive of teacher's professional world of practice. Teachers' praxeologies evolve due to a double dialectic (Arzarello et al., 2014) that encapsulates two interrelated processes: (1) a first dialectic, at the didactic level, in the classroom, highlighted through the analysis of teachers' practices; (2) a second dialectic, at the meta-didactic level, within PD programmes, engendered through the shared reflections, developed by teachers and researchers when interpreting the first dialectic.

Method

The PD programme

The PD programme consists of six meetings which took place in the Mathematics Department of Turin University. One of the authors was the teacher educator of the programme who interacted with the teachers. Other two researchers took part to the meetings as observing participants. The first meeting was devoted to presenting theoretical aspects to frame mathematics teaching methodologies. The second meeting was devoted to a "ready made activity", aimed at supporting the teachers' appropriation of the theoretical aspects shared in the first meeting, involving them in a classroom activity where they played the role of students, while the teacher educator played the role of the teacher. The task for the teachers for the next meeting was to work in pairs or groups of three to design a scenario on a given mathematical task for students (see Figure 1). This work has been done using a software to create comics (Lesson Sketch, https://www.lessonsketch.org).

Task for teachers

You are requested to work in pairs or groups of three.

- Design a scenario for implementing the task for students in your class, writing an hypothetical excerpt of a classroom discussion, to be developed starting from possible students' answers.
- The focus of the teacher's interventions should be on fostering a comparison between students' answers, supporting their learning processes, highlighting and discussing their difficulties, and activating reflections .
- Use the Lesson Sketch software to construct the scenarios.
- Share your prepared scenarios with the researchers.

Task for students

On a segment AB, whose length is 10 cm, choose a point P, between A and B, and construct two squares on the same side with respect to the line AB: one with side AP, the other with side PB.
Explore how the perimeter of the entire figure obtained from the two squares varies as the position of P changes on the segment AB.

Figure 1: The task for teachers and the task for students

The third meeting was devoted to discuss the scenario prepared by the teachers. Between the third and the fourth meetings, teachers were requested to re-design the scenarios according to the feedback given during the third meeting. Also, the fourth and fifth meetings were devoted to a collective discussion on the re-designed scenarios. After the fifth meeting, the activities were implemented by teachers in their classes and the final meeting (the sixth) was devoted to a discussion about the way the final scenario mirrored with what really happened when the teachers used it as a script for their classrooms' activities and to final reflections about the PD process.

Participants, data collection and data analysis

Four teachers from scientific oriented secondary school (9 and 10 grades) and one teacher from middle school (6-8 grades) participated in this study. All the teachers, except for the middle school teacher, were familiar with this kind of PD programmes in which researchers and teachers share and discuss didactical ideas together. The mean teaching experience of the participating teachers is 25 years. The group met six times, from October to June. The meetings, conducted by three researchers and teacher educators, were video-recorded and the videos were transcribed. The written scenarios were also collected and analyzed. In the last meeting, the teachers were interviewed and they were asked to reflect on their experience on collaborative scenario design and on the comparison between the scenario design and the actual implementation of the task in their classes. The interview was also recorded and transcribed later. To analyse the data, we describe the changes of the designed scenarios in the course of the PD programme in order to infer the evolution of teachers‘ praxeologies. Moreover, we refer to the teachers‘ reflections at the end of the programme to strenghten our inferences.

Results

The data we are going to analyse refer to the third, fourth and sixth meetings of the researchers and the teachers who attended the PD programme. With regards to the third and fourth meetings, we will concentrate on the discussion developed on the scenario designed together by two teachers, Paola and Silvia, who planned to implement the activity in a 10-grade class (upper secondary school with a scientific orientation). Because of space constraints, we will focus on the discussion developed on some scenes of the scenario and on the re-design of these scenes by the teachers.

As stated above, before the third and fourth meetings, the teachers' meta-didactical praxeology is referred to the task of designing a scenario. The teachers do not have a meta-didactical praxeology for scenario design at the beginning of the PD programme (since they never engaged with this kind of task), so we can assume that the techniques at their disposal, after the first two meetings, are the initial criteria for the scenario design (see the task for teachers in Figure 1). In the same way, we can assume that the theoretical part of their initial praxeology is composed by the theoretical frame shared during the first and second meetings (theoretical reflections on the design of tasks to support students' inquiry and on the role played by the teacher in implementing this kind of activities).

At the beginning of the discussion developed during the third meeting, the teachers refered to their initial scenario as a "a pre-draft", because they were aware that it should be improved to become in tune with what was required by the researchers (task for teachers, Figure 1). The teacher educator displayed, through the video-projector, the different scenes of Paola and Silvia's scenario, organized in a PowerPoint presentation.

In *scene three* the teacher poses to the class the question in the task for students (see Figure 1), and, in *scene four*, a pair of students observe: “If P moves away from A, then the perimeter decreases”, and the teacher asks to the rest of the class: “Do you agree?”. Paola declared that they chose to start with this students’ intervention because it reflects the possible result of an initial partial exploration of the relation between the perimeter and the length of the segment AP.

In *scene five*, the students answer with “yes” or “no” to the question posed by the teacher in the previous scene (“Do you agree?”). Specifically, four students answer “yes”, highlighting that they do not grasp that the perimeter is not always decreasing, and only one student answers “no”. The teacher educator posed a question about what the teacher should do if the answers given by the students were not in tune with this hypothesis: “*if there isn’t this ‘No’, what can you do? And if they all say ‘no’*?”. In the subsequent discussion the teachers agreed that the scenario should be constructed as a sort of ‘*tree of possibilities*’, with the aim of identifying possible different ways of developing the classroom discussion, according to the actual interventions of the students. This was the first new criterion for the scenario design that was shared during the third meeting. The teacher educator also suggested a possible strategy to make the students discuss answers that were not given by them: the teacher can say that, in another class, a student proposed the answer that she wants to discuss.

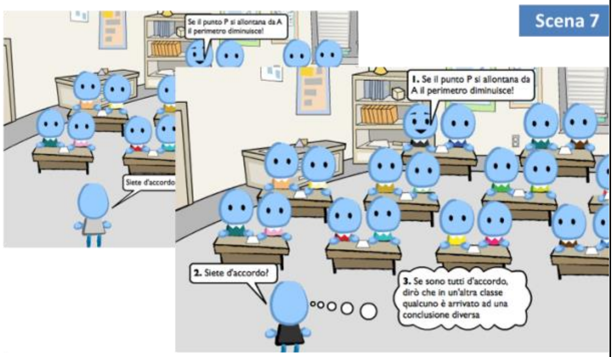
In *scene seven*, Paola and Silvia envisioned different students’ interventions to support the “yes/no” answers given in the previous scene. Among these interventions, in particular, they hypothesized that a group of students may construct an incorrect symbolic expression “40-2*x”*, to describe the relationship between the length of AP (*x*) and the perimeter of the figure, and may add: “*if x increases, the perimeter decreases*”. The discussion of this scene focuses on two aspects: the importance of making students discuss their interpretations of the algebraic expressions and the ways to involve the students, who did not use an algebraic approach, in the discussion.

In the subsequent part of the third meeting, another important focus was on the need of making explicit the motivations associated to the different interventions of the teacher, according to the aim of the classroom discussion. As a result of these reflections, a second new criterion for the scenario design was shared, that is the idea of inserting ‘*thought balloons*’ to highlight the reasons why the teacher proposes specific interventions during the discussion: “*…we could insert both what the teacher says and what she thinks*”. The aim of this choice is to to make explicit what is implicit, to foster the sharing, at a meta level, of the scenarios.

The third meeting ended with a task for the subsequent meeting: going on in the scenario design and re-design, taking into account all the reflections developed during the discussion between teachers and researchers. The influence of this discussion was evident in the re-design of Paola and Silvia’s scenario, discussed during the fourth meeting.

A first evidence of this influence was that Paola and Silvia inserted, in the re-designed scenario, new scenes aimed at highlighting the thinking of the teacher, in particular with reference to choices about the teaching methodology. For example, the thinking of the teacher is envisioned in *scene seven* (*scene four* in the previous version of the scenario), where the teacher thinks about what to do in case all the students agree with an incorrect solution proposed by a group of students (“If P moves away from A, then the perimeter decreases”). The thinking of the teacher displays another important effect of the discussion developed in the third meeting about the ways in which the teacher should react to specific students' interventions in order to focus their attention in discussing the most important elements that could support their understanding (in Table 1, the evolution of scene seven).

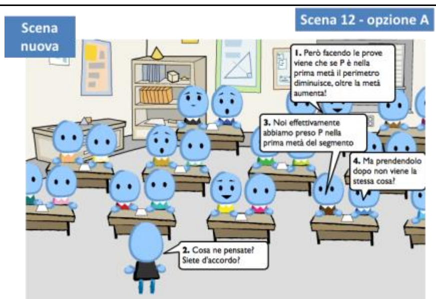
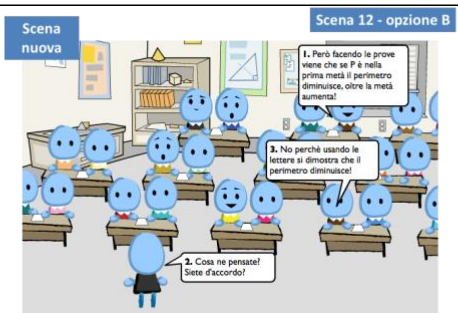
Table 1: The evolution of scene seven

<table>
<tr><th>The two versions of scene seven</th><th>Translation of the balloons</th></tr>
<tr><td>
</td><td>*First version of the scene*:
Group: If P goes away from A, the perimeter decreases.
Teacher: Do you agree?

Second version of the scene:
Group: If P goes away from A, the perimeter decreases.
Teacher: Do you agree?
Thinking of the teacher: If they do all agree, I will say that in another class some students reached a different conclusion.</td></tr>
<tr><td colspan="2">**Acknowledgments:** The scenes have been constructed by teachers using the depicted tool and the ThExpians B character set in LessonSketch environment (www.lessonsketch.org). Intellectual property for LessonSketch, Depict and the ThExpians B character set belongs to The Regents of the University of Michigan.</td></tr>
</table>

Silvia and Paola also adopted the other new criterion, discussed during the third meeting: designing the scenarios as '*trees of possibilities*'. *Scene 12*, for example, is a new scene introduced by them and represents a first example of '*ramification*' within the scenario. In fact, the two teachers designed two versions of scene 12 (see Table 2), in which the students comment on the results obtained by the group who proposes the expression 40-2x as a representation of the relation between the perimeter and the length of AP (in *scene seven* in the previous version of the scenario).

Table 2: An example of 'ramification'

<table>
<tr><th>Scene 12 – Option A</th><th>Translation of the balloons</th></tr>
<tr><td>
</td><td>1. Making the numerical examples, we obtained that in the first half (of the segment AB) the perimeter decreases, while in the second half it increases.
2. Teacher: What do you think about it? Do you agree?
3. We actually chose P in the first half of the segment.
4. But, if we choose P in the second half, isn't it the same?</td></tr>
<tr><th>Scene 12 – Option B</th><th></th></tr>
<tr><td>
</td><td>1. Making the numerical examples, we obtained that in the first half (of the segment AB) the perimeter decreases, while in the second half it increases.
2. Teacher: What do you think about it? Do you agree?
3. No, because using the letters we proved that the perimeter decreases!</td></tr>
<tr><td colspan="2">**Acknowledgments:** The scenes have been constructed by teachers using the depicted tool and the ThExpians B character set in LessonSketch environment (www.lessonsketch.org). Intellectual property for LessonSketch, Depict and the ThExpians B character set belongs to The Regents of the University of Michigan.</td></tr>
</table>

In *scene 12 - option A*, one student of the group who directly tried to work on a symbolic expression understands the main problem connected to the incomplete representation they constructed, that is the lack of reference to the whole range of examples. In *scene 12 - option B*, the students who proposed the incorrect expression both rely on their construction, without analysing the expression in depth. Paola and Silvia insterted also other examples of '*ramifications*' (also ramifications with three branches) in their re-designed scenario. Reflecting about the possible 'ramifications' within the scenarios could represent a way to foresee, at the meta-didactic level, gaps between the teacher's intention and what comes to the students' minds (namely, similar to what Aldon (2014) defines bifurcations at the didactical level).

The evolved scenarios include many aspects connected to teaching-learning processes: mathematical aspects (envisioning students' learning processes when facing the task for students and their possible answers), aspects related to the teaching practice (envisioning possible ways in which the students and the teacher could interact), and aspects connected to teachers' justifications of specific didactical choices (making the thinking of the teachers explicit).

The analysis of the evolution of the scenario designed by Paola and Silvia highlights a corresponding evolution of their meta-didactical praxeology referred to the task of carrying out a scenario design. In particular, the technique-component (initial criteria for the scenario design) evolves within the space shared by teachers and researchers during the meetings: as a result, new criteria (the use of 'thought baloons' and the idea of scenario as a 'tree of possibilities') are integrated in the initial criteria, hence the technique-component evolves. Moreover, teachers modified their scenarios taking into account new elements that correspond to ways of questioning themselves when they face the scenario design: in which ways could the discussion be prompted? What could be the best starting point for the discussion? How could I react to specific students' answers? What should I do if this intervention is not proposed by students?

Also the task-component and the technology and theory – components evolve throughout the whole PD programme. An evidence of the evolution of these components of the teachers' praxeology could be highlighted if we focus on the reflections on the scenario design that the teachers proposed during their final meeting (the sixth). The new personal justifying discourses introduced by teachers, in fact, pose the task at a different meta-level. On one hand, they reflected on the role played by the scenario design in supporting their actual interaction with students, as witnessed, for example by this reflection: "*...while the students were working, I re-read my notes, going to see what I had written that I would do, ... it was useful for me to have clear what I wanted to ask and where I wanted to go*".

On the other hand, teachers also reflected on the role of scenario design as a tool to support their professional development: "*... we have acquired a different way of working, an ease in working, even on topics that have not been analysed or experimented in depth, in a different way, improvising a little more*". This demonstrates that teachers are aware that they interiorized the way of questioning themselves that characterises the practice of scenario design and that this new habit of mind enables them to become more flexible during their teaching practice.

Final discussion

In this paper we have shown how the collaborative work between teachers and researchers on the scenario design has fostered the evolution of teachers' meta-didactical praxeologies. The task, the techniques and the theoretical components of teachers' praxeology evolved within the space shared by teachers and researchers during the meetings, both in terms of changing, and of new-entering. This result has been specifically confirmed through the analysis developed in the previous section and by the reflections introduced by teachers during the last meeting, which highlight their awareness of this process. This enables researchers to develop further reflections about the role of scenario design as a methodological tool. The teachers, in fact, said that, thanks to their work on scenario design, they become able to better govern it, because the scenario evolved through their work on it, as the following reflection highlights: "*Having designed everything together makes you feel the activity yours. You handle it well. It is not like replicating something that has been built by others"*. This observation shows that teachers, working together, acquired new competences in developing the scenario design. This fact, from the research point of view, means that their praxeology components passed from external to internal ("yours" in teacher's word). Similar results were highlighted when the same PD programme was implemented in Australia, where the process of scenario design entailed teachers' reflections on their teaching practices while systematically studying the development of scenarios.

The reflections developed by the teachers during the last meeting of the PD programme highlight also their awareness about the fundamental role of their collaboration in the scenario design, as testified in this reflection: "*We also worked together on it. This is the richness of this work… It is different to share with someone and find out more elements*". Moreover, they also declared that an important added value was their collaboration with researchers because "…*from the interventions researchers do, you see that there is a different eye because it is that of someone who sees things from above*".

Another reflection concerns the complexity of the task of scenario design for teachers. All the teachers, in fact, declared that this task was really challenging for them, in terms of both time and mental commitment. As researchers, we have to think that, if, from one side, engaging teachers in scenario design prompts a shared (meta-didactical) praxeology, from the other side, there should be a corresponding didactal praxeology to be used in the class. In fact, scenario design is a process directed to transform design in actions. We are convinced that scenario design is not a guarantee that every scene will be played by the teachers in their classrooms. A pilot study we conducted in Israel aimed at examining the adaption of the scenarios in the classrooms revealed that playing the scenarios several times by the teachers, and reflecting on them after each time, improves the adaption between what is played and what is designed. Furthermore, we also learned that the teaching experience of the teachers determined the number of times the teacher may play the scenario to improve its adaption.

From the researchers point of view, this work on scenario design suggests further theoretical researches. One of these concerns the project of extending the well known idea of instrumental genesis to something that could be called 'meta-instrumental genesis', basing on the fact that the scenario design aims at teachers' education and not at direct students' learning.

Essentially, scenario design can be framed basing on the theoretical lens of the instrumental approach (Rabardel & Bourmand, 2005; Guin, Ruthven, & Trouche, 2005), which, as it is well known, distinguishes between an artefact, available for a given user, and an instrument that is developed by the user through the processes of instrumentation and instrumentalization. A scenario design is a process through which the teachers and the researchers interact with each other, representing, through the scenario (as artefact), what may happen in the classroom when introducing a certain didactical situation. The scenario design allows forecasting and commenting possible narratives in the way students and teachers can interact with each other while facing a didactical situation in the classroom. The teachers' reflections on these narratives engender a double-level dialectic that supports an evolution of scenarios and, simultaneously, of the techniques used to develop the scenario design, which corresponds to an evolution of the teachers' meta-didactical praxeologies during the PD programme. This process, as the process that occurs in the instrumental genesis, has a dual nature. On the one side, teachers and researchers guide the way the scenario is built and used (instrumentalization). On the other side, the effect of the scenario design is to dynamically transform what could be a usual a-priori or a-posteriori analysis (Artigue, 2009) into the elaborations of fresh scenario's utilization schemes for the didactic situations at stake. Hence the collaborative work on the scenario design stimulates a process of instrumentation by both teachers and researchers.

The further step of our research will be, therefore, to investigate the idea of meta-instrumental genesis, referring also to notions already introduced to analyse the process of instrumental genesis that characterize the teachers' work on artifacts/instruments (double instrumental genesis, by Haspekian, 2014) and on resources/documents (documentational genesis, by Gueudet & Trouche, 2009). Moreover, we aim to study how the relationship between meta-didactical transposition and didactical transposition could support the investigation on the relationship between meta-instrumental genesis and instrumental genesis.

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DISCUSSING 'LESSON STUDY' AND 'DIDACTICAL SUITABILITY CRITERIA' AS TOOLS DESIGNED FOR TEACHER COLLABORATION IN MATHEMATICS

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Lesson Study and the Didactical Suitability Criteria are two major professional tools for teachers to critically reflect on their practice to improve it. This paper aims to discuss how these two tools can complement each other when teachers collaborate in designing an interdisciplinary lesson embedded in a Professional Training Program addressed to Pre-K and Primary pre-service teachers, in a High Education (HE) institution. We use a 'design research' approach to conduct the study. We use the indicators developed under the frame of the Didactical Suitability Criteria to assess teachers' competence in "planning and managing" a lesson, which is part of the 'Lesson Study' process. We conclude that this study provided insight for pre-service teachers to develop some fundamental competencies concerning their teacher training abilities.

This paper aims to discuss how 'Lesson Study' (LS) and the 'Didactical Suitability Criteria' (DSC) can complement each other when teachers collaborate in designing interdisciplinary mathematics lessons. We discuss how teachers engage in collaborative work using these two tools. Drawing on LS and DSC, teachers become critically aware of different sets of 'mathematics teachers' competences,' which can be assessed and fostered by combining those two professional tools.

Theoretical Framework

There is an increasing interest in looking for avenues of dialogue between researchers and teachers to improve teachers' teaching effectiveness. During the last decades, experiences such as lesson study (Fernández & Yoshida, 2012) or teachers acting as researchers (Muijs & Reynolds, 2017) have been the more and more common all over the World. Even successful organizations such OECD have included the teachers' perspective within their surveys, as demonstrated by the delivery of TALIS (Teaching and Learning International Study) since 2008. Professional development appears to be a relevant aspect of teachers' effectiveness in teaching mathematics (Díez-Palomar, 2017). However, as stated by Kennedy (2016), not all professional development initiatives promote effective learning. To avoid the adverse effects of adopting ineffective actions, we need to base the design of our professional development programs on robust and reliable research findings.

Many different approaches in teachers' professional development have emphasised the important role that professional discussion on teachers' practices have on improving their teaching. Schön (1992) argues about the necessity of training reflective professionals, able to critically think on their practices, while schools should become more open to encourage teachers' reflective practices as a crucial element for teachers' professional training opportunities. Breda, Font, and Pino-Fan (2018) report on different theoretical instruments oriented towards this aim, including LS (Fernández and

Yoshida, 2012), which is a research activity (Ponte, Baptista, Velez and Costa, 2012) leaning to the development of teachers' reflective competence. Besides, Ontosemiotic Approach of Mathematical Knowledge and Instruction (OSA) (Godino, Batanero & Font, 2007) provides us with the DSC and their components and indicators, as a methodological tool to structure teacher's critical thinking.

LS (Isoda, Stephens, Ohara and Miyakawa, 2007) has been a methodology used to improve the teaching and learning of mathematics that has become more and more present in professional training programs, all over the World (Adler & Alshwaikh, 2019; Dudley, 2015; Ebaeguin & Stephens, 2014; Huang et al., 2018; Inprasitha, 2011; Lewis & Tsuchida, 1998; Murata et al. 2012). Although LS can be used to create tasks or good practices, some studies are pointing out that this is not its primary objective. Actually, according to Yoshida (2012), the main concern upon using this methodology is the fact that teachers "outside of the lesson study communities will implement these products or materials without understanding the intentional goals and expected student outcomes, because of their lack of understanding of content and pedagogy and/or lack of skills to observe and assess student learning in the classroom." (p. 141) Yoshida complains that many teachers without a strong understanding of content, pedagogy, and curriculum design will fail in using LS to engage in designing mathematics lessons.

Combining LS with the use of the DSC may open the possibility to cope with Yoshida's concerns and provide a methodology to improve teachers' content knowledge, pedagogical content knowledge, and curriculum knowledge, in Shulman terms (Shulman, 1986). The DSC is intended to be a partial response to the following questions: What criteria should teachers use to design a lesson, which will allow them to evaluate and develop their students' mathematical skills? And What changes should they make in their lesson design to help their students to meet those mathematical skills? DSC can first serve as a guideline for teachers to design their lessons and, secondly, assess the implementation of that lesson, to re-design it after the discussion of the results of that implementation. The DSC includes the following suitabilities (Font, Planas, and Godino, 2010):

(1) Epistemic suitability, to assess whether the mathematics being taught are "good mathematics" or not.

(2) Cognitive suitability, to assess if what is intended to be taught is at a reasonable "distance" from what the students' know, or not; and after the implementation if the lessons learned are close to what is intended to learn, or not.

(3) Interactional suitability, to assess whether the interactions between the teachers and the students (or students-students) solve students' questions and difficulties, or not.

(4) Mediational suitability, to assess the appropriateness of the resources used in the lesson.

(5) Emotional suitability, to assess the involvement of students during the lesson.

(6) Ecological suitability, to assess the appropriateness of the lesson to the educational project of the school, the curricular guidelines, and the conditions of the social and professional context.

The usability of the DSC methodology requires defining a set of observable indicators for each of the six components above mentioned, which will allow teachers to assess the degree of suitability of their practices. In this paper, we discuss how using the DSC methodology in the second stage of the LS (which is 'Collaborative Planning' on the lesson conducted) may help participants in that process to achieve the goals of using LS to improve the teaching practices through the development of a set of indicators addressed to further develop teachers' competences.

Methodology

This study is framed within an ARMIF research project (Castelló, Giménez, Godall, Puig, & Tilló, 2017), and the RTD research project funded by the Spanish Ministry of Science, Education and Universities titled *Use of the Lesson Study and the Concept of Didactic Suitability in the Development of the Competence in Analysis and Didactical Intervention in the Frame of Mathematics Teachers' Training*. In this context, an LS unit was developed in a 'Professional Training for Pre-K and Primary Pre-service Teachers Program' for mathematics teachers, held in a High Education (HE) Spanish Institution. Table 1 summarizes the stages of the LS designed, evidencing the collaboration and the use of resources.

Table 1: Structure of the 'Lesson Study' sessions

1. Setting the goals	
Session 1: Introduction to the ARMIF framework. General aspects and specific contributions. Agree on operating rules, norms, and schedule. Session 2: Introducing the selection of articles within newspapers. Discussion of the tentative concepts that may be used to plan the lesson.	Collaboration between the associate professor and pre-service teachers. Resources: ppt projection and planning for the LS Participants: associate professor and pre-service teachers. Resources: news in newspapers
2. Collaborative Planning	
Session 3: Design of a classroom session. Draft. The professor discusses the draft with each group of pre-service teachers.	Participants: associate professor and pre-service teachers. Resources: Template for lesson planning
3. Implementation	
Implementation of the lesson designed.	Participants: pre-service teachers. Resources: Didactical material (see Figure 1)
4. Reflection and Debrief	
Session 4 and 5: Presentation of the final version of the lesson implemented and its results. Self-evaluation and collaborative discussion with the whole group. * After the last session, each group of pre-service teachers presents their final lesson, adding the contributions from the collaborative discussion conducted in session 5 (lesson re-designed)	Participants: associate professor and pre-service teachers. Resources: ppt presentations, didactical material and lesson planning; rubric for evaluation

Participants in this study were pre-service teachers of mathematics. They worked in small groups. In the first stage of the LS ('Setting the goals'), the ARMIF project was introduced. Pre-service teachers were asked to use articles published in newspapers as "inspiring resources" to find a topic suitable for designing an interdisciplinary (mathematics and science) lesson. Drawing on this focus, they had to create a lesson unit integrating both mathematics and sciences, addressed to pre-K and K students. During stage 2 of the LS ('Collaborative Planning'), a design research approach was adopted by the pre-service teachers participating in this study.

In each lesson, some key competencies were assessed at least in two different moments (at the beginning -session 1- and during the design -session 2 or 3-). All sessions included time for reflection were participants from each small group had to identify their progress in designing their lessons. At the last LS stage (sessions 4 and 5), all participants analyzed their work regarding the planned lesson qualitatively. Students' presentations and focus group discussions were used to conduct such an analysis. Each group of students presents their work to the mainstream. In such a presentation, the future teachers presented not only the tasks for early childhood children but also the Science and Math approaches and their relationship. Some different chosen topics were challenging and original for Kindergarten's children.

Figure 1. Images are given by the group presenting the theme "Archeological time and human evolution."

After hearing each of the projects, all students (as well as the teacher) used a rubric to evaluate their peers' (students) work. They discussed what kind of indicators can we identify to assess the development of teachers' key competencies.

Results

All pre-service teachers were able to design a lesson integrating mathematics and science in their designs. They combined both components (mathematics and science) to produce innovative STEM lessons (e.g., the Solar System, the Volcanoes, the Earthquakes, the Honeybees, etc.) addressed from Pre-K to Primary pupils. Using this "dialogue" between Mathematics and Science, the pre-service students were able to create situations and contexts for effective mathematics learning.

One aspect that pre-service teachers think that it is crucial to plan their didactic sequences is the contextualization, using resources such as the newspapers (mediational suitability). Thus, for example, we find statements such as:

> In particular, we have started with a piece of news to be able to explain a set of mathematical contents, so that all students would be motivated, and the tasks are contextualized in a familiar and real context. (Group "The marine turtles").

On the other hand, we find reflections that, in addition to highlighting the importance of contextualization, allude to the need to address innovative and complex issues in early childhood education:

> On the one hand, the choice of the text from a newspaper allows us to deal with a breaking news topic, which usually is not included as a lesson in the regular classroom in early childhood education; but we believe that it is possible to use it satisfactorily if we adapt it to that age stage. Besides, that piece brings us closer to the discovery of a new species drawing on the bones found in South Africa. This fact has allowed us to devise activities that relate the adaptation and evolution of species with their physical and functional changes, specifically the bones and the use of different instruments to build tools. (Group "Archeological time and human evolution").

This fact (innovation) was explicitly connected by the pre-service teachers to the epistemic suitability of the lesson designed. In the example of the study about evolution, we analyzed that the group proposes to discover a treasure of archeological materials (different bones, stones, glasses to travel through time, and so on). Using the six components of the DSC (Table 1), the pre-service teachers were able to produce a rich lesson covering important aspects related to mediational, cognitive, and epistemological suitabilities. They planned a task addressed to 5 years old children, asking them to classify the shapes of the archeological pieces, putting them on an evolution table representing the evolution timeline. The following tables 2 and 3 summarize the indicators developed by the research team to evaluate students' contributions, in line (mainly) with the epistemological and cognitive components of the DSC methodology. Table 2 is focused on the evaluation of the competence "planning and management of the lesson."

Table 2: Indicators to evaluate the competence 'Planning and management of the lesson.'

Dimensions	Categories	Indicators
Structure and strategic thinking	The relevance of the structure	Analysis-synthesis: Coherence, correct identification of the key ideas
	Strategic action	Integration of structures and systems of meaning. Use of schemes to explain new relations of knowledge
	Complexity	Addressing complex situations
Adaptability and regulative control	Re-addressing	Be able to design proposals to improve
	Self-criticism	Learn and improve from their own mistakes
	Transference	Face new situations
Decision making and applicability	Coherence	Exchanging ideas in complex cases, in specific moments of the work timeline
	Capacity of reaction	Acceptance of the mandatory changes to re-address the situation
	Applicability in real situations	Carrying out the proposed methods

In the evaluation that the students carry out on their work and one of their peers, they recognize different aspects as presented in Table 2. Statements such: "The report has addressed a whole series of aspects related to science and logic-mathematics fields, such as classification, comparison, relationships, measurements, logical reasoning, time and change, human beings, the discovery of natural elements in the environments, among others." (Group "Archeological time and human evolution"), together with the own design of the activities addressed to the children, are evidence of the pre-service teachers' ability to recognize the category: relevance of the structure (which is aligned with the epistemological design of the lesson). This happens because pre-service teachers can identify key ideas and coherence among the proposals.

Table 3, in turn, is devoted to analyzing the competence "Creativity, management and implementation."

Table 3: Indicators to evaluate the competence 'Creativity, management and implementation.'

Dimensions	Categories	Indicators
Creative learning	Innovative opening	Generation of new situations of ideas Proposal for breaking ideas regarding the established procedures
	Flexibility	Using different kinds of strategies
	Originality	The contribution of original ideas to the problems presented based on the resources that are known
	Integration and formal expression	The appropriate approach to generate ideas
	Creative application	Recognition of innovative ideas
Management capacity	Project design	Prioritization of mid-term and long-term objectives, undertaking corrective actions if necessary. Flexible planning and coordination of collaborative tasks
	Global management	Application of quality levels. Analysis of the context to define specific objectives as a response to innovative challenges that s/he proposes
Entrepreneurial and innovative capacity	Leadership	A global vision of the reality that surrounds him An evaluation of positive and negative aspects of the innovative proposals
	Implementation	Active positioning and ability to convince A proposition of improvements needs
	Analysis of risks and benefits	Planning possible actions that will be undertaken

An interesting example is a work about "Swirls." Since the beginning, the future teachers engaged in the activity asked the trainer: "We think it is an interesting theme for Kindergarten but, we do not know how to introduce the mathematics in this theme?" After several discussions, they finally proposed the spiral shapes in three dimensions, which is unusual for activities addressed to children of this age. In many other examples, prospective teachers are aware to use extramathematical connections (Vanegas & Gimenez, 2018) but they had difficulties to identify mathematical activities. They have at least two different moments of collaboration. First when they are asked to talk about the other groups ideas; second, when they present the final group reflection.

> "In this activity, we decided to work the mathematics with patterns ... It was difficult for us to understand that in this we did mathematics, so that we initially saw only the scientific work. We greatly helped us our colleagues' comments" (*wax worms* group).

Such an approach means the use of reflective collaborative process to understand the need for adapting mathematiscal knowledge to be understandable without forcing the mathematical objects (epistemological dimension of the DSC).

In addition to the recognition of the possibility of tackling unusual topics, the group of pre-service teachers that proposed the theme “swirls” implemented one of the activities of its didactic sequence with some children.

We also found reflections suggesting the progress of the pre-service teachers in understanding the dimension "Entrepreneurial and innovative capacity," which demonstrate their capacity to differentiate proposals and evaluate positive and negative aspects of the innovative proposals.

Discussion

Previous research has provided enough evidence on the positive impact that teacher collaboration has in improving lesson design and teachers’ practices (Schleicher, 2015). Tools and methodologies such as LS (Yoshida, 2012), teacher noticing (Jacobs, Lamp, & Phillip, 2010), frameworks of student mathematical thinking (Carpenter et al., 2014), identification of relevant episodes (Schoenfeld, 1987), cycles of reflection (Smyth, 1991), the model of reflection-and-action within communities of practice (Parada, Figueras, & Pluvinage, 2011), among others, have demonstrated that teacher collaboration to reflect on teachers’ practices have a significative impact on improving the work that we do in our classrooms. However, as the ‘ICMI-25 Study Discussion Document’ points out, there are still many questions around this topic that make our understanding of how this type of collaboration work is still limited. The work that we are reporting here has the potential to contribute to clarifying "on the specific mathematics knowledge and pedagogy" that we have learned after all the research conducted on teacher collaboration, in the last decades. Combining LS and DSC, we have found concrete indicators to assess (and even measure, as in some work in progress) mathematical concepts (drawing on the epistemological suitability), ways to teach/learn them (connected to the cognitive suitability), as well as resources that teachers can use to make that happen (linked to the mediational suitability). Aspects such as innovation, creativity, management, etc., emerge from using those tools in teachers’ collaboration practices. The DSC offers yet additional components teachers, for their collaborative work, as the interactional suitability and the ecological suitability, which increases the potentiality of the DSC to highlight and make explicit relevant aspects of the mathematics knowledge and its pedagogy. LS is a great tool to provide some structure to the process of teacher collaborative reflection. Both approaches are conceptual tools emerging from teachers collaboration, that have been validated for previous research (Adler & Alshwaikh, 2019; Breda, Font, & Pino-Fan, 2018; Ebaeguin, & Stephens, 2014). The results obtained here (as well as in previous studies) suggests that such a combination may contribute as available tools to support teachers' collaboration. However, still, we will need further research to clarify how teachers engage in using LS and DSC. Here the framework was a training process in a Professional Training Program, which shows the potential of using LS and DSC as tools in the formative stage of pre-service teachers. However, questions such as how to keep using such tools in real school-settings, in which we need to consider the institutionalized and normative aspects of teaching, or the “readiness” of in-service teachers to learn how to use those tools (which assumes that they are “open” to critically reflect on their own practice), need to be further explored.

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THE GESTINV DATABASE: A TOOL FOR ENHANCING TEACHERS PROFESSIONAL DEVELOPMENT WITHIN A COMMUNITY OF INQUIRY

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In this study we present a new model to design activities for mathematics teachers' professional development. The model results from the interaction of the Mathematics Teacher Specialized Knowledge (MTSK) design with a new tool (Gestinv, a structured and interactive database) within a community of inquiry. We follow a developmental approach to teachers' awareness of their beliefs, and convictions regarding mathematical knowledge and pedagogical content knowledge. We frame communities of inquiry in an Activity Theory perspective.

Rationale

Mathematics teacher training has become a research topic that entails theoretical, methodological and educational issues. Several researches (Shulman, 1986; Wenger, 1998; Ball, Thames, & Phelps, 2008, Jaworski, 2006; Da Ponte & Chapman, 2006; D'Amore & Fandiño Pinilla, 2009; Bartolini Bussi et al. 2017; Carrillo-Yañez et al. 2018) have enhanced different theoretical perspectives and methodological design that allow teachers to foster their specialised knowledge in order to implement in their school practice effective and forefront teaching methodologies and to create new cultures of mathematical activity. Our study develops within this line of research focusing both on a theoretical perspective for the design of effective methodologies and on the introduction of tools used and designed for teacher collaboration. Our methodological design results from the networking of two robust theoretical perspectives in the field of teacher training: Jaworski's notion of Community of inquiry (2006) - within a fallible stance towards knowledge it conceptualizes mathematical knowledge and knowledge in teaching as resulting from inquiry, intertwining the knower and the knowledge - and the Mathematics Teacher's Specialised Knowledge (MTSK) model (Carrillo-Yañez et al., 2018). We believe that these perspectives take into account two defining characteristics of a teacher training model. On the one hand we need to conceptualize appropriate social interaction within a community of practice between teachers and between teachers and didacticians. We consider teacher training a developmental process that entails *teacher change* (Guskey, 2002) in terms of a transformation of beliefs, convictions (D'Amore & Fandiño Pinilla, 2004), *Weltanshauungen* regarding mathematics, teaching-learning processes, the students and the political and social role of the education system. Such a change cannot be a solitary, individual and autonomous process, instead it is constitutively a sociocultural activity whose outcome is the transformation of the individual's identity as a teacher. On the other hand we need to conceptualize and outline the specific knowledge and professional skills that we would like teachers to achieve as a result of the change they undergo in their training process. We are referring to a wide range of knowledge that includes mathematics, epistemology, pedagogy, didactics, psychology etc. Sociocultural development within a community of practice requires tools that mediate the activity, contribute to the interpersonal exchange within its actors and bring to the fore cultural and conceptual objects.. As highlighted by the *ICMI 25 - Discussion Document*, within *activity theory* (Wertsch, 1981) perspective, Grossman, Smagorinsky and Valencia (1999, p. 14) make a distinction between *conceptual tools* and *practical tools*. The former are "principles, frameworks, and ideas about teaching [and] learning … that teachers use as heuristics to guide decisions about teaching and learning". The latter are classroom practices, strategies, and resources such as daily and unit plans, textbooks, and instructional materials that "do not serve as broad conceptions to guide an array of decisions but, instead, have more local and immediate utility”. Our study brings into play a new tool that a group of mathematics education researchers on behalf of the Italian National Evaluation Institute for the School System (INVALSI) introduced in 2014 in order to create a friendly tool for teachers, researchers and all the stakeholders involved in the education system (Bolondi, Ferretti & Gambini, 2017). The Gestinv database collects in a structured way a broad range of information regarding the national mathematics test that INVALSI issues annually and that involves all Italian students of grades 2, 5, 8, 10, 13. This tool allows users to carry out focused and cross researches concerning the national tests, available from 2008, according to mathematical contents and their relationship with the National Guidelines, the results of the tests and the related percentages - percentage of correct, incorrect, invalid and missing answers and, for multiple choice task, the percentage of each option - school level, content keywords and statistical features (characteristic curves, distractor plots, ITN). Our experience as teacher trainers shows that Gestinv is a resource that intertwines conceptual *and* practical tools (Grossman, Smagorinsky and Valencia, 1999). On the one side, the profound theoretical framework (INVALSI, 2017) that informs both the construction and the selection of the items and the complex structure behind Gestinv triggers the use of mathematical content knowledge, mathematics education theoretical tools, ideas about teaching and learning that altogether amount for a conceptual tool. On the other side, Gestinv is a tool that can be implemented in the classroom, with a local and immediate utility, as an instructional material for mathematics teaching and learning that requires heuristics and strategic thinking on the part both of the students and the teacher. In this sense we can consider it also a practical tool. We stress the fact that in our study the use of Gestinv allows teachers to use the INVALSI standardized test not as a means of assessment but as a tool useful for them both in their pre-service and in-service training, which, within mathematics education research, can improve the teaching-learning processes of mathematics. The aim of our study is to present a new model for pre-service and in-service teacher professional development. This model is based on the affordances of Gestinv database used within a community of inquiry, which can affect mathematics teacher’s specialised knowledge by critically reflecting on the complexity of standardized assessments according to the constructs of mathematics education and mathematics curriculum.

A model for mathematics teachers’ professional development

The model we propose stands on three legs, the Community of inquiry, the MTSK model and the use of standardized assessment in the mathematics education field. As we mentioned in the introduction, we believe that teacher training has to take into account three dimensions that contribute to the formation of a mathematics teacher identity: social activity, tools that allow individuals to enact and materialize social practices, mathematical knowledge and teaching knowledge that constitute the teacher’s specialized knowledge.

Community of inquiry

Social interaction within a community of practice accounts for the construction of subjectivity as a teacher. Sociocultural perspectives in mathematics education (Radford, 2008; Sfard, 2008) have shown the role of social-communicative practices in a cultural-historical context both on the learning processes and the construction of identity, two sides of the same coin. We believe that we can extend and adapt these research findings to mathematics teacher’s professional development, since their training can be envisaged as an objectification-subjectification within the learning of mathematical knowledge and pedagogical content knowledge. Furthermore, within the sociocultural perspective we are advocating here, mathematical knowledge and knowledge for teaching are not fixed a priori entities that must be taken for granted. They are continuously reflected and refracted in social and communicative activity that allow us to make sense of cultural-historical constructs and we call this sense making process learning. Teacher’s professional development cannot disregard this feature of thinking and knowing. Jaworski (2006, 2014) adds an important feature that characterizes a community of practice (Wenger, 1998) that is, *inquiry*, which unfolds in terms of critical thinking, questioning, doubting, bringing new points of view ecc.

> "The transformation of a community of practice to a community of inquiry requires participants to look critically at their practices as they engage with them, to question what they do as they do it, and to explore new elements of practice. Such inquiry-based forms of engagement have been called "critical alignment" (Jaworski 2006). Critical alignment is a necessity for developing an inquiry way of being within a community of inquiry. " (Jaworski, 2014, p.77).

Therefore, within this conception of a community of practice, we can think of "inquiry as a way of being" in which teachers take on the mantle of inquiry as central to how they think, act, and develop in practice and encourage their students to do so as well." (Jaworski, 2014, p.77). We see how belonging to a community of inquiry results in a special attitude, a mode of being and becoming that defines the way teachers act, feel, think, learn and teach. This new attitude has important implications on the way teachers are going to handle the complexity of Chevallard's triangle whose vertices, knowledge, pupil and teacher (Chevallard & Joshua 1984), are inseparably intertwined. An attitude of inquiry allows the teacher to be tuned with an intrinsically unpredictable, uncontrollable, fluid and flexible situation, i.e. the mathematics classroom, which requires constant interpretation and reinterpretation in order to design and carry out activities, make decisions and handle social interaction. The subjectivity that the teacher realizes in his professional development in a community of inquiry cannot be separated from his mathematical knowledge and his knowledge for teaching. In the following section, we will look at the other leg of our model, that is the knowledge for teaching that a teacher acquires in a community of inquiry. We will turn to the construct of Mathematics Teacher Specialized Knowledge (MTSK) introduced by Carrillo-Yañez and colleagues (2018).

The MTSK model

The importance of knowledge for teaching, concerning a specific school subject, is internationally acknowledged; already in the mid-80s, Shulman (1986; 1987) focused on the concept of Subject Knowledge for Teaching and proposed a model aimed at outlining the areas of knowledge that teachers should possess, in terms of Pedagogical Content Knowledge (PCK). His innovation was the outlining of a "new knowledge of the content", specific to teaching. Within this line of research, over the last few years several works have tackled different aspects concerning both mathematical knowledge, and the specific knowledge for teaching (PCK) (i.e. Depaepe et al., 2013). To investigate teachers' knowledge, these studies did not set off from the contents listed in school curricula, but focused on empirical approaches in order to understand the mathematical content needed for teaching by investigating its basis, role and relevance. These studies - one of the most relevant is Ball, Thames, & Phelps (2008) - not only contributed to the improvement of the PCK by identifying its related subdomains of skills necessary for teaching, but also provided a framework for the conceptualization of the mathematical content knowledge, thereby harmonizing PCK and the mathematical content into the so called Specialized Content Knowledge (SCK). The results obtained by Ball and colleagues (2008) have been developed into a broader model to frame teachers' professional development. Carrillo-Yañez and colleagues (2018), introduce the Mathematics Teacher Specialized Knowledge (MTSK). MTSK coordinates two extensive areas of knowledge, the Mathematical Knowledge (MK) and the Pedagogical Content Knowledge (PCK) that meet and intersect in the teacher's system of beliefs. MK is the knowledge possessed by a mathematics teacher in terms of a scientific discipline within an educational context and PCK is the knowledge relating to mathematical content in terms of teaching-learning. Beliefs about mathematics and its teaching and learning lie at the "center" of the model (fig. 1) to "underline the reciprocity between beliefs and knowledge domains" (Carrillo-Yañez et al., 2018, p. 240). In the model, MK and PCK are divided into three sub-domains. The MK is composed of Knowledge of Topics (KoT), Knowledge of the Structure of Mathematics (KSM), Knowledge of Practices in Mathematics (KPM). In the MTSK model, the PCK "is a specific type of knowledge of pedagogy which derives chiefly from mathematics." (Carrillo-Yañez et al., 2018, p.

246). The three subdomains of PCK are the Knowledge of Mathematics Teaching (KMT), the Knowledge of Features of Learning Mathematics (KFLM) and the Knowledge of Mathematics Learning Standards (KMLS).

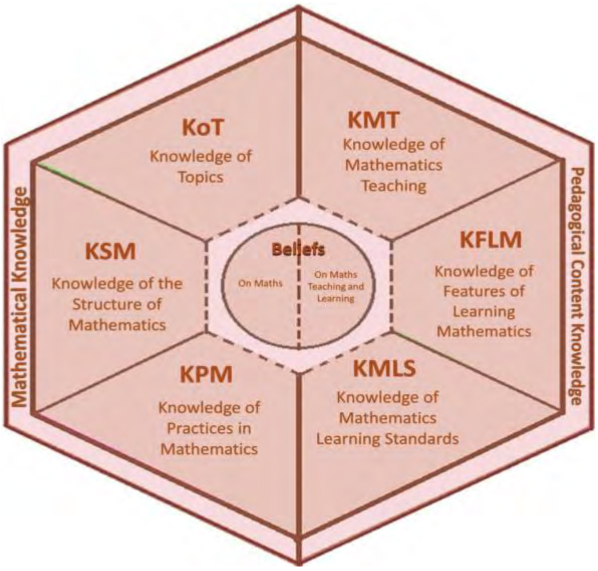

Figure 1: The MTSK model (Carrillo-Yañez et al., 2018, p. 241)

We would like to draw the attention of the reader on the fact that in our model the features of the inquiring practice, in which the teachers engage and align, are entangled with the change and construction of the system of beliefs that is at the core of the MTSK model. In our understanding, the core of the MTSK model is triggered by questioning, doubting, discussing, exploring, investigating etc. both MK and PCK within the community of inquiry. So far, our model provides a framework for activity and its outcome in terms of MTSK. We need a tool that bridges the gap between the community of inquiry and the MTSK and, furthermore, mediates and materializes/condenses the practice of inquiry. We are ready to introduce the third leg of our model: the database Gestinv.

The database Gestinv

In recent years more and more studies in mathematics education pinpoint the relevance of standardized evaluations and their impact on school systems (De Lange, 2007). As pointed out by Looney (2011), one of the main objectives of research regarding the role of standardized assessment is to find an effective way to merge its results, methods, theoretical frameworks and tools – that are designed in order to impact at a systemic level – into the local actions of teachers and schools. Therefore, teachers play a crucial role in this process that involves their professional development and system of beliefs. In order to meet this pedagogical demand, a group of mathematics education researchers (ForMATH Project), in collaboration with computer scientists, on behalf of the National Evaluation Institute for the School System (INVALSI) introduced in 2014 a new tool that teachers could use in order to bring the standardized assessment into their school practice and professional development. We are referring to Gestinv, a database with structured information regarding Italian standardized assessment that contains 1718 test items, spanning 10 years of INVALSI activity. The impact of the Gestinv database has been assessed both quantitatively and qualitatively, through standard indicators such as the number of registered users (more than 16.643), the number of accesses (on average, 200 every day), the time spent on the website and other parameters. These data, along with its structured information, promote Gestinv as a tool to implement in the design of teacher training models. Given the teachers' acquaintance with such a tool, it could be easily used both in the training sessions and their everyday practice, thereby bridging the gap between teachers' educational and school practices. There are many ways you can use the database; inside you can in fact carry out different forms of searches. The database has been devised both for Italian and Mathematics. Entering the Mathematics section you can search according to:

- the National Guidelines, that is, the Learning Objectives at the end of the third grade of Primary School; the Learning Objectives at the end of the fifth grade of Primary School; the Goals for the competences development at the end of Primary School; the Learning Objectives at the end of the Lower Secondary School; the Goals for the competencies development at the end of the Lower Secondary School; the National Guidelines for Liceo; the Guidelines for Technical and Professional High Secondary School; the transversal High Secondary School Cultural Axis);
- Keywords (there are about 200 keywords that identify the main topic for each item);
- "Full Text": the database allows you to find the full text of an item by typing in the search record one or more of its words;
- the vertical cognitive processes outlined by the INVALSI theoretical framework - six both for Primary Level and Secondary Level;
- national rates of correct/incorrect/invalid answers
- types of test questions (multiple choice, open questions, ecc..)
- guided search - it is possible to carry out a cross search (with and/or logical connectors) involving all the parameters mentioned above.

We believe that, in our model, Gestinv plays an important role in providing teachers and didacticians with an *interactive* tool to access a broad range of information and feedback regarding both the learning and teaching processes. Information are available not only in terms of global rates (measured by statistical models) but also to precise occurrences that we can observe in the answers to the items. In particular, the results of the INVALSI tests highlight and quantify relevant macro-phenomena, which can be interpreted according to methods and results of mathematics education research. On the one hand, the articulated structure and richness of information provided by Gestinv bridges the practice carried out in the community of inquiry with the subdomains of the MTSK model, acknowledging both the mathematical knowledge and pedagogical content knowledge. On the other hand, Gestinv serves as a mediator of and materializes the activities of the community of inquiry. In fact, an inquiring activity carried out in the *interaction* with Gestinv could trigger a change in the teachers' beliefs about mathematics and mathematics teaching and learning, by questioning, doubting, focusing, discussing etc. on mathematical contents and their development throughout the school levels, modes and results of assessment, teaching strategies and methodologies, cognitive processes and vertical difficulties. We can conclude that, in our model, Gestinv plays the role of both a conceptual and practical tool. Below a diagram that synthesizes our model for mathematics teachers professional development that pivots around Gestinv.

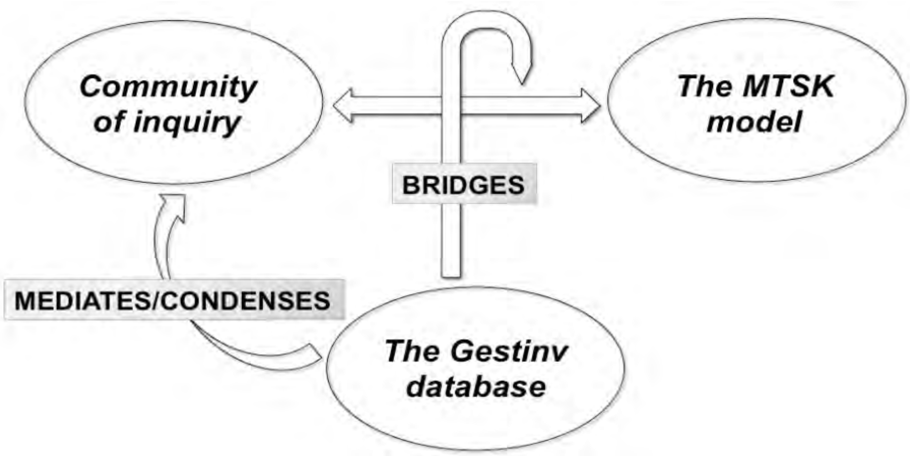

Figure 2: The model of mathematics teachers' professional development using Gestinv

The model used with teachers' professional development courses

We implemented the model we described above in several courses that involved both in-service teachers (almost 1300 in the last 4 years), at Italian national level, and pre-service teachers (almost 800 in the last 3 years) that attended the laboratory of mathematics education at the Free University of Bolzano-Bozen (the activity are conducted inside the *Multilab Project* of the Faculty of Education). The courses were designed according to the rules and expectations of a community of inquiry, that is, questioning - asking and seeking to answer questions (Jaworski, 1994; Mason, 2001) - doubting, problem solving, discussion, exploring and investigating, thus accomplishing learning on mathematical knowledge and pedagogical content knowledge. We embrace a developmental approach that pushes teachers and didacticians "into a deeper awareness of their own actions, motives and goals" (Jaworski & Goodchild, 2006, p. 353) The teacher professional development courses are conceived with the aim of constructing professional communities of learning in which groups of teachers engage in inquiring about the teaching and learning of mathematics (DuFour, 2004). According to our model for teachers' professional development, teachers engaged in activities that required an interactive use of Gestinv and exploited its richness of information and the structure of the search tools that we described in the previous section. The practices were designed in order to enhance and accomplish teachers' awareness on both subdomains of MTSK model. Special attention was devoted to the changing of beliefs pursued through the inquiry attitude, which underpins the community of practice. The community of practice worked according to the principles of group learning, thus teachers were divided in sub-groups of maximum 5 persons. The activity plan followed the following schema:

- Introduction of the activity. The didacticians discuss with the teachers the mathematics education constructs, also looking at learning difficulties, that will be useful for the activity. Moreover, the didacticians present some of the functions of Gestinv that the teachers will use in their inquiry. Last but not least the didacticians address the mathematical content selected for the activity (for example, algebra) from a conceptual and epistemological point of view. The mathematical content is discussed with the teachers according to some of the principles of Bartolini Bussi (1996) *mathematical discussion*.
- Analysis of an example. The didacticians discuss with the whole group of teachers, according to the rules of a community of inquiry, a didactical macro-phenomenon (as a paradigmatic example) using Gestinv within the MTSK model stressing the mathematical knowledge and/or the pedagogical content knowledge.
- Group activity. The group divides into sub-groups of maximum 4/5 members. The didacticians in charge of the course assign a task covering a mathematical content, a learning difficulty, a cognitive process related to one or more sub-domains of the MK and the PCK that are developed according to objectives and goals of the National Guidelines. The small group activity is carried out according to the rules of a community of inquiry, and the members of the group strongly interact with Gestinv. In-service teachers usually work on the same topic, whereas pre-service teachers sub-groups usually work on different ones. The group activity aims at the construction of a multimedia product, an artifact, the design of an activity for students etc., which should highlight teachers' reflections, convictions and beliefs. For example, as regards algebra, teachers are invited to inquire Gestinv, according to the preparatory activies mentioned above. It results that some of the lowest percentages of correct answers concerns items that require symbolic manipulations of powers. The activity on Gestinv prompts an attitude of inquiry - based on a working-sheet and the study of specific research materials regarding the historical-epistemological and didactical aspects of algebra and powers - that brings to the fore beliefs, convictions, reflections, emotions, agentivity etc., which will inform their multimedia product.
- General discussion. The sub-groups present their materials in a written or oral form to the big group. Each presentation is discussed within the community of inquiry in order to highlight beliefs and convictions, tackle doubts, difficulties and unclear contents regarding both the MK and the PCK and outline the subdomains of the MTSK that emerged from the activity. Another setting for this final phase requires each sub-group to prepare a written presentation that is exchanged so that each sub-group presents orally to the big group the material of another sub-group. The final discussion, based

on the oral presentations is performed with the same characteristics of a community of inquiry described above.

We conclude this paragraph presenting some extracts, which show significative changes in beliefs regarding the subdomains of the PCK, taken from a focus group carried out at the end of a teacher course.

Knowledge of Mathematics Teaching - *KMT*

T_05: In this period I saw a new possibility in getting inspiration from the database in order to prepare tests for my students.

T_06: I did the same especially for tests involving argumentation and reasoning.

Knowledge of Features of Learning Mathematics - KFLM

T_05: I never thought I could get so much information about the INVALSI tests. Under this perspective they can be really useful. D: What do you mean? T_05: we can really access information regarding the learning of our students and single out didactical paths for our activities.

T_06: Exactly, I found extremely useful to look for the items with the lowest percentage of correct answers. I was amazed and I proposed some of the questions we analyzed together to my students because I did not believe it could be so. Instead they gave the same incorrect answers like the other students!

D: Can you give some examples?

T_06: For example, fractions. I could not believe in the results after all the hours devoted to this topic. Instead I was not surprised with the results regarding area and perimeter.

Knowledge of Mathematics Learning Standards - KMLS

D: As regards the link to the National Guidelines do you think that the INVALSI tests can be useful to exemplify them?

T_01: Yes, there is an objective that I searched in the database, and it gave me hints to design an activity with my students.

These data result from a first explorative implementation of the model and are insufficient to verify the effectiveness of our model in teachers' professional development. Further empirical investigation is necessary to bring evidence regarding the efficacy of our model both for in-service teacher training and undergraduate university pre-service teachers. This process is started and we have already collected new experimental data that have to be analyzed and show that the model can be effective in teachers' professional development.

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USING STORYCIRCLES TO INQUIRE INTO THE SOCIAL AND REPRESENTATIONAL INFRASTRUCTURE OF LESSON-CENTERED TEACHER COLLABORATION

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*We build on the proposal that lessons are containers for the knowledge base of the teaching profession. We describe the Story*Circles *process for teachers' collaborative work on lessons and how it perturbs the usually individualistic culture of teaching in the United States. StoryCircles may thus support teachers' learning about lessons and from each other, as well as help develop the professional knowledge base. Further, we anticipate how Story*Circles *creates conditions that reveal elements of the social and representational infrastructure of teacher collaboration.*

This paper describes a project in which we investigate the role that teacher collaboration in lesson representation can play in representing and disseminating professional knowledge. Story*Circles* (Herbst & Milewski, 2018) is a process in which groups of teachers engage in cycles of scripting, visualizing, and arguing about moves in a lesson. Story*Circles*' focus on lessons is founded on Hiebert, Gallimore, and Stigler' s (2002) argument that lessons are containers of the knowledge base of the teaching profession. Those authors also argue that teachers' knowledge can be transformed into more usable forms for inclusion in the knowledge base for teaching by the development of a system that: (1) treats ideas for teaching as objects to be parsed, studied, and reflected on; (2) shares those ideas publicly in order to be vetted and improved; and (3) stores and accumulates those ideas in ways that enable the transmission of knowledge from one generation to the next. Hiebert et al. (2002) suggest that lessons offer one potential means for teachers to engage in such work—as the analysis of lessons offers teachers a way to work collectively with others to improve their own individual practice while simultaneously developing resources useful towards developing a system for representing lessons capable of building the broader professional knowledge base for teaching.

Japanese lesson study (Lewis, Perry, & Murata, 2006) shows an example of such a professional activity and tradition which includes lesson documentation, professional learning, and public study and scrutiny of lessons. Hiebert and his colleagues (e.g., Hiebert & Morris, 2009; Morris & Hiebert, 2011; Morris, Hiebert, & Spitzer, 2009) have developed another example of such a lesson-based system in the context of their elementary teacher education program: a system in which teachers collectively agree on problems of practice they commit to work on and then work on those problems by developing, exchanging, refining, and annotating lesson plans with specific learning goals. Whereas lesson study has been transplanted to the US in some locales, its implementation at scale is challenged by many practical problems, including geography, teacher schedules, and individualism in American teaching culture. (Lewis, 2002; Lieberman, 2009; Yoshida, 2012). But technology in the last 20 years has offered important advances that support the documentation and sharing of lessons

[1] This work has been supported by a grant from the James S. McDonnell Foundation (Teachers as Learners program).

through streaming digital video, computer-supported collaborative work among professionals (using video-conference software and internet forums), and the study and learning of practice using tools to represent, parse, and annotate practice in the context of practice-based teacher development (Herbst et al., 2016). Could a lesson-based system for the preservation and development of professional knowledge be constructed using those technologies?

Story*Circles*: Transacting teaching knowledge through the study of virtual lessons

In this paper, we speculate on the role that Story*Circles* might play in developing the lesson-based system envisioned by Hiebert et al. (2002). Herbst and Milewski (2018) have described Story*Circles* as a process that gathers groups of teachers with a facilitator and a web-based storyboarding tool to create representations of a lesson through cycles that include scripting a lesson, visualizing the lesson as a storyboard, and arguing about possible alternatives to what is seen in the storyboard (see Figure 1). Herbst and Milewski (2018) describe examples of Story*Circles* that use a variety of social, representational, and technological resources. Milewski, Herbst, Bardelli, & Hetrick (2018) have shown in more detail how one Story*Circle* operated over several weeks, communicating through synchronous videoconference and asynchronous forums, using as resources curricular tasks and records of student work, and using the services of a dedicated storyboarder to help them visualize the lesson they were scripting. The two-year project *EMATHS through Story*Circles—from which the records used in that paper came—shows that the Story*Circles* process can be used to support collective work on lesson documentation by teachers; and we have evidence that this process can be associated with teacher learning (Herbst, Ko, and Milewski, in press). Our goal in this paper is to describe a design research approach that involves shaping StoryCircles to enable groups of teachers to participate in the study and documentation of lessons while developing their instructional capacity.

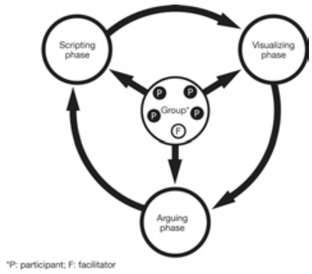

Figure 1. StoryCircles (from Herbst & Milewski, 2018, p. 331)

Our project is inscribed within a larger effort (the Teachers as Learners program, from the James S. McDonnell Foundation) to use design research to investigate teacher learning in the context of discussion based instruction. To that larger effort, our project brings a focus on high school algebra and geometry instruction. Our design research project involves the interplay of three components: documenting mathematics teaching knowledge, supporting teacher collaboration, and promoting teacher learning. We elaborate in this paper how the work we are doing helps investigate research questions that contribute to the question of how StoryCircles can support those three components.

The work of Story*Circles* can be usefully framed by Carroll and Mumme's (2007) professional development triangle (see Figure 2a) which they describe as an iteration over the Cohen, Raudenbush, and Ball's (2003) instructional triangle (see Figure 2b). The instructional triangle contains several implicit claims that are important to highlight as they are reused in the professional development triangle. In particular, the triangle highlights relationships that are ternary—students and teacher relate through the content. It also highlights the non-inert nature of the content—representations of content (e.g., in curriculum or technology) have features that enable those relationships. Whereas other representations of mathematics instruction may better address some complexities of the phenomenon, the simplicity of the instructional triangle makes it serviceable to support the analogy at play in practice-based professional development. The arrows in Figure 2a represent claims equivalent to those in the case of the instructional triangle. In the professional development triangle, instruction itself, represented by a copy of the instructional triangle in the lower left, plays the role of content. The professional development triangle can be used to frame Story*Circles* and this helps pose questions and conjectures about the interactions one might observe therein.

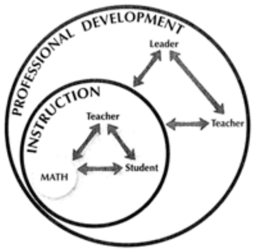

Figure 2a: The professional development triangle (from Carroll & Mumme, 2007, p. 11)

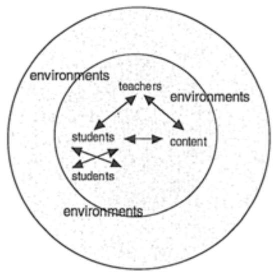

Figure 2b: The instructional triangle (from Cohen, et al., 2003, p. 124)

The canonical cases of Story*Circles* (Herbst & Milewski, 2018) involve teacher participants using some resources (e.g., a task statement, records of student work) to script a lesson where, say, the class engages in the problem provided or where the student work considered emerges and is addressed. These interactions can be represented in terms of the bidirectional arrow between participating teachers and instruction: the lesson, represented by the problem or by student work, influences the scripting work of the participating teachers; the storyboard they create adds to the representation of the lesson. Story*Circles* also involve a facilitator, who makes some resources available to create the script (e.g., chooses samples of student work) and uses them to frame a task for the participating teachers. Not only the facilitator impacts how instruction is represented in Story*Circles* by choosing what resources they make available initially, but also the representations of instruction impact the facilitator's work: The script and storyboard the participants create provides context and may create challenges for the facilitator's continued facilitation of the interaction, as when the script represents a teacher providing an explanation that might evince participants' limited mathematical knowledge. The facilitator impacts the participants by the suggestions or requests made, but also the participants impact the facilitator, for example when the nature of the participants' social dynamics calls the facilitator to intervene and shape the interaction. Most importantly, as in the instructional triangle though not apparent in Figure 2a, participants of Story*Circles* influence each other as they contribute to a shared representation of a lesson and argue about alternatives to what is represented.

Research with Story*Circles*

When we developed Story*Circles*, our goal was to create a context for teachers to bring in what they know from practice into a shared task in which they might learn from each other about practice. As such, Story*Circles* encourages participants to bring in their professional case knowledge; yet, the goal is not merely to share. Rather, the goal is to create a common artifact, a representation of a lesson. The social processes involved in such work are somewhat uncharted: Practitioners in the United States rarely have professional work contexts in which they have to argue with each other about the meaning of students' productions or about the wisdom of the actions that could be taken. In the context of such professional work, one may ask what demands on social and representational infrastructure are elicited in Story*Circles*.

The goal of making a collective product draws inspiration from Papert's (1991) *constructionism*, whereby learning happens in and through the construction of artifacts. Along those lines, teacher learning can happen as groups of practitioners pursue the canonical goal of Story*Circles,* which is to collectively create a lesson, through the cycles of scripting, visualizing, and arguing. At the same time, the resources available to such construction, coming both from the representations of the lesson available in a repository (e.g., collections of student work) and possibly contributed by participants (e.g., through recollections of how the lesson went when they taught it, possibly recorded in videos), are not only useful to create a new representation of a lesson but also can challenge the sense that this lesson will be unique. Indeed, we hypothesize that Story*Circles* may help promote the notion that a lesson is actually a multiverse[2] of possibilities, each of which depends on contingent actions and speech by students and decisions and actions by a teacher. This sense of lesson as multiverse is an important element of the knowledge to be learned in Story*Circles*, even if the artifact created is a single instance.

Disrupting social and representational infrastructure

Two aspects of the work of Story*Circles*—learning from interactions with colleagues and learning from the construction of artifacts—highlight the importance of two kinds of infrastructure upon which StoryCircles are deployed: social infrastructure and representational infrastructure. There is a social infrastructure that supports conversations about practice among teachers and a representational infrastructure which is used in making instruction an object of transactions in such conversations. Hall, Stevens, & Torralba (2002) note that elements of infrastructure become visible when representation is disrupted in communication. We surmise that Story*Circles* create such disruptions.

The social infrastructure of teacher work groups is disrupted by Story*Circles*'s expectation that participants will create together a representation of a lesson. As noted above, this is not common among American teachers who tend to work rather independently on a day-to-day basis (de Lima, 2003) and whose checks on each other's practice only happen through curriculum coverage and final exams (MetLife Foundation, 2009). Outside those larger decisions, teachers make many moment-by-moment decisions that are rarely subjected to much scrutiny even though they have important implications for students (Horn & Little, 2010). Story*Circles* can provide teachers with opportunities to uncover the shared categories of perception and appreciation and the logic of argument underlying their differing views on the possibilities for a lesson. When confronted with alternative ideas about what to do in the same moment, teachers need to offer some forms of justification for one alternative

[2] We appeal to the quantum physics notion of multiverse or multiple universes (Deutsch, 1998) as a metaphor, to summon the notion that the many contingencies and decisions available to make in a lesson generate a multiplicity of possible lessons from which one will take place each time the lesson is taught.

over another. This disruption in the discourse of teachers creates opportunity to reveal and broaden their social infrastructure.

The representational infrastructure of teacher work groups is disrupted by Story*Circles*'s expectation that lessons be visualized as storyboards with cartoon characters. The cartoon characters and the sense of time inherent in comics and storyboards make this medium a very different form of representation of lessons than traditional ones such as written lesson plans or video records. While lesson plans might be easily shared, stored, and accumulated, as is the case with many teaching-focused websites and social media platforms, they do not present lessons in the interactive detail needed to support all of the activities Hiebert et al. (2002) name as crucial. Much of what is involved in the meaning of practice (Nicolini, 2012), such as the demands of physical location, timeliness, embodiment, and language, is elided in lesson plans. Video records, on the other hand, represent those aspects of practice very well, yet they also contain incidental and idiosyncratic information, which challenges the capacity of video records to be generative of alternatives. As video records are records of action that did take place, this often deters considering what else could have happened.

Since 2005 we have been working with digital, nondescript, anatomically simple cartoons to represent teaching and use those representations to elicit data from teachers to feed our inquiry on the rationality of teaching practice (Herbst, Nachlieli, & Chazan, 2011). We have seen those cartoons not as unique characters displaying their personal richness in character-driven stories, but rather as a cast of characters enacting a practice. As we developed that work, we have also contended with the question of whether and how digital cartoons can be organized and used to document and publicly archive the knowledge base of the profession. The use of nondescript images to represent practice is jarring to some, but others see in it an opportunity to build up a sign system to represent practice, a system onto which practitioners and others may project the meanings of actual practice (Chazan & Herbst, 2012). The expectation that cartoons be used to represent a lesson brings forth assumptions and expectations about what signs mean and what signs needs to be developed to enhance the representational infrastructure.

The social and representational infrastructure of teacher work groups

Inquiring further into social and representational infrastructure, we are interested in four aspects in which Story*Circles* may support the collective work of teachers. One of them is the social infrastructure of teachers' work groups as this is disrupted by Story*Circles*. Rather than being taught by the facilitator and treated like students, each participant in Story*Circles* is a knowledgeable professional, but activities therein require open argumentation and consensus development about moves in instruction. One set of questions we have is on the nature of the argumentation: What sources of justification do participants bring to the work of figuring out what the lesson will look like? Additionally, we expect Story*Circles* may reveal three aspects of representational infrastructure. As we think of these elements of infrastructure, it is useful to note that while infrastructure needs to be functional at some basic level to enable activity, the disruptions occasioned in the activities present opportunities for emerging solutions (as Hall et al., 2002 illustrate).

A conceptual vocabulary including a notion of lesson and lesson features. What we mean by *lesson* is an element of infrastructure. In prior versions of Story*Circles* such as the ones reported by Milewski et al. (2018), lessons were identified by the mathematical problem being used in a course of studies (algebra or geometry). The process of anticipating how the task would unfold in such a course quickly brought up other features of the lesson, such as the assumptions made about what had been previously studied and what the instructional goal for the task would be. Likewise, as the goal of Story*Circles* was to document how teachers used the previously developed problems, it was quite conceivable that different users of the problem could bring to the Story*Circle* their experiences with different student responses to the problem, not all of which would have taken place in the same classroom, but all of which could be considered possible features of the lesson. The mathematical conceptions that a problem could elicit in a given course of studies are arguably features of the lesson: Knowing those conceptions would support a teacher's capacity to manage the lesson.

A representational infrastructure thus includes a conceptual vocabulary that includes words like *lesson*, *problem*, *prior knowledge*, *course of studies*, *instructional goal*, or *student conception*. Their uses in contexts where there is high demand on social interaction are likely to occasion disruptions due to ambiguity and conflicting uses. We are interested in how facilitator and practitioners resolve such impasses, in the work they do to negotiate a professional language to enable communication oriented to scripting a lesson. This conceptual vocabulary infrastructure may provide resources for looking at a lesson from a distance, enabling the notion that a lesson is a multiverse, that could be instantiated in many related but divergent stories.

A semiotics of practice. A definitional element of Story*Circles* is that lessons are scripted by participants and visualized in the form of a storyboard that puts together various scripted moves. To create such storyboards, participants or an assistant storyboarder use a set of cartoon graphics. These graphics are emblems of the notion of an emergent semiotics of practice that serves as part of the representational infrastructure. We use *semiotics* as short for *social semiotics* following the systemic approach by Halliday (see Halliday and Matthiessen, 2004), whereby signs are seen as social resources for meaning making; while the systemic approach started with language, many examples are available that expand it to other modalities including graphics and embodiment (O'Halloran & Smith, 2012). The cartoons presently used can be very poor signs to represent some practices. Indeed, in prior work with teacher educators we have seen that the expectation that classroom interaction be represented using cartoons has called for systematic development of the representational infrastructure.

In contrast with the conceptual vocabulary, this semiotics of practice seems particularly serviceable to close up in the representation of practice, to render what particular instances of a lesson might look like. While we have done much work toward that end, the semiotics of practice is still evolving, as it attempts to satisfy the needs of practitioners to represent practice. The original cast of characters used in the *Depict* software included only blue-skinned characters, which made it hard for some teacher educators to represent the experiences of racialized students when moving from one class to another (see Herbst et al., 2017). The incorporation of skin-tones that enabled pre-service teachers to see students by skin color was instrumental for a teacher educator to communicate what he had seen in his class. Likewise, we wonder what demands on the representational infrastructure will ensue from teachers' need to represent what they can foresee happening in a lesson. This is of particular interest to us in regard to student conceptions and teachers capacity to indicate how they represent the multimodality of student conceptions.

A navigation system. The notion that teacher learning might result from the consideration of alternative representations of lessons suggests that a navigation system will be needed to support that activity—the navigation between a lesson as alluded to through the use of a conceptual vocabulary and a lesson as represented with a semiotics of practice. When Story*Circles* have been occupied in producing a single storyboard, this could be visualized with a simple slideshow player, but if a lesson involves a multiverse of alternatives spawning from decisions and contingencies and each of them is represented in storyboards that need to be visualized and discussed, the user will need more sophisticated navigation controls, records of their navigation, and ways of annotating their considerations of them. This navigation system might call for a technological solution eventually, but the search for this solution will involve a bricolage of existing technologies and customary signs, with the research effort needed for discovering what facilities and functionalities participants call forth to support their study of a lesson.

Furthermore, the scripting of those alternatives will need more sophisticated prompts than "what do you envision you could do next?" or "what do you think students would say?" To support the scripting of lessons, we have collected records of classroom instruction involving dozens of copies of student work and multiple recordings of the same lessons taught by experienced teachers. An aspect of the navigation system we are investigating concerns how to bring that material out along with other resources from the literature on student conceptions or classroom discussions. Specifically, we explore an additional element in Story*Circles*. Participants are dealt key frames of stories that they could use as playing cards, bidding to include them in stories: For example, a card could depict a student showing a specific solution in the document projector, another card could have the teacher asking "who did this in a different way?" Participants could insert these cards as they script the stories. Our interest here is to investigate what sorts of cards support teachers' consideration of alternative storylines and how they do so. We are also interested in the activity structures that can be used to support teachers in collectively considering multiple ways in which the cards could be inserted into a storyline.

Conclusion

Story*Circles* was first conceived as a way to focus the work of a group of teachers on the anticipation of a lesson. As we take on the challenge of thinking of one lesson as containing a multiplicity of alternative instantiations, spawn from multiple decisions and contingencies, and such representation of a lesson as an object of archive and study by the profession, questions arise for Story*Circles*. Specifically, questions concerning what elements of social and representational infrastructure are perturbed and require to be addressed in using StoryCircles for teachers to study lessons.

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LEARNING TO IMPLEMENT RESEARCH-INFORMED TEACHING OF EQUIVALENT FRACTION THROUGH LESSON STUDY IN CHINA

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This study aimed to explore how a mathematics educator and a group of teachers collaboratively worked together to develop a research-informed exemplary lesson through Lesson Study (LS) and what participating teachers learned through the multiple cycles of LS. The goal of the LS was to develop an exemplary lesson on equivalent fractions based on the notion of hypothetical Learning Trajectory (LT) (Simon, 1995). From a Community of Practice (CoP) perspective, the boundary objects of research lesson between the researcher and teacher communities were examined to identify possible patterns of transformation. A fine-grained analysis of the data set including lesson plans, videotaped lessons and debriefs, and interviews with teachers found that (1) there was tension between the LT-based instruction and participating teachers' understanding of instruction; (2) the research lessons progressed toward LT-based instruction through three phases: hesitation, experiment, and enactment. The practicing teachers deepened their understanding of teaching equivalent fractions that promotes student learning.

Introduction

How to link research to practice is a longstanding unsolved issue in education (Cai et al., 2017; Silver & Lunsford, 2017). Research has identified factors resulting in this issue (Kieran, Krainer, & Shaughnessy, 2013; Silver & Lunsford, 2017) and suggested pathways to building connections between research and practice (Kieran et al., 2013; Silver & Lunsford, 2017). Lesson study (LS) provides an ideal collaborative setting on which both researchers and practitioners can work together to improve practice while conducting research (Kieran et al., 2013; Silver & Lunsford, 2017). Specially, some researchers explored how LS can contribute to the linkage of research and practice (Huang, Gong, & Han, 2016; Ding, Jones, & Sikko, 2019). However, little is known about the mechanism and process of how research findings could be effectively implemented in practice through LS. To this end, this study aims to uncover the process through exploring the implementation of research-informed teaching of equivalent fractions through LS.

Theoretical Framework and Literature Review

LS and teacher learning. LS is usually deemed as a job-embedded, practice-based, and student-focused professional development approach which includes a prototype: (1) Goal setting; (2) Planning; (3) Conducting a research lesson; and (4) Reflecting on the research lesson (Lewism 2016). With the popularity of LS as a teacher professional development approach internationally, numerous studies evidenced positive effects of LS (e.g. Huang & Shimizu, 2016; Xu & Pedder, 2014). These include improving teaching, student learning, teacher learning, implementing new curriculum and building connections between research and practice. By being involved in LS and interacting with researchers, participating teachers have many opportunities to contact with powerful research products and ideas that, in one way or another, may find their way to their practice (Thorsten, 2015). Huang et al. (2016) examined how a LT of division of fractions was used to guide the design of research and how the cycles of LS could contribute to the refinement of the LT. In the context of China, typically, a LS is facilitated by master teachers, mathematics specialists or researchers. Researchers further argued that LS in China is a research methodology to link research to practice (Huang et al., 2017).

Learning across communities within the context of LS. The boundaries between different communities make it difficult to exchange knowledge across communities. When researchers and teachers work together during LS, the LS creates a space in which these two communities come together. This space is conceptualized as a “boundary encounter”. Participants from separate communities who are involved in boundary encounters negotiate meaning of effective teaching of a particular topic and /or an idea both “across the border” with another community and within their original community. By creating these boundary encounters, LS provides an opportunity for members from separate communities to communicate about, collaborate around, and potentially transform practice. Thus, LS as a boundary encounter allows for the negotiation of meanings of effective teaching and learning of mathematics between and within both the research and the teaching communities. Both specialists and mathematics teachers who come together during LS are “boundary brokers”: community members who introduce elements of practice from one community into another (Wenger, 1998). Boundary brokers support connections across communities and create opportunities for new meanings to merge in different communities. Boundary encounters are organized around brokers who learn together and from each other, and then return to their own communities with new practices that potentially transform that community. When brokers from two communities come together in a boundary encounter that generates boundary practices, they work around representations of knowledge that convey meanings across multiple communities. These representations are “boundary objects”, which inhabit intersecting worlds and satisfy particular requirements from each of them (Akkerman & Baker, 2011). During LS, research lessons with students’ learning artifacts represent boundary objects that manifest the meaning of mathematics teaching and learning across research and teaching communities. They allow for the emergence of shared boundary practices. Researchers and mathematics teachers work together using the research lesson to make sense of curriculum ideas and enact these ideas in the classroom for student learning.

LT of equivalent fractions. This study adopted the notion of hypothetical LT developed by Simon (1995) and other scholars (Clements & Sarama, 2004; Sztajn, et al., 2012). Research shows that the use of LTs can support teachers’ knowledge growth and instructional decision making, allow teachers to focus on students’ thinking, and eventually improve students’ achievement (Clements et al., 2011; Wilson et al., 2015). Understanding equivalent fractions is fundamental to addition and subtraction of fractions, some approaches to division of fractions and thinking about the relationship between fractions and decimals (Petit, Laird, Marsden, & Ebby, 2016; Lamon, 2012). Understanding equivalent fractions includes understanding two aspects: two different fractions can represent the same amount and the relationship among equivalent fractions. Studies showed that many students can use the algorithm to find equivalent fractions, but they do not know why it works (Wong & Evans, 2007). Thus researchers have proposed different models and strategies for interpreting and making sense of equivalent fractions. For example, two instructional approaches of operative (e.g., partitioning the fraction into subparts) and figurative (e.g. showing same length or same area) were proposed (Kamii, 1994; Simon, 2006). In most curriculums, approaches to equivalent fractions are compatible with a part-whole interpretation (e.g., m/n is m parts of a unit that has been divided into n equal parts) or measurement interpretation (m/n is m iterative measures of unit fraction of 1/n). (Kara, Simon, & Placa, 2018). Researchers argued that establishing equivalent fractions empirically (e.g, using diagrams, number lines, or manipulatives) does not help students' understanding of the logical necessity of the equivalence (Kara et al., 2018; Simon, 2006). It was suggested that adopting recursive partitioning and measurement interpretation could help students truly understand the relationship among equivalent fractions (Kara et al., 2018; Petit et al., 2016).

Research Questions. Considering the presentation sequences in the Shanghai textbook, it is clear that in Shanghai, the equivalent fractions are addressed by an empirical approach based on a part-whole interpretation. Based on literature and the textbook in Shanghai, we make a LT of equivalent fractions as follows: (1) observing why two fractions are equivalent using visual models; (2) exploring the relationship among equivalent fractions using a part-whole model through recursive partitioning; (3) exploring the co-variation relationship between denominators and numbers among equivalent fractions using part-whole models, or iteration of unit fractions in measurement models through recursive partitioning. The purpose of this study is to explore the how the LT-informed instruction could be implemented in a normal classroom that promotes students' understanding through a LS approach. Specifically, the research questions are: (1) how researchers (first author) and practitioners (team teachers) can resolve differences between them for how to design a lesson based on a LS approach? (2) Which factors have played key roles in this process? To address the research questions, a CoP perspective is adopted to examine the transformation of the boundary objects of research lesson, and the changes of brokering throughout the LS.

Data collection and analysis

The participants of the LS included a university researcher and 12 teachers seven elementary schools (Grades 1 to 5) and two junior high schools (Grade 6 to 9). A total of four intensive discussions were conducted in the LS to form the final classroom instruction design (Table 1). The two teachers who designed and implemented the research lessons in Grade 4 were from two different schools, and they had six to seven years of teaching mathematics experience. Ms. Wang is from a primary school in a suburb of Pudong New Area in Shanghai, and Ms. Chen is from an elementary school in the Huangpu District, which is a city center in Shanghai. All the collaborative meetings (two lesson planning sessions and two debriefing sessions) and observed lessons (two rehearsals and two enactments) were video recorded throughout the LS (see Table 1 for detail). Four research assistants transcribed the videos in Chinese with quoted transcriptions being translated into English. In addition, the two teachers were interviewed after completion of LS in order to understand the factors influencing the major changes over the process. According to the theoretical framework, teachers' learning mainly happens within the boundary encounter (e.g. LS process) through boundary crossing of the two communities of research and teaching. Thus, the boundary objects (e.g., research lesson) were the focus of data analysis. By examining research lesson transcripts and debriefing transcripts, the turning points (Huang & Han, 2015) of boundary objects were identified. Furthermore, the findings were triangulated with interview data.

Table 1: Events and contents in the LS

Event	Content
Collaborative Meeting for Goal Setting of the Research Lesson [LP1]	The researcher introduced the purpose and intention of the research, proposed a LT-based instruction of equivalent fractions, and discussed the teaching objectives and strategies in the research lesson.
Collaborative Meeting for Planning of the Research Lesson [LP2]	Students' learning readiness in Grade 4 (including previous knowledge and cognitive requirements) was discussed. The research lesson plans were explained by Ms. Wang and Ms. Chen respectively and then discussed collaboratively.
Implemented Lesson to Observe [RL1] and Post-Lesson Meeting for Reflection [DB1]	The research lessons rehearsed by Ms. Wang and Ms. Chen were observed separately, then discussed together.
Improved Lesson to Observe [RL2] and Post-Lesson Meeting for Reflection[DB2]	Two revised research lessons taught by the both teachers were observed and discussed again.

Results

Turning point 1: Making the boundary objects accessible to teachers. The researcher first explained that the purpose of LS was to develop a lesson on equivalent fractions from the perspective of LT. The teachers shared their experience in implementing lessons on equivalent fractions with part-whole models. They did not require students to explain equivalent fractions with recursive partitioning in the part-whole models. The researcher hopes teachers to adopt the approach: using models to help students interpret equivalent fractions, rather than visually see that two fractions represent the same amount. Thus he shared theories and studies related to equivalent fractions with the teachers. However, all the teachers, based on their teaching experience and discussion with each other, argued that such teaching requirements are beyond the students' cognitive abilities. They believed that the teaching goal should be to help students understand that two equivalent fractions represent the same amount (length or size) using part-whole models. However, the researcher proposed a different way to construct equivalent fractions with recursive partitioning in measurement model, while maintaining his stance. After getting the idea from the researcher, all teachers had a deep discussion about that. Some teachers agree with the researcher that they thought this approach would be an innovative way to learn equivalent fractions. But other teachers thought that this approach would let students have a misunderstanding of the teaching content. This approach drew teachers' attention actually, but they were still worried that students were not able to understand recursive partitioning and were hesitant to take this risk. The researcher continued to use specific examples and visual models to illustrate his approach. Although most teachers still insisted on their views, some teachers were willing to try the new approach.

Teacher 1: We can try this as an extended part in the lesson.

Teacher 2: Students can understand the half of one third is one sixth.

Teacher 3: Several years ago, a child talked this in my class.

Teacher 4: We can try first, looking at children's responses and then think how to do.

In the meeting, they said that iterating unit fractions in the measurement model is easier to understand than co-variation involved in the part-whole model. Simultaneous change of numerator and denominator is a process of co-variation, where two changes occur at the same time, which is difficult for students. In the measurement model, firstly, the new unit fraction (change of the denominator) is obtained from recursive partitioning the known unit fraction, and then the equivalent fractions are constructed through iterating the new unit fraction, where the previous simultaneous co-variation process is decomposed into two successive processes. Teachers' understanding the benefits of the new approach is the first turning point, which makes teachers' learning across boundary of the two communities possible. In order to convince more teachers to accept this new approach, the researcher collected students learning data by taking a pre-test (about part-whole, measurement and partitioning). One question was related to measurement model and recursive partitioning, where students are asked to use the tape model to generate a unit fraction of 1/4 or 1/6 by participating the existing unit fraction of 1/2. In Ms. Wang's classroom, more than 90% of the students were able to find the correct answers. This evidence also strengthened the researcher's confidence and encouraged Ms. Wang to take this new approach.

Turning point 2: Making the boundary objects feasible in lessons. Ms. Wang was willing to adopt the new approach in her lesson. She explained in the interview that the LS was carried out in her school, so she participated in many coordination works. The researcher also had a phone or online discussion with her, so she had more opportunities to understand the researcher's intention and the research. At the same time, the students' pre-test performance strengthened her confidence in this approach. During the interview, Ms. Wang also talked about her experience.

> Ms. Wang: Several years ago, I observed a lesson in which the teacher used a unit fraction to generate fractions. At that time, I thus recognized the importance of unit fraction, which is like the unit [one] in the integer set.

Due to these factors, she participated in the CoP more actively and took a central role. After the observation for the class, during the second meeting for planning the lesson, one teacher agreed with her lesson plan and further suggested that they must rely on specific models, not just by language, but also with visual models when asking students to explain. Three teachers raised their concerns. Mr. Lu believed that the explanation is too abstract for students in elementary school. Ms. Chen argued that students could only perceive implicitly the co-variation of the numerator and denominator in equivalent fractions. Ms. Gu argued that students will not generate a new unit fraction from existing unit fractions, rather than they will recognize the new fraction as part of a whole. As can be seen from the teacher's discussion, not all teachers were willing to try to use the new approach at first. Although the pre-test evidence shows that students have the ability to find equivalent fractions in both part-whole and measurement models, a majority of the teachers were still worried about whether students would be able to explain equivalent fractions. Some teachers felt that students might return to the original whole when constructing a new unit fraction and would not be able to generate a new unit fraction directly based on existing unit fractions. From this perspective, the new approach as a boundary objects is not so transparent for these teachers to understand.

Turning point 3: Making the boundary objects effective. In her first lesson, Ms. Wang used the measurement model to construct a new unit fraction based on the existing unit fraction through recursive partitioning, thus generating an equivalent fraction by iterating the new unit fraction. It was found from the lesson that the students' explanations were not very smooth at the beginning, but after several rounds of probing and re-explanation, their explanation became more and more fluent. In the post-lesson debriefing meeting, it was expected that some of the teachers would question the use of the new approach. However, all focused on how to effectively implement the new approach. They believed that there was too much repeated practice such as generating 2/8, 3/12, 4/16, and 5/20 based on 1/2, and there was too much direct intervention– requiring the students to follow a detailed instruction by filling out blanks to explain equivalent fractions. Taking these suggestions in her second lesson, Ms. Wang encouraged the students to explain with their own oral language. Moreover, she gave them time to explore different equivalent fractions after students created 2/8 and 3/12 based on 1/4. In the interview, she explained.

> Ms. Wang: Explaining the equivalent fractions with iterating unit fractions had not been used in previous lessons, so, I provided detailed instruction and several similar practice problems because of worrying about students' ability to explain.

Turning point 4: Transferring the boundary objects to peripheral participants. In the first lesson, Ms. Chen asked the students to construct equivalent fractions by folding a paper. Actually, some students also found that the numerator and denominator of these fractions have a double relationship (i.e., 1/2=2/4). She asked to explain. When the students explained that it was because of folding, she did not continue to probe. In the second lesson, after the students constructed equivalent fractions through folding, Ms. Chen further asked, "What changes will the fraction occur if they fold again?" This elicited students' discussion about the relationship between two equivalent fractions, which was not explored in the previous lesson. Then, the teacher posed a question: "Well, yes. So, you see, fold once, fold twice, just verified. If you fold three times, each fold produces a new equivalent fraction by doubling (both numerator and denominators of the previous fractions). Why is it doubled?" This question requires students to consider the reasons for the simultaneous change of the numerator and denominator of the fraction. In the interview, Ms. Chen explained the reasons for the change in her lessons.

> Ms. Chen: In the previous lesson, the students found the equivalent fractions by multiplying the numerator and the denominator by 2. However, they drew this conclusion depending on the numerical pattern.

Ms. Chen explained that it was the reason that she changed in the second lesson, so that the students could consider the whole (equal divided into parts), then the shaded parts, and finally the whole and part. Obviously, in the second lesson, Ms. Chen has promoted the cognitive requirement of explaining the equivalent fractions in the previous lesson, where she elicited the students to explain by recursive partitioning in part-whole models.

Conclusion and discussion

Boundary objects (e.g. research lessons and relevant student learning artifacts) play a key role in transferring theoretical ideas into practical classrooms by eliminating the differences between the researcher and the teachers. The study found four *turning points* of the transformation of the boundary objects: teachers' access the research ideas brokered by the researcher, to exploring new the research-informed approach in teachers' classrooms, then to finding effective strategies of implementing the research-informed new approach, finally other teachers' adopting the research-informed teaching approach. It is a critical step that Ms. Wang attempted the research-informed lesson in her classroom. The research lesson served as boundary objects to link the two communities and attracted other teachers to try. Through the collaborative planning, teaching, reflecting and revision, the research lesson was improved regarding students' engagement and explanation, which helped participating teachers realize the value of implementing this new approach in their classrooms. Thus, the peripheral participating teachers changed their teaching approach toward research-informed one.

The *major factors* contributing to teachers' learning across communities included teachers' knowledge and beliefs, evidence of students' learning readiness and salient learning performance in classrooms, and reflection on the development of the research lesson. Ms. Wang was an active broker due to her personal working relationship with the researcher and previous relevant experience in observation of the use of unit fractions in teaching. This special identity made her more confident in the approaches of the researcher and more willing to try learning across communities. For Ms. Chen, her transformation was later and slower, which is the result of her reflecting on her practice and learning from Ms. Wang's successful exploration of equivalent fractions with an operative approach. She did not give up the approach of recursive partitioning in a part-whole model, but she also pushed students to recognize co-variation in equivalent fractions.

According to Wenger (1998), the boundary object is important for expansive learning from crossing boundaries. Within the context of lesson study, a research lesson as the boundary object has been progressed through the cycle of planning, implementation and reflection. It was found that finally teachers accept the researcher's idea through the observation for class and the discussion between all teachers. But more importantly, the joint effort during the lesson study process to make the research lesson effective, which is valued by both communities, is fundamental for promoting teacher's learning across communities. Teachers changed their mind and learned a lot from the discussion and observation, including accepting an innovative way to teach mathematics. Theoretically, this study provides evidence that LS is a powerful platform on which teachers can learn ideas from research and implement research-informed teaching in their classrooms. In China, there is a job-embedded, nationwide teaching research system (Yang, 2009). Within the system, LS takes place at multiple levels from school-based to cross-district level (Huang et al., 2017). If researchers serve as knowledgeable others in LS, then, instructional products such as annotated lesson plan developed by one LS group could be further adopted and enriched by another LS group, thus it will be possible to implement research-informed teaching on scale. Practically, this study also indicates that when researchers try to implement a new idea or approach in classrooms, in addition to the necessary transparent explanation, they should find a teacher agency to broker across the border between research and teaching communities.

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APPROACHING RESOURCE SYSTEM STRUCTURE IN COLLECTIVE WORK: FROM TEACHER SCHEMA TO RESOURCES DICTIONARY

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This research is situated at a particular moment of reform of the Algerian educational system. It originates from the difficulties of implementing this reform from a point of view, for mathematics education, of the competency-based approach and the integration of ICT. We ask the potential of the resources of Sésamath, a French association of teachers, for supporting the work of teachers facing this reform. Anchored in the documentary approach to didactics, we present in this article results of one teacher followed in the college: in her regular classes and in a mathematics workshop where Sésamath resource was integrated. It particularly considers teacher resource systems by analyzing their structure and evolution, aiming to modeling these systems

Introduction

This article presents the resource system of a teacher followed over two years and more specifically the structure of this system, we are approaching its modeling and we present our theoretical and methodological contribution to the documentary approach.

Our research is situated at a particular moment of reform of the Algerian education system, marked by a skills approach and an introduction of ICT at all levels. Our question is rooted in the difficulties created by the implementation of this reform, both from the point of view of the competency-based approach and from the point view of the integration of ICT, especially for the teaching of mathematics.

In this article, we consider in particular the schematic representation of teacher's resource system, named Meriam, she teaches mathematics in Arabic language and for grades six and five. For the analysis of its resource system which is construct collectively. Our method is inspired by systemic analysis (Le Moigne, 1977) and covers two aspects: a static or structural analysis which consists to define the elements of the system their relations and under the functional or dynamic aspect marked by action rules and events that act on these elements.

Theoretical frame

In this section, we present the theoretical frameworks mobilized, we are focused on the documentary approach to didactics where we give particular attention to the notion of resources by presenting our theoretical contribution to this framework.

Documentary approach to didactics

Gueudet & Trouche (2008) give particular attention to the professorial activity and the documentation of mathematics teachers, in particular from the point of view of their collective documentary work and their professional development. They consider that the teacher's work feeds on the resources available in the collective to build what is necessary to do his job. They also consider that the teacher, in his documentary work, has a set of resources of various natures that will give, for a given class of situations and during a documentary genesis, to a document. The documentary work of the professor is considered as the engine of a documentary genesis, which develops a new resource (composed of a set of selected, modified, recombined resources). Any documentary genesis, for a teacher, is a carrier of professional development. In the sense that the teacher acquires new knowledge, new skills and new practices (Gueudet & Trouche, 2008). The documentary approach sees itself as a relevant theoretical framework to understand the professor's work around his resources and even around his resource system.

Resource, taxonomy of resource versus resource system. The documentary approach distinguishes the resource from the document. By resource we designate all the elements of the set (materials, digital, notebooks and manuals of Sésamath). These elements are the ingredients, inputs that the teacher needs to create his own document, his output. This conception takes place in a finite cycle that can return to the actions and / or resources the input.

The resources constitute the available for the teacher he appropriates, transforms them, adapts them to build his document. Unlike the resource, the document has a teaching objective, a didactic intent, to deal with a class of situations adapted to a context (for example to integrate dynamic geometry activities using a dedicated software). Pédauque (2006) defines a document by its use, intention and information:

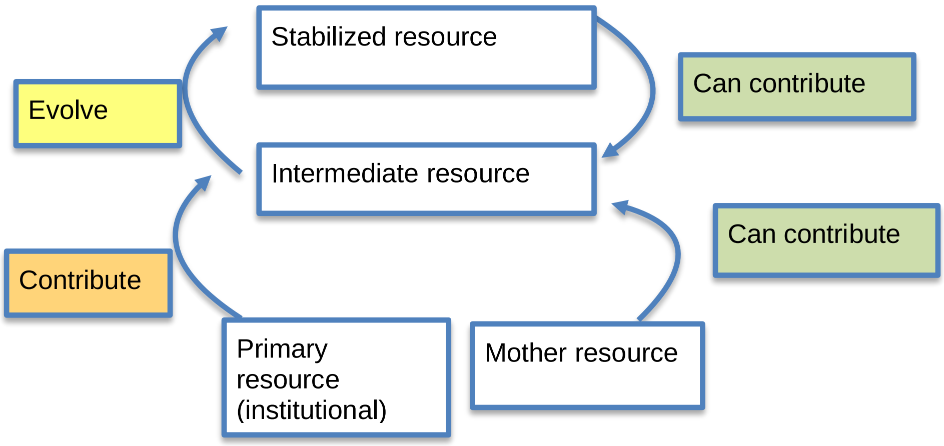

Figure 1: Taxonomy of resources

During his documentary work, the teacher has a variety of material and / or digital resources that combine in different contexts, give rise to a document (s) during a documentary genesis. The document resulting from this transformation must meet a didactic intention, the needs of the teacher, rules organizing its use.

It is therefore necessary to collect these resources as much as possible during the monitoring period: the ones they used, and those they have developed as they work. We contribute to documentary approach and we try to give a taxonomy of teacher's ressource (see Figure 1):

- As primary resource we consider any institutional resource such as a textbook, an accompanying guide;
- As resource mother, any starting resource that the teacher mobilizes to prepare a given course (Hammoud, 2012), we retain this definition but we consider that these resources are not institutional

from Algerian authority but from other institutions. We take as an example Sésamath's resources, foreign textbooks, results of didactic research etc, teachers are more confident using these ressources;

- As intermediate resource, any resource that dynamically evolves in the system, newly created, modified, recombined, adjusted in an individual and / or collective context;
- As stabilized ressource we mean each ressource that its up date become at a a long time such as some notes courses.

The primary ressource contribute to create or update intermediate resource (such as the curriculum), at the same time some mother ressources (as Sésamath ressource, external textbook) and stabilized ressources can contribute to this action. In this system an intermediate ressource can evolve to a stabilized resource (such as worksheet for a specific lesson).

Resource states : active, standby. Guin et Trouche (2008) consider as resource, any active entity, but it is also possible that some resources, remain inactive for a period, in the sense, where they do not undergo any action, then we consider them as resource on standby. We also attribute this concept for any entity in the system that the teacher discovers before use. A standby resource can therefore pass to an active resource and vice versa, following a triggering event. What we call an internal event can be a decision of the teacher regarding the resource or an external event (such as an institutional decision, a colleague's opinion) and which allows to change the status of the resource, (see Figure 2).

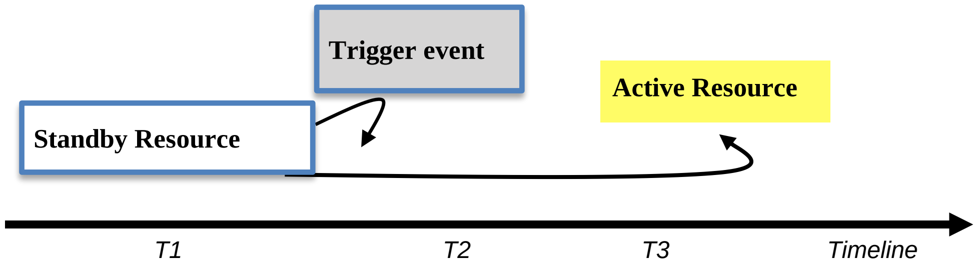

Figure 2- Our vision resource's transition, from standby state to active state (between T1 and T3)

This article tries to approach the structure of this system, the characteristics of these constituents and their relations, we want to answer the question:

To what extent does the analysis of teachers' resource names reveal the structure description of this resource?

How the taxonomy resource's definition can contribute to modeling the resource system of the teacher?

Methodology

Gueudet and Trouche (2008) developed a method for the analysis of the teacher's documentary work, based on a continuous monitoring of his activities over a significant period of time but mainly focused on the principle of the reflexivity of data collection. The teacher is considered to be the main actor in this collection and in his reflective return on his own practices. We draw inspiration from this methodology for monitoring and analyzing the documentary work of our experimental sites. We emphasize that reflexive inquiry focuses on the individual aspect of documentary work, in the context of our work, we are interested in the collective work of teachers. We consider the collective components of this work and we take into account the activity systems of the mathematics teacher in the various communities. Our methodology is therefore based on a reflexive principle of resource collection used or produced by the teacher during his documentary work, individual or community resources.

We present our method of analysis of the resource system, in two aspects: static to approach the structure of the system and in the dynamic aspect to approach the activity of the teacher with his resources (Sayah, 2017) based on the notion of schema.

The teacher's resource system is an open system, define by inputs that represent the primary resources, intermediate, stabilized resources and mothers resources, this system is subject to constraints, rules of events whose output elements are also intermediate resources, which can enter in a reflexive way, into the evolution cycle of the this system. Analyzing mathematical teacher's resource system, is to analyze the structure of this system. We consider that a system is a set of elements in dynamic interactions, organized, according to a goal. This definition can be extended as : "a system is something (identifiable); who does something (activity (s)); and that has a structure; which evolves in time; in something (environment); for something (finality (s)) » (Le Moigne, 1977).

We draw on the system analysis method (Nanci & Espinasse, 2001) to analyze our resource system. This method of analysis proceeds first to the analysis of the flow of information exchanged between the main actor and his colleagues at the level of a structure to carry out his activities.

The Schematic Representation of Ressource System (SRRS) (Gueudet et Trouche, 2010), is the main tool used to identify the ressource system structure, define its elements, its properties and the relation between its elements. This representation combined with other tools resulting from the reflexive investigation methodology allows us to define the entries of the resource dictionary and to define its elements. For each identified resource we retain from the teacher: his name as it was transmitted to us and in the language used by the teacher, its definition this will allow us to classify the resource according to our taxonomy (mother resource primary, intermediate and stabilized). To have a detailed look on its resource system, we decompensated the activity of the teacher interviewed in process, we analyze each process in its own right (example process lesson preparation process, preparation of a control, process implementation of a resource Sésamath in an institutional class. This will allow us to collect the maximum resources mobilized by the teacher to ensure its function. From these resources we develop « the resource dictionary » to identify and structure them around resources (primary, mother, intermediate, stabilized etc.) (Sayah, 2018).

From this dictionary and the semantic rules connecting the different resources, we elaborate the conceptual resource model (RCM) by ignoring their use (answering the question of what is needed?), This constitutes the aspect static system: define its constituents, properties and relationships. We hold that no entity can be isolated in the system. We go on to describe the different processes using the Resource Use Conceptual Model (MCUR) in trying to answer the questions (what to do, how and where). Thus we locate in an individual and collective context both the resources of the teacher while approaching its activities by identifying its different processes.

Data analysis: From schematic representation of ressource system to ressource dictionary (SRRS to DicRS)

We present Meriam's one year follow-up data, individual and community follow- up. Meriam is a member of the Cop (Sésamath) community for the selection, adaptation and translation of Sésamath resources into institutional classes. We followed her documentary work at home and college in her institutional classes, her interactions with the collective where we focused our attention on her appropriation of Sésamath resources at different times. During this period, we highlight the evolution of its system, or even analyze the appropriation of Sésamath resources.

Meriam's resource system (see Figure 3) is perceived as a set of objects (set of material resources such as textbooks, guide, Sésamath resource) and a relationship between these objects that represent interactions, interviews also are crossed with Meriam's SRRS and analyzed to structure her resource system. We asked Meriam to describe us, as much as possible she can, her schematic representation of the resource system (SRRS) and we asked her to read it without any interruption on our part. Its resource system depends on several actors:

- From the pedagogical responsible, Meriam receives her primary resources (officials such as: textbooks, teaching guides);
- With the coordinator, Meriam validates her control subjects (homework and composition);

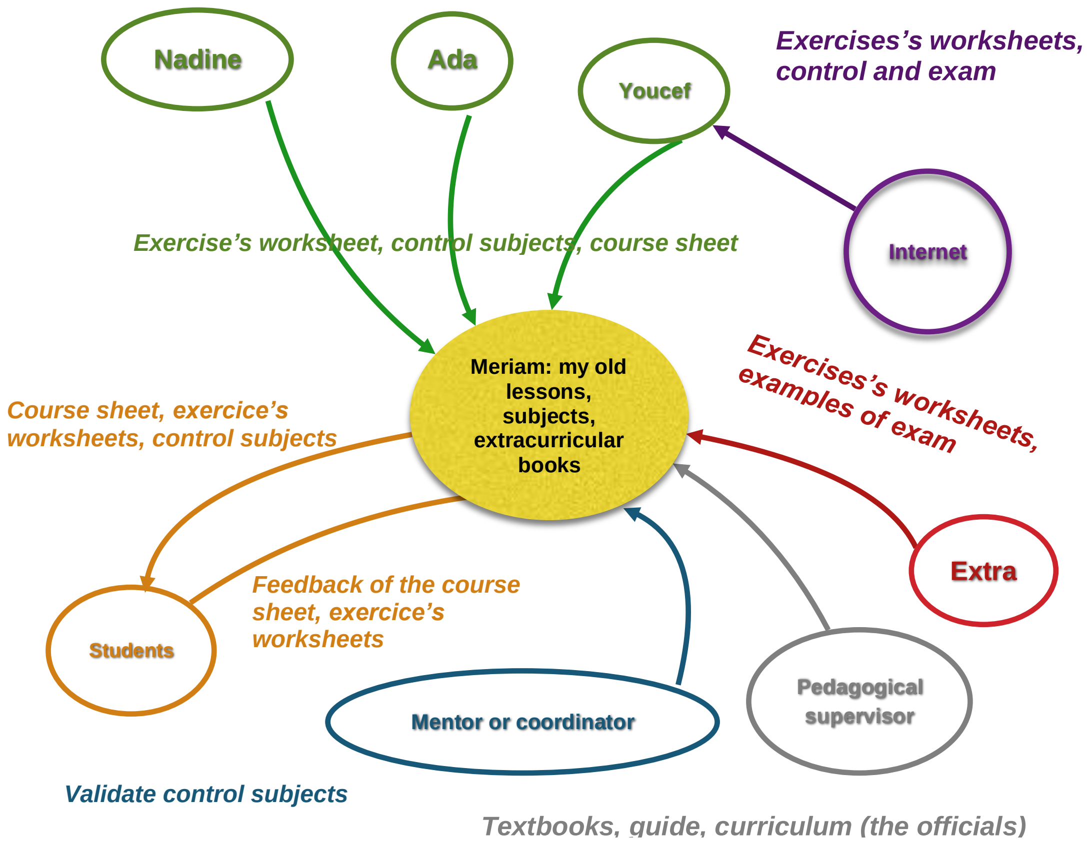

Figure 3- Meriam's Schematic Representation Resource System (SRRS)

- With Nadine and Youcef, she exchanged drills of exercises and subjects of controls;
- With Adam as a private colleague, being part of both a public college and his college, she exchanges subjects of control;
- With her students, Meriam considers that the feedback of the courses and exercises in class, is a source of resources for her system;

- The Internet is part of his system but not directly as she has no connection, neither at home and at the college at the beginning of our project but she receive resources through her colleague Youcef;
- Finally, she has resources from some other colleges (extra) that are generally series of exercises and controls.
- Pointing out with her finger on his schematic representation, Meriam adds at the end:
  Meriam: « and here I have my old documents, subjects, lesson cards, extracurricular books .." ».

This representation describes the resource system as a whole we consider it as a flow of resources exchanged between actors and shows in no case the relations or the structure of the resources involved. We use this schematic representation, the interviews conducted during its follow-up (before and after the videos class observations) and the observations of coordination sessions to approach the structural aspects of the Meriam resource system.

These different tools allowed us to define the elements of its resources dictionary. We report for each resource its name and its definition as appointed by the teacher, and that we translate into English. On the basis of these definitions, we classify each resource (primary, mother, intermediate or stabilized), we classify it as collective ressource or individual ressource, and we identify the rules of their uses which determine their relations. (see Table 1).

الموارد	التعريف بالمورد	Resource name, translate by the researcher	Resource collective/ individual	Descriptions
		Ressources primaires		
البرنامج	نقصد به المعلومات و المعارف التي يجب تلقينها للطفل خلال فترة معينة	Program	Individual	Represents information and knowledge to be transmitted to the student for a period of time.
منهاج	يشمل كل العمليات التكوينية التي يساهم فيها التلميذ بتأطير من المدرسة في فترة تعلم الرياضيات لقسم الثالثة متوسط	Curriculum	Indivual	Includes all the training processes in which the student contributes with the supervision of the school during the learning period.
التدرج السنوي	توزيع الجوانب التعليمية ومجمل نشاطات التعلم وفق مبدأ تدرج الزمن الدراسي حسب مستويات التعليم	Progressio n	Collective	The distribution of educational aspects and all activities according to the period of learning and the level of education.
		Ressources intermediaires		
المذكرة	ورقة عمل لسيرورة الحصة توضح تنظيم الوقت استتبق الاحداث، تفاصيل الدرس ، نشاط البحث، خلاصة المعرفة و التقويم	Worksheet course	Individual or collective	Worksheet of the course, specifies time management, anticipations of events, details of the lesson, research activity, synthesis of knowledge and evaluation.

Table 1: Meriam resource dictionary

On the basis of these definitions, we classify each resource (primary, mother, intermediate or stabilized), we classify it as collective ressource or individual ressource, and we identify the rules of their uses which determine their relations. (see Table 1).

Discussion and conclusion

For the analysis of its resource system, we have based on the various schematic representations, its interviews and observations to approach its system of resource on its static and dynamic aspect. The static aspect is marked by the description of its resource dictionary which feeds from one schematic representation to another by new resources. Initially conceived around its primary resources (institutional documents), its intermediate resources (old course sheets, old subjects) and certain stabilized resources (either own control subjects or from colleagues). Some resources may remain in standby state for a period of time (such as a series of exercises for remediation). These resources can only go into the active state on a triggering event such as "student assessment and decision to take back the concepts taught". Based on this resource taxonomy and the relationships between these resources, we established its conceptual resource model where we noticed new instances of the proposed taxonomy. Reading this model has led us to identify some relationships between these resources (see Table 2):

Relation of our Conceptual Ressource Model	Instance Relation examples (CRM)
Primary ressource contribute to create an *intermediate resource.*	The *pedagogic guide* contribute to create *course note.*
Mother ressource can contribute to create or update an *intermediate ressource.*	GeoGebra ressource contribute to create intermediate ressource « geometry in space ». Her old exams subjetcs contribute to create new one.

Table 2: Relation instance in Meriam's Conceptual Resource Model

Finally understanding system ressource teacher needs to describe at first the elements of this system, define their properties and finally identify their relations. The dictionary ressource can be a step to approching this system not only in Algerian context but in other context to be crossed.

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BETTER TOGETHER: A CASE STUDY OF COLLABORATIVE LEARNING

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"I think we are better together, if there is a motto and like I would even extend this to my online professional learning community that I have, I'm way better with them…" (William, Interview, April 22, 2015). This contribution provides a description of a case study situated within a large scale professional development initiative for teacher collaboration that involved the joint efforts of professional, institutional and governmental organizations. This paper focuses on this single case study to explore the area and questions of Theme D and offers a description of some of the tools and resources designed for, and resulting from, teacher collaboration in mathematics education in Ontario, Canada.

Introduction

Research in education points to the role of the teacher as having the most impact on student achievement (Hargreaves & O'Connor, 2018). Some suggest that "what a teacher knows is one of the most important influences on what is done in classrooms and ultimately on what students learn" (Fennema & Franke, 1992, p.147). Designing effective professional development that will support teacher learning is therefore important and research suggests "collaborative professional development is related to a positive impact on teachers' instructional strategies; their self-esteem and self-efficacy; and student learning processes, motivation and outcomes" (Borko & Potari, 2019, p. 2). Documenting and understanding ways that teachers engage in collaborative professional development, the resources they engage with, how they use these resources, and the ways they learn is therefore critical. This paper provides a description of one professional learning community (PLC) over an extended period of time in order to highlight some of the ways that resources can be used to sustain and scale up collaboration. In my discussion of professional learning resources used by the PLC I use Adler's (2000) descriptions and definitions for different resources used in mathematics education and also consider her meaning of resources to include both material and non-material resources. In my discussion of the PLC itself, I lean on Wenger (1988) for he discusses communities of practice and the ways they can accumulate and diffuse knowledge through an organization. Wenger claims that communities of practice are "nodes for the exchange and interpretation of information, they retain knowledge in 'living ways', they steward competencies, and they provide homes for identities" (p.5). These characteristics were observed in the study described in this paper.

The Grade 9 Applied Mathematics Inquiry Project

Secondary school in Ontario begins when students enter Grade 9 and are required to study mathematics following either an Applied or Academic pathway. The Academic pathway "develop[s] students' knowledge and skills through the study of theory and abstract problems" (OME, 2005, p. 6) while the Applied pathway "focus[es] on the essential concepts of a subject, and develop[s] students' knowledge and skills through practical applications and concrete examples" (OME, 2005, p. 6). The mathematics community in Ontario and the Ontario Ministry of Education (OME) had identified the achievement of students in Grade 9 Applied Mathematics as an area of concern. Data from schools and provincial assessments demonstrated that students in Grade 9 Applied Mathematics were not achieving at similar levels as students in Grade 9 Academic Mathematics (EQAO, 2015). In the Spring of 2014 the OME approved funding for a large-scale initiative that would help to address this concern. This initiative, the Grade 9 Applied Mathematics Inquiry Project (the Project), was a joint endeavour between several stakeholders in Ontario. Funding was provided by the OME to the Ontario Association for Mathematics Education (OAME) to develop and pilot a professional learning initiative for ten high school professional learning communities (PLCs) across the province. OAME partnered with a research team from the University of Ottawa, led by Christine Suurtamm, to design an effective professional learning program that would focus on helping teachers to enhance their understanding of the Grade 9 Applied Mathematics curriculum as well as on implementing the curriculum in ways that would best meet the needs of their students. My involvement with this project was as a research assistant (RA) assigned to school sites to support and observe the work that PLCs engaged in.

The project had two foci, a professional development focus as well as a research focus (Suurtamm, 2017). The goals for the professional development component included: increase student achievement in Grade 9 Applied Mathematics; increase teacher knowledge of the curriculum, both in terms of mathematics content knowledge and mathematics knowledge for teaching; identify effective teaching and learning strategies; share implementation strategies with stakeholders; and foster teacher leadership in mathematics. The research focus was identified by the Principal Researcher and aimed to address the following research questions: What role does teacher collaboration play in professional learning and classroom practice with respect to Grade 9 Applied Mathematics? How can teachers enhance their understandings of the interactions between Grade 9 Applied Mathematics curriculum, pedagogy, and students' needs?

Fields High School PLC

During the two year project I was assigned to two research sites, the Fields High School PLC for the full two years of the project and the Parks High School PLC for the second year of the project. The Fields High School PLC was the case study that I worked with as a graduate student research assistant and over the course of the two years I participated in 19 full day PLC meetings, 6 project wide full day meetings, 10 individual interviews with members of the PLC, 16 research team meetings, and 2 province wide professional learning conferences planned with the project team participants. This was also the focus of my master’s thesis (McKie, 2016). This long-term observation, both in and out of the classroom, and wide range of data collected with close involvement of the participants (Gueudet & Trouche, 2012) allowed me to develop an in-depth understanding of the ways that the PLC worked, the resources they used, and observe the outcomes of their collaboration. An important component of the data collection was the more than 150 hours of audio-recordings of teacher conversations. Gueudet & Trouche (2012) define the non-material resource of teacher conversations as an integral component of the resource system that are often difficult to capture. These conversations provide the often missing voice of the teacher in research reports that can provide insights to teacher learning (Jaworski et al., 2017).

The members of the PLC that were part of the math-focused OAME Grade 9 Applied project included Dolores, the Principal, Wayne, the Mathematics Department Head, William and Tasha, teachers of

Grade 9 Applied Mathematics, and Rachel, co-head of student services with a focus on special education. It is important to note that one of the criteria for acceptance into this project was the participation of both a school administrator and a special education teacher.

Resources *For* Teacher Collaboration

Adler (2000) states that the two critical elements in professional learning are curriculum content and pedagogical strategies. It was evident through both observation of the PLC monthly meetings and the individual interviews with the PLC members that these were the two driving forces behind the work at Fields. The PLC was focused on the specific content of the Grade 9 Applied Mathematics curriculum and the collaboration focused on selecting pedagogical strategies that would best meet the needs of their students and enhance their learning.

Material resources. When describing the types of material resources important in mathematics education Adler (2000) includes technologies, which can range from chalkboards to computers, mathematics materials, including manipulatives and textbooks, mathematical objects, which can be as simple as a physical representation of a geometric shape or as complex as a mathematical theorem, and everyday objects like money or stories. Material resources that I observed the PLC at Fields using for collaboration included technologies, most often shared computer drives and social media platforms, and mathematical materials such as books and research articles.

The teachers at Fields High School used technology as a tool to support and facilitate collaboration within their own PLC and between other PLCs in the project. Lesson plans, assessment rubrics, and teacher notes were all stored on a shared Google drive that allowed teachers to collaborate on a single document or create a copy of someone else's lesson and alter it to better meet the needs of their specific classroom. As the project progressed and project PLCs met during face to face, project-wide meetings, the different PLCs began to make connections and collaborate. Different school PLCs were at different stages of their professional learning and when they met with other PLCs were exposed to pedagogical strategies with which they may be unfamiliar. For instance, the use of non-permanent, vertical surfaces to engage all students in problem solving, favoured by the PLC at Fields, were starting to catch on with other PLCs through the sharing of resources at project-wide meetings. As another example, some PLCs were unfamiliar with using open ended mathematical activities and therefore the research team provided time to grapple with these types of problems when the groups met. Through engaging with these problems and talking with the other PLCs in the project that used these strategies in their classroom many decided to introduce them in their own classroom. The PLCs stayed connected outside of the project-wide meetings using email, social media platforms such as Twitter and Facebook, and sharing resources in a shared drive.

Dolores provided several books and articles to staff members throughout the project and used staff meetings to engage in book studies focused on the principles and practices outlined in these resources. Some of the resources that the PLC referenced during the project included the work of Stephen Covey, Ginny Newman, Daniel Pink, Carol Dweck, Jo Boaler, John Hattie, Lucy West, Sandra Herbst, Margaret Smith, Mary Kay Stein, and Peter Liljedahl. These resources were often present when the PLC members gathered to plan and design lessons for their students during PLC meetings. One particular resource, John Hattie's *Visible Learning*, formed the foundation of several lessons designed by the PLC using the lesson study model they engaged with. Under the leadership of Dolores, who had recently attended a leadership conference led by John Hattie, the PLC chose to focus on students' metacognition and designing lessons that would reveal to students the different ways that they think and ideally alert the students to learning behaviours that would lead them to success in the mathematics classroom.

Human and socio-cultural resources. The human and socio-cultural resources (Adler, 2000) were perhaps the most important tools employed by the PLC for supporting and facilitating collaboration. Adler (2000) defines human resources as the participants and their knowledgebase and processes including collegiality "for maintenance of the practice as well as change" (p. 212). I include the socio-cultural resources of verbalisation, communication and time under the same heading as I believe that they are interconnected. In order to discuss the ways that the human and socio-cultural resources supported and facilitated collaboration at Fields it is important to discuss the individual members, their shared beliefs, and how the collegiality of the group evolved and how the community of practice (Wenger, 1998) at Fields evolved.

As Wenger (1998) suggests "communities of practice develop around things that matter to people" (p. 2). Through both observation of the PLC's monthly meetings and individual participants' interviews it became clear that the PLC at Fields High School was developed around the shared belief that the teaching and learning of mathematics should be learner-centred (Adler, 2000). Both Wayne and Tasha shared that their individual experiences as learners in high school were negative and they knew that the traditional model of high school mathematics teaching did not benefit the types of learners that they were teaching in the Grade 9 Applied classroom. "As a kid that's how they taught me and I knew that didn't work...when I finished high school, I couldn't remember a single thing, I knew that is definitely not the way kids learn…they need to understand it" (Tasha, Interview, April 22, 2015).

William, on the other hand, shared that high school mathematics was a positive experience for him. He excelled in high school and throughout his mathematics degree in University. His teaching career continued to be a positive experience until he struggled teaching a class of students in a Grade 10 Applied Mathematics course. "I was losing classes, I was teaching applied level classes and completely losing them, having zero success with them so a lot of things happened that made me think about changing" (William, Interview, April 22, 2015). He credits Wayne with pushing him to rethink his teaching strategies and they began to collaborate in between classes and after school and completely shifted the way they taught the Grade 9 and 10 Applied Mathematics courses.

It was the casual collaboration of Wayne and William several years earlier that was the beginning of the PLC and with the support of Dolores they began to seek opportunities for professional learning outside of their school. Wayne and William reported that a mathematics education conference they attended had an immense impact on their professional learning. They attended a session on non-permanent vertical surfaces by Dr. Peter Liljedahl (2015) and they were convinced that these strategies would be a powerful addition in their own classrooms. Tasha explains that "Wayne and William were so excited about the idea that on Monday they came in and got these whiteboards and just started implementing it and it's been wonderful" (Tasha, Interview, April 22, 2015).

This exposure to new ideas was brought back to the school and became one of the conceptual tools that the PLC members used "to guide decisions about teaching and learning" (Grossman, Smagorinsky, & Valencia, 1999, p. 14). They began to design, plan, observe, and debrief on lessons that involved tasks that engaged in the principles of the “thinking classroom” (Liljedahl, 2015) and used the vertical, non-permanent surfaces to shift the way students interacted with mathematical objects and ideas. The thinking classroom model involves engaging students in rich mathematical tasks on visible, vertical surfaces in groups that have been randomly chosen each class. Research using this model suggests that students are quicker to engage in the problem, and stay engaged longer, when the class is set up using these principles (Liljedahl, 2015). This example of dynamic and evolving knowing illustrates Wenger’s (1998) point of the importance of “the participation of people who are fully engaged in the process of creating, refining, communicating, and using knowledge” (p. 2) and is one of the defining characteristics of a community of practice (Wenger, 1998).

A crucial human resource, and integral component of a community of practice, is leadership (Loucks-Horsley & Matsumoto, 1999; Wenger, 1998; Hord, 2004) and all of the tools and processes I have described as being employed at Fields were buoyed by the support and leadership of Dolores. She described her job as creating opportunities for those that were willing and interested in collaborating. Interestingly she admitted to me that she did not feel comfortable in the mathematics and science domain. Instead she described her role within the PLC as that of a “mosquito”, buzzing ideas into people’s ears rather than dominating or directing the flow of discussions during PLC meetings. She feels there is a risk of a principal becoming too overbearing in a PLC and chose instead to stay on the periphery.

Adler (2000) argues that “innovation requires enough people who are willing and capable of overcoming inadequate resources to support educational change” (p. 206). This sentiment is echoed by Dolores when she describes her method of managing PLCs at her school “the biggest challenge for a PLC is the naysayer...they just submarine the whole thing. I don’t do that. I don’t spend a lot of energy on people who are pushing back” (Dolores, Interview, April 27, 2015). She is clear that participation in any PLC is voluntary and it is important that members want to be there. When two teachers decided to leave the PLC prior to the beginning of this project it made room for others who were eager to participate, such as Rachel, the Student Success Teacher “I didn’t know if that was even a possibility for me to be able to get involved with it but I’m really happy that I had the chance to be able to do that” (Interview, May 26, 2015). This excitement was shared by others involved in the project and when the project funding was renewed for a second year it created the opportunity for the PLC to involve more people “everyone is so excited to be a part of it, everybody wants to stay involved. [They] want to stay with the group that we’re in and we want to keep that positive rolling” (Rachel, Interview, May 26, 2015).

In speaking with the different members of the PLC individually it was clear that Dolores was an integral part of the PLC and responsible for creating that excitement. They described how they felt they could take risks, that she would support them, that she had their back. They are describing what Wenger (1998) refers to as “legitimizing participation” (p. 7) in which the work of the community is acknowledged and recognized as bringing value to the organization and by providing the time and space for the community to work. (Wenger, 1998).

This was evident when they shared that they felt she was proud of the work they were doing and this acknowledgment of their effort to improve the teaching and learning of mathematics at Fields High School spurred the work on.

> If you are looking at doing things differently and changing the way you teach [she] will support you right… she's totally willing to let you, if you think it's good. Well I think she is proud of what has happened here, right? She should be proud of what's happened here it's been, it's probably one of the most collaborative environments you can work in if you choose to collaborate, if you choose to jump in (William, Interview, April 22, 2015).

> Another big thing about [our open door policy] is the support of the Principal and she is all about supporting us and allowing us to take chances and take risks and it's a huge deal and more of us are willing knowing we are not going to get burned (Wayne, Interview, April 22, 2015).

One of the most important resources for collaboration is time (Kruse, Louise, & Bryk, 1995; Roberts & Pruitt, 2009), and in this case, the teachers felt conflicted over the amount of time they spent collaborating through different PLC activities as well as professional learning opportunities outside of the school. Each PLC meeting meant a day out of their classrooms, away from the students they are trying to support. William shared "that's a big struggle, move teaching forward, move instruction forward but at the expense of the kids that are really right in front of me this semester" (William, Interview, April 22, 2015). For Wayne the situation was similar:

> I feel like I'm short changing my students…That's hard to accept because here you are doing all this work to try to improve things and your kids are suffering…So here I'm doing this work for the good of education so to speak and yet I'm ignoring my students. So that's interesting because all these programs...they are all about our most vulnerable kids and yet we are not in the room right? So that for me would be the toughest part of the whole business (Wayne, Interview, April 22, 2015).

As the project continued, this challenge was often discussed and different solutions were attempted to help reduce the amount of time teachers spent away from their classroom. Dolores shared with me that in her opinion both William and Wayne should be given teaching credit for all the professional development support they provided at different schools around the province. This would allow them to teach at Fields as well as help to mobilize the professional learning generated without having to miss teaching days in their own classroom.

Resources *From* Collaboration

The lesson study model that the PLC employed was a framework that allowed the teachers to reflect on student thinking and design lessons intended to engage students in specific skills. Their methods were similar to what Carpenter, Fennema, Franke, Levi, and Epsom (2015) refer to as "cognitively guided instruction" (p. 134) in which teachers focus on choosing problems that best fit the needs and skill level of their students and responding to student thinking rather than choose lessons or activities that will satisfy a mathematical curriculum. A typical PLC meeting at Fields started with the focus on one participant's class and the students in that class. The teacher would describe each student on their class list and share any specific learning challenges or exceptionalities that a student may have. This stage of their process provides a clear understanding of the group of students the PLC will be designing a lesson for. Following that the teacher will share the concept or lesson that they hope to design with the group. One example of a lesson planning session involved a bag of t-shirts William had come across. He thought the t-shirts would provide an interesting point of curiosity to design a lesson around. After several hours of exploring different activity ideas the group designed a lesson that would have students suggesting things they noticed and wondered about the giant pile of t-shirts in the middle of the classroom and the teacher leading the questioning towards strategies students could use to estimate the volume of the pile and how long it might take the class to fold all of the t-shirts in the pile. The underlying mathematical concepts would be estimation strategies and unit rates.

The following week William taught the lesson with the other PLC members observing. After the lesson the teachers spent the afternoon debriefing what they observed. This second component of the lesson study process employed teacher noticing (Jacobs, Lamb, & Philips, 2010) through observing the lesson that the PLC designed, with participants paying attention to the students' interactions with the lesson and specific teacher moves that produced different student reactions. The debrief of this process following the observed lesson allowed for adjustments to the lesson as participants shared the different actions and reactions they noticed and discussed ways to improve or alter the lesson to maximize the learning for the students, resulting in a more focused lesson plan. In this way the PLC was creating a catalogue of PLC designed and tested lessons that could be used by others in the school and beyond.

During the second year of the project I was assigned to a second PLC in a different city in Ontario, at Parks High School. The participants from Parks High School had many questions about the different activities of the PLC at Fields High School. I suggested that Parks might want to visit Fields High School for a day to share resources, observe a lesson, and debrief and discuss what they saw. As a consequence of that day the two PLCs had created a new shared Google folder where they could share lesson and unit plans, activity ideas, and assessment strategies. One teacher from Parks High School claimed that the sharing of resources across different PLCs, or what he referred to as the "cross-pollination" aspect of the project provided what his school board needed in terms of accessing different pedagogical strategies. Interestingly, although the project officially ended in the summer of 2016 the participants from the two PLCs are still accessing the resources on the shared Google drive and the participants interact with each other on the social media platform Twitter.

Concluding Comments

The project as a whole incorporated different resources for collaboration including material, human, and socio-cultural as well as generated resources both in terms of practical tools such as lesson plans, teaching strategies and conceptual tools for the individual participants and PLCs. A wealth of resources was created through the organization of a larger network between the various PLCs. At the end of year 2, the project received additional funding from the OME to hold several summer conferences that allowed the resources created by individual PLCs and cross-collaboration between PLCs to be shared with educators outside of the project. In the summer of 2016 over 400 new participants attended conferences that were managed by the research team and OAME. Conference sessions and workshops were led by the PLC participants both in their original teams and with members from other PLCs that they made connections with around shared interests. Topics of sessions included using rich tasks, assessment, and learning progressions across different grades. I was tasked with creating an online resource (math4thenines.ca) that would highlight, and make accessible to the rest of the province, workshops, sessions and other resources created from the work of the PLCs in the project.

As I begin my doctoral studies I am still very much connected to the network of mathematics educators that participated in the Grade 9 Applied project. We share resources online through Twitter and shared Google drives, we connect at provincial mathematics education conferences, and local professional development sessions. I am interested in revisiting the Grade 9 Applied project and addressing the important question from Theme D, "Which resources can be used (and how) to sustain and scale up collaboration over time?" I know that many participants are still connected to one another but are they still collaborating and if so what does that collaboration look like now, how has the collaboration evolved? What tools are the participants using to facilitate and support any collaboration and what resources created from the project collaboration are still being used?

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COLLABORATIVE DESIGN OF LEARNING ENVIRONMENT THAT FOSTERS REIFICATION OF A MATHEMATICAL OBJECT: THE CASE OF FUNCTION

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This research aims at designing learning environment that fosters reification of a mathematical object by knotworking actors who participated in a design research with different roles. Based on the Activity Theory, researchers of different expertise, elementary and secondary teachers collaboratively designed learning environment on functions in Japanese elementary and secondary classroom from 6th to 9th grades. The team consists of researcher of task design heuristics, commognitive analysis, ICT specialist, Lesson Study specialists. The learning environment includes resources of coherent discourse and a dynamic ICT environment, where coherent discourse consists in resource for collbaboration and dynamic ICT environment consists in resource from collaboration.

Introduction and Background

This research addresses to one of the key questions of Theme D: How are teachers and researchers engaged in the design of resources in collaboration? What are the outcome of these collaborations. In this paper we focus on the resources that play significant role in collaborative task design. There are substantial and research in this theme. ICMI Study 22 on Task Design (Watson & Ohtani, 2015) includes case studies that illustrate proactive roles of researchers and teachers in collaborative task design. Ponte, Mata-Pereria, Henriques, and Quaresma (2013) considers “exploratory task” as resource for collaboration between researcher and teachers in Lisboa, Portugal. New mathematics curriculum for basic education required teachers to develop and use exploratory tasks that may support students’ mathematical reasoning. With such institutional context, developmental work on exploratory task design was conducted with the close collaboration of research expertise and classroom teaching expertise.

Aims and Rationale

In line with recent research tradition, we also aim at designing resources that constitute opportunities for collaborations of researchers and teachers. In this article, we chose specific mathematical concept of “function” as a research topic. And we cover both elementary and junior secondary schools and pay special attention to “process of reification”. By reification we mean a transition from operational procedure into a mental object. As resources for teacher-researcher collaboration, we will take into account of “learning environment” which includes “classroom discourse” and “dynamic ICT environment”, where “classroom discourse” consists in “resource for” and “dynamic ICT environment” consists in “resource from” collaboration.

Since 2000, we create collaborative team of heterogeneous voices: researcher of task design heuristics (Ohtani), “commognitive analysis” (Hino), ICT specialist (Nunokawa), and expert teachers of both elementary and junior secondary schools (Nakamura and Kanno) in Kanazawa, Japan. commognitive analysis we mean specific approach of discourse analysis. Commognition is the portmanteau of communication and cognition and the focal notion of the approach to learning grounded in the assumption that thinking can be usefully conceptualized as one's communication with oneself. Mathematics is a historically established discourse, and learning mathematics means becoming a participant in this special form of communication.

We adopt Cultural Historical Activity Theory (Engeström, 1990; Leont'ev, 1975) so as to describe a resources developed by various actors within and across communities could productively engage in expansive activities. The actual dynamism of collaboration is well captured through the lens of elaborated "Activity System" (Engeström, 1990: 79).

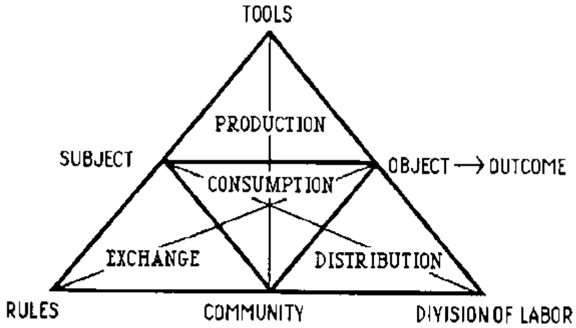

Figure 1: Activity System

The top triangular section represents individual(s) tool or resource mediated activity by Vygotskii (1984). In this diagram, subject is researchers and practitioners of above mentioned collaborative team. Tools are both theoretical frameworks (e.g. reification and commognitive perspective) and teacher's professional knowledge (e.g. "Pedagogical Content Knowledge" in relation to function). Object and outcome are learning environment (e.g. shared didactical and methodological choices on teaching function with "Dynamic Digital Environment"). Tool-mediated relationship between subject and object (or outcome) has meaning only in relation to the components at the bottom of the triangle: this includes other people who share the same object (community); social norms and conventions (rules); and the division of actions among members of the community (division of labour). This includes variety of actors play specialized role in communal design of learning environment for a meaningful approach to function in ordinary school under prescribed national curriculum. Further, roles, identities, and interactions of various participants are identified in correspondence between top and bottom triangles: various participants in community appropriate or internalize new idea through dialogue (consumption); researcher of different expertise make unique contribution to making lesson plans (distribution); and any decision on didactical and methodological choices are made as shared within team members (rules).

Challenge Confronting our Team

Understanding function is difficult for students even in the last 9th grade of secondary school. According to the national survey of achievement for 9th graders (NIER, 2012), one of problems to be resolved is to develop students' understanding relations between two quantities as direct, inverse, and linear functions (ibid: 27). For example, to a basic question "A person walks a road of 1500m long. The person walked *x* m and *y* m is still remaining to go. What is the relation between *x* and *y*", 9th grade students chose the following items: "y is direct proportion of x" (13.4%); "y is inverse proportion of x" (23.7%); "y is linear function of x" (36.3%); and "Neither of these" (25.5%).

We create a team in order to design learning environment that may contribute to resolve deep-routed problem of teaching and learning function in elementary and junior secondary schools in Japan. In view of three levels of curriculum (intended, implemented, and attained), the results of NIER shows current situation of the attained curriculum in relation to function. When we design learning environment, it is important for our team to take into account intended and implemented curriculum. As for intended curriculum, we will look at Course of Study and approved textbooks by Ministry of Education. As for implemented curriculum, we will look at research in classroom discourse on function.

Course of Study and Textbooks

In the Japanese curriculum, properties of direct and inverse proportion are learnt in upper elementary (grades 5 and 6) while investigating change and correspondence of two co-varying quantities in numerical tables, algebraic representations, and graphs. In secondary mathematics (grades 7 to 9), the concept of function is formally defined and direct, inverse proportion, linear and simple quadratic functions ($y=ax^2$) are formulated as algebraic equations. Learning function at junior secondary level is characterized as transition from mere collection of properties to defining-character that will support reification (van Hiele, 1986). During team meeting, we found that even expert teachers have no clear idea why we teach direct and inverse proportion in grade 6 and 7. In view of van Hiele level, it has much to do with transition from the second (descriptive) to the third (local deduction) level as well as range of variables expand from Q_+ (non negative rational numbers) to Q which necessarily redefine direct and inverse proportions from co-variation to algebraic expression.

Nunokawa (2014) undertook comparative analysis of how Japanese textbooks introduce a definition of function is and how it is described throughout the unit. There was substantial inconsistency in statements about functions, and this can cause unintentional difficulties for students in understanding function as a reified mathematical object represented by tables, algebraic expressions, and graphs. All team members experienced big controversy over the conception of function: one-to-one correspondence; relation between two variables, algebraic formula, black box, operator, and so forth. We all think it crucial that ambiguity or multimodal meaning of function has serious effect on the part of students. Thus, we think it necessary to observe actual lessons and investigate in what's happening in classroom.

Classroom Discourse on Function

According to Sfard (2011), discourse is a specific type of communication which has interrelated characteristic features: special keywords; visual mediators; distinctive routines; and generally endorsed narratives (ibid, 2). Mathematics is conceived as a discourse that creates its own mathematical objects and students would reify mathematical objects through participating in mathematical discourse in which they talk about an object and the existence or nature of it.

Nachilieli & Tabach (2011) analyzed 7th grade students as they were making their first steps in the discourse of functions for nearly two months. Their analysis showed that the students were able to participate in the discourse without ever dealing directly with this as-yet nonexistent object. Classroom function discourse was characterized as 'lower' level (ibid; 25) - based on concrete calculation and operations on quantities given in a table, expression, or graph. In order to support students to reify function as a mathematical object it is crucial for the teacher to identify a leading discourse which guides students to proceed from the talk on function as 'inter-discursive' or 'discourse-for-others' to 'intra-discursive' or 'object-driven' (ibid, 25-26).

Font et.al (2010) found inconsistency in classroom discourse about functions, for example: function is one-to-one correspondence between two variables; a linear function y=2x+1; a function is represented by an algebraic expression. The second statement identifies a function *with* an algebraic expression but the last statement distinguishes a function *from* an algebraic expression. This illustrates a possible inconsistency in classroom discourse. Ensuring consistency in the classroom discourse about functions might enable a student to participate in learning functions in a more sensible way and might also affect reification of function as a mathematical object.

Therefore, we set out to design a coherent teaching unit for students to purposefully investigate co-varying quantity, represent its properties, and talk about functions as existing objects.

Design Principles

We met regularly in order to share theoretical references and to take advantages of exchange and discussions of heterogeneous voices, which helped to share theoretical tools and to perform didactical and methodological choices among. Through intensive discussion we consequently adopt the following three heuristics as design principles.

Leading Discourse on Function

We made a distinction between function and its tabular, algebraic, and graphical representation. Further, we adopt a definition of function not as an abstract one-to-one correspondence, but as a concrete quantity. This is the definition that was formulated by Euler (1775/2000) in his textbook on differential calculus: "Hence, if x designates the variable quantity, all other quantities that in any way depend on x or determined by it are called its function." (ibid, vi)

We conceive numerical tables, algebraic expressions, and graphs as traces or shadows of the function itself. We expect students to grasp properties of particular function from bits and pieces of the shadows.

Observing and Telling Properties of Changing Quantity in Dynamic ICT Environment

In order to emphasize function as a concrete quantity that depends on or is determined by a given quantity, the design research made use of the dynamic mathematical software 'GeoGebra'. We expect students to observe the changing quantity while manipulating a tablet PC with GeoGebra applets. The GeoGebra applets were developed by Nunokawa.

The Applet 'Counter' exhibits a numerical expression in which independent and dependent variables change simultaneously. The Applet 'Distinction' exhibits both a cartoon character and its shadow moving simultaneously on coordinate plane which suggests natural distinction between a function and its graphic representation. In addition, the dynamic environment allows students to observe a qualitative aspect of 'rate of change' for a linear function and also for inverse proportion on coordinate plane. In contrast to a linear function, the rate of change of inverse proportion is not constant and varies dramatically as x comes close to the origin. Further, it is expected that working with tablet PC in a dynamic environment encourages students to have more opportunities for making conjectures and talking about properties of the varying dependent quantity.

Algebraic and Geometric Treatments of Function

In all Japanese eighth grade textbooks, linear function is defined by algebraic expression '$y=ax+b$' and is presented as static mathematical object: an algebraic expression of a graph that contains two points; a common point of two graphs by solving a system of equations in two unknowns. Graphs of linear functions are recognized as straight lines, and relations between them are described by geometrical properties.

Premature introduction of linear functions as static objects, and treatment of both algebraic and geometrical discourses on linear function causes inconsistency with the characterization of function as a concrete quantity that depends on or is determined by a given quantity. In this experiment, we engineer a progressive change of discourse from the dynamic aspect of co-varying quantities towards the static aspects of reified objects.

Designing a Unit and Environment

The team of researchers and expert teacher collaboratively designed a teaching unit and environment which contains the intended leading discourse and the ICT environment.

Telling Function: anthropomorphize function as Japanese "Ninja"

Based on the research on how the role of discourse affects reification of mathematical objects, we decided to distinguish function from its numerical, algebraic, and graphical representation and to talk of function coherently as the varying quantity y that is somehow determined by x. Further, in accordance to Hino's suggestion, we decided to anthropomorphize a function as a Japanese "Ninja" and to conceive of an algebraic expression, a numerical table, and a graph as shadows of a "Ninja" who gave a glimpse of its existence. The traditional image of Ninja is associated with stealth and invisibility. We expected the existence of Ninja and its images to help students to reify function as object and to conceive of its representations as shadows of the object.

In the lesson, the teacher and the students presupposed that there are many kinds of function that are named after Ninja: Ninja of 'direct proportion', 'inverse', and 'linear function' and so forth. Every Ninja will have a particular move; different Ninja will move differently. Though a function itself is stealthy and invisible, we can detect properties of change of the function and discern two linear functions from a glimpse of shadows.

In the course of lesson, we expect the students to have some sense of the virtual existence of functions. Further, we expect the students to say something like: 'the inverse proportion Ninja moves at different paces. Indeed, the Ninja moves much slower when far from the origin of coordinate plane.'; 'this linear function Ninja moves faster than this linear function Ninja.'

Observing Quantity Changing in Dynamic ICT Environment: GeoGebra Applets

We designed several kinds of GeoGebra applets for teacher's demonstrations and students' observations. For example, in order to find an intersection point of two straight lines that represent respective linear function, we designed an applet in which two Ninja move on the y-axis at different paces, at the same time, two corresponding cumulative shadows appear on the coordinate plane (see

Figure 1). With this applet, ‘two Ninjas exchange secret messages’, we ask the students to find out the meeting point and time in the coordinate system by applying properties of graphs of linear functions. The students are expected to make the distinction between a function and its graphic representation easily. Hence, the applet supports the student to find the coordinates using the idea of ‘rate of change’ instead of the algebraic equation, thus retaining a consistent discourse of function as co-varying quantities.

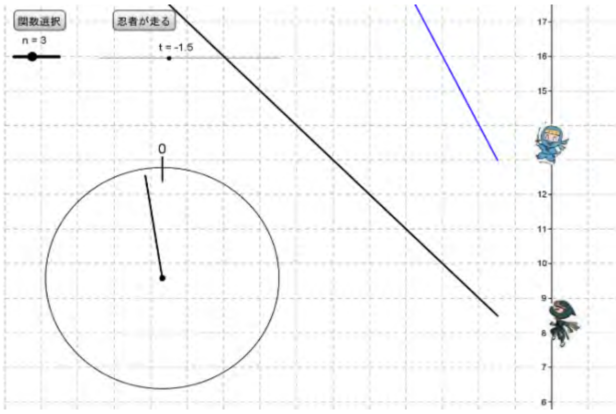

Figure 2. Two Ninjas Exchange Secret Message

Introduction of Discourse on Function as Algebraic Approach: “Ninjutsu”

In addition to finding the intersection of two straight lines, there are tasks that ask for finding algebraic expression of a linear function given the coordinates of one point and gradient of graph, or two points. We extended the quantity approach to these tasks, which ask students to draw graphs and apply the property of rate (or pace) of change to find the parameters *an* and *b* in $y=ax+b$. Then, we asked students for a solution without graphing. Hence, we introduced discourse on function as an algebraic approach associated with the Ninja instruction: ‘Find answer using ‘Calculation Ninjutsu’’. By ‘Ninjutsu’ we mean a specific magical strategy used by Ninja. This algebraic approach is effective when the coordinates of intersection point of two straight lines consists of rational values. We recommended that students 'master Ninjutsu' through apprenticeship.

Members of this research team decided the baseline of the unit plan together. Kanno developed worksheets and Nunokawa the GeoGebra applets. The team discussed again the balance in the use of applets between teacher’s demonstrations and students’ hands-on observations.

Design Experiment and Results

The teaching experiment on linear function was implemented during July 3 to October 20 2014 by Ms. Kanno with whom we have collaborated over 17 years. She taught all the lessons for eleven lower achieving students. Every lesson, three times a week, was video recorded for later analysis. The private utterances and written work of four students were video recorded by micro camera fixed on their desk. Written documents are also collected and scanned.

An Illustration of Classroom Discourse that Fosters Reification of Mathematical Objects

Preliminary analysis of the entire corpus of data generates a provisional typography of discourse, characterized by level (higher/lower) and referent (object/non-object).

As preliminary results, we found discourse that refers to a function itself is apparent in the second sub-unit where function was personified as Ninja and its representations are characterized as shadows. We conceived the discourse that affects objectification of function to be at the higher level and referring to specific characteristics of co-varying quantities such as 'rate of change'. Such aspects of discourse were discerned in sub-unit two. In the following, we illustrate this from days 5 and 9.

Day 5: students compare 'rates of change' of linear function and inverse proportion. The lower discourse level was dominant for the first part of the lesson when the students calculated rate of change with given numerical data. Higher and object-reference discourse occurred while the students observed a GeoGebra applet which shows rate of change dynamically. They observed that for inverse proportion 'pace varies', 'at first, the pace suddenly declines', 'rate of decline goes gradually smaller' and so on. These utterances refer to specific characteristics of varying quantity and thus are categorized as object–oriented discourse on function.

Day 9: teacher demonstrated moving Ninjas along y-axis and the corresponding graphs. Students themselves operated this with tablet PC. We could not detect higher and object-referred discourse in the whole classroom episodes. However, one of the selected students (TM) described a characteristic of co-varying quantities on his work sheet (See Figure 2). TM invented an idea of quantity changing of two Ninja in terms of 'degree and speed'. The students were asked to 'write down your method of distinction between the shadow graph 3 and 4' (these are represented in $y=2x+2$ and $y=x+2$ respectively). TM wrote that '④ about 80°and the speed is fast.', '③ about 60°and compared with 4, the speed is slow.'. Since the idea of speed has nothing to do with the graph, we can conclude that TM is describing changing quantity.

Discussion

Through setting appropriate contexts and using dynamic and interactive representations, we directed students' attention to features of changes of variables and succeeded in promoting discussions whose main topics were those features. Since changes of variables are critical features of functions, it can be said that our unit could construct a discourse where functions were the objects of talk, even for these low attaining students.

However, our collaborative team experienced unexpected student's activity. First, as illustrated above student TM constructed idiosyncratic meaning to Ninja movements. We could not expect to which aspect the students pay attention to GeoGebra applets. Every team member could have learned from student sense-making activity and negotiation of meaning with students is essential. Second, the teacher sometimes experienced difficulty to consciously keep consistent discourse while indicating GeoGebra applets. We observe that "double stimulation" (Vygotskii, 1984) hindered teachers discourse.

We had not asked the teacher to intentionally direct the students' attention to changes of variables in the latter part of the unit, because we expected this focus to be maintained. Contrary to our expectation, the class did not mention changes of variables and examined only surface relationships between those expressions and graphs. This suggests that the intentional construction of the discourse through explicit mention of changes of variables should have been maintained even in the latter part of the unit. Nachlieli & Tabach (2011) claim: "students should have probably spent more time getting acquainted with the three lower-level discourses before the subsuming discourse on function was introduced" (p. 25). Our research provides an alternative route: highlighting changes of variables and encouraging students to talk about them. In reality, it is hard for the teacher, ideally, continuation of this idea throughout the whole unit may improve effectiveness.

Concluding Remark

In order to develop resources *for* and *from* productive collaboration across various communities, a wise strategy will be to establish a transparent context between researcher and practitioner. Krainer's (2006) notion of "stakeholder approach" might avoid privileging theory over practice in educational design. It is crucial for developing a stakeholder approach for collaboration among diverse communities so as to facilitate mathematical learning with rich resources. Researchers are responsible for taking a step to get theoretical knowledge (eg. commogniton and reification) known, used and reflected upon by teachers and that researchers should highlight teachers' Pedagogical Content Knowledge and collaboratively design practical resources (eg. classroom discourse with dynamic ICT environments) that encourage teachers to get interested in research. In view of Activity Theory, the design team is to some extent characterized by flexible networks or '*knotworking*' in which "collaboration between the partners is of vital importance, yet it takes shape without rigid, predetermined rules or a fixed central authority" (Engeström, 2008: 20). Creating flexible network that enables developing resources is of crucial importance in Theme D.

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STUDYING PRIMARY AND SECONDARY TEACHERS' COLLABORATIVE DESIGN OF RESOURCES FOR ALGEBRA

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This paper reports research on primary and secondary teachers' collaborative design of resources aiming at fostering students' algebraic thinking. The study took place in the context of a professional development program implemented in France and Greece aiming to explore the issue of algebra teaching and learning in primary and secondary mathematics classrooms. Analysis of selected examples of interactions at the boundaries of the two educational levels highlights the evolution of teachers' collective documentation work as well as their professional learning.

Introduction

The reported study took place in the context of the PREMaTT[1] project targeting the development of algebraic thinking in primary and secondary mathematics classrooms through the collaboration of teachers and researchers. The project focused on the processes of designing and sharing digital and non-digital resources as new forms of teachers' professional development (PD) towards better understanding of what algebra is and what kinds of tasks foster its development in primary and secondary grades. This resource approach to teacher collaboration raises issues such as how teachers from both educational levels conceptualize what algebra is, how these conceptualizations are brought to the fore through interactions between sets of resources coming from different collectives and how they are imprinted in the designs resulting from teachers' collaboration (Gueudet & Trouche, 2012). Our focus is on the process of collective design and implementation of resources by two communities of researchers, primary teachers and secondary mathematics teachers from different schools established in the context of project implementation in France and Greece.

Over the years, teaching and learning of algebra has been figured as a prominent research area in mathematics education. Considering arithmetic as a prerequisite for algebra, algebra has been traditionally taught in secondary school. Thus, algebra has been excluded, until recently, from many primary mathematics curriculums around the world. In light of students' difficulties at the secondary level and the need to address them, it has beed suggested a progressive introduction to algebra in primary grades (Carraher & Schliemann, 2007). Thus a need has arisen to characterize the nature of algebraic thinking and consider tasks in which younger students could be engaged. Radford (2014) suggests three features of algebraic thinking: (1) indeterminacy: the existence of unknown quantities (e.g., variables, parameters); (2) denotation: the need to name and symbolize

1 http://ife.ens-lyon.fr/ife/recherche/groupes-de-travail/prematt

these indeterminate quantities in different ways (with algebraic symbolism, alphanumeric signs, natural language, gestures, or a mixture of these); and (3) analyticity: the manipulation of indeterminate quantities (e.g., addition, multiplication) as if they were known. Therefore, algebraic symbolism is neither a necessary nor a sufficient condition for algebraic thinking. At the same time, growing research on early algebra has recently raised issues such as: characterization of "early algebraic thinking" as there is no clear-cut break between early algebra and algebra; forms of curricular activity that support early algebraic thinking; nature of PD programs that support teachers' capacity to foster early algebraic thinking; and use of digital tools in the teaching and learning of early algebra (Kieran et al., 2016). Existing research focuses mainly on early algebra at primary level (e.g., tasks, teaching approaches, students' difficulties) as well as on the introduction of algebra at secondary level. However, the transition from arithmetic and early algebra to algebra within and between the two levels remains problematic as (a) the teachers at one level have limited knowledge of the issues related to the other and (b) PD programs struggle to engage teachers from the two levels. Therefore, how primary and secondary teachers collaborate in PD programs to develop resources aiming to develop students' algebraic thinking is an area that requires further research. In our study, we use the boundary crossing approach (Akkerman & Bakker, 2011) to explore primary and secondary teachers' collaborative work to develop resources fostering the development of students' algebraic thinking, as well as their professional learning.

Theoretical Framework

We use the documentational approach to didactics (Gueudet & Trouche, 2009) to study teachers' PD, focusing on choice, appropriation and transformation of resources, either individually or collectively by groups of teachers. The term resource describes a variety of artifacts: textbook, software, discussions with colleagues etc. Teachers modify existing resources to adapt them to their pactices/contexts, giving birth to documents that can further evolve over time. This process integrates practice and knowledge and combines elements of stability and evolutions (adapting to new context, new curriculum…). The process of gathering, creating and sharing resources within a community, called community documentational genesis, results in a community documentation composed of "shared repertoire of resources and shared associated knowledge (what teachers learn from conceiving, implementing, discussing resources)" (Gueudet & Trouche, 2012, p. 309).

Since our study involved teachers from both primary and secondary education, between which a clear boundary exists, we use also the boundary crossing approach (Akkerman & Bakker, 2011). A boundary is defined as "a socio-cultural difference leading to discontinuity in action or interaction" (ibid., p. 133). Boundary crossing refers to an individual's transitions/interactions across sites in professional situations where participants may need to enter onto unfamiliar territories/contexts, by negotiating and combining their ingredients so as to create hybrid situations (Suchman, 1994). Boundary objects are artifacts facilitating crossing of boundaries, fulfilling a bridging function (Star, 2010). Situations requiring crossing of boundaries can generate learning through four mechanisms: (a) *identification* of a boundary, entailing "a questioning of the core identity of each of the intersecting sites, that leads to renewed insight into what the diverse practices concern" (Akkerman & Bakker, 2011, p. 142). This can be achieved by a dialogical subprocess of "othering", i.e., defining one practice in light of another and highlithging differences between them; (b) *coordination* of activity flow: partners find means/procedures allowing diverse practices to cooperate efficiently, dialogue between partners is established only as far as necessary to maintain the work flow; (c) *reflection* on the specificities of two sites and the existence of a boundary between them involving processes of perspective making (i.e., redefining one's own perspective in relation to the other) and perspective taking (i.e., taking a new look at one's own perspective through the other's eyes); (d) *transformation*: actors from different sites engage in constructive activity by crossing boundaries and addressing boundary objects. Transformation processes start with a confrontation/problem "that forces the intersecting worlds to reconsider their current practices and the interrelations" (ibid., p. 146) often resulting in recognizing a shared problem space. This, in turn, engages participants in the process of hybridization by combining ingredients from different contexts into something new and unfamiliar. Our research question thus is: How does the teachers' collaborative work at the boundary between primary and secondary education (1) influence their documentational work targeting the introduction to algebra, and (2) contribute to the development of a shared view of what algebraic thinking is through the design and use of digital and non-digital resources?

Methodology

Our methodology refers to design-based research (Cobb et al., 2003), which is a collaborative and iterative approach to the design (in our case of educational resources) conducted in ecological conditions (close to teachers' practices). The collaboration between and with the teachers is at the heart of our research. In the Greek context, the collaboration consisted of PD meetings and classroom implementations including (a) initial discussions on what algebra is, how it is introduced in the curriculum and textbooks (with respective examples) and ideas for developing classroom tasks, (b) design of digital and non-digital resources individually or in subgroups, presentation and discussion of the designs, (c) classroom implementation, and (d) reflection and possible adaptations. In the French context, the collaboration took two different forms. At the level of schools (primary and secondary) involved in the project, groups of teachers accompanied by one or more researchers, called 'factories', worked on the design of classroom activities, their implementation and re-design. At the project level, specific sessions were organized, managed by a pedagogical engineer mobilizing agile methodologies fostering collaboration. During these sessions, all members brought their expertise to the common endeavour of designing resources that respond to teachers' needs. At this level, more general issues of interest for all factories were discussed, such as how to orchestrate classroom activities, what kinds of strategies can be expected from students at various school levels, or what algebra is. The collected data in both contexts are: designed resources (lesson plans, worksheets, digital files); video records/transcripts of PD meetings or collaborative work sessions; teachers' e-mail messages; teachers' interviews; and videos from classroom implementations.

In the following section, we analyze episodes selected from two different contexts - Greek and French - as representative of interactions between primary and secondary teachers occurring during collaborative task design and community meetings. Our analysis attempts to identify the evolution of the collective documentation and utterances indicating the activation of learning mechanisms.

Analysis of Two Cases

The Greek Case

Context. The Greek PREMaTT community involves a group of 7 secondary, 2 primary teachers working in different public schools and 4 researchers. Both primary and secondary teachers were experienced and the majority of them had been involved in several research projects and PD programs. Although algebraic thinking is targeted in the primary mathematics curriculum through specific tasks (e.g., patterns in the last grade, 12 year-old pupils), most of the primary teachers consider algebra as an 'unknown' terrain belonging exclusively to secondary education. This view, held also by secondary teachers, is reinforced by the fact that primary (6-12 years old, 1st-6th grades) and secondary (13-18 years old, 7th-12th grades) school levels constitute two distinct cycles and there are not official initiatives promoting their connection (e.g., through PD programs). In this paper, we focus on a subgroup of two qualified teachers: Anna (PhD in teaching mathematics with digital tools, primary) and Tom (Master degree in mathematics education, secondary) who collaborated in the design and implementation of one task involving the use of eXpresser (Noss et al., 2009), a specially designed microworld that had been presented in the PD meetings. eXpresser allows students to create figural patterns through repeated building blocks of square tiles, to use (iconic) variables to reproduce their constructions for different number of repetitions, to express generalization and to check their correctness through appropriate feedback.

A case of reflection and transformation (Anna and Tom). In the initial group discussions, identification and coordination constituted the dominant mechanisms. Teachers exchanged views as to how algebra appears in different grades of primary or in the fisrt grade of secondary education. The presentation of concrete examples from curriculums and textbooks triggered the emergence of coordination across the two sites. Reflection (mainly perspective making) and transformation processes were brough to the fore as the teachers were progressively engaged in developing their didactical designs through analyzing, justifying and adapting their choices. The case we analyse below highlights such processes.

Perspective making and taking around curricular and digital resources. During the first PD meeting, Anna presented the 'flower bed' task from the primary school textbooks (6th grade) in which students have to design the next instance of a geometric pattern representing a flower bed (the green part) framed by square yellow tiles and fill in a table (Fig. 1). Inspired by Anna's presentation, Tom chose this task (perspective taking) and adapted it for the 7th grade (first grade of secondary education). By making explicit the rationale of his choices (perspective making), Tom presented his first version of the task in which students were asked to: (a) calculate the number of the blue square tiles for instances with 1, 2, 3, 10, 100 red tiles (Fig. 2); (b) find out how many tiles would be red and how many would be blue if the pattern was created with 150 tiles in total; and (c) answer the previous questions by creating the pattern in eXpresser. It seems that Tom's approach for introducing algebraic notions is decontextualized and rather abstract while it is not clear how students are supported to build bridges between early algebraic thinking and algebraic generalization. The use of the digital tool only in the final task indicates that it is exploited to support students in expressing the generalization formally using the eXpresser's notation (Fig. 2). This approach is consistent with typical teaching approaches followed by the majority of the Greek secondary mathematics teachers.

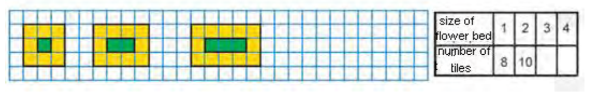


Figure 1: The 'flower bed' task

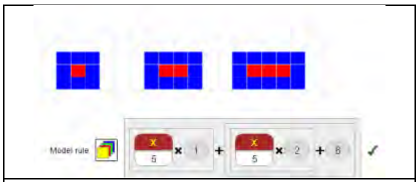


Figure 2: Tom's initial design

Transformation: Confrontation of task design as a space of negotiation. The design of Tom operated as a boundary object that helped him transform his documentational work taking into account Anna's perspective and crossing the boundaries between primary and secondary education. Reflecting on Tom's design, Anna brought to the fore aspects of her knowledge and her assumptions concerning early algebraic thinking by proposing to him to: (a) contextualise his tasks engaging students in constructing kinesthetically the first instances of the pattern with the use of eXpresser; (b) promote a more recursive view of the pattern – at least in the first instances - that would scaffold students' attempts to identify regularities and work with functional situations; (c) give students the chance to express mathematical concepts and relations verbally in everyday language before moving to symbolisation and abstraction. Anna's constructionist perspective towards the use of digital tools became explicit by her suggestion to Tom to develop together a pattern in eXpresser that would be given to students to build on it, change it or decompose it. This way students would not start working on eXpresser from scratch and it would be easier for them to focus on the pattern's properties. The digital tool here operates as a boundary object forcing the two teachers to reconsider their practices.

In parallel, Anna and Tom discussed how to use tables of values in worksheets to guide students' exploration and help them discern the pattern. Tom had initially thought of using tables to guide students' exploration and help them investigate relations among sets of numbers. By reflecting with Anna on the role of table in students' solutions/approaches, he finally decided to exclude it so as to preserve the openness of the task. It seems that in the process of recognizing a shared problem space around the use of tables the dialogical mechanisms of perspective taking and making were also present. Reflecting around boundary objects such as the use of digital resources and the integration of tables in the task, they both came to know what the diverse practices are, they recognized shared problem spaces while boundaries were encountered and contested. This is more evident in Tom's redesigned activities.

Hybridization through collective documentation. Tom redesigned the task by attempting to bridge the different perspectives. Specifically, he: (a) used a realistic frame according to which the students had to help a mayor to colour the pavements of a town; (b) introduced the whole activity through the use of eXpresser by engaging students in constructing specific instances of the pattern; (c) asked students to generalise through the construction of the general rule and the use of algebraic notation in eXpresser. This way the digital tool is meant to be used by students both as an experimentation space and a means of generalization. Tom's new worksheet comprises only two questions: (a) How many blue square tiles are needed in a pavement with 500 red square tiles? and (b) If 1557 square tiles of both colours are used in total in a pavement, how many tiles are blue and how many tiles are red? In both questions students had to explain their line of thought.

The collaborative task design by Anna and Tom brought to the fore various boundaries such as: a more constructive/geometric vs a more instructive/arithmetic approach to patterns; a step-by-step approach of generalization vs a formal approach towards the general rule. Although these discontinuities are still present, the evolution of Tom's design indicates a process of community documentation. This process led to a shared body of resources and shared associated knowledge at the boundaries of primary and secondary education related to how algebraic thinking can be approached at each educational level. Anna's and Tom's professional knowledge influenced the design of activities and at the same time their collective documentation work operated as a boundary object facilitating the extension of their existed knowledge.

The French Case

Context. The French PREMaTT community involves a group of secondary mathematics teachers from three different schools, groups of teachers from three primary schools, several researchers and a pedagogical engineer. The project took place in a period of a curricular reform that resulted in establishing four cycles of schooling. Cycle 3, comprising grades 4-6 (last 2 years of primary and first year of secondary school, 9-11 years-old), creates a 'bridge' between primary and secondary education and this transition poses a new challenge for teachers from the two levels.

The group of four secondary teachers who formed the SESAMES factory (S2 in the sequel), is involved for several years in research on teaching algebra in Cycle 4 (last 3 years of lower secondary school, 12-14 year-old). It has produced a number of resources shared on a platform and is involved in PD of mathematics teachers. S2 joined the project to reflect on the transition within the Cycle 3 and between Cycle 3 and Cycle 4 in relation with (early) algebraic thinking. Lamartine factory (L1 in the sequel) comprised two experienced primary (Grade 5) teachers, one having a university degree in mathematics. The analysis reported below focuses on S2 and L1.

Adaptation of an existing resource: coordination, reflection and transformation. From the outset, S2 presented some of their resources designed for teaching algebra in Cycle 4. One of the resources (Fig. 3) was chosen for adaptation to different school levels, from Grade 4 to 9.

With matchsticks, I construct triangles as in the figure. 	How many matchsticks are needed: at stage 1 (to construt 1 triangle)? At stage 2? At stage 5? At stage 10? At stage 100? At stage 265? Find a way to calculate the number of matchsticks needed depending on the number of triangles constructed.

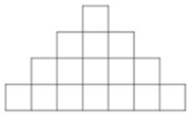
Figure 3: Resource offered by S2 chosen for the first design and implementation

This task was implemented by all factories (coordination). Sharing experiences of these first implementations sparkled a reflection about patterns: what is/what is not a pattern, and consequently what is/what is not related to algebra. Patterns thus acted as a boundary object that led teachers from both levels to reflect on cognitive processes involved in solving pattern tasks and how they connect to algebraic thinking. Questions were raised about stages of algebraic thinking development that students at different shool levels can reach, delineating a shared problem space recognized collectively (transformation): when are they able to create formulas? How pattern generalization manifests itself in primary pupils not yet introduced to algebraic symbolism?

Design of a new resource: coordination, identification and reflection mechanisms. Reflecting on patterns led the teachers to suggest other examples. The patterns considered were often of the order of recurrence. To avoid installing a didactical contract following which the pupils are excpected to manipulate symbols, a new problem was brought by a teacher from S2: a pattern of 'pyramids'. A collaborative session was organized to study the potential of this problem (coordination). Whereas L1 found the problem relevant with respect to the generalization, S2 was not convinced as they anticipated the difficulty to reach the formula, which is n^2 (n being the number of cubes in the middle column), that requires deconstructing and reconstructing the figure. Other possible solutions are 1+3+5+…+(2n-1) when considering stages of the pyramid, or 1+2+…+(n-1)+n+(n-1)+…+1 when considering its columns; these solutions are complex as they require establishing a link between the rank of the pyramid and the number of cubes in the middle column. This session was a key moment of the project: from a same problem, the opposite views about it led to the design of two different activities by L1 and S2 respectively (Fig. 4).

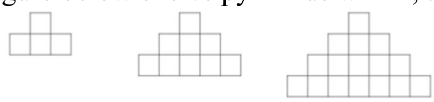
Figure 1 Figure 2 Figure 3

How many cubes are there in figure 4? Explain.



Phase 1. Find different ways how to calculate the number of squares needed to construct this pyramid.

Phase 2. The figure below shows pyramids with 2, 3, and 4 stages.



1. Calculate the number of squares for pyramids with 2, 3, 5 stages.
2. How many squares for a pyramid with 10 stages?
3. How many squares for a pyramid with 100 stages?

Figure 4: Different tasks designed by L1 (top) and S2 (below) from a same problem

Instead of looking for commonalities in the sequence of pyramids, S2 decided to start by focusing on the structure of the pyramid with four stages and search for differents ways of calculating how many squares it comprises (phase 1). The goal was to let students (Grades 6-8) to manipulate, modify the structure of the pyramid, and accept that the new figure has no relation to the initial problem anymore. The emphasis on the calculation (rather than counting) led to a more focused work on numbers and their properties. Pattern generalization came in phase 2 with a question about the number of squares in a pyramid with a quite big number of stages (100) so that the students are not able to represent or draw the pyramid and count the squares but are forced to reason. L1 rather chose to progressively guide younger students (Grades 4-5) toward generalization by working with pyramids having smaller number of stages (1, 2, 3, 4 and for the 'quick' ones 6 and 10). These designs brought to the fore various boundaries (identification): different routes toward algebraic thinking - through figure construction and pattern generalization (L1) vs figure deconstruction and working on the structure of the figure (S2); different stance to the level of difficulty in problem solving – choice of variables to guide pupils toward the solution and avoid failure (L1) vs choice of variables to let pupils face difficulty leading to the evolution of strategies (S2). Identification of such boundaries was critical in reflecting on the essence of algebraic thinking.

Conceptualization of algebraic thinking. At the end of almost two years of collaboration, a work session was organized aiming at coming back to the project central issue: what agebraic thinking (embracing early algebra) is. Based on experiences of all factories, groups mixing researchers and primary and secondary teachers were invited to express and share their ideas (perspective making). These discussions revealed the necessity to clarify and agree upon meanings of words such as modelling, generalizing or pattern (identification of boundaries and coordination). Only then the community was able to come up with two directions leading toward algebraic thinking: (1) pattern generalization (independent of symbolic writing) requiring modelling the structure of figures and allowing prediction, and (2) working on numbers: structure, equivalence, relations, and properties.

Conclusion

In this paper we focused on the boundaries and learning mechanisms activated in the context of primary and secondary teachers' community documentation targeting students' algebraic thinking. We studied two distinct contexts: French, where a recent curricular reform established a cycle bridging primary and secondary levels; and Greek, where a clear cut between the two educational levels exists. In the Greek case, the two qualified teachers were 'ready' to work with digital resources. The analysis brough to the fore various boundaries (e.g., step-by-step vs formal approach to generalization) but indicated the potential of collaboration between teachers of the two levels: the collective documentation operated as a boundary object favoring (a) the recognition of a shared problem space related to how algebra can be approached by each one of them and (b) the development of hybrid documents resulting from teachers' collaboration that combined views/ingredients from both educational levels. In the French case, collaborative design of resources revealed boundaries in perception of algebra (i.e., primary teachers were novices whereas secondary teachers, although having already been teaching algebra, had limited insight on processes of algebraic thinking). Collaboration among the teachers raised the need to make these perceptions explicit and agree upon common definitions of key terms like modelling, generalizing or patterns. These terms acted as boundary objects enabling negociation of meaning and resulted in both the design of classroom activites for Grades 4-8 and a shared view of algebraic thinking and the kinds of tasks fostering its development since primary grades. In both cases, resources from primary and secondary teachers' collaboration appeared as a lever for their professional development. This design was orchestrated and monitored by researchers. Their role and effects on the reported outcomes will be analyzed at the next stage.

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Supporting Teachers in the Development of Noticing Children's Mathematical Thinking with a Web-Based Tool

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We introduce a web-based resource, the Collaborative Inquiry Tool, designed to support teachers in organizing self-facilitated learning sessions as a supplement to their participation in formal professional development focused on children's mathematical thinking. The broad goal is to provide teachers with a bridge between professional development and their classrooms by encouraging structured conversations with colleagues regarding the mathematical thinking of their students. We illustrate the Collaborative Inquiry Tool by describing the conversation for one session in which teachers (a) collectively engaged with an exemplar case in the tool, (b) worked together to articulate the thinking of their own students in written work for a story problem linked to the case, and (c) decided next instructional steps based on their students' understanding. We conclude with a brief discussion of how the tool supported teachers working together and potential next steps for the tool.

For this paper we present a web-based tool, the Collaborative Inquiry Tool (Empson, Jacobs, & Pynes, 2016), created to support upper-elementary teachers in self-facilitated collaborative-inquiry in developing expertise in the professional noticing of children's mathematical thinking. We illustrate how teachers are envisioned to use the tool to engage in collective noticing (Pynes, 2018) by exploring one teacher group's engagement in a single session. To engage in collective noticing teachers meet to examine and discuss student work for a common story problem they each posed to their own students. Professional noticing of children's mathematical thinking refers to the practice of, almost simultaneously, making sense of children's mathematical thinking during instruction and deciding how to respond to that thinking (Jacobs, Lamb, & Philipp, 2010). Therefore, this practice is foundational to teaching in ways that are responsive to children's mathematical thinking, or instruction that builds on children's understanding. While noticing children's mathematical thinking is fundamental to teaching that is responsive to students, researchers have documented that even though not all teachers demonstrate expertise in noticing (Dreher & Kuntze, 2015; Jacobs et al., 2010), their capacity to notice can be supported and developed (Goldsmith & Seago, 2011; Jacobs et al., 2010; van Es & Sherin, 2008). However, research also suggests that the practice of noticing children's mathematical thinking is a learnable, but complex skill that takes time — often years — to develop (Jacobs et al., 2010). Supporting the development of this practice, both inside and outside of the classroom, is an important goal when supporting teachers in learning how to teach in ways that are responsive to children's mathematical thinking.

Professional noticing is a generative teaching practice in that teachers have opportunities to continually refine both what and how they notice the mathematical thinking of their students on the basis of their own noticing, inside and outside of the classroom. When teachers meet together outside of classroom teaching to discuss students' written work, they have an opportunity to reflect on their noticing by making explicit what they notice about a student's mathematical thinking, as represented by written work, not only to their partner teacher, but also to themselves. In addition, when multiple teachers discuss the same piece of student work each teacher has an opportunity to voice her or his perspective of the student work and consider a range of interpretations based on the mathematical details which could allow for richer professional noticing or discussions. Participation in collective noticing affords teachers an opportunity to develop their own practice of noticing.

Purpose of the Collaborative Inquiry Tool

The Collaborative Inquiry Tool is an online tool designed to support teachers in school-based teams to develop their expertise in noticing children's mathematical thinking. The initial design was to supplement a three-year face-to-face workshop developed for the NSF-funded project Responsive Teaching in Elementary Mathematics (RTEM). The Collaborative Inquiry Tool was designed to support teachers in connecting the conceptual frameworks of teaching and learning introduced during sessions to the instructional decisions they make in classroom. The tool reinforced research-based frameworks on children's mathematical thinking of rational numbers (Empson & Levi, 2011) and developed teachers' capacity to notice children's mathematical thinking (Jacobs & Ambrose, 2008; Jacobs et al., 2011) that were introduced during professional development sessions.

During the professional development, teachers analyzed student thinking through the use of videos and written work from children selected by the professional development facilitators, analyzed written work from their own students, and interviewed students from the surrounding school districts. During these activities teachers were introduced to 1) research-based frameworks of children's mathematical thinking of rational numbers and problem types to support this development (Carpenter et al, 2014; Empson & Levi, 2011) and 2) questioning moves to support and extend children's mathematical thinking (Jacobs & Ambrose, 2008; Jacobs & Empson, 2015).

Expectations for Collaborative Inquiry Sessions

Teachers were asked to attend the professional development with at least one additional colleague in order to continue conversations related to professional development sessions on their campus. Twice a semester, over a three-year period, teachers were asked to meet with their professional development colleague(s) to participate in the recommended Collaborative Inquiry Module. To facilitate the discussion, teachers were asked to pose a common story problem to their students, provided by the module, and schedule 45 to 60 minutes for the Collaborative Inquiry session. For the illustrated session, the teacher group met for about 50 minutes and engaged in each segment of a Collaborative Inquiry session.

Structure of a Collaborative Inquiry Session

The structure of a session is illustrated with one teacher group from Tiger Swallowtail Elementary. The collaborative inquiry group was made up of one fourth-grade and two third-grade teachers all participating in their first year of professional development. The teachers engaged in their third module during the Spring semester (between January and May) after they had completed the first year of professional development (8.5 days). The teachers from Tiger Swallowtail Elementary agreed to audio record all 12 of their sessions and send images of the student work discussed.

The tool currently consists of 12 collaborative inquiry sessions based on research on how children think about and solve problems. Each session was designed to engage teachers in face-to-face focused inquiry regarding children's thinking of key mathematical relationships through four main segments.

These four segments included Prepare, Video or Written Work, Discuss Own Students, and Next Steps.

Segment One: Prepare

In order to prepare for the collaborative inquiry session, teachers are asked to log in to the tool for the purposes of downloading a problem that was written to reflect the focus of the module. For example, the focus of Module 3 was for teachers to view and discuss early, or emergent, strategies for solving equal sharing problems. An equal sharing problem is a type of partitive division story problem that results in a fractional amount and is the first fraction story problem teachers are encouraged to pose to their students.

Teachers can download the problem to copy for their students as either a portable document file (PDF) or a word processing file. The word processing file allows teachers the opportunity to adjust either the problem context or number selection in a way that is responsive to their students. For example, one problem includes the context of children equally sharing churros and the teachers from Tiger Swallowtail Elementary decided churros would be unfamiliar to their students, so they decided to change the problem context to children sharing apples. Other teachers chose to remove additional number sets from the printed problem.

For Module 3, teachers are asked to pose the following problem: *There are ___ pancakes for ___ kids to share equally. How much pancake does each kid get?* For this module, teachers are given a choice of two number sets, 4 share 11 and 8 share 5. Additional text informs the teachers that during the session they will have an opportunity to watch a video of a child solving a similar problem with 8 share 5.

The Prepare tab provides some suggestions for teachers to enact while their students solved the problem. For example, it is usually suggested that teachers unpack the problem, or introduce the problem context to the students and ensure the students understood what the problem was asking. In addition, the tool suggests teachers walk around and pose questions to students during the problem-solving task. In Module 3, the tool suggests that teachers ask students how they decided to partition the pancakes. Lastly, after the students solve the problem, the teachers are prompted to choose six pieces of student work to discuss with their colleagues.

Segment Two: Video or written work

To begin the session teachers are presented with a video of a student or written work from several students who had solved the same problem that the teachers posed to their students. The teachers are asked to view and discuss what they noticed about the mathematical thinking of the module's focal student or students. The video is chosen by the team to highlight particular problem types and features of children's mathematical thinking.

In Module 3, the teachers from Tiger Swallowtail Elementary watched a fifth grader named Ryan solve the equal sharing problem 8 share 5 pizzas. In this video, Ryan solved the problem and a teacher asked questions to elicit Ryan's thinking about the problem (see Figure 1 for an image of how Ryan solved problem. Ryan initially solved the problem as if it was 4 children sharing 5 pizzas. Through the interaction, Ryan solved the problem by partitioning five circles into eighths and finding an answer of the 5/8. After the teacher posed questions about Ryan's strategy, she asked a follow-up problem to elicit how Ryan understood 5/8 as a quantity in comparison to both one whole and 1/2. After a long wait time, when it seemed that Ryan was not going to be able to answer the problem, Ryan stated that 5/8 was 1/8 more than 1/2. To see a video of this interaction, visit https://soe.uncg.edu/rtem/.

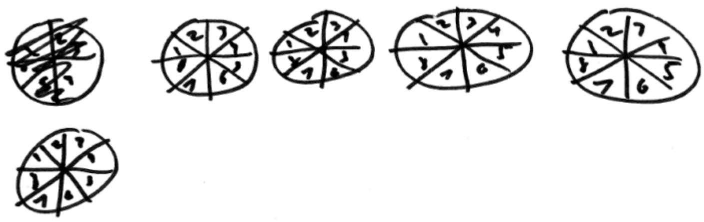

Figure 1. Image of Ryan's Strategy for 5 kids share 8 pizzas

After the video concluded, the teachers from Tiger Swallowtail Elementary discussed the details of Ryan's strategy and what they thought he understood about solving an equal sharing problem. The teachers noted Ryan's valid strategy of partitioning each pizza into eighths, however they were curious about Ryan's initial strategy until one teacher stated, "I think he was thinking four [children] instead of eight. When the teachers began to again describe Ryan's valid strategy, they noted that Ryan was not ready to provide a final answer until "he saw every piece of the puzzle," or had to directly model the problem situation.

The teachers also observed how the video teacher's questions elicited and supported Ryan's thinking, allowing him to persist in problem solving. The teachers noted that Ryan's teacher challenged his thinking beyond "what he got on the paper" and through her questioning they observed that Ryan was able to compare 5/8 to a half, stating that he "understands his comparison." The teachers also noted that Ryan's teacher "held back and let him struggle," which "speaks a lot to giving kids time to think." The teachers stated they would be interested in seeing what other numbers Ryan would be able to work with, for example how would he divide some number of pizzas with 7 sharers, or if he would be able to combine 1/2 and 1/8 if he had initially divided the pizzas by a size other than 1/8.

Segment Three: Explore your Students' Work

After teachers discuss the focal student or students, they are asked to review the written work of their own students that they brought to the meeting. Teachers are prompted to discuss what they noticed about the mathematical thinking of at least one student from each of their classrooms, and if possible, choose student work that reflects a range of student thinking. Specifically, teachers are asked to describe each student's strategy in detail and discuss the potential understandings of the student as revealed by the strategy. To help facilitate this discussion, the module includes a written description of some things that the project team noticed about the focal student's mathematical thinking. These descriptions are meant to be illustrative and do not include all of the possible ideas that could have been noticed.

For example, in Module 3, the project team highlighted Ryan's emergent understanding of 5/8 as a sum of 1/2 and 1/8. Teachers were able to interpret Ryan's understanding through the use of the questions the teacher in the video posed to Ryan in order to elicit his understanding of five-eighths as greater or less than one-half, and a follow-up question of how much greater five-eighths was than one-half.

The teachers from Tiger Swallowtail Elementary discussed ten pieces of student work. Each teacher initially shared one piece of student work from their own classroom and by the end of the session Teacher 1 shared work from three students, Teacher 2 shared work from one student, and Teacher 3 shared work from six students. The following is an excerpt from their conversation around Brayden's strategy (Figure 2) for 4 kids share 11 pancakes from Teacher 3's classroom. Brayden used a direct modeling strategy and represented the four sharers as circles and the eleven pancakes as dots within a circle marked "pancakes." Brayden distributed the items, most likely one at a time, until he had three dots remaining. Brayden then redrew the three dots as larger circles, partitioning two into fourths and one into eighths, to share equally with the four children, for a final answer that could be symbolically represented as 2+1/2+2/8. Note that Brayden distributed two 1/4-sized pieces and represented this value as 1/2.

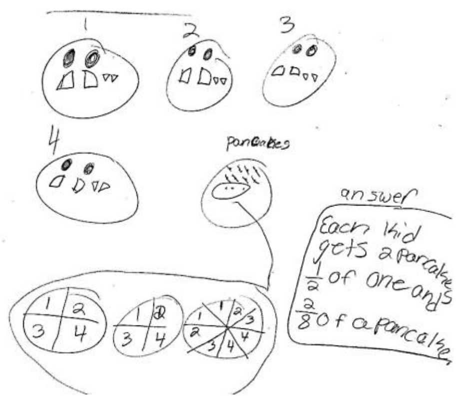

Figure 2: Brayden's strategy for 4 kids share 11 pancakes

T3: Brayden started out, and I was surprised by this because he's a pretty, he's pretty advanced too and his thinking was different on this. He started here and he drew the circle and put 11 dots and they were the pancakes. And I kind of-.

T1: He just wanted to see 'em.

T2: Yeah, pretty much all my kids had to draw out the kids and the pancakes. That was one thing I noticed being-. You know, they had to see everything.

T3: And then one two three four kids. And so then he put a dot here and he'd mark one out. Put a dot here and mark one out put a dot here mark one out put a dot here mark one out. Dot dot dot dot.

T1: Representing I passed out one pancake.

T3: And then he got to the three and he came down here and he drew those three pancakes.

T2: Oh yeah.

T3: And he started with fourths and so each one got a fourth a fourth a fourth a fourth, a fourth a fourth a fourth a fourth, then he split this one into <<laughs>> eighths. <<laughs>> 1-eighth 1-eighth 1-eighth and so on. So his answer is two pancakes, a half of one, and 2-eighths of a pancake.

T2: Okay.

T3: So I wrote that out on the board 2 plus 1-half plus 2-eighths

T2: Interesting. I wonder why he did that.

T3: I know.

T2: Why do you think he did that?

T3: I don't know. <<laughs>>

T1: Instead of just another fourth?

T2: Hmm, interesting.

During this episode, the teachers discussed all of the mathematical details in Brayden's strategy, stating how Brayden represented the context and solved the problem. While most of the details are described by Brayden's teacher, Teacher 3, the following description of their interaction shows how the partner teachers also engaged in noticing by elaborating on the details and asking questions about the student's process. This episode demonstrates how working with colleagues to engage in collective noticing can support a teacher in noticing the mathematical details of a student's strategy and consider the potential mathematical understandings the student has demonstrated by the details.

In particular, Teacher 3 stated that the dots in Brayden's strategy represented pancakes and the four circles represented the kids. Teacher 3 then shared how Brayden most likely marked one dot at a time in to each of the circles representing the kids, marking out each dot as it was distributed, and Teacher 1 added that this process represented Brayden passing out each pancake. Teacher 3 then moved to discuss the three large circles at the bottom of the strategy representing the three remaining pancakes. Her description suggested Brayden most likely partitioned the ninth pancake into fourths and distributed each fourth and repeated this step for the tenth pancake. Teacher 3 then noted Brayden partitioned the 11th pancake into eighths and distributed two-eighths to each kid for a total of two wholes, two fourths, and two eighths. Partitioning this last circle into eighths might be considered atypical, as the child had already partitioned the ninth and tenth pancake by the number of sharers, and this is recognized by both Teacher 1 and Teacher 2 asking why he partitioned the pancake into eighths rather than an additional fourth.

During this excerpt, Teacher 3 had an opportunity to verbalize the details of Brayden's strategy to Teacher 1 and Teacher 2. Teacher 3 connected the details of Brayden's strategy to the process Brayden most likely used, sharing what the shapes represented within the context of the story and in connection to the numerical quantities. When describing the strategy in this way, Teacher 3 had an opportunity to engage with Brayden's problem-solving process, or to think as one of her own students. The group's opportunity to notice is further demonstrated when a strategy detail that might not be considered typical was described. While Teacher 3 described how Brayden partitioned the last pancake, she laughed, which could indicate she thought it was unusual; however it was Teacher 2 who made this part of Brayden's reasoning more explicit through her questioning. To notice a detail like this suggests the teachers engaged with a student's informal strategy and recognized that students typically partition wholes while considering the number of sharers, making a connection between the researched-based frameworks for how children solve problems and how their own students solved the problem.

Segment Four: Next steps for own students

For the last segment of the module, teachers are asked to use what they learned about one student's mathematical thinking to design a follow-up problem that they could pose within the following week. To support teachers in this task, the project team provides sample follow-up problems with articulated reasoning based on what was noticed about the mathematical thinking of the module's focal student.

For Module 3, the project team noticed that Ryan's partitioning was not automatic and wondered how Ryan would partition if the number of sharers was an odd number, which matched the thinking of one of the teachers as they discussed their own students. When the teachers read the suggested problem, *5 children share 4 sandwiches*, the teachers began to anticipate Ryan's process. They wondered how he would model the context of the problem. Teacher 3 asked if Ryan would start by partitioning the sandwiches into halves while Teacher 2 suggested he might partition each into fifths because he solved the pizza problem this way. Teacher 3 disagreed and referred back to the text, stating "it was not automatic, we noticed he was able to partition into eighths but it was not automatic." The teachers then turned their attention to their own students and began to anticipate how their own students would solve the suggested problem. Teacher 3 stated that her students would not consider dividing one sandwich by the number of sharers and this was a strategy she had not seen prior to the professional development. Teacher 2 challenged this idea asking, "didn't some of your kids do that?," and found the work of two students who had divided by the number of sharers from the pile. This interaction demonstrates how teachers a) use the module example to consider the mathematical thinking of their own students, and b) how partner teachers can request evidence to support teachers' claims.

In this session, Teacher 3 initially considered a follow-up question she would pose to a student about the original pancake problem. The student had partitioned the 11 pancakes into fourths and answered that "each girl would get ¼ out of each [pancake]." Teacher 3 stated she would like to ask the student how she would name that quantity, if she would name it as 11/4, and she would also ask the student if the quantity is more or less than a whole pancake. This question may be related to Ryan's discussion of how 5/8 related to one whole.

While the module asks the teachers to create a new problem based on the understanding of one student, Teacher 1 and Teacher 2 considered more general instructional decisions without articulating the connection to the current mathematical understandings. Teacher 3 then reread the module prompt, and chose a student who responded "2 remainder 3," indicating the student only distributed wholes and had not partitioned the remaining pancakes. This selection prompted Teacher 1 to suggest a problem, such as 4 share 9, to provide a context that would leave only one whole remaining that students could then partition by the number of sharers. Teacher 1 thought this would be a good problem because her own students had not worked with dividing remainders. Teacher 2 agreed this would be a good next step for their students, and Teacher 1 reiterated a number set with one whole remaining would be foundational for students. As a collective, the teachers were able to use the potential mathematical understanding of one student to select a problem that could elicit or support the mathematical thinking of other students in their classes.

Discussion

The Collaborative Inquiry Tool is an example of a new resource developed to support teachers in discussing the mathematical thinking of their students with colleagues and consider instructional decisions that are built on articulated evidence. The tool supports teachers in the practice of noticing children's mathematical thinking by integrating the mathematical thinking of students that are not familiar to the teachers as well as artifacts from their own classrooms. Descriptions of children's mathematical thinking are provided as possible, but not exhaustive, examples and written by people with extensive experience. Teachers can consider these descriptions with their own perspectives in ways that may confirm or extend their own noticings. Teachers may also use these noticings to consider if their own students demonstrate similar mathematical understandings, similar to Teacher 3 wondering how her student would compare 11/4 to a whole, or if the details of a student's strategy suggest a different understanding. In addition, working with teachers in collective noticing allows for multiple perspectives around the same piece of student thinking to be shared, particularly when details may be confusing. Although the tool is currently 12 sessions, it provides a structure that could support teachers in sustaining this collaboration outside of tool. In addition, while this tool was developed as a supplement to a professional development, the project team is considering how to expand the tool to audiences outside of the professional development and potentially using other research-based frameworks on children's mathematical thinking.

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ICMI Study 25
TEACHERS OF MATHEMATICS WORKING AND LEARNING IN COLLABORATIVE GROUPS
Theme D
Lisbon, Portugal, 3-7 February 2020

EVOLUTION OF CRITERIA FOR REPRESENTATIONAL ADEQUACY FOR TEACHING INTEGERS THROUGH COLLABORATIVE INVESTIGATION

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Representations have been considered as an important tool for doing and teaching mathematics but there is paucity of research about the criteria used by teachers to select, use evaluate and generate representations. This paper discusses the criteria for representational adequacy that teachers evolved while engaging in collaborative investigation workshops. Analysis of the teachers' talk during the workshops indicated evolution of three distinct criteria, such as translatability between representations, meaningfulness and coherence of representations with the nature of mathematics. Analysis has helped to arrive at a grounded framework for criteria used by teachers to assess representational adequacy. The implications for how collaborative investigation can help in developing the teachers knowledge of representations and about representations has been discussed.

Rationale

Representations are one of the important tools used for teaching. They can be in various forms like symbolic, visual or contextual in nature. Teachers' knowledge, goals and beliefs have been considered important to understand selection and use of representations in the classroom (Thompson, 1992; Shoenfeld, 1999). However little research has been done on how teachers select and use representations (Stylianou, 2010) and to identify the underlying criteria that guide this selection. This study explores teachers' engagement with representations in the context of collaborative invesitigation which led to emergence of criteria for determining representational adequacy at two different levels.

Literature review and theoretical framework

Pape and Tchoshanov (2001) state that "representation is an inherently social activity" (p.120) allowing learners to construct and interpret both internal and external representations individually as well as in social contexts. Study by Cobb, Yackel and Wood, (1992) illustrated that representations are not "transparent" and communication using representations requires significant amount of negotiation among members, to be able to develop shared meanings. It is not easy for a person to see the "mapping" between a concrete material and an arithmetic operation without elaborating and showing how they are connected (Pape & Tchoshanov, 2001). Several studies have identified the challenge that teachers face in transforming mathematical ideas into representations (Ball, 1990, 1992) thus pointing towards a possible knowledge gap in making "translations" between multiple representations. Knowledge of representations and their affordances and limitations has been considered as part of mathematical knowledge for teaching (Ball, Thames and Phelps, 2008). Teachers' use of representations is also considered as part of "transformation" of the knowledge which involves "representation of ideas to learners… in the form of analogies, examples, illustrations, explanations and demonstrations" (Rowland, Huckstep and Thwaites, 2003, p.2).

Though the importance of teachers' knowledge and use of representation is acknowledged in research literature, this paper contributes a grounded framework of teachers' criteria for representational adequacy that identifies ideas considered important by teachers in selecting, evaluating and using the different representations for teaching. I argue that knowledge of these criteria is a part of "knowledge about representations" which is distinct from "knowledge of representation". The former is closely related to the construct of "meta-representational competence" in the research literature (diSessa, 2004). For example, knowing the area and column representation of multiplication is knowledge of representation, while knowing in which context using a particular method makes sense is the knowledge about representation. In this paper, a theoretical framework has been arrived at as a result of the analysis of teacher talk during the collaborative investigation (Smith and Bill, 2004) of the topic of integers in the workshop.

Collaborative activities like lesson study, professional learning communities have been identified as fruitful in developing teacher's knowledge. Studies indicate that collaborative discussion through anticipating and discussing student thinking while making lesson plans in lesson study leads to substantial teacher learning (Fernandez, 2005). Silver, Clark, Ghousseini, Charalambous and Sealy (2007) have reported how collaborative conversations helped to connect different pedagogical, content and student related issues. Meyer and Wilkerson (2011) elaborate on how opportunities to develop teachers' knowledge arise through the discussion of concepts and instructional strategies prior to making a lesson plan rather than through the use of an existing lesson plan and focusing on its implementation. This indicate the potential for collaborative investigation for teacher learning. However, in Indian context, there exists no institutional structure or support for teachers to work collaboratively and teachers are expected to "follow" the facilitator or circulars issued by administrators about what to teach and how to teach. In this reported study, the space for collaboration among teachers and between teachers and teacher educators was established through collaborative investigation activity in workshops.

The study

This study involved 4 in-service middle school math teachers (more than 15 years experience) working with a team of 3 researchers and a faculty. These teachers were nominated by their principals as "effective teachers" to participate in the study. Hence the sample was purposive in nature. All the teachers who participated had more than 15 years experience of teaching and were in the age range of 39 to 50 years. Teachers engaged in collaborative investigation through six one-day workshops spanning a period of four months for designing representations for teaching integers. During these workshops, teachers shared the explanations and representations used by them for teaching integers at sixth grade, shared the common errors they have seen among students. Teachers then collaboratively used a framework of integer meanings to analyse textbook chapter and think of examples of contexts and models for teaching integer arithmetic and analyse them for appropriateness. Research team also collaborated with teachers by suggesting ideas as well as giving critical inputs about the meanings of representations and whether it will make sense to students. After a collaborative discussion on a ten day plan for teaching integers, teachers made individual unit plans. After teaching, they shared their experience of teaching integers with each other. The nature of collaboration among teachers was discursive in nature as teachers built on thoughts and ideas shared by them and research team in the process of evaluating and designing representations for teaching integers. The main research question addressed in this paper is: What are the criteria for representational adequacy that teachers used to select, evaluate and generate representations for teaching integers through collaborative investigation?

Methodology

The researcher was a participant observer in the workshops. Around 40 hours of audio data from all 6 days of the workshop were transcribed. Transcripts were then read and preliminary codes were developed from the data through open coding by the researcher as described by Miles and Huberman (1990). She also wrote memos summarizing the findings of each day while making analytical notes about the codes and identifying the significant events which denoted evidences of teachers' reflection or learning. These were discussed with a faculty member and in the second round of transcript reading, the researcher and the faculty member coded each turn in the transcript independently using the following categories: speaker, mathematical purpose, pedagogical purpose, integer meaning, operation meaning, type of representation, and specific model/context. After initial coding, codes were reviewed by the researcher and faculty member to resolve ambiguities. Through analysis of these codes recurrent themes which illustrated the criteria used by teachers to evaluate representations were identified like translatability, meaningfulness and consistency. The events belonging to these themes were compared and essential features were identified, and framework was developed as illustrated in Table 1 and 2 in this paper. In each turn of the teacher talk, where a representation was being discussed, the implicit and explicit reasons given by a teacher to evaluate a representation positively or negatively was coded as one or more of the criteria of translatability, meaningfulness and consistency based on initial analysis. The analysis transcripts related to criteria revealed that there exists two levels of criteria application -surface level and deeper level. The integration of two or more criteria reflected a deeper level application of criteria thus allowing it to be distinguished from surface level application.

Findings

Through analysis of transcripts from the workshop, three criteria for representational adequacy were identified which are translatability, meaningfulness and consistency with the nature of mathematics. These criteria were used by teachers to evaluate different types of representations including contexts, models and symbolic representations for teaching integers.

Translatability criteria refers to the feasibility as perceived by the teachers of translating one form of representation into another, for example, from symbolic form to a model. Using the criteria of meaningfulness requires one to acknowledge that representations are not transparent and that misconceptions about these meanings may exist among students. It may refer to different meanings ascribed in different contexts and may also refer to the required revision in meaning when learning a mathematical concept of a higher level. For example, students need to revise their understanding of whole numbers to include fractions and integers on a number line. Mathematical consistency indicates the consistent nature of mathematics and the way it is exhibited in the discourse. This can be related to concerns for consistency in explanations whether using smaller numbers or bigger numbers. Teachers often did not prefer certain explanations as they felt that they can be used to explain operations with smaller numbers (like 3- 4) but cannot be used to explain for example, 335 – 448 as they did not know how to represent such big numbers.

During the one-day workshop, teachers discussed the different representations used by them for teaching integers in sixth grade. They shared that they generally use contexts for only introducing the concept of integers, neutralisation model for addition of integers, number line model for both addition and subtraction and finally moving on to use of rules with symbolic expressions for addition and subtraction. The analysis of the prescribed textbook indicated that the sequence and the use of representations by teachers exactly matched what was there in the textbook.

Analysis of interactions of the first workshop indicated that teachers preferred symbolic representations and use of rules over other types of representations for teaching integers. Table 1 below presents a few example of excerpts/descriptions of teachers' talk during the workshops and the implicit criteria inferred from analysis of the teachers' talk.

Table 1: Examples of Use of Criteria in Teachers' Talk During the Workshops

No.	*Representation used*	*Excerpts from workshop*	*Criteria*
1	Symbolic expression	"For subtraction (of integers), they have to first convert it into addition (3 – 4 = – 4 +3), which children forget to do" (Day 1)	Translatability: surface level
2	Neutralisation model	"Here [with coloured buttons] they can see +1 and –1. They make it 2. They don't consider it zero… (how to explain) why it becomes zero?" (Day 1)	Translatability + Meaningfulness: deeper level
3	Number line model	"If (while adding)negative integer (as addend) then move in left direction on the number line. If subtraction of negative integer then move in opposite direction (right)" (Day 1)	Meaningfulness-surface level
4	Neutralisation model	"We are calling them for buttons [i.e asking them to work on it].... Ultimately we should tell them rules otherwise big numbers they will face problems" (Day 1)	Mathematical consistency: surface level
5	Symbolic expressions	"You cannot take away 5 from 2.... This is also a problem.... We did that so that they avoid mistakes when they are young" (Day 1)	Meaningfulness + Mathematical consistency: deeper level
6	Context	"But negative numbers are not there... marks scored in [suppose] five class tests are there. Suppose in the first test [one gets] 15, second test 17 [and then] 14 and 20 so if the child scores 2 marks more in the second test than in the first so +2 then in the third test scoring 3 marks less so [–3. So it is a] change question." (Day 3)	Translatability+ Meaningfulness + Consistency: deeper level

In row 1 of Table 1, a teacher discussed how they teach students subtraction of integers using numeric expressions by asking them to convert the subtraction problem into an addition problem. The teacher made the claim of translating one form of expression to another as one would get the same answer but did not justify why it should be done or the equivalence of the expressions. It was also clear that students are told this as a procedure rather than discussing the equivalence. This is a case where the criterion of translatability has been used at a surface level. On the other hand, the excerpt in row 2 of Table 1 indicate the use of translatability criterion at deeper level since the teacher is not only concerned about translating the actions done using neutralisation model with buttons to numeric expression but also about the meaning of addition of positive and negative integers being clear to students. The teacher realised that models are not transparent as students could not understand why +1 and –1 should cancel each other and thus some explanation is required. She implicitly understood how meanings held by students are not consistent with addition as neutralization as they view addition as increment based on their past experience and thus counted all the 1s irrespective of the signs The discussion then moved to considering the positive and negative integers as representing increase or decrease in quantity. When the increase and decrease is equal, there would be no change in the state of the quantity. Teachers felt that this could be a worthwhile idea to explore with students by using the red and black buttons in the neutralisation model to represent the increase or decrease in quantity.

In row 3 of the Table 1, the operation of addition and subtraction on Number line was translated by a teacher as moving towards positive and negative integers respectively by identifying the + and the – sign in the symbolic expression. The instruction usually given to students was in form of rules to be memorised about the direction in which to move when given a particular symbolic problem. The instructions were different when performing subtraction of positive versus that of a negative number. In reconstructing the the meaning of addition and subtraction as movement towards right and left direction on the number line, no consideration has been made for the meanings held by students about addition as increase and subtraction as decrease in quantity as a result of engagement in whole number arithmetic. Another issue in this explanation is the confusion that might arise due to not differentiating between the sign of the operation with the sign of the integer, as will be in the case of 3 – (– 4). Teachers used number line as a tool for communication about how to solve numerical problems rather than a tool for expanding students' understanding of numbers, addition and subtraction.

The row 4 of table 1 has an excerpt from a teacher from the beginning of the workshop where in the teacher indicated his preference of rules over the use of other representations like models and context. The reason for his faith in telling rules was that he felt that rules would work for even larger numbers while he had seen use of models and contexts for only few easy numbers. His preference was also due the speed at which one is able to solve the integer arithmetic problems using the rules while use of models and contexts required more time. It is thus inferred that the teacher used the criterion of mathematical consistency at surface level to consider use of rules as consistent without having the knowledge of how even the models and contexts can be used consistently with different types of numbers.

In row 5 of Table 1, teachers realised that the emphasis during teaching in the primary grades that bigger number cannot be subtracted from smaller number may lead to students reversing the minuend, that is, write 5 – 2 when asked to subtract 5 from 2. The teacher's discourse shifted from attributing students' error to students' lack of understanding to recognising that instruction too can lead to errors when due consideration of the mathematical concepts is not done while teaching in earlier classes. Earlier, teachers were insisting that students need to be told to reverse the order while writing the subtraction problem in numbers. Thus, the shift in discourse is also in terms of translating the problem

"Subtract 5 from 2" into symbols to looking at what meaning could be made of this problem by students. Teachers discussed how the symbolic representation of, 2 – 5 does not correspond to the 'take away' meaning that students are familiar with. This explained why students would reverse the numbers in a subtraction problem to conform to the take away model of subtraction that they knew. This brought the meaning of subtraction into focus as teachers became aware of the inconsistency of "take away" meaning of subtraction for subtraction with integers. The meaning of subtraction as 'finding the difference' is consistent for both whole numbers as well as integers and thus could probably be used to build students' understanding of integer subtraction. Another issue that gets highlighted in this mistake is of considering the minus sign as indication of subtraction operation and not as integer. This is another difficulty faced by students which teachers realised through this discussion. These discussions related to meanings attributed to minus sign, numbers and operations created the need to reflect on the meanings ascribed by students as well as teachers themselves and explore the alternative meanings of integers and their operations. On the other hand, it also made them aware of the consistency that needs to be maintained in the meanings of subtraction and the distinction between the subtraction operation and the integer. Thus, it was considered that the teacher was using the criteria of meaningfulness and mathematical consistency at much deeper level during this conversation.

In row 6 of the Table 1, a teacher is discussing a proposal of using scores in tests to be represented as integers. She identified that it would not involve use of negative integers as signifying state and then used change meaning to think of a way where change in marks could be represented using unary integers which could be combined and represented using addition. The consistency in context description was established by finding referents first for integers and then for operations of integers, thus making a distinction between use of minus sign to denote integers and to denote the subtraction operation. For example, there are different referents for negative integer (score of teams) and minus sign for subtraction refers to the process of finding the difference between scores of teams making the meaning of minus sign representing integers and that of subtraction as distinct. This led to consistency in use of minus sign as integer and that for subtraction. This excerpt shows how the teacher has internalised the criteria of translatability, meaningfulness and consistency by being able to translate the actions in the context into numeric expressions using both sign of operations as well as for integers and being able to meaningfully depict integers and their addition through the meaning of integers as 'change'.

Based on analysis of excerpts like above, a framework was arrived at to identify the features of using a criterion at surface level or at deeper level which is presented in Table 2. The discursive nature of collaboration among the teachers as well as researchers pushed the discourse in the workshops towards use of criteria at deeper level. Although it is not possible within the scope of this paper to illustrate how the discourse changed over the course of workshops, the nature of interactions in the workshop were such that surface level application of criteria for a representation was either countered by teachers or research group members with a deeper level application. Each of the teachers shared their ideas about what would be an appropriate representation to use for teaching integers for the particular topic. This was followed by evaluation or elaboration by other members either in support or against the use of the particular representation. Initially teachers had divergent opinions about the use of representations and favoured teaching the rules. The initial talk indicate overt concerns for translatability of representations. However, over the course of collaboration teacher talk indicated the emergence of other criterias of meaningfulness and consistency as well as integration of criteria in their voiced concerns about representations. When teachers reported their use of representations for teaching integers using the plan developed in the workshops, their talk indicated more extensive use of contexts and models than before. Their shared experience also included their reflections about the use of representations and their role in developing understanding of integers and their operations.

Table 2: The Surface and Deeper Level Application of Criteria of Translatability, Meaningfulness and Mathematical Consistency for Representational Adequacy

Criteria for representational adequacy	Surface level application	Deeper level application
Translatability criteria	-getting same answer -correspondence between symbolic procedure and operation in representation -Translation is rule based - No need felt for justification	-Translations are conceptual based - Structures and processes in representation have some meaning -Justifications made explicit for why representation works
Meaningfulness criteria	-Representation as a tool for communication -Meanings are under explored, may not be explicitly discussed --Meaning held by students not explored or revised	-Awareness of non-transparency of representations -Awareness and recognition of range of meanings that correspond to the concept - Meaningful connections among translations -Representation as a tool for exploration of meaning
Mathematical Consistency	- Equivalence between representations is assumed -Usage of symbols not consistent with meaning	- Mathematically consistent -Equivalence between representations established

Discussion and Conclusion

Teacher talk and criteria used for evaluating representations indicated that teachers tried translations between models and symbolic representations through arbitrary rules which did not have any justification and indicated surface level concerns for representational adequacy. These criteria indicated the beliefs that teachers held about representations, about mathematics and about teaching and learning of mathematics. Teachers believed that symbolic representations are more efficient than other more concrete representations like models or contexts which has been reported by studies elsewhere (Cai, 2006) as concrete or visual approaches were believed to be not useful for representing larger numbers. Other studies have also found that teachers considered symbolic and numerical representations as more central to learning and doing mathematics as compared to visual which are termed as "informal" (Stylianou, 2010; Bergquist, 2005). Teachers' collaborative exploration of models and contexts during the workshops made them experience how it is possible to represent larger numbers and operations using models and contexts and develop deeper levels of concern for representation selection through the arguments and justification given by their peers and teacher educators. As discussed, teachers exhibited translatability criterion initially but developed the criterion of meaningfulness and consistency only within the professional development workshop. This is perhaps due to the professional development setting having different culture from what teachers usually experience in their schools making them give rationale for the use of certain representation. The nature of interactions in collaborative investigation supported teachers in making shifts in their discourse and criteria for determining representational adequacy. Thus collaborative investigation has implications for how teachers' criteria for representational adequacy can be extended through engaging teachers in collaboratively evaluating and designing representations for teaching.

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RAMZOR – A DIGITAL ENVIRONMENT THAT CONSTITUTES OPPORTUNITIES FOR MATHEMATICS TEACHERS COLLABORATION

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The multi-phase study described in this paper is a part of an R&D project aimed at examining the development and the implementation of a digital environment named RAMZOR for constituting opportunities for professional collaboration among mathematics teachers and supporting their efforts to plan and realize their complex daily tasks. The results of the study indicate that by providing a suitable framework for teacher collaboration and sharing of knowledge, a digital environment such as RAMZOR has the power to facilitate the development of mathematical and didactic knowledge of teachers who document their plans ahead or share their experience retrospectively, as well as of those teachers who are inspired by their colleagues' documentation and implement them. The study also indicates the potential of RAMZOR to serve as the pillar of the mathematics teachers community of practice. The examination of this potential and the conditions necessary to make RAMZOR an environment that nurtures an independent and sustainable national community of mathematics teachers is the focus of our on-going research.

Introduction

The rapidly increasing use of web media such as Wikipedia, social networks like Facebook, and databases with search engines such as Google, inspired us to consider the idea of constrcting a suitable web-platform for preserving, accumulating, and continuously improving mathematics teachers' practical knowledge through applying collaborative efforts in designing lesson plans (LPs), periodic or thematic teaching programs, evaluation items and assessment tests. We took upon the challenge and gradually developed a collaboration-facilitating computer environment which we named RAMZOR. This name means "a stoplight" in Hebrew, as metaphorically, this site is supposed to serve the ordinary mathematics teacher so that s/he can do the following on a daily basis: STOP to explore and consider other colleagues' ideas; PREPARE for teaching a specific class; and then GO to the classroom, implement and possibly reflect upon the implementation of the plan, or share it with others (Movshovitz-Hadar, 2018). In what follows, we describe RAMZOR, its rationale and features, and the attentive process of its design. In addition, we provide findings from evaluation studies and formative assessment ones that accompanied the development and refinement of RAMZOR throughout its employment by mathematics teachers.

Background for the construction of RAMZOR

Barber and Mourshed (2007) phrased a most appropriate moto for our study: *"the quality of an educational system cannot exceed the quality of its teachers*" (ibid p. 11).

It has become well known that teaching is an extremely complex profession. Teachers need to possess a wide range of skills and various types of knowledge, e.g. pedagogical knowledge that relates to teaching materials and methods, knowledge about students' learning, the capability of analyzing reflectively their actions and their impact, and much more (Shriki & Lavy, 2012). But above all, they should be able to integrate these skills and knowledge and employ them when they prepare their LPs and other teaching materials. In fact, the daily design of LPs is at the heart of teachers' professional work. However, in most cases (at least in our country) teachers prepare their LPs "in mind", and the preparation of detailed LPs is considered to be an "unnecessary burden" required only in pre-service teacher education. Even after teaching a certain lesson, LPs is not recorded and, at best, notes are written in the textbook for future reference. As a result, at the individual level, drawing conclusions is limited, and at the professional community level, there is a lack of sharing practical knowledge with colleagues (Movshovitz-Hadar, Shriki & Zohar, 2014). This stands in contrast to the recognized benefits of sharing knowledge through joint lesson planning: "*we discovered the magic of effective joint lesson planning… The expectation of teachers is not only that they should develop and employ effective practices in the classroom, but that they should share them throughout the whole system. Best practice therefore quickly becomes standard practice, adding to the pedagogy"* (Mourshed et al., 2010, p. 77).

Indeed, as evident from McKinsey report (Barber & Mourshed, 2007), one of the main factors that contribute to the success of any reform is nurturing teacher cooperation and the next generation of system leaders to ensure long-term continuity in achieving reform goals. To that end, teachers must be treated as trusted professional partners, and need to be given the tools and responsibility to lead change (OECD, 2011). Acknowledging that the "professional wisdom" lies, first and foremost, at the hands of the teachers, who actually carry out the task of teaching, we believe that they are the ones who ought to develop and improve cooperatively the teaching and learning materials. The idea underlying the design of RAMZOR as a digital platform that facilitates collaboration among teachers anytime, anywhere, was to provide teachers with a communal environment that would enable them to preserve their resources, share and discuss their practical knowledge and daily experience, develop and jointly improve materials for teaching and learning, and more. In other words, the ultimate goal of RAMZOR is to support the development of mathematics teachers' community of practice (TCoP).

Professional learning communities (PLC) and community of practice (CoP)

The two notions, PLC and CoP, share similarities as well as differences in the context of the models they are based on (Blankenship & Ruona, 2007; Vangrieken, Meredith, Packer, & Kyndt, 2017). This is true in particular with respect to issues related to membership, leadership, organizational culture, and knowledge sharing. While the PLC models draw from learning organization theory, CoP draws from situated cognition, social learning theory, or knowledge management theory. Thus, whereas PLC models address team or group learning focusing mainly on students' needs and increasing students' achievement, CoP models address the need for alignment to the organization strategy and they are focused mainly on the improvement of practice. In addition, PLC models highlight the role of an external leader while the CoP models favor leadership from within the community. Nonetheless, both models provide a theoretical framework for a study towards better understanding of ways to facilitate teachers' learning and collaboration processes as well as their specific effect on teachers' practice and students' learning.

According to Wenger (1998), in order for a community to be recognized as a CoP, three conditions must be met: (i) The domain: A CoP is identified by a common domain of interest; (ii) The community: A CoP consists of members who are engaged in joint activities and discussions, assist each other, share information, and nurture relationships that enable them to learn from one other. However, they do not necessarily work together regularly; (iii) The practice: Members of a CoP are practitioners. They develop a shared repository of resources and ways of addressing frequent problems, thereby engaged in the process of mutual learning. In general, national communities of mathematics teachers, conform to Wegner's first two characteristics: they share an interest in mathematics, its teaching, and learning, many of them meet in professional conferences, read professional journals, and share a professional language enabling them to learn from one another. However, the third characteristic, to a large extent, is still missing in many national communities of mathematics teachers, and the one in Israel in particular. Although a vast majority of them teach mathematics on a regular basis, only a small number of mathematics teachers develop their own repertoire of resources and fewer share them with the entire community (Shriki & Movshovitz-Hadar, 2011). While conceiving RAMZOR, our intention was to develop a dedicated "home address" for the entire professional community of mathematics teachers in Israel, viewing the nurturing of TCoP as promoting the share of knowledge thus supporting the improvement of teachers' practice.

Features of RAMZOR digital environment

RAMZOR digital environment for teachers (http://RAMZOR.sni.technion.ac.il) was developed by "Omnisol Information Systems LTD". It was originated as a "structured Wiki" platform – a Wiki that manages structured objects. Some of its capabilities are similar to "MediaWiki" which include a dedicated text editor, discussion page, versions control, personal user pages alerts and more, while its "structured" capabilities include advanced search competencies, monitoring and permissions management. RAMZOR allows collaborative development and improvement of teaching and learning materials of various types and encourages professional interaction among teachers.
Writing the details is done on a pre-determined Word file templates (for LP, Thematic or periodic program, assessment items or evaluation tests) which are preceded by several descriptive details. For instance, the preceding details for a LP are: the title of the lesson, prior knowledge required, students' grade and study level, behavioral objectives of the lesson, anticipated difficulties and how to manage them, and more; The LP template includes a detailed reference to each part of the lesson, the time allotted to them, the flow of the lesson in a form of teacher-students dialogue, and more. In addition, auxiliary materials can be attached, such as student worksheets, assignments for homework and links to relevant applets. RAMZOR also enables users' comments to each entry, feedback and elaboration on it. Each teacher has his/her personal area and can, for the purpose of respecting privacy, choose who to share the materials with, from keeping a draft for personal viewing to the sharing with the entire community.

RAMZOR: three preliminary design studies

In designing RAMZOR, we implemented a multi-phase study process that included three preliminary studies aimed at examining the contribution of collaborative writing of teaching materials to the professional development of mathematics teachers as well as evaluating the suitability of RAMZOR's features to teachers' needs (Segal, Shriki, Movshovitz-Hadar, 2016).

Preliminary study 1 - My favorite math LP. This study focused on teachers' willingness to share their LPs with colleagues. Via an e-mail, we approached about 400 middle and high school mathematics teachers, asking them to send their favorite LP, written according to specific guidelines provided in advance. They were asked to approve uploading their LPs into a designated open web site. The teachers were told that among the senders there would be a lottery with monetary prizes. Four rounds of raffles took place in 6 months intervals. Only 10-15 LPs were sent to each round. This first step left us not only disappointed but with many open questions related to teachers' responsiveness and motivation to share their LPs with their colleagues.

Preliminary study 2 - Joint lesson planning on MediaWiki system. Eleven graduate students, experienced high school mathematics teachers, participated in a semester-long activity in which they collaboratively designed LPs on a MediaWiki system. At the time this experiment was carried out, MediaWiki seemed to us as the best available platform for facilitating collaborative group work aimed at developing a dynamic repository of LPs and discussing educational ideas. Results of a study that followed the teachers' experience (Shriki & Movshovitz-Hadar, 2011) indicated that the process of joint lesson planning supported the development of the participants as a small TCoP that interact on a daily basis, discuss ideas, and share LPs and other professional resources. The results also pointed at many concerns of the participants, categorized as social and technical ones.

These results led us to recognize the need for teachers to arrive at agreed-upon social norms for managing a shared repository of learning and teaching resources as a preliminary necessary condition for nurturing teacher cooperation. we also realized that the technical concerns related to MediaWiki make it an inappropriate platform for accumulating, preserving, and improving mathematical LPs. To develop a more appropriate environment, we approached Omnisol Information Systems Company and started the development of RAMZOR digital environment mentioned above.

Preliminary study 3 - A 3-day summer school for joint lesson planning. With the insight gained from study 2, we organized a 3-day summer school for two consecutive years in which two groups of selected teachers designed LPs (individually or in pairs/small groups), provided feedback to peers' LPs (orally or in writing), improved their own LPs subsequent to receiving feedback, and reflected upon the entire process. Data was gained through questionnaires, interviews, transcripts of small groups discussion and whole group ones, and content analysis of the LPs and feedbacks (Movshovitz-Hadar et al., 2014). Our findings indicated that ongoing processes of collaboration and sharing are rare in schools. According to the teachers, this situation is a result of several causes, among them: (1) Heavy workload that leaves no time for interaction; (2) Mathematics teachers' tendency not to consult their colleagues for fear of being perceived as having insufficient mathematical knowledge; (3) In small schools there is often only one mathematics teacher for certain grades/levels of teaching, and therefore has no colleague to consult with; (4) A lack of awareness to the benefits of cooperating and sharing knowledge. The LPs were written using the early version of RAMZOR digital environment. This enabled participants to relate to the LPs and enabled us to witness shortcomings of the digital environment, thus, to extend our R&D efforts towards improving the suitability of RAMZOR as a tool for managing professional knowledge. The teachers' reflections indicated that they had developed an awareness of their personal gains from writing LPs and receiving peers' feedback. The teachers also pointed out that writing LPs and sharing knowledge strengthened their self-efficacy and contributed to empowering them as members of the TCoP, and in particular, as expressed by one of the participants "*the fact that academia finally understands that teachers are the ones who possess the professional knowledge, and the ones who can be trusted to chart the way, has increased our professional stature*".

In summary, two main observations were reached through the three preliminary studies, on the basis of which we kept developing the digital environment: (i) Although teachers recognize the benefit of being exposed to other teachers' materials, they do not rush into the opportunity to share their own LPs, unless they are put in a framework that makes them do it; (ii) Once provided with a digital environment that enables lesson planning, teachers become aware of the major role of planning their lessons in details, in terms of the effectiveness of teaching, improvement of the teaching quality, and deepening of learning.

As typical to R&D projects, we put less emphasis on developing, testing and advancing theory (OECD, 2004). Rather, our focal points are concerned with the design of a long-term effort, situated in the real-life school settings, which involves a larger sample of mathematics teachers using RAMZOR for planning their work and sharing their experiences.

RAMZOR in practice: The main study

Following the aforementioned preliminary studies, during 2014-2017 we conducted a three-year intervention, in 20 high schools. In the background, there was a sharp decline between the years 2006-2014 in the number of students taking matriculation exams in mathematics at the highest possible level (called in Israel - 5 units), which resulted, among other things, from a severe shortage of teachers willing to teach mathematics at this high level. There was a need to support the development of a professional and independent CoP of mathematics teachers who teach 5 units.

The intervention was based upon three central pillars: **(i) Mentoring.** An experienced mathematics teacher ("the mentor") was responsible for one or two teachers from the same school ("the mentee"). Altogether there were 20 mentors and 24 mentees from 20 high schools. The mentoring included: mutual observations in classes, preparation of teaching programs (periodic or thematic), and designing LPs and evaluation items. The research literature provides an extensive discussion of the advantages of the mentoring model, in particular of school mentoring (e.g. Kadji-Beilran, Zachariou, Liarakou & Flogaitis, 2014); **(ii) Utilizing RAMZOR**. The mentors and mentees were engaged in collaborative writing of LPs, teaching programs and assessment items for various topic included in the curriculum. Teachers implemented the LPs in their classes as well as those written by their peers, provided feedback, and wrote "parallel" versions adapted to their classes; **(iii) Professional development workshops** for mentors and mentees. During each of the three years of the project, there were periodic online and face-to-face meetings with the project staff. The online sessions were held separately for the mentors and the mentees, while the face-to-face meetings were for both the mentors and mentees.

Research questions. The study that followed the 3-year intervention project addressed several research questions. In this paper, we focus on two of them: What is the contribution of collaborative writing of teaching materials using RAMZOR: (i) To the personal professional development of the participant (mentors and mentees)? (ii) To the development of the participants as members of a mathematics-TCoP?

Research instruments. Four main research instruments were used to gather information: (i) A monthly formative assessment questionnaire; (ii) Recording of the online and face-to-face meetings; (iii) An examination of the LPs, teaching programs and evaluation materials written in the RAMZOR; (iv) Semi-structured interviews.

Data analysis. A qualitative research paradigm was employed. First, the data were analyzed through a process of open and axial coding to identify the main categories and sub-categories (Corbin &

Strauss, 2008). At the second stage, a triangulation process was implemented, comparing the categories obtained from each research tool (Miles & Huberman, 1994). The third phase of the analysis consisted of reviewing the findings obtained at the first and second stages while focusing on data related to the participants' professional development and the development of mathematics-TCoP. **Results.** Due to space limitations, we focus on partial findings that relate to the contribution of using RAMZOR to the teachers' mathematical and didactic knowledge, as indicators of professional development, as well as to their sense of belonging to TCoP. In the questionnaires distributed at the end of each year, the participants were asked to express their consent with the statements: "Reviewing and employing other teachers' LPs contributed to my mathematical knowledge" and "Reviewing and employing other teachers' LPs contributed to my didactic knowledge". Table 1 presents the distribution of the degree of teachers' agreement with the statements on a five-level Likert-type scale, both for mentors and mentees, at the end of the third year of the project.

Table 1: Distribution of mentors' and mentee's degree of agreement with the statements: Reviewing and employing other teachers' LPs contributed to my: mathematical knowledge (Stat. A); didactic knowledge (Stat. B), in percentage

Teachers	Degree of Agreement									
	A very large extent		A large extent		A medium extent		A little extent		Not at all	
	Stat. A	Stat. B	Stat. A	Stat. B	Stat. A	Stat. B	Stat. A	Stat. B	Stat. A	Stat. B
Mentees (N=24)	22	22	64	71	14	7				
Mentors (N=20)		25	54	44	46	31				

Table 1 indicates that both the mentors and the mentees perceive the reviewing and employment of other teachers' LPs as contributing to their mathematical/didactic knowledge at least to a medium extent. In addition, the mentees perceive this contribution as more meaningful than the mentors. It should be remembered that for the mentees it was the first experience in teaching high-level mathematics. Therefore, one could expect that reviewing and practicing the detailed LPs in RAMZOR that were written collaboratively by mentors and mentees will be more beneficial to the mentees, in terms of mathematical and didactic knowledge, than for the mentors. The following are several typical quotations from the participants' entries (translated from Hebrew). As for **mathematical aspects**, participants wrote: "*There were LPs that exposed me to new ways of proving statements that involve geometry locus. For example, I learned that homothetic transformation enables the preservation of the loci features*" (a mentee*);* "*In one of the LPs, there was a clear visual explanation for the meaning of the integral formula. It opened my eyes, and I realized that understood (and taught) it technically*" (a mentee)*; "The very fact that I had to write a LP about conic sections made me delve into the little details I was not sure about in the case of the hyperbola. When you teach, you can sometimes "smooth things out", but not when you know your colleagues will read and use it" (a mentor).* As for **didactic aspects**, participants wrote: "*I applied the LP on similar triangles with the problem-posing approach, and students' problems actually help me to understand how students understand the topic and process information*" (a mentee)*;* "*I tried the LP on presenting the solution of distance word problems graphically, and I realized that it helped many students* *understand the verbal formulation and identify the connections between the givens"* (a mentee); "*I understood how a thoughtful integration of technology can respond to students' difficulties. I applied the LP where there was an applet demonstrating the relationship between the tangent slope and the derivative of a given function, and since then I look for LPs in RAMZOR that integrate technology"* (a mentor); "*My mentee found a LP in trigonometry with 3D applications. Together we analyzed this LP and thought it might help many students to see spatial information. We tried it in class, and for many students it was kind of a breakthrough. They were finally managed to see things*" (a mentor).

The participants were also asked to express their consent with the statement: “Sharing my lesson plans with other teachers contributed to my sense of belonging to the teacher community of practice”. Table 2 presents the distribution of the degree of teachers’ agreement with the statement, both for mentors and mentees, at the end of the third year.

Table 2: Distribution of mentors’ and mentee's degree of agreement with the statement: Sharing my lesson plans with other teachers contributed to my sense of belonging to teacher community of practice, in percentage

Teachers	Degree of Agreement				
	A very large extent	A large extent	A medium extent	A little extent	Not at all
Mentees (N=24)	29	43	28		
Mentors (N=20)	31	38	23	8	

As evident from Table 2, most of the mentors and the mentees reported that sharing LPs with their peers contributed to their sense of belonging to TCoP at least to a medium extent. The following are typical quotations from the participants' entries (translated from Hebrew): "*When I write a lesson plan in RAMZOR, I get constructive feedback from colleagues. It is really important because we don't have opportunities to share our ideas and products and get feedback on what we do. For me, this is the meaning of a community*" (a mentee); "*When I share my lesson plans, I feel that I am contributing not only to my mentee but to the entire community. It is like remaining your legacy. Otherwise, it would disappear. It really makes me happy and proud*" (a mentor).

Discussion and conclusions

The multi-phase study described in this paper indicates that spontaneous processes of sharing practical knowledge among teachers are not common. Lacking feedback on their work, many teachers feel a sense of isolation. Therefore, it seems that RAMZOR digital environment address a real need of teachers, as it provides a suitable environment for teacher collaboration. The results of the main study indicate that RAMZOR became a set-up for teacher collaboration and for sharing of knowledge. The fact that the repository includes detailed LPs and not merely anecdotal information facilitated the development of mathematical and didactic knowledge both of mentors and of mentees, as composers as well as consumers of LPs. In addition, the study points out the potential of mathematics teachers’ collaborative work with RAMZOR to support the development of mathematics-TCoP. According to the definition of Wenger (1998), the teachers in this study had interest in a common domain: improving their math teaching in 5 units level, they were engaged in joint activities and discussions, assisted each other, shared practical knowledge and learned from one another. All these were made possible due to the collaborative nature of RAMZOR environment. However, it should be noted that the work of the mentors and mentees was carried out as part of a project in which we, as the managers of the project, were involved in direct connection with the participants. Therefore, in order to examine the feasibility of TCoP development without our direct involvement, while maintaining RAMZOR as the pillars of teachers' collaborative work, in September 2017 we started a new 3-year R&D project. Thirteen high schools in one of the largest cities in our country are taking part in this project. Two teachers from each school are working with the project team staff on a regular basis, while at the same time they work with their entire math teachers school staff aiming at the establishment of a local school as well as an overall municipal independent and sustainable mathematics-TCoP. Obviously, our ultimate goal is to nurture such a national mathematics-TCoP, turning RAMZOR digital environment into an accessible fruitful repository for the benefit of all teachers. Further research is needed to examine the conditions necessary to achieve this goal.

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TEACHERS INVESTIGATING THEIR PRACTICE COLLABORATIVELY

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A proposal of collaborative teacher in-service education is presented. The proposal was conducted with fifteen primary and secondary mathematics teachers, who plan, observe, and collaboratively question their teaching practice within the framework of a master's program. The study is qualitative whose objective was to interpret the appropriation of the inquiry methodology by the teachers during their teaching practice, mediated by the planning and implementation of a didactic unit. The design is based on self-observation where the class sessions were video recorded, with a prior planning of a didactic unit structured on didactic situations. The results report that teachers recognize the complexity of the educational activities and act on it in three categories: didactic sequence, scientific competence, and interactivity.

The Colombian Ministry of Education (MEN, 2013), affirms that the evaluations applied to in-service teachers inform on weaknesses both in the disciplinary knowledge and in the didactic knowledge inherent to the discipline itself, which would explain the difficulties that the students show. Rico (2007) states that “teachers do not have adequate and sufficient conceptual tools from which to make good planning” (p.53), this statement motivates the question about tools that teachers should know and use to perform their work efficiently.

Reflection on the practice, being considered as a critical competence for the improvement of teaching, is also a strategy for professional development, especially if it is a guided reflection as a process of self-inquiry, development, and learning. Llinares and Kainer (2006) state that reflective practice offers a perspective on student learning and provides information on changes in the teaching of mathematics.

THEORETICAL FRAMEWORK

The reflection on the practice was conducted with didactic units designed according to the inquiry methodology (Harlen, 2013) and the didactic situations of Brousseau (2007). The analysis of the teaching practice considers categories and subcategories. The categories are: didactic sequence, scientific competence, and interactivity (González-Weil et al., 2012). In what follows, the categories and subcategories are explained, for the sake of clarity, categories are underlined, while subcategories are written in italics.

The first category is <u>The didactic sequence</u> answers the question: what activities are carried out in the classroom and how are they structured?, the sequence study the relationship between the situation posed and the content, the reorientation of the practice in the classroom having in mind students needs, the use of didactic resources and the strategies to articulate knowledge. Composed of two subcategories: *core activity* and *moments of the flexible class*, which are interpreted through inquiry in their phases: Trigering Fact that refers to the approach of a contextualized problem, previous knowledge, open and participatory class planning; Exploration that refers to hypothesis search and collaborative Exploration and the Resolution which refers to the evaluation of the solution proposed by the students (Bustos, 2011).

The core activity "is organized around experiences of direct access to learning such as laboratory work or field trips, which includes the use of varied resources, where students are the protagonists in knowledge construction" (Sanmarti, 2000). Defined by the codes, 1A-1: the teacher develops the themes, through problems, in real contexts; 1A-2: the teacher relates the contents with everyday situations, and 1A-3: the teacher uses various resources for knowledge construction. Each code is composed as follows: the first number defines the category, the letter indicates the sub-category and the second number, the consecutive number in the items. For example, 1A-2: indicates that the category of study is the 'didactic sequence', A: sub-category 'core', and the number '2' indicates the second item of the grid.

Moments of the flexible class, are related to the adjustments that teacher makes, during the development of the class, between the planned and the executed, according to the students' needs, to favor individual and collaborative work, the communication of processes and results (González-Weil et al., 2012). This sub-category was analyzed according to items 1B-4: the teacher makes his strategy more flexible according to the students' learning needs; 1B-5: the teacher plans and builds the teaching process step by step successively and cumulatively; 1B-6: the teacher supports students knowledge' construction.

The second category, Scientific competence answers the question: what areas of scientific competence does it implement in the classroom? refers to the mathematical and didactic knowledge that the teacher has about the mathematical object to be taught. This competence is defined by *two subcategories*: *promotion of knowledge, skills and attitudes*, and *the teaching of discipline competences*; interpreted by the codes: Exploration: EX-teacher individual explanation. Integration: INT-systematization of ideas, INT-joint construction. Resolution: RE-confirmation and analysis of the explanations.

Promotion of knowledge, skills and attitudes, are those actions that the teacher conduct during the development of problem situations; the teacher does not offer definitive answers, but raises new questions. This characteristic is recognized during teaching practice, by the codes: 2A-7: the teacher asks guiding and challenging questions; 2A-8: the teacher's response is consistent with the students' concerns; 2A-9: the teacher proposes strategies that promote communication; 2A-10: the teacher promotes argumentation about the student's process to solve a problem; 2A-11: the teacher applies strategies that promote the articulation of previous knowledge with the new learning; 2A-12: the teacher requests the explanation of the solution processes carried out; 2A-13: the discipline language used by the teacher is appropriate to students' cognitive development; and 2A-14: the teacher exhibits strategies that refer to inquiry, argumentation, dialogue and modeling of learning.

The teaching of discipline competencies refers to the teaching ability to adapt mathematical discipline knowledge to contextualized situations. Interpreted by the codes: 2B-15: the teacher proposes strategies for students to conceptualize from the processes performed; 2B-16: the teacher manages didactic situations presented during the teaching; 2B-17: the teacher designs activities that promote a plan of action to solve problems; 2B-18: the activities carried out by the teacher recognize the students' cognitive development; 2B-19: the teacher promotes interest, attention, and participation; and 2B-20: the teacher asks questions that lead to results' discussion.

The third category, Interactivity, answers the question: what characteristics does the teacher-student interaction have and how does it support learning? The collaborative work that supports learning is studied by asking questions related to the students' doubts. It is defined as "the articulation of the actions of the teacher and the students around a specific learning task or content" (Coll, Colomina, Onrubia, & Rochera, 1992, p.204). It is made up of two subcategories: *Negotiation and Construction with students*, and *Scaffolding* based on student needs; which are interpreted as: Triggering Fact: HD-involve the student and Exploration: EX-construction of meanings.

Negotiation and construction with students are characterized by the exchange of information or knowledge between student and teacher. The codes are 3A-21: the teacher favors collaborative work by classroom activities; 3A-22: the teacher uses strategies that promote autonomous learning; and 3A-23: the teacher promotes shared construction of meanings.

Scaffolding based on student needs, consists of a form of assistance that the most advantaged students provide to support and guide peers, who would not be able to solve the tasks on their own. It is recognized by the codes: 3B-24: the teacher integrates the previous knowledge with the new learning; 3B-25: the teacher offers help for the construction of knowledge; 3B-26: the teacher gives clear instructions on the process to be performed; and 3B-27: the teacher stimulates and regulates learning.

INQUIRY METHODOLOGY

The inquiry methodology is a strategy in which the teaching discourse, the question, the self-evaluation and the feedback in the classroom "motivates students to get involved in their learning and tries to equip them with strategies to think and act" (Wells, 2001, p.136).

In this research the interpretation of the appropriation of the characteristics of the inquiry methodology in teachers' practice was made through the phases of the practical investigation: triggering event, exploration, integration, and resolution (Bustos, 2011) and its items, which are shown in Tables 1, 2 and 3. Triggering event refers to the research conducted by teachers that intends to involve the student, and it "is characterized by directing the activity toward the understanding of the problem and the search for potential explanations or hypothesis" (Bustos, 2011, p.102). In the exploration, the student, individually, poses alternative solutions to a proposed problem, then collectively discusses the solutions. In the integration, students validate their statements argumentatively, finally in the resolution intervenes the teacher who promotes consensus.

The teacher reflects on what he does in the classroom, while the student values what he learned. Self-evaluation and co-evaluation are features that help the student to strengthen the self-regulation of learning (Harlen, 2013). The inquiry methodology is a strategy in which the teaching discourse, the question, the self-evaluation and the feedback in the classroom "motivates students to get involved in their learning and tries to equip them with strategies to think and act" (Wells, 2001, p.136).

CONTEXT AND METHOD

The participants were fifteen in-service teachers enrolled in a Master in Mathematics Methods, at the Pereira Technological University, Risaralda Province, Colombia. The students were awarded scholarships by the Colombian Ministry Education of Colombia and participated in the macro project "The inquiry methodology in the teaching and learning of mathematics", in which teachers collaborate with researchers and with colleagues to observe and question their teaching practices.

The master courses are taken over two years. In some seminars, the didactic units were designed, discussed, and validated. The research was qualitative, descriptive, and interpretative (Hernández, Fernández, & Baptista, 2010). The design was based on Grounded Theory (Strauss & Corbin, 2002), which considered three moments: In the first, the retrospective was made, corresponds to the characterization of the teachers' practice before their post-graduate education. In the second, along with the seminars, the didactic units were designed, planned and validated, based on the methodology of the inquiry (Bustos, 2011); as in the theory of didactic situations of Brousseau (2007). In the third moment, the didactic units were implemented, and the observation, systematization, and analysis of the practice of each one of the participants were conducted. The teachers, in pairs, observed their practice, filmed and analyzed it according to the instruments of observation and analysis.

Two instruments were used for observing and systematizing the teaching practice: Grid for Observation of Teaching Practice through the three categories, and the Analysis Matrix. Researchers in mathematics metnods validated both instruments, and their construction was done in the Seminar of Mathematics Didactics of the Technological University of Pereira, later they were adjusted, by teachers of the mathematics macro project, with the participation of the inservice teachers. The second instrument, Analysis Matrix, was built based on the phases of practical inquiry.

The practice of the teachers during the implementation of the teaching unit was recorded, later transcribed, and with the help of Atlas.ti software the most repetitive actions, co-occurrences, were quantified. Each transcription segment was analyzed and assigned a code according to the Practice Observation Grid; afterward, the process was repeated in the Analysis Matrix. The subjectivity in the interpretation was reduced by collectively constructing a dictionary with the meanings of each item explained in the two instruments.

ANALYSIS AND FINDINGS

In this section, the analyzes, summarized in three tables, are presented, and the findings are discussed. The data was compiled from fifteen reports corresponding to each participant, and the average of concurrence was taken for each item that characterized the teaching practice. Table 1 summarizes the findings related to the teaching practice during the implementation of the didactic unit using the didactic sequence category.

Table 1. Percentages of co-attendance of the 15 teachers.

			Práctical Inquiry					
			Triggering event: Approach to the contextualized problem	**Triggering event: Exploration of prior knowledge**	**Exploration: Hypothesis search**	**Exploration: Collaborative exploration**	**Triggering fact: Open and participatory class planning**	**Resolution: Evaluation of the proposed solution**
Teaching Practice: Didactic Sequence	Core activity	1A-1	38%	9%	7%			
		1A-2	47%	15%	14%			
		1A-3	16%	14%	14%			
		1B-4				18%	11%	6%

Flexible class moments	1B-5	16%	22%	8%
	1B-6	16%	25%	15%

Fuente: Datos de la investigación Source: Research data

The characteristic of the methodology of inquiry with more significant appropriation happened when co-occurring in 47%, between the strategies focused on the contextualized situations approach, with the core activity subcategory, in regard to a teacher that relates the contents with daily situations, 1A-2; and in 15% with the exploration of previous knowledge, as a triggering event of the practical investigation. Similarly, a 14% co-confidence can be seen with strategies in which exploration is proposed in the search for hypotheses, and solutions to the proposed situations are discussed. Some evidence is illustrated with the following fragments: ‘Today the poster contest will take place in the hall, each team must desing a poster taking into account the following instructions: The poster must be divided into eight equal parts. Each part must have a different color. In one of the parts should go the teams’ name. In each of the other seven parts you should put an image that represents the seven foods or foods that you like’. This is a situation proposed for teaching fraction representation as part-whole. While for the teaching of arithmetic operations the bazaar was used: ‘Since in the school bazaar: ‘There will be 20 stalls selling different products, advertising and promotion will play a critical role, for this reason, grade 6A must design advertising for the lemonade sale’.

Table 2 presents the percentages of simultaneous occurrences of the methodology of the inquiry’ characteristics, concretized in the practical investigation, and in the items that allowed to observe, systematize and analyze the teachers’ teaching practice through the category of scientific competence.

Table 2. Co-occurrence percentages of the 15 teachers.

			Práctical Inquiry			
			Exploration: Teacher explanation (individual contribution of ideas)	**Resolution: Confirmation and analysis of explanations**	**Exploration: Teaching explanation (int joint construction)**	**Resolution: Confirmation**
Teaching Practice: Scientific competence	Promotion of knowledge, skills and attitudes	2A-7	14%	13%		
		2A-8	17%	12%		
		2A-9	9%	14%		
		2A-10	9%	30%		
		2A-11	10%	10%		
		2A-12	9%	15%		
		2A-13	15%	20%		
		2A-14	15%	23%		
	Teaching disciplinary competences	2B-15			20%	19%
		2B-16			15%	9%
		2B-17			12%	12%
		2B-18			11%	12%
		2B-19			23%	18%
		2B-20			24%	22%

Source: Research data

It is inferred that the most reiterative characteristic during the implementation of the didactic units refers to the student argumentation promoted by teachers to solve a problem; 2A-10, co-occurring at 30% when they promoted the confirmation and analysis of the explanations; Resolution-confirmation and analysis of the explanations. For example, during the teaching of the additive structure, the teacher promoted the argument, as shown in the following passage:

D: What operations did we do to organize the participating teams in the municipal championship?
E: Many,
D: Which?
E: Addition and subtraction,
E: Yes, in the sum several teams were grouped in each category and in the subtraction some teams were suspended due to problems in the payroll which caused several changes that were not foreseen when organization of the categories.
D: What is the average goals per game?
E: The average goals are 2 per game,
D: With these answers I invite you to build the tables of the addition and subtraction to verify the information, therefore the matrix was delivered on a grid sheet.

Table 3 shows the most reiterative characteristics, observed through the subcategories of interactivity to co-occur with the phases of practical inquiry, were given when the student construction of meanings is promoted; 3A-23, by involving them with the triggering event: involve the student in the construction of meanings, promote and regulate learning; Exploration: construction of meanings with 27% and 26%, respectively.

Table 3. Percentages of co-occurrence of the observation of the teaching practice.

			Práctical Inquiry	
			Trigering fact : Involve the student	**Exploration: Construction of Meanings**
Teaching Practice: Interactivity..	Negotiation and construction with the studen	3A-21	19%	
		3A-22	19%	
		3A-23	27%	
	Scaffolding based on student requirements	3B-24		17%
		3B-25		16%
		3B-26		10%
		3B-27		26%

Source: Research data

The Figure 1gives evidence of interaction:

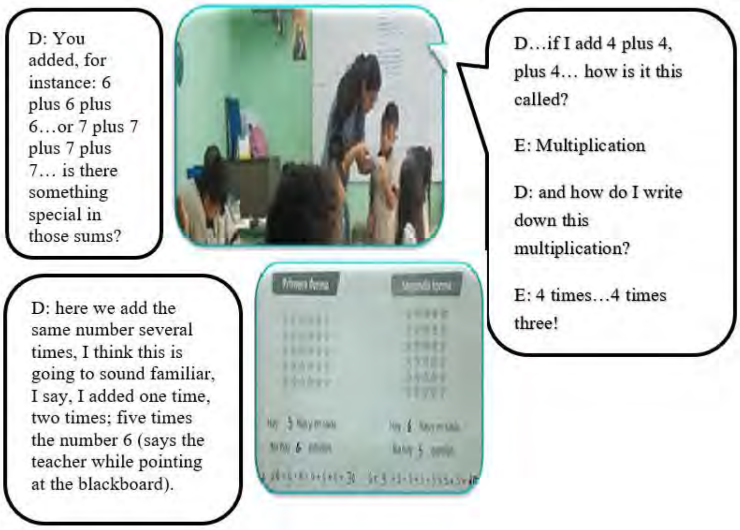

Figure 1: Teacher interacting with student

Concerning collaborative exploration, an 18% relationship was found with actions in which the teacher adapts his strategy according to the students learning needs. 25% of the teacher's actions were intended to promote individual and group work, formulation of strategies along with the students, blackboard' use by students, formulation of questions, students' proposals to convince, both their classmates and the teacher. The strategies that sought resolution co-occurred with 15% of the teaching actions to regulate the teaching and learning process. Student participation was encouraged during mathematics classes, by the promotion of self-assessment and participation.

17% of the teaching interventions refer to answers consistent with the students' concerns, with the intention of building knowledge and promoting student attitudes. 17% of the interventions promoted the exploration of mathematics ideas. The recognition of the student's knowledge is important in the inquiry methodology, for it helps the teacher 'institutionalize' mathematical knowledge that arises during the discussions; with the aforementioned actions the teacher not only helps students to understand but also motivates them. It was also found that 30% of the teaching interventions were characterized by the re-confirmation and analysis of the student explanations. 24% of the teaching actions were oriented towards the joint construction of knowledge. 22% of the strategies are located in the phase of integration or progressive systematization of ideas, which were in co-occurrence with the formulation of questions to institutionalize findings, the questions have a central role in the classroom, those formulated both by the teacher and by students. 26% of the teaching actions aimed at the construction of meanings in a joint manner, in which not only was the triggering event provided or the conditions for the solution but also through examples and questions, the students made decisions to continue working on the task.

CONCLUSIONES

This paper presents an experience of teacher education in which researchers and participants study classroom teaching practices, through the inquiry methodology and didactic situations, to study the scientific teaching skills. The results report on the teachers achievements when they collaboratively study their classroom practice. Although the work of observation and analysis is demanding, the teachers were able to assume a scientific attitude in the observation of their classes, and consider it as composed of different layers that require identification, analysis, collective discussion, and systematization. A fundamental principle of inquiry methodology is the interactivity, the construction 'with the other'. Features such as: promoting student participation towards the conceptual construction of the mathematical object are strengthened, using questions in order to reach class' objectives, formulates strategies to confront students' previous knowledge with new knowledge, to recognize the feedback as way to knowledge joint construction, to foment the use of self-evaluation and co-evaluation to strengthen the self-regulation of learning. The class analysis turns out to be an experience of professional teacher development and could have effects on the scientific training and performance of the students. The latter, however, is a matter of an ongoing investigation.

The teachers also designed class sequences for a specific mathematical object, whose criterion of choice was the low students' performances in that specific mathematical object. The experience of reflection serves not only the professors but also the students.

It is notable that these professors do not have specific training in mathematics, but the experience of using the methodology of the inquiry, with the development of didactic sequences to be observed and analyzed during their implementation, gives evidence of the adequacy of this methodology to be used, even in cases where teachers are not experts in the discipline. Teachers recognize the value of reflection on their practice, which has led to learning communities in their educational institutions. Evidence of these statements could be offered in an extended version of this paper.

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THE REALIZATION TREE ASSESSMENT (RTA) TOOL AS A REPRESENTATION OF EXPLORATIVE TEACHING

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We offer a new representation of mathematics teaching – the Realization Tree Assessment (RTA) tool – and exemplify how it may be productive for teachers' collaborative discussions on explorative teaching. The RTA assists in explicating the mathematical object that is at the core of a task together with its different realizations. It was originally used for assessment of lessons and opportunities for explorative participation, where it enabled the examination of the extent in which different realizations of the mathematical object were exposed and who authored the narratives about the mathematical object (students or teacher). In this paper, we describe the affordances of this tool as a representation of teaching which affords access to the mathematical aspects of Explorative Pedagogical Discourse. We exemplify these affordances with a discussion of pre-service teachers working in collaborative groups on RTA images.

Representations of teaching

Preparing pre-service teachers (PSTs) for engagement in "reform", student-centered instruction is a complex process (Cohen, 1990; Santagata, Kersting, Givvin, & Stigler, 2011). Teacher educators need to use various representations of teaching practices to help teachers to see and understand teaching practices that afford opportunities for students' conceptual engagement (Grossman et al., 2009). Representations of teaching are the different ways that teaching is represented and what these representations make visible to novices (ibid, p. 2058). A variety of representations of mathematics teaching have been used in the context of teachers' learning. For example: written cases (Merseth, 2003), video records (Karsenty & Arcavi, 2017) and animated classroom stories (Chieu, Herbst, & Weiss, 2011). However, these representations of teaching are often not sufficient for making visible to novices the full mathematical opportunities for learning that are available in a task or a lesson. Evidence for this is supported by literature on teacher noticing, and more specifically, on teacher learning to notice specific aspects of instruction that are aligned with student-centered, "reform" instruction (Van Es & Sherin, 2008). These studies report that teachers who were exposed to video recording as representation of teaching, were often more sensitive to classroom management and social actions than to the mathematical aspects of students' understanding. These findings echo the wider reported phenomena of teachers' appropriating relatively "shallow" aspects of reform-based, explorative instruction (Heyd-Metzuyanim, Smith, Bill, & Resnick, 2018) such as group-work or inviting students to present their ideas, while the more complex mathematical aspects such as the need to clarify mathematical ideas and derive conceptual connections from discussions remain neglected (Stein, Correnti, Moore, Russell, & Kelly, 2017).

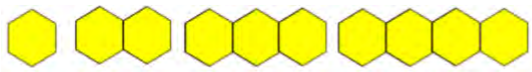
In this paper, we suggest a new representation of mathematics teaching – the Realization Tree Assessment (RTA) tool, that was initially developed to assess the opportunities given to students to participate exploratively, namely, construct narratives about mathematical objects (Weingarden, Heyd-Metzuyanim, & Nachlieli, 2019). It has not, however, been examined as a tool for discussing lessons with teachers. Our goal in this paper is to explore the opportunities that the RTA offers for making PSTs aware of the more nuanced mathematical aspects of instruction that encourage explorative participation.

The Realization Tree Assessment (RTA) tool

The RTA is a tool inspired by the Commognitive theory (Sfard, 2008), and specifically, by its idea of objectification as the essence of mathematical learning. Mathematical objects are discursive entities, claims Sfard (ibid), which do not exist anywhere outside the discourse. In order to participate in the discourse about these objects, the student needs to reify, alienate and "same" different realizations of the mathematical object. For example, when learning about functions, the student initially communicates about tables, graphs and algebraic expressions as different entities. Eventually, through saming and objectifying, these three realizations come to be talked about as one object – the function (Nachlieli & Tabach, 2012).

Tasks and the lessons implementing them can offer more or less opportunities for saming different mathematical objects. The RTA assists in mapping these opportunities and in explicating the mathematical object that can be discussed through the task. For example, in the case of a task called "the hexagons task" (see figure 1), the task's affordances lie in the opportunities to connect different algebraic expressions to a single visual mediator (the perimeter of the hexagons' train). Thus, the

Trains 1, 2, 3, and 4 (shown below) are the four trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



1. Compute the perimeter for each of the first four trains.
2. Draw the fifth train and compute its perimeter.
3. Determine the perimeter of the 25_{th} train.
4. Write a description that could be used to compute the perimeter of any train in the pattern. Explain how you know it will always work.

Figure 1: The hexagons task

task provides opportunities for saming different algebraic expressions that reify the procedures for counting the hexagon's sides. For example, 4n+2 describes counting top and bottom sides, then the 2 edges external edges; 6n-2(n-1) reifies counting all the hexagon edges, then omitting the internal ones; and there are many more such possible expressions.

The RTA presented in figure 2 details the various realizations of the mathematical object "the perimeter of the n-th hexagon train", which are reasonable to expect from middle school learners. Based on watching a video recording of a lesson (particularly the whole-classroom discussion part), RTA coding proceeds by marking the realizations and the links between realizations that were mentioned during the lesson, either by the teacher or by the students.

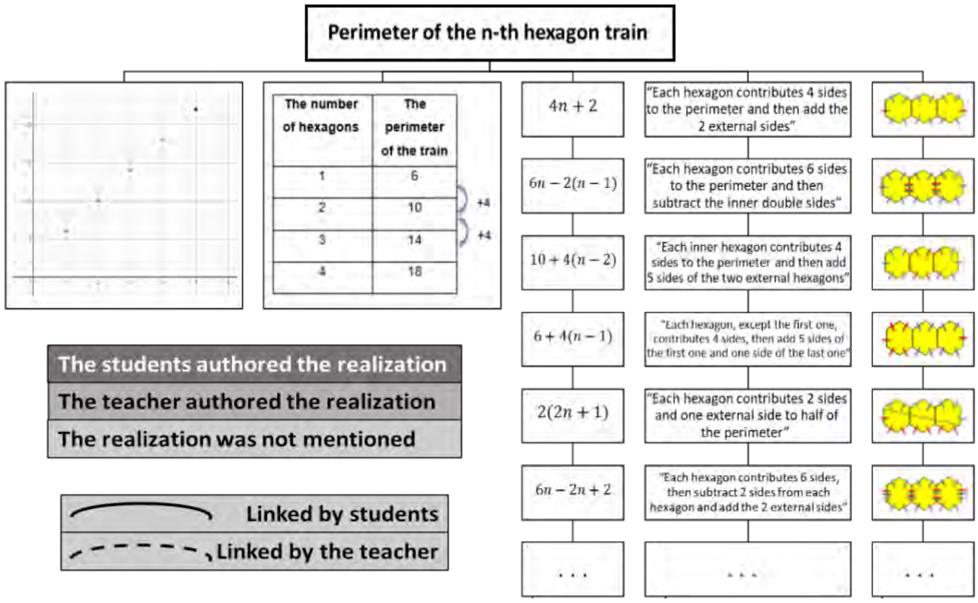


Figure 2: The Realization Tree of the hexagons task

Up until now (Weingarden & Heyd-Metzuyanim, 2018; Weingarden et al., 2019) we used the RTA as an assessment tool for mapping mathematics lessons and the extent to which they afford students opportunities for explorative participation. However, the process of coding, discussing and presenting multiple RTAs has led us to believe that this tool can be a valuable resource for teacher learning explorative teaching practices. To theorize this potential, we now move to explain how we see this learning to teach exploratively as becoming a participant in a certain Pedagogical Discourse.

Explorative Pedagogical Discourse (EPD)

Previous research has shown that although teaching for explorative participation in its various constructions ("ambitious", "dialogic", "reform-oriented" teaching) has been promoted by teacher-educators for more than three decades, classrooms are often still dominated by ritual learning opportunities, namely opportunities for imitating procedures demonstrated by the teacher (Nachlieli & Tabach, 2018). Moreover, even teachers who undergo professional development towards teaching for explorative participation often do not apply the practices that promote explorative participation in their classrooms (Heyd-Metzuyanim et al., 2018; Santagata et al., 2011; Spillane & Zeuli, 1999).

In our recent works (Heyd-Metzuyanim, accepted; Heyd-Metzuyanim & Shabtay, 2019), we have conceptualized the process of teachers' learning to teach exploratively as a process by which teachers become initiated into a particular discourse – the Exploration Pedagogical Discourse (EPD). The EPD has historically been contrasted with "traditional" forms of teaching (Munter, Stein, & Smith, 2015), which we name the Acquisition Pedagogical Discourse (APD). The APD, which is still common in the public sphere as well as in schools, frames learning as a matter of "acquiring" skills and concepts and teaching as a matter of "delivering" these elements into the students' minds.

Studies applying the APD vs. EPD conceptual framework (Heyd-Metzuyanim, accepted; Heyd-Metzuyanim & Shabtay, 2019) show that teachers who have been accustomed to the APD have relative ease with noticing the social or non-mathematical aspects of the EPD, such as affording students opportunities to speak and discuss, inviting them to present their work on the board or organizing them to work in groups. In contrast, adopting the mathematical aspects of the EPD, often called "explicit attention to concepts" (Hiebert & Grouws, 2007) or providing opportunities for objectification (Heyd-Metzuyanim & Shabtay, 2019) is much more difficult for teachers both to perceive in model lessons, and to adopt to their own practice. For this cause, the RTA may be a productive representation of teaching, since it focuses precisely on the mathematical aspects of the teaching for explorative participation.

The question of our research is thus: what is the potential of the RTA to be an effective means for introducing teachers to the EPD?

Context and participants

The study reported here took place during a course for pre-services mathematics teachers (PSTs). During the course, 15 PSTs participated in a RTA workshop that lasted four academic hours and included three main parts: (1) introducing some theoretical aspects of the RTA; (2) coding an empty RTA based on a short video of a hexagon's lesson; (3) comparing and discussing different images of shaded RTA images. In this paper, we discuss only the third part. The workshop was videotaped and the plenary discussion was transcribed. In what follows, we describe the design of the RTA workshop, including its pedagogical goals. In addition, we exemplify how each pedagogical goal was accomplished during the workshop, including initial indications for how the RTA facilitated adopting the mathematical aspects of the EPD.

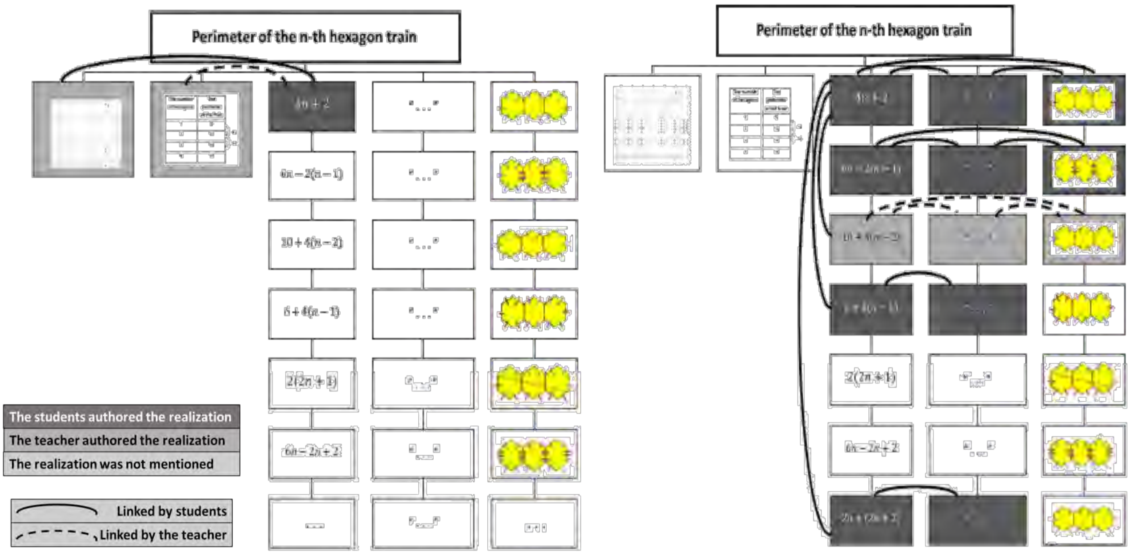
The RTAs as representation of teaching

Three pedagogical goals guided us during the planning of the workshop: (1) helping PSTs identify the mathematical object that can be exposed through the hexagon's task; (2) turning PSTs' attention to the different types of links that can be made between realizations; (3) relaying the importance of students' authority in mathematics lessons. These three pedagogical goals were pursued by the selection of 3 pairs of RTA images that, by way of contrast, would turn PSTs' attention to the aspects mentioned above. All RTA images were selected from a database of RTAs previously coded for our assessment-related studies (Weingarden et al., 2019).

First goal: attending to the central mathematical object afforded by the task

The idea behind the selection of the first pair of RTAs (figure 3) was to offer the PSTs opportunities to identify very different ways by which the hexagons task could play out in a lesson, and to discuss what was, in fact, the mathematical object that was most probably constructed in such a lesson. The RTA in figure 3a represents a lesson where the algebraic, graphic and ordered-pairs realizations of a linear function were mentioned and linked during the lesson, mainly by the teacher. The branch of the algebraic realizations in this RTA is empty, meaning no alternative realizations to 4n+2 were mentioned in the lesson. Moreover, this algebraic realization was not linked to the hexagons pattern. This lack of linking signals that the lesson probably missed the affordances of the task to same the different algebraic realizations of the "perimeter of the n-th hexagon train" and more broadly, engage with the idea of equivalent algebraic expressions.

In contrast, the RTA in figure 3b shows a lesson where multiple algebraic realizations were authored by students, as well as multiple links made. These two contrasting RTA images have the potential to engage PSTs with the mathematical aspects of EPD since the main difference between these images lies in the mathematical object and its realizations. While the lesson depicted in RTA 3b affords ample opportunities to discuss the object "perimeter of n-th hexagon train", the lesson depicted by RTA 3a has very little opportunities for doing that. Notably the lesson of RTA 3a may offer some opportunities for discussing the object "linear function", yet this object is not directly related to the hexagon's visual mediator.



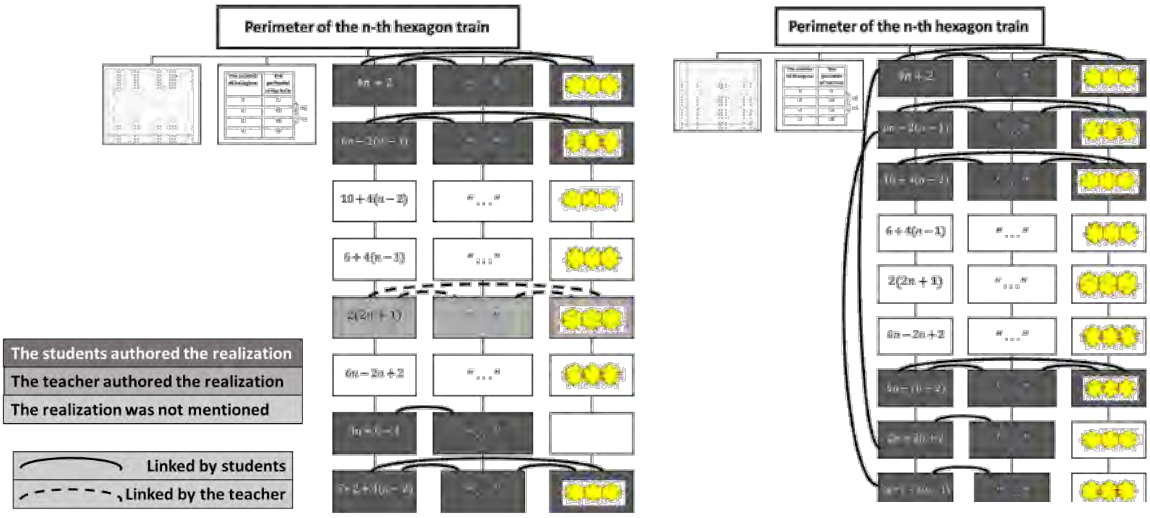
a. linear function **b. equivalent algebraic expressions**

Figure 3: The RTA images for the first goal: attending to the central mathematical object

The discussion following this pair of RTA images among the PSTs indeed revolved around issues of the mathematical object discussed by the task. The PSTs first recognized that the two lessons were different. Then, one PST recognized that this lesson was about another object, saying "It's a lesson about functions". Other PSTs added: "You could say that this *(figure 3b)* is the perimeter of the n-th place, but it seems that on the left [figure 3a] it focused more on the function, even though it is a discrete function". Another PST also referred to the teacher's goal for the lesson depicted by RTA 3a saying: "maybe this was a preparation for the topic of linear functions."

Second goal: turning attention to links between realizations

The second pair of RTAs (figure 4) was selected with the goal of turning PSTs attention to the different types of links that can be made between realizations. These RTAs depicted two lessons with a similar number of realizations and a similar level of students' authority. The significant contrast which we wanted PSTs to notice was in the links made between the realizations. In figure 4a, although there was a substantial number of links between the algebraic, verbal and visual realizations, there were virtually no links between different algebraic realizations mentioned during the discussion. The discussion thus had a "show and tell" feeling, where each student presented his or her solution (usually resulting in an algebraic expression) but links between realizations were not made. This, in contrast to the lesson depicted in figure 4b, which offered multiple opportunities for the students listening and participating in the discussion to link between the different algebraic realizations.



a. links between verbal-visual-algebraic realizations **b. links between algebraic realizations**

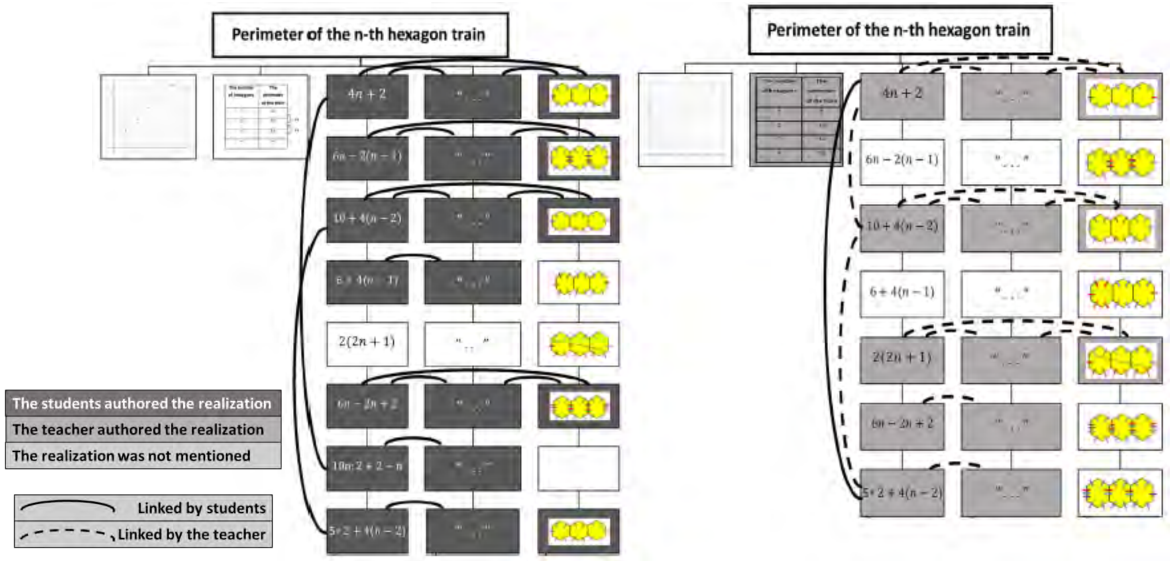
Figure 4: The RTAs for the second goal: types of links

We wanted the contrast between these two RTA images to turn PST's attention to the different types of links that can be afforded by the task. We were aiming for discussion to take place around questions such as which links are more important? Which afford more understanding? And why? Some evidence from the discussion among the PSTs shows that this goal was at least partially accomplished. The PSTs immediately identified the difference between the images: "[in the left RTA] there are no links between the algebraic realizations". Then they talked about the different role of each type of links. They declared that the links between the different algebraic realizations (the vertical links) in RTA 4b are more important than the links between each algebraic realization to its visual and verbal realizations (horizontal links), represented in RTA 4a, saying: "It *(RTA 4a)* does not have an added value. He [the teacher] does not tell anything, he did not talk about linking".

Third goal: students' authority

The third pair of RTAs (figure 5) was selected to fulfil the goal of highlighting the value of students (and not the teacher) producing narratives about mathematical objects. The lessons represented in figure 5, have more or less a similar number of realizations, similar number of links and moreover, they include both type of links -- the links between each algebraic expression and its visual and verbal realizations (horizontal links) and the links between different algebraic realizations (vertical links). Therefore, the opportunities for saming in these two lessons were pretty much the same for any student who was listening to the discussion (although for those who actively participated they were obviously different). However, the lessons differ from each other mainly in the level of students' authority.

The discussion following this pair of RTA images among the PSTs indeed shows that the goal of discussing students' authority was met. The PSTs easily identified the difference between the images. They then proceeded to talk, guided by the first author's questions, about the importance and usefulness of students' authority in mathematics lessons. One of the PSTs referred directly to the third goal while asking: "Are we striving for the fact that this right [lesson] *(figure 5b)* is



a. high level of students' authority **b. low level of students' authority**

Figure 5: The RTA that represent the third goal: students' authority

undesirable or is it okay?" This question led the discussion to the issue of which lesson is "better"? One of the PSTs valued the "teacher-centered" lesson (figure 5b) while claiming that: "I think it's *(figure 5b)* all right… We are in the real world. You cannot make all lessons be explorative lessons where the students have all the initiative… Sometimes you have to get stuff done". In response to this opinion some other PSTs claimed the lesson in figure 5b was "extreme" in that "the teacher did a lot" in it. However, in contrast to the first two pairs, this pair of RTAs did not afford significant opportunity for the mathematical EPD. The PSTs were more engaged in the social aspect of the discourse, namely who does the talking in the lesson and its general organization.

Discussion

Our goal for this paper was to suggest the RTA tool as a representation of teaching. Three pedagogical goals guided us during the designing of the RTAs as a representation of teaching: (1) helping PSTs identify the mathematical object that is exposed through a lesson; (2) turning PSTs attention to the different types of links that can be made between realizations; (3) relaying the importance of students' authority in mathematics lessons. These pedagogical goals were pursued by the selection of specific contrasting RTA images and engaging PSTs with discussion about them.

Previous research has shown the importance of using representations of teaching in the context of teacher education: it stimulates PSTs to bridge theory and practice (Herbst, Chazan, Chen, Chieu, & Weiss, 2011); it helps PSTs to see and understand various teaching practices (Grossman et al., 2009); and it produces a platform by which knowledge for teaching becomes public and can be communicated among colleagues in collaborative discussions (Hiebert, Gallimore, & Stigler, 2002).

The initial results presented in this study show that the RTA as a representation of teaching afforded PSTs in our course a unique platform to communicate about different implementations of one task and the ways by which this implementation provides students opportunities for objectifying and saming different realizations. In this way, the RTA assisted in making the mathematical aspects of the lesson visible, public and communicated among the PSTs. This result is important since it has been found that engaging teachers with the mathematical aspects of EPD is more difficult than engaging them with the social aspects of this Pedagogical Discourse (Heyd-Metzuyanim, accepted).

However, this study has several limitations. It describes an initial attempt to use the RTA as a representation of teaching in PSTs collaboration. Therefore, great efforts have been devoted in designing the workshop and presenting the method of using the RTA as a representation of teaching, rather than assessing its effectiveness. Future studies are needed to examine more deeply the PSTs' discourse (APD vs. EPD) and the learning process afforded by using the RTA.

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